Hexagonal splicing P system

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Abstract

In the frame work of P systems introduced by Paun (1998), generation of rectangular arrays and hexagonal arrays have been studied in the literature. In this paper, we introduce a new P system generating a family of hexagonal array languages. We compare this new family with the existing families of hexagonal array languages.

Keywords: bio-inspired computing, P system, splicing, hexagonal arrays.
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1 Introduction

Splicing system is one of the models based on biological phenomena introduced by Tom Head [4, 6] which enriched both formal language theory and life science with major developments. The splicing systems make use of a new operation, called splicing on strings of symbols. This operation was extended to rectangular arrays by Helen Chandra et al. [7] and a new splicing system called H array splicing system is introduced.

On the other hand, a new class of distributed and parallel computing device inspired by biochemical considerations was introduced by Paun (1998), and is called P system. This system consists of four basic features namely, membrane structure, objects, evolution rules and communication. The membrane structure consists of several membranes arranged in a hierarchical structure inside a main membrane called skin membrane and delimiting regions. With each region of a membrane, a set of objects which can be symbol or strings or arrays over a given alphabet and a set of evolution rules are associated. The objects evolve according to the evolution rules which can produce new objects and can be sent out of a particular membrane or to any inner membrane using target indicators. The evolution rules are applied in a maximally parallel manner at each step. All objects which can evolve should evolve. Many variants of P system have been introduced and extensively studied by various researchers which can be seen in [9, 10]. In particular P systems generating hexagonal picture languages have been initiated in [1, 2].

Siromoney et al. (1976) introduced hexagonal arrays which have several applications, especially in picture processing and image analysis. A new method of splicing on hexagonal arrays was introduced by Hemalatha et al. [5] to generate hexagonal arrays.

In this paper, we introduce a new P system with hexagonal arrays as objects and arrow head splicing operation on hexagonal arrays as evolution rules. We compare the family of hexagonal picture languages generated by the proposed system with the existing families of hexagonal picture languages like hexagonal array languages, local and recognizable hexagonal picture languages.

2 Preliminaries

We review here the notions of hexagonal pictures, hexagonal picture languages and splicing operation on hexagonal arrays [3, 5, 11, 12]. For the basic definition and generation about P system, we refer to [8, 9].

Definition 2.1. A hexagonal picture p over an alphabet V is a hexagonal array of symbols of V. The set of all hexagonal arrays over V is denoted by V^{**H}.

A hexagonal picture p over the alphabet $\{a, b, c\}$ is given in Fig. 1.

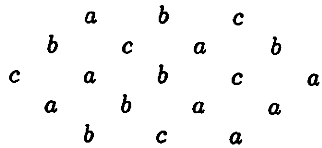


Figure 1: Hexagonal array

Definition 2.2. If $p \in V^{**H}$, then $p^{\#}$ is the hexagonal array obtained by surrounding p with a special boundary symbol $\# \notin V$.

Instead of a regular hexagon, here we also consider some arrowhead hexagonal arrays namely East (E), North West (NW) and South West (SW) arrow heads as shown in Fig. 2.

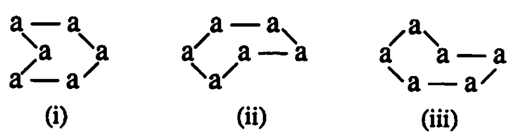
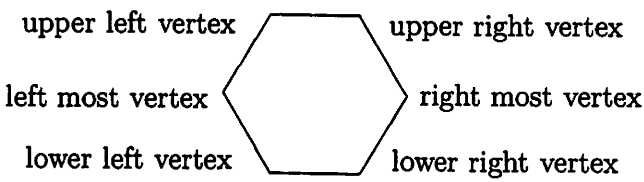


Figure 2: (i) East, (ii) North West and (iii) South West arrow head

Definition 2.3. We consider hexagons of the type :



Definition 2.4. Given a picture $p \in V^{**H}$, let $l_1(p)$ denote the number of elements in the border of p from upper left vertex to left most vertex in the direction $\swarrow$ called x direction, $l_2(p)$ denote the number of elements in the border of p from upper right vertex to right most vertex in the direction $\searrow$ called y direction and $l_3(p)$ denote the number of elements in the border of p from upper left vertex to upper right vertex in the direction $\rightarrow$ called z direction.

