ON SYMMETRIC f BI-DERIVATIONS OF LATTICES

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ABSTRACT. Recently Ozbal and Firat [22] introduced the notion of symmetric f bi-derivation of a lattice. They give illustrative examples and they also characterized the distributive lattice by symmetric f bi-derivation. In this paper we define the isotone symmetric f bi-derivation and get some interesting results about isotoneness. We also give the relations between the distributive, modular and isotone lattices by symmetric f bi-derivation.

1. INTRODUCTION

The symmetric bi derivation was introduced by Maksa in [20]. It was shown in [21] and [28] that symmetric bi derivations are related to general solutions of some functional equations. Vukman in [26] and [27] studied the symmetric bi derivation and commutativity properties of rings. In [25] the author introduced the notion of lattice derivation and gave interesting results. Also in [13] the author studied this lattice derivation. Xin and et al. [29] improved derivation for a lattice and discussed some related properties. They gave some equivalent conditions under which a derivation is isotone for lattices with a greatest element, modular lattices and distributive lattices. Ceven and Ozturk [8] gave a generalization of derivation on a lattice which was defined in [29]. In the paper they gave the notion of $f-$derivation for a lattice and got some properties about $f-$derivation of lattices. Also Ceven [9] introduced the symmetric bi derivations on lattices. The author investigated some related properties. He characterized the distributive and modular lattices by the trace of symmetric bi derivations. For the important role and applications of lattices one can see [1, 2, 6, 7, 14, 18, 23, 24, 30]. Asci and Ceran [4] defined the symmetric bi-(σ,τ) derivations on prime and semi prime Gamma rings and proved some results concerning symmetric bi-(σ,τ) derivations on prime and semi prime Gamma rings. In [3] and [5] they generalized the notion of symmetric bi-(σ,τ) derivations.

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In this paper we define the isotone symmetric f bi-derivation and get some interesting results about isotoneness. We also give the relations between the distributive and isotone lattices by symmetric f bi-derivation.

2. Preliminaries

Definition 1. [15] *Let L be a nonempty set endowed with operations $\wedge$ and $\vee$. If $(L, \wedge, \vee)$ satisfies the following conditions for all $x, y, z \in L$*

(1) $x \wedge x = x$, $x \vee x = x$

(2) $x \wedge y = y \wedge x$, $x \vee y = y \vee x$

(3) $(x \wedge y) \wedge z = x \wedge (y \wedge z)$, $(x \vee y) \vee z = x \vee (y \vee z)$

(4) $(x \wedge y) \vee x = x$, $(x \vee y) \wedge x = x$

then L is called a lattice.

Definition 2. [15] *A lattice L is distributive if the identity (5) or (6) holds.*

(5) $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$

(6) $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$

Definition 3. [23] *A lattice L is called modular if it satisfies the following conditions for all $x, y, z \in L$*

(7) If $x \leq z$, then $x \vee (y \wedge z) = (x \vee y) \wedge z$

Definition 4. [15] *Let $(L, \wedge, \vee)$ be a lattice. A binary relation $\leq$ is defined by $x \leq y$ if and only if $x \wedge y = x$ and $x \vee y = y$.*

Definition 5. [15] *Let L and M be two lattices. The function $g : L \to M$ is called the lattice homomorphism if it satisfies the following conditions for all $x, y \in L$.*

(8) $g(x \wedge y) = g(x) \wedge g(y)$

(9) $g(x \vee y) = g(x) \vee g(y)$.

Lemma 1. *Let $(L, \wedge, \vee)$ be a lattice. Define the binary relation $\leq$ as in the Definition 4. Then $(L, \leq)$ is a poset and for any $x, y \in L$, $x \wedge y$ is the g.l.b. of $\{x, y\}$ and $x \vee y$ is the l.u.b. of $\{x, y\}$.*

Definition 6. [11] *Let $P_1, P_2, ..., P_n$ be ordered sets. The Cartesian product $P_1 \times P_2 \times ... \times P_n$ can be made into an ordered set by imposing the coordinatewise order defined by*

$$(x_1, x_2, ..., x_n) \leq (y_1, y_2, ..., y_n) \Leftrightarrow (\forall i)\ \ x_i \leq y_i \text{ in } P_i.$$

Let L and K be lattices. Define $\vee$ and $\wedge$ coordinatewise on $L \times K$, as follows:

$$(l_1, k_1) \vee (l_2, k_2) = (l_1 \vee l_2, k_1 \vee k_2) \tag{2.1}$$

$$(l_1, k_1) \wedge (l_2, k_2) = (l_1 \wedge l_2, k_1 \wedge k_2) \tag{2.2}$$

$L \times K$ satisfies the identities (1)-(4) so $L \times K$ is a lattice.