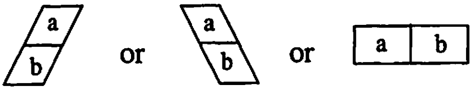
The directions are fixed with origin of reference as the upper left vertex, having coordinates $(1, 1, 1)$. The triple $(l_1(p), l_2(p), l_3(p))$ is called the size of the picture p.

Given a hexagonal picture p of size (l, m, n), for $g \leq l, h \leq m$ and $k \leq n$, we denote by $B_{g,h,k}(p)$ the set of all hexagonal subpictures (called hexagonal blocks) of p of size (g, h, k). Each member of $B_{2,2,2}(p)$ is called a hexagonal tile.

Definition 2.5. A hexagonal picture language $L \subseteq V^{**H}$ is called local if there exists a finite set Δ of hexagonal tiles over $V \cup \{\#\}$ such that $L = \{p \in V^{**H} / B_{2,2,2}(\hat{p}) \subseteq \Delta\}$. L is denoted by $L(\Delta)$.

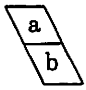
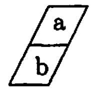
The family of local hexagonal picture languages will be denoted by $HLOC$.

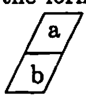
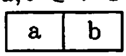
Definition 2.6. Let V be an alphabet. $\#, ¢$ and \$ are three special symbols not in V. The xyz dominoes of the hexagonal domino system [2] are shown in Fig. 3.

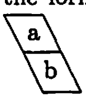
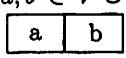


where $a, b \in V$

Figure 3: xyz dominoes

Definition 2.7. An East Arrow Head (EAH) domino splicing rule over V is R_e consisting of domino rules of the form r_1 or r_2, $r_1 = \alpha_1 ¢ \alpha_2 \$ \alpha_3 ¢ \alpha_4$ with α_i, $i = 1,2,3,4$ of the form  for some $a, b \in V \cup \{\#\}$ and $r_2 = \beta_1 ¢ \beta_2 \$ \beta_3 ¢ \beta_4$ with β_i, $i = 1,2,3,4$ of the form  for some $a, b \in V \cup \{\#\}$.

A North West Arrow Head (NWAH) domino splicing rule over V is R_{nw} consisting of domino rules of the form r_3 or r_4, $r_3 = a_1 ¢ a_2 \$ a_3 ¢ a_4$ with a_i, $i = 1,2,3,4$ of the form  for some $a, b \in V \cup \{\#\}$ and $r_4 = b_1 ¢ b_2 \$ b_3 ¢ b_4$ with b_i, $i = 1,2,3,4$ of the form  for some $a, b \in V \cup \{\#\}$.

A South West Arrow Head (SWAH) domino splicing rule over V is R_{sw} consisting of domino rules of the form r_5 or r_6, $r_5 = c_1 ¢ c_2 \$ c_3 ¢ c_4$ with c_i, $i = 1,2,3,4$ of the form  for some $a, b \in V \cup \{\#\}$ and $r_6 = d_1 ¢ d_2 \$ d_3 ¢ d_4$ with d_i, $i = 1,2,3,4$ of the form  for some $a, b \in V \cup \{\#\}$.

Two hexagonal arrays can be spliced with the help of above domino splicing rules. Two hexagonal arrays H_1 and H_2 are cut so that there result a convex hexagonal array and an arrow head hexagonal array from each of H_1 and H_2. The cutting is guided by suitable dominoes. The convex hexagonal array resulting from H_1 (H_2) is pasted with the arrow head hexagon resulting from H_2 (H_1).

3 Hexagonal Array Splicing P System

In this section, we define a new splicing P system with hexagonal arrays as objects and the arrow head domino splicing rules as evolution rules. We compare the family of hexagonal array languages generated by this system with the existing families of hexagonal picture languages.

Definition 3.1. A hexagonal array splicing P system is a construct

$$\Pi = (V \cup \{\#, \$, ₵\}, \mu, L_1, L_2, \ldots, L_n, R_1, R_2, \ldots, R_n, i_0)$$

where

1. V is a finite alphabet; $\#, \$, ₵$ are special symbols not in V.
2. μ is a membrane structure consisting of n membranes labeled with $1, 2, \ldots, n$.
3. L_i is a finite subset of $(V \cup \{\#\})^{**H}$ are called axioms representing the bordered hexagonal arrays initially present in the region i, $1 \leq i \leq n$.
4. R_i, $1 \leq i \leq n$ are finite sets of evolution rules associated with the regions $1, 2, \ldots, n$ of μ in the following form: $(\{r\}, tar)$ where $r \in R_e \cup R_{nw} \cup R_{sw}$ as defined in Definition 2.7 and $tar \in \{here, out\} \cup \{in_j / 1 \leq j \leq n\}$.
5. i_0 is the output membrane.

When an object is present in a region of our system, it is assumed to appear in arbitrarily many copies.

Any n-tuple $(H_1^{\#}, H_2^{\#}, \ldots, H_n^{\#})$ of bordered hexagonal array languages over V is called a configuration of Π. For any two configurations $(H_1^{\#}, H_2^{\#}, \ldots, H_n^{\#})$, $(H_1'^{\#}, H_2'^{\#}, \ldots, H_n'^{\#})$ we write $(H_1^{\#}, H_2^{\#}, \ldots, H_n^{\#}) \Rightarrow (H_1'^{\#}, H_2'^{\#}, \ldots, H_n'^{\#})$ if we can pass from $(H_1^{\#}, H_2^{\#}, \ldots, H_n^{\#})$ to $(H_1'^{\#}, H_2'^{\#}, \ldots, H_n'^{\#})$ by applying the arrow head domino splicing rules from each region of μ in parallel to all possible bordered hexagonal arrays of the corresponding regions, and following the target indications associated with the rules. More precisely, if $X\#, Y\# \in L_i \subset (V \cup \{\#\})^{**H}$ and $\{\{r = \alpha_1 ₵ \alpha_2 \$ \alpha_3 ₵ \alpha_4\} \in$

$R_e \cup R_{nw} \cup R_{sw}, tar\} \in R_i$, such that we can have $(X\#, Y\#) \vdash^r W\#$, then $W\#$ will go to the region indicated by tar. If $tar = here$, then the generated bordered hexagonal array remains in i^{th} membrane, if $tar = out$, the generated array is moved to the region immediately outside the membrane i, if $tar = in_j$, then the generated array is moved to membrane j, provided that this is immediately below i^{th} membrane; if not, the rule cannot be applied.

A sequence of transitions between configurations of a given P system Π, starting from the initial configuration $(L_1, L_2, \ldots, L_n)$ is called a computation with respect to Π. The result of a computation consists of all hexagonal arrays over V which are sent to the membrane i_0 (output membrane) at any time during the computation. We denote by $L(\Pi)$, the language generated by Π.

The family of all hexagonal arrays generated by this system is denoted by HAHSPL.

We now give an example of hexagonal array splicing P system generating hexagonal chess patterns.

Example 3.2. A hexagonal chess pattern is shown in Figure 4 where w, g, b stands for white, grey and black hexagons in the hexagonal chess pattern.

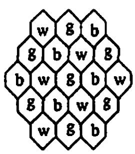


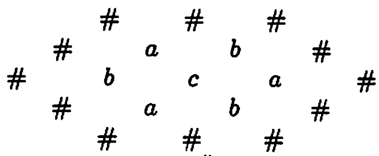
Figure 4: Hexagonal chess pattern

Let a, b and c represents white, grey and black hexagons of the chess pattern respectively. Then the corresponding hexagonal array is given in Figure 5.

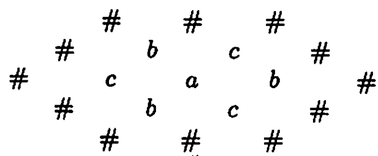
$$
\begin{array}{ccccccccc}
 & & a & & b & & c & & \\
 & b & & c & & a & & b & \\
c & & a & & b & & c & & a \\
 & b & & c & & a & & b & \\
 & & a & & b & & c & &
\end{array}
$$