Definition 7. [29] *A function $D : L \to L$ on a lattice L is called a derivation on L if D satisfies the following condition*

$$D(x \wedge y) = (Dx \wedge y) \vee (x \wedge Dy)$$

The abbreviation Dx is used for $D(x)$ in above definition.

Definition 8. [29] *Let L be a lattice and D be a derivation on L*
(i) If $x \leq y$ implies $Dx \leq Dy$ then D is called an isotone derivation,
(ii) If D is one to one then D is called monomorphic derivation,
(iii) If D is onto then D is called epimorphic derivation.

Definition 9. [8] *Let L be a lattice. A function $D : L \to L$ is called an $f-$derivation on L if there exists a function $f : L \to L$ such that*

$$D(x \wedge y) = (D(x) \wedge f(y)) \vee (f(x) \wedge D(y))$$

for all $x, y \in L$.

Definition 10. [9] *Let L be a lattice and $D : L \times L \to L$ be a symmetric mapping. Then D is called symmetric bi derivation on L if it satisfies the following condition*

$$D(x \wedge y, z) = (D(x, z) \wedge y) \vee (x \wedge D(y, z))$$

for all $x, y, z \in L$.

Definition 11. [22] *Let L be a lattice, a function $D(.,.) : L \times L \to L$ is called symmetric f bi-derivation of $L \times L$ if there exists a function $f : L \to L$ such that*

$$D(x \wedge y, z) = (D(x, z) \wedge f(y)) \vee (f(x) \wedge D(y, z)) \tag{2.3}$$

for all $x, y, z \in L$.

Proposition 1. [22] *Let L be a lattice and $D(.,.)$ be a symmetric f bi-derivation on L. Then the following identities hold for all $x, y \in L$.*
(i) $D(x, y) \leq f(x)$ and $D(x, y) \leq f(y)$
(ii) If L has a least element 0 then $f(0) = 0$ implies $D(0, z) = 0$.

3. Main Results

We begin giving a good example for the symmetric f bi-derivation on Lattices.

Example 1. *Let L be the lattice* **2** *and* $L \times L$ *is also a lattice which is given in Figure 1. and define the function* $D(.,.)$ *on* L *by*

$$D(x,y) = \begin{cases} 2, & (x,y) = (0,0) \\ 2, & (x,y) = (0,1) \\ 2, & (x,y) = (0,2) \\ 2, & (x,y) = (1,0) \\ 1, & (x,y) = (1,1) \\ 1, & (x,y) = (1,2) \\ 2, & (x,y) = (2,0) \\ 1, & (x,y) = (2,1) \\ 1, & (x,y) = (2,2) \end{cases}$$

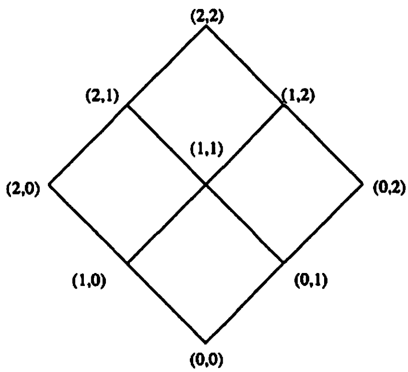

Fig. 1 The Lattice in Example 1.

Then $D(.,.)$ *is not symmetric bi derivation of* L*. Because*

$$\begin{aligned} D(0 \wedge 1, 1) &= D(0,1) \\ &= 2 \end{aligned}$$

also

$$\begin{aligned} (D(0,1) \wedge 1) \vee (0 \wedge D(1,1)) &= (2 \wedge 1) \vee (0 \wedge 1) \\ &= 1 \vee 0 \\ &= 1 \end{aligned}$$

As a result from the definition of symmetric bi derivation it is seen that $D(.,.)$ *is not a symmetric bi derivation. Now if we define a function* $f : L \to L$ *such that*

$$f(x) = 2, \quad x = 0,1,2$$

then $D(.,.)$ *defined above is a symmetric* f *bi-derivation of* L*.*

Proposition 2. *Let L be a lattice and $D(.,.)$ be a symmetric f bi-derivation on L. Then the following identities hold for all $x, y, z \in L$.*
(i) $D(x,z) \wedge D(y,z) \leq D(x \wedge y, z) \leq D(x,z) \vee D(y,z)$
(ii) $D(x \wedge y, z) \leq f(x) \vee f(y)$.