Figure 5: Array representing hexagonal chess pattern

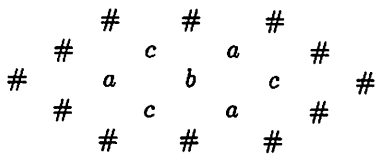
Now we give a splicing P system generating the above patterns. Let $\Pi = (V \cup \{\#, \$, ¢\}, \mu, L_1, L_2, L_3, L_4, R_1, R_2, R_3, R_4, 4)$ where $V = \{a, b, c\}$, $\mu = [_4[_3[_2[_1]_1]_2]_3]_4$, $L_1 = \{H_1^\#, H_2^\#, H_3^\#\}$ where



$H_1^{\#}$

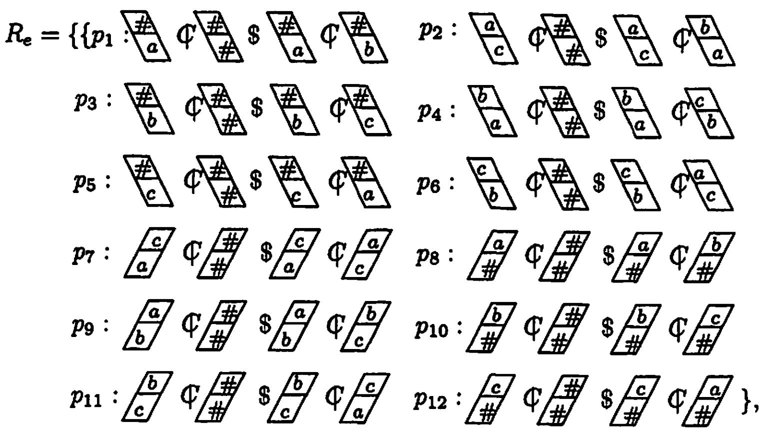


$H_2^{\#}$



$H_3^{\#}$

$R_1 = R_e \cup R_{nw} \cup R_{sw}$ with $R_{nw} = R_{sw} = \phi$ and



$\{here, out\}\}$

$L_2 = \phi$, $R_2 = R_e \cup R_{nw} \cup R_{sw}$ with $R_e = R_{sw} = \phi$

$R_{nw} = \{\{p_1 :$ $\begin{smallmatrix}a\\b\end{smallmatrix}$ ¢ $\begin{smallmatrix}c\\a\end{smallmatrix}$ \$ $\begin{smallmatrix}\#\\\#\end{smallmatrix}$ ¢ $\begin{smallmatrix}c\\a\end{smallmatrix}$ $p_2 :$ $\begin{smallmatrix}b\\\#\end{smallmatrix}$ ¢ $\begin{smallmatrix}a\\\#\end{smallmatrix}$ \$ $\begin{smallmatrix}\#\\\#\end{smallmatrix}$ ¢ $\begin{smallmatrix}a\\\#\end{smallmatrix}$

$p_3 :$ $\begin{smallmatrix}b\\c\end{smallmatrix}$ ¢ $\begin{smallmatrix}a\\b\end{smallmatrix}$ \$ $\begin{smallmatrix}\#\\\#\end{smallmatrix}$ ¢ $\begin{smallmatrix}a\\b\end{smallmatrix}$ $p_4 :$ $\begin{smallmatrix}c\\\#\end{smallmatrix}$ ¢ $\begin{smallmatrix}b\\\#\end{smallmatrix}$ \$ $\begin{smallmatrix}\#\\\#\end{smallmatrix}$ ¢ $\begin{smallmatrix}b\\\#\end{smallmatrix}$

$p_5 :$ $\begin{smallmatrix}c\\a\end{smallmatrix}$ ¢ $\begin{smallmatrix}b\\c\end{smallmatrix}$ \$ $\begin{smallmatrix}\#\\\#\end{smallmatrix}$ ¢ $\begin{smallmatrix}b\\c\end{smallmatrix}$ $p_6 :$ $\begin{smallmatrix}a\\\#\end{smallmatrix}$ ¢ $\begin{smallmatrix}c\\\#\end{smallmatrix}$ \$ $\begin{smallmatrix}\#\\\#\end{smallmatrix}$ ¢ $\begin{smallmatrix}c\\\#\end{smallmatrix}$

$p_7 :$ [a|b] ¢ [c|a] \$ [#|#] ¢ [c|a]

$p_8 :$ [b|c] ¢ [a|b] \$ [#|#] ¢ [a|b]

$p_9 :$ [c|#] ¢ [b|#] \$ [#|#] ¢ [b|#]