Proof. (i) From the equation (2.3)

$$D(x,z) \wedge f(y) \leq D(x \wedge y, z)$$

and

$$f(x) \wedge D(y,z) \leq D(x \wedge y, z).$$

From Proposition 1(i) we have $D(x,z) \leq f(x)$ then we obtain

$$\begin{aligned} D(x,z) \wedge D(y,z) &\leq f(x) \wedge D(y,z) \\ &\leq D(x \wedge y, z). \end{aligned}$$

We know that

$$D(x,z) \wedge f(y) \leq D(x,z)$$

and

$$f(x) \wedge D(y,z) \leq D(y,z)$$

then it is obvious that

$$D(x \wedge y, z) \leq D(x,z) \vee D(y,z).$$

It completes the proof.
(ii) Since

$$D(x,z) \wedge f(y) \leq f(y)$$

and

$$f(x) \wedge D(y,z) \leq f(x)$$

then

$$D(x \wedge y, z) \leq f(x) \vee f(y).$$

□

Definition 12. *Let L be a lattice and $D(.,.)$ be a symmetric f bi-derivation on $L \times L$, if $x \leq y$ implies $D(x,z) \leq D(y,z)$ then $D(.,.)$ is called an isotone symmetric f bi-derivation,*

Proposition 3. *Let L be a lattice with a greatest element 1 and $D(.,.)$ be a symmetric bi $f-$derivation on L and $f(1) = 1$. Then the following identities hold;*
(i) If $f(x) \leq D(1,z)$ then $D(x,z) = f(x)$
(ii) If $f(x) \geq D(1,z)$ then $D(x,z) \geq D(1,z)$

Proof. (i) Since

$$\begin{aligned} D(x,z) &= D(x \wedge 1, z) \\ &= (D(x,z) \wedge f(1)) \vee (f(x) \wedge D(1,z)) \\ &= D(x,z) \vee f(x) \end{aligned}$$

we have $f(x) \leq D(x,z)$. From Proposition 1 (i), we obtain $D(x,z) = f(x)$.

(ii) Since

$$\begin{aligned} D(x,z) &= D(x \wedge 1, z) \\ &= (D(x,z) \wedge f(1)) \vee (f(x) \wedge D(1,z)) \\ &= D(x,z) \vee D(1,z) \end{aligned}$$

we have $D(x,z) \geq D(1,z)$. □

Proposition 4. *Let L be a lattice and $D(.,.)$ be a symmetric f bi-derivation of L. If f is an increasing function, $y \leq x$ and $D(x,z) = f(x)$ then $D(y,z) = f(y)$.*

Proof. Suppose f is an increasing function and $y \leq x$, then $y = x \wedge y$. Thus

$$\begin{aligned} D(y,z) &= D(x \wedge y, z) \\ &= (D(x,z) \wedge f(y)) \vee (f(x) \wedge D(y,z)) \\ &= (f(x) \wedge f(y)) \vee D(y,z) \\ &= f(y) \vee D(y,z) \\ &= f(y) \end{aligned}$$

□

Proposition 5. *Let L be a lattice and $D(.,.)$ be a symmetric f bi-derivation of L. Then for any $x, y, z \in L$ the followings hold:*

(i) If $D(.,.)$ is isotone then $D(x,z) = D(x,z) \vee (f(x) \wedge D(x \vee y, z))$.

(ii) If $f(x \vee y) = f(x) \vee f(y)$ then $D(x,z) = D(x,z) \vee (f(x) \wedge D(x \vee y, z))$.

(iii) If f is an increasing function then $D(x,z) = D(x,z) \vee (f(x) \wedge D(x \vee y, z))$.

Proof. (i) Since $D(.,.)$ is an isotone symmetric f bi-derivation then we have

$$\begin{aligned} D(x,z) &= D((x \vee y) \wedge x, z) \\ &= (D(x \vee y, z) \wedge f(x)) \vee (f(x \vee y) \wedge D(x,z)) \\ &= D(x,z) \vee (f(x) \wedge D(x \vee y, z)) \end{aligned}$$

completes the proof.

(ii) From Proposition 1 (i) $D(x,z) \leq f(x) \leq f(x) \vee f(y)$ we obtain

$$\begin{aligned} D(x,z) &= D((x \vee y) \wedge x, z) \\ &= (D(x \vee y, z) \wedge f(x)) \vee (f(x \vee y) \wedge D(x,z)) \\ &= (D(x \vee y, z) \wedge f(x)) \vee ((f(x) \vee f(y)) \wedge D(x,z)) \\ &= D(x,z) \vee (f(x) \wedge D(x \vee y, z)) \end{aligned}$$

completes the proof.