$p_{10} :$ [c|a] ¢ [b|c] \$ [#|#] ¢ [b|c]

$p_{11} :$ [a|#] ¢ [c|#] \$ [#|#] ¢ [c|#]

$p_{12} :$ [b|#] ¢ [a|#] \$ [#|#] ¢ [a|#] $\}, \{here, out\}\}$

$L_3 = \phi$, $R_3 = R_e \cup R_{nw} \cup R_{sw}$ with $R_e = R_{nw} = \phi$

$R_{sw} = \{\{p_1 :$ $\begin{smallmatrix}\#\\a\end{smallmatrix}$ ¢ $\begin{smallmatrix}\#\\\#\end{smallmatrix}$ \$ $\begin{smallmatrix}\#\\a\end{smallmatrix}$ ¢ $\begin{smallmatrix}\#\\b\end{smallmatrix}$ $p_2 :$ $\begin{smallmatrix}c\\b\end{smallmatrix}$ ¢ $\begin{smallmatrix}\#\\\#\end{smallmatrix}$ \$ $\begin{smallmatrix}c\\b\end{smallmatrix}$ ¢ $\begin{smallmatrix}a\\c\end{smallmatrix}$

$p_3 :$ $\begin{smallmatrix}\#\\b\end{smallmatrix}$ ¢ $\begin{smallmatrix}\#\\\#\end{smallmatrix}$ \$ $\begin{smallmatrix}\#\\b\end{smallmatrix}$ ¢ $\begin{smallmatrix}\#\\c\end{smallmatrix}$ $p_4 :$ $\begin{smallmatrix}b\\a\end{smallmatrix}$ ¢ $\begin{smallmatrix}\#\\\#\end{smallmatrix}$ \$ $\begin{smallmatrix}b\\a\end{smallmatrix}$ ¢ $\begin{smallmatrix}c\\b\end{smallmatrix}$

$p_5 :$ $\begin{smallmatrix}a\\c\end{smallmatrix}$ ¢ $\begin{smallmatrix}\#\\\#\end{smallmatrix}$ \$ $\begin{smallmatrix}a\\c\end{smallmatrix}$ ¢ $\begin{smallmatrix}b\\a\end{smallmatrix}$ $p_6 :$ $\begin{smallmatrix}\#\\c\end{smallmatrix}$ ¢ $\begin{smallmatrix}\#\\\#\end{smallmatrix}$ \$ $\begin{smallmatrix}\#\\c\end{smallmatrix}$ ¢ $\begin{smallmatrix}\#\\a\end{smallmatrix}$

$p_7 :$ [b|c] ¢ [#|#] \$ [b|c] ¢ [c|a]

$p_8 :$ [c|a] ¢ [#|#] \$ [c|a] ¢ [a|b]

$p_9 :$ [a|#] ¢ [#|#] \$ [a|#] ¢ [b|#]

$p_{10} :$ [a|b] ¢ [#|#] \$ [a|b] ¢ [b|c]

$p_{11} :$ [b|#] ¢ [#|#] \$ [b|#] ¢ [c|#]

$p_{12} :$ [c|#] ¢ [#|#] \$ [c|#] ¢ [a|#] $\}, \{here, out\}\}$

In membrane 1, the initial axiom arrays $H_1^{\#}, H_2^{\#}, H_3^{\#}$ are present in multiple number of copies. Using the east arrow head splicing rules p_3, p_4, p_9 and p_{10}, the axiom hexagonal arrays $H_1^{\#}$ and $H_2^{\#}$ are spliced to give $H_4^{\#}$ which will be sent out to membrane 2.

$$H_1^{\#} = \begin{array}{ccccccc} & & \# & & \# & & \# \\ & \# & & a & & b & & \# \\ \# & & b & & c & & a & & \# \\ & \# & & a & & b & & \# \\ & & \# & & \# & & \# \end{array} \quad H_2^{\#} = \begin{array}{ccccccc} & & \# & & \# & & \# \\ & \# & & b & & c & & \# \\ \# & & c & & a & & b & & \# \\ & \# & & b & & c & & \# \\ & & \# & & \# & & \# \end{array} \quad H_4^{\#} = \begin{array}{ccccccccc} & & \# & & \# & & \# & & \# \\ & \# & & a & & b & & c & & \# \\ \# & & b & & c & & a & & b & & \# \\ & \# & & a & & b & & c & & \# \\ & & \# & & \# & & \# & & \# \end{array}$$

Similarly using the rules p_5, p_6, p_{11} and p_{12}, the axiom hexagonal arrays $H_2^{\#}$ and $H_3^{\#}$ are spliced to give $H_5^{\#}$ which will be sent out to membrane 2.

$$H_5^{\#} = \begin{array}{ccccccccccc} & & \# & & \# & & \# & & \# \\ & \# & & b & & c & & a & & \# \\ \# & & c & & a & & b & & c & & \# \\ & \# & & b & & c & & a & & \# \\ & & \# & & \# & & \# & & \# \end{array}$$

Also using the rules p_1, p_2, p_7 and p_8, the axiom hexagonal arrays $H_3^{\#}$ spliced with $H_1^{\#}$ to give $H_6^{\#}$. This will be sent out to membrane 2.

$$H_6^{\#} = \begin{array}{ccccccccccc} & & \# & & \# & & \# & & \# \\ & \# & & c & & a & & b & & \# \\ \# & & a & & b & & c & & a & & \# \\ & \# & & a & & a & & b & & \# \\ & & \# & & \# & & \# & & \# \end{array}$$

In membrane 2, using the north west arrow head splicing rules p_1, p_2, p_7, p_8 and p_9, $H_4^{\#}$ and $H_6^{\#}$ are spliced to give $H_7^{\#}$. $H_7^{\#}$ will be sent out to membrane 3 using the target 'out'. At the same time, using the rules p_3, p_4, p_8, p_{10} and p_{11}, the hexagonal arrays $H_5^{\#}$ and $H_4^{\#}$ are spliced to yield $H_8^{\#}$. Again this will be sent out of the membrane.
In the same way, using the rules p_5, p_6, p_7, p_{10} and p_{12}, the hexagonal array $H_6^{\#}$ and $H_5^{\#}$ are spliced to give $H_9^{\#}$ which will be sent to membrane 3.

$$H_7^{\#} = \begin{array}{ccccccccccccc} & & \# & & \# & & \# & & \# \\ & \# & & a & & b & & c & & \# \\ \# & & b & & c & & a & & b & & \# \\ & \# & & a & & b & & c & & a & & \# \\ & & \# & & c & & a & & b & & \# \\ & & & \# & & \# & & \# & & \# \end{array}$$

$$
H_8^{\#} = \begin{array}{cccccccccccc}
 & & \# & & \# & & \# & & \# & & & \\
 & \# & & b & & c & & a & & \# & & \\
\# & & c & & a & & b & & c & & \# & \\
 & \# & & b & & c & & a & & b & & \# \\
 & & \# & & a & & b & & c & & \# & \\
 & & & \# & & \# & & \# & & \# & &
\end{array}
$$

$$
H_9^{\#} = \begin{array}{cccccccccccc}
 & & \# & & \# & & \# & & \# & & & \\
 & \# & & c & & a & & b & & \# & & \\
\# & & a & & b & & c & & a & & \# & \\
 & \# & & c & & a & & b & & a & & \# \\
 & & \# & & b & & c & & a & & \# & \\
 & & & \# & & \# & & \# & & \# & &
\end{array}
$$

In membrane 3, we have only south west arrow head splicing rules. $H_8^{\#}$ is spliced with $H_7^{\#}$ using the rules $p_6, p_2, p_4, p_7, p_8, p_{10}$ to yield $H_{10}^{\#}$ which is sent to the output membrane. Similarly $H_9^{\#}$ and $H_8^{\#}$ are spliced using the rules $p_1, p_5, p_2, p_7, p_8, p_{12}$ to give $H_{11}^{\#}$ which is sent out to the output membrane. Also $H_9^{\#}$ and $H_7^{\#}$ are spliced to give $H_{12}^{\#}$ which is again sent to the output membrane.