(iii) Since f is an increasing function and $x \leq x \vee y$ then $f(x) \leq f(x \vee y)$ so;

$$\begin{aligned} D(x,z) &= (D(x \vee y, z) \wedge f(x)) \vee (f(x \vee y) \wedge D(x,z)) \\ &= D(x,z) \vee (f(x) \wedge D(x \vee y, z)) \end{aligned}$$

□

Proposition 6. *Let L be a lattice, $D(.,.)$ be an isotone symmetric f bi-derivation and f is a decreasing function (or $f(x \vee y) = f(x) \vee f(y)$). If $D(x,z) = f(x)$ and $D(y,z) = f(y)$ then $D(x \vee y, z) = f(x) \vee f(y)$.*

Proof. We have $x \leq x \vee y$ and $y \leq x \vee y$ and since $D(.,.)$ is isotone, then $D(x,z) \leq D(x \vee y, z)$ and $D(y,z) \leq D(x \vee y, z)$.As a result we have $f(x) \vee f(y) = D(x,z) \vee D(y,z) \leq D(x \vee y, z)$. Since f is decreasing then $f(x \vee y) \leq f(x) \vee f(y)$. By using $D(x \vee y, z) \leq f(x \vee y) \leq f(x) \vee f(y)$ we get $f(x) \vee f(y) = D(x \vee y, z)$. □

Theorem 1. *Let L be a lattice with greatest element 1 and $D(.,.)$ be a symmetric f bi-derivation of L and $f(x \wedge y) = f(x) \wedge f(y)$. Then the following conditions are equivalent;*

(i) $D(.,.)$ is an isotone symmetric f bi-derivation

(ii) $D(x,z) \vee D(y,z) \leq D(x \vee y, z)$

(iii) $D(x,z) = f(x) \wedge D(1,z)$

(iv) $D(x \wedge y, z) = D(x,z) \wedge D(y,z)$

Proof. (i) ⇒ (ii) Suppose that $D(.,.)$ is an isotone symmetric f bi-derivation. We know that $x \leq x \vee y$ and $y \leq x \vee y$. Since $D(.,.)$ is an isotone then $D(x,z) \leq D(x \vee y, z)$ and $D(y,z) \leq D(x \vee y, z)$ Finally we obtain $D(x,z) \vee D(y,z) \leq D(x \vee y, z)$

(ii) ⇒ (i) Suppose that $D(x,z) \vee D(y,z) \leq D(x \vee y, z)$ and $x \leq y$. Then we have

$$\begin{aligned} D(x,z) &\leq D(x,z) \vee D(y,z) \\ &\leq D(x \vee y, z) \\ &= D(y,z) \end{aligned}$$

Then $D(.,.)$ is isotone.

(i) $\Rightarrow$ (iii) Suppose that $D(.,.)$ is an isotone symmetric f bi-derivation. We have $D(x,z) \leq D(1,z)$. It is known that $D(x,z) \leq f(x)$ from Proposition 1. Then we get $D(x,z) \leq f(x) \wedge D(1,z)$. From Proposition 5 (i) for $y=1$ we have

$$\begin{aligned} D(x,z) &= D(x,z) \vee (f(x) \wedge D(1,z)) \\ &= f(x) \wedge D(1,z) \end{aligned}$$

(iii) $\Rightarrow$ (iv) Assume that (iii) holds. Then

$$\begin{aligned} D(x \wedge y, z) &= (f(x \wedge y)) \wedge D(1,z) \\ &= f(x) \wedge f(y) \wedge D(1,z) \\ &= (f(x) \wedge D(1,z)) \wedge (f(y) \wedge D(1,z)) \\ &= D(x,z) \wedge D(y,z) \end{aligned}$$

(iv) $\Rightarrow$ (i) Let $D(x \wedge y, z) = D(x,z) \wedge D(y,z)$ and $x \leq y$. Since $D(x,z) = D(x \wedge y, z) = D(x,z) \wedge D(y,z)$ we get $D(x,z) \leq D(y,z)$. That is $D(.,.)$ is isotone. □

Theorem 2. *Let L be a modular lattice and $D(.,.)$ be a symmetric f bi-derivation of L. Then;*

(i) $D(.,.)$ is isotone if and only if $D(x \wedge y, z) = D(x,z) \wedge D(y,z)$

(ii) If $D(.,.)$ is isotone and $f(x \vee y) = f(x) \vee f(y)$ then $D(x,z) = f(x)$ implies $D(x \vee y, z) = D(x,z) \vee D(y,z)$