$$
H_{10}^{\#} = \begin{array}{ccccccccccccc}
 & & & \# & & \# & & \# & & \# & & & \\
 & & \# & & a & & b & & c & & \# & & \\
 & \# & & b & & c & & a & & b & & \# & \\
\# & & c & & a & & b & & c & & a & & \# \\
 & \# & & b & & c & & a & & b & & \# & \\
 & & \# & & a & & b & & c & & \# & & \\
 & & & \# & & \# & & \# & & \# & & &
\end{array}
$$

$$
H_{11}^{\#} = \begin{array}{ccccccccccccc}
 & & & \# & & \# & & \# & & \# & & & \\
 & & \# & & b & & c & & a & & \# & & \\
 & \# & & c & & a & & b & & c & & \# & \\
\# & & a & & b & & c & & a & & b & & \# \\
 & \# & & c & & a & & b & & c & & \# & \\
 & & \# & & b & & c & & a & & \# & & \\
 & & & \# & & \# & & \# & & \# & & &
\end{array}
$$

$$
H_{12}^{\#} = \begin{array}{ccccccccccccc}
 & & & \# & & \# & & \# & & \# & & & \\
 & & \# & & c & & a & & b & & \# & & \\
 & \# & & a & & b & & c & & a & & \# & \\
\# & & b & & c & & a & & b & & c & & \# \\
 & \# & & a & & b & & c & & a & & \# & \\
 & & \# & & c & & a & & b & & \# & & \\
 & & & \# & & \# & & \# & & \# & & &
\end{array}
$$

Theorem 3.3. *The class HAHSPL intersects the class HLOC.*

Proof. Consider the hexagonal array splicing P system

$$\Pi = (V \cup \{\#, \$, \text{¢}\}, \mu, L_1, L_2, R_1, R_2, 2)$$

where $V = \{a\}$

$\mu = [_2[_1]_1]_2$

$$L_1 = \left\{ \begin{array}{ccccccccc} & & \# & & \# & & \# & & \\ & \# & & a & & a & & \# & \\ \# & & a & & a & & a & & \# \\ & \# & & a & & a & & \# & \\ & & \# & & \# & & \# & & \end{array} \right\}$$

$R_1 = R_e \cup R_{nw} \cup R_{sw}$

where

$R_e = \{\{p_1 : \begin{bmatrix} \# \\ a \end{bmatrix} \text{¢} \begin{bmatrix} \# \\ \# \end{bmatrix} \$ \begin{bmatrix} \# \\ a \end{bmatrix} \text{¢} \begin{bmatrix} \# \\ a \end{bmatrix} \quad p_2 : \begin{bmatrix} a \\ a \end{bmatrix} \text{¢} \begin{bmatrix} \# \\ \# \end{bmatrix} \$ \begin{bmatrix} a \\ a \end{bmatrix} \text{¢} \begin{bmatrix} a \\ a \end{bmatrix}$

$p_3 : \begin{bmatrix} a \\ a \end{bmatrix} \text{¢} \begin{bmatrix} \# \\ \# \end{bmatrix} \$ \begin{bmatrix} a \\ a \end{bmatrix} \text{¢} \begin{bmatrix} a \\ a \end{bmatrix} \quad p_4 : \begin{bmatrix} a \\ \# \end{bmatrix} \text{¢} \begin{bmatrix} \# \\ \# \end{bmatrix} \$ \begin{bmatrix} a \\ \# \end{bmatrix} \text{¢} \begin{bmatrix} a \\ \# \end{bmatrix} \}, \{here, out\}\}$

$R_{nw} = \{\{p_5 : \begin{bmatrix} a \\ \# \end{bmatrix} \text{¢} \begin{bmatrix} a \\ \# \end{bmatrix} \$ \begin{bmatrix} \# \\ \# \end{bmatrix} \text{¢} \begin{bmatrix} a \\ \# \end{bmatrix} \quad p_6 : \begin{bmatrix} a \\ a \end{bmatrix} \text{¢} \begin{bmatrix} a \\ a \end{bmatrix} \$ \begin{bmatrix} \# \\ \# \end{bmatrix} \text{¢} \begin{bmatrix} a \\ a \end{bmatrix}$

$p_7 : \begin{bmatrix} a & a \end{bmatrix} \text{¢} \begin{bmatrix} a & a \end{bmatrix} \$ \begin{bmatrix} \# & \# \end{bmatrix} \text{¢} \begin{bmatrix} a & a \end{bmatrix}$