Proof. (i) Suppose that $D(.,.)$ is an isotone symmetric f bi-derivation. Since $x \wedge y \leq x$ and $x \wedge y \leq y$ we get $D(x \wedge y, z) \leq D(x,z) \wedge D(y,z)$. Also using Proposition 1 and the fact that $D(x,z) \wedge f(y) \leq D(x,z) \leq f(x)$, we get

$$\begin{aligned} D(x,z) \wedge D(y,z) &= (D(x,z) \wedge D(y,z)) \wedge (f(x) \wedge f(y)) \\ &\leq (D(x,z) \vee D(y,z)) \wedge f(x) \wedge f(y) \\ &= ((D(y,z) \vee D(x,z)) \wedge f(y)) \wedge f(x) \\ &= (D(y,z) \vee (D(x,z) \wedge f(y))) \wedge f(x) \\ &= ((D(x,z) \wedge f(y)) \vee D(y,z)) \wedge f(x) \\ &= ((D(x,z) \wedge f(y))) \vee (f(x) \wedge D(y,z)) \\ &= D(x \wedge y, z) \end{aligned}$$

Conversely let $D(x \wedge y, z) = D(x,z) \wedge D(y,z)$ and $x \leq y$. Since $D(x,z) = D(x \wedge y, z) = D(x,z) \wedge D(y,z)$ we have $D(x,z) \leq D(y,z)$.

(ii) Suppose that $D(.,.)$ is isotone and $D(x,z) = f(x)$. Using Proposition 5 and since L is a modular lattice we have

$$\begin{aligned} D(y,z) &= D(y,z) \vee (f(y) \wedge D(x \vee y, z)) \\ &= (D(y,z) \vee f(y)) \wedge D(x \vee y, z) \\ &= f(y) \wedge D(x \vee y, z) \end{aligned}$$

Hence using hypothesis we obtain

$$\begin{aligned} D(x,z) \vee D(y,z) &= D(x,z) \vee (f(y) \wedge D(x \vee y, z)) \\ &= (D(x,z) \vee f(y)) \wedge D(x \vee y, z) \\ &= (f(x) \vee f(y)) \wedge D(x \vee y, z) \\ &= f(x \vee y) \wedge D(x \vee y, z) \\ &= D(x \vee y, z) \end{aligned}$$

□

Theorem 3. *Let L be a distributive lattice and $D(.,.)$ be a symmetric f bi-derivation of L where $f(x \vee y) = f(x) \vee f(y)$. Then the followings hold;*
(i) If $D(.,.)$ is isotone then $D(x \wedge y, z) = D(x,z) \wedge D(y,z)$
(ii) $D(.,.)$ is isotone if and only if $D(x \vee y, z) = D(x,z) \vee D(y,z)$

Proof. (i) Since $D(.,.)$ is isotone we know that $D(x \wedge y, z) \leq D(x,z) \wedge D(y,z)$. From the Proposition 1 we get

$$\begin{aligned} D(x,z) \wedge D(y,z) &= (D(x,z) \wedge f(x)) \wedge (D(y,z) \wedge f(y)) \\ &= (D(x,z) \wedge f(y)) \wedge (f(x) \wedge D(y,z)) \\ &\leq (D(x,z) \wedge f(y)) \vee (f(x) \wedge D(y,z)) \\ &= D(x \wedge y, z) \end{aligned}$$

Hence $D(x \wedge y, z) = D(x,z) \wedge D(y,z)$

(ii) Let $D(.,.)$ be isotone, we have $D(x \wedge y, z) = D(x,z) \wedge D(y,z)$ from (i). Then from Proposition 1 and Proposition 5 we get

$$\begin{aligned} D(y,z) &= D(y,z) \vee (f(y) \wedge D(x \vee y, z)) \\ &= (D(y,z) \vee f(y)) \wedge (D(y,z) \vee D(x \vee y, z)) \\ &= f(y) \wedge D(x \vee y, z) \end{aligned}$$

Similarly

$$D(x,z) = f(x) \wedge D(x \vee y, z)$$

then we obtain

$$\begin{aligned} D(x,z) \vee D(y,z) &= (f(x) \wedge D(x \vee y, z)) \vee (f(y) \wedge D(x \vee y, z)) \\ &= (f(x) \vee f(y)) \wedge D(x \vee y, z) \\ &= f(x \vee y) \wedge D(x \vee y, z) \\ &= D(x \vee y, z). \end{aligned}$$

Conversely suppose that $D(x \vee y, z) = D(x, z) \vee D(y, z)$ and $x \leq y$. Then since $D(y, z) = D(x \vee y, z) = D(x, z) \vee D(y, z)$, we have $D(x, z) \leq D(y, z)$ □

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