$p_8 : \begin{bmatrix} a & \# \end{bmatrix} \text{¢} \begin{bmatrix} a & \# \end{bmatrix} \$ \begin{bmatrix} \# & \# \end{bmatrix} \text{¢} \begin{bmatrix} a & \# \end{bmatrix} \}, \{here, out\}\}$

$R_{sw} = \{\{p9 : \begin{bmatrix} \# \\ a \end{bmatrix} \text{¢} \begin{bmatrix} \# \\ a \end{bmatrix} \$ \begin{bmatrix} \# \\ \# \end{bmatrix} \text{¢} \begin{bmatrix} \# \\ a \end{bmatrix}$

$pl0 : \begin{bmatrix} a \\ a \end{bmatrix} \text{¢} \begin{bmatrix} a \\ a \end{bmatrix} \$ \begin{bmatrix} \# \\ \# \end{bmatrix} \text{¢} \begin{bmatrix} a \\ a \end{bmatrix}$

$pl1 : \begin{bmatrix} a & a \end{bmatrix} \text{¢} \begin{bmatrix} a & a \end{bmatrix} \$ \begin{bmatrix} \# & \# \end{bmatrix} \text{¢} \begin{bmatrix} a & a \end{bmatrix}$

$pl^{-} : \begin{bmatrix} a & \# \end{bmatrix} \text{¢} \begin{bmatrix} a & \# \end{bmatrix} \$ \begin{bmatrix} \# & \# \end{bmatrix} \text{¢} \begin{bmatrix} a & \# \end{bmatrix} \}, \{here, out\}\}$

$L_2 = \phi$, $R_2 = \phi$.

The system works as follows: Initially membrane 1 contains the axiom array given by L_1. This array is spliced with itself using one of the three type of rules. Suppose if we apply the domino splicing east arrow head rules given by $R_e = \{p_1, p_2, p_3, p_4\}$, the left part of the hexagonal array in L_1 is pasted with the right part of the hexagonal array as given in Figure 6.

The resultant hexagonal array can be sent out of the membrane by using the target *out*. If it remains in the same membrane, we can again apply the rule R_e. By applying the rules R_e iteratively, we can increase the growth of the hexagonal arrays in z-direction.

On the other hand, we can apply the rules R_{nw} or R_{sw} initially instead of R_e so that the hexagonal array can grow in x-direction or y-direction

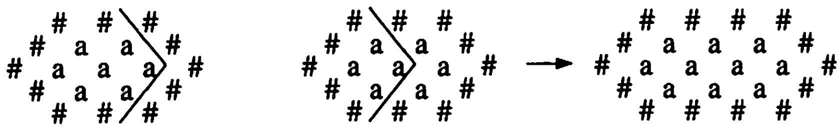


Figure 6: Spliced hexagonal array

respectively. By applying the rules R_{nw} or R_{sw} iteratively we can generate all sizes of hexagonal arrays. Hence the language generated by the system is the set of all hexagonal arrays over $\{a\}$ of size (ℓ, m, n), $\ell \geq 2$, $m \geq 2$, $n \geq 2$, which is local. □

Theorem 3.4. *The class of regular controlled table 0L hexagonal array languages CT0LHAL and HAHSPL are incomparable but not disjoint.*

Proof. Consider the language

$$L = \left\{ \begin{array}{ccccc} & a & & a & \\ a & & a & & a \\ & a & & a & \end{array} , \begin{array}{ccccccc} & & a & & a & & a & & \\ & a & & a & & a & & a & \\ a & & a & & a & & a & & a \\ & a & & a & & a & & a & \\ & & a & & a & & a & & \end{array} , \ldots \right\}.$$

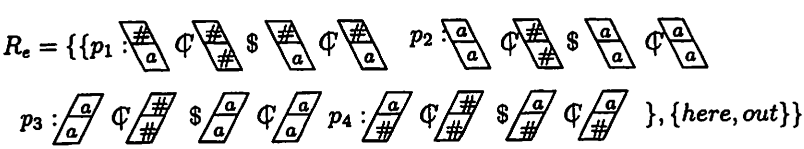
The above language can be generated by a regular controlled table 0L hexagonal array grammar [11]. But this cannot be generated by a hexagonal splicing P system, since the domino arrow head splicing rules cannot control the equality in the values of (ℓ, m, n).

Consider the hexagonal splicing P system

$$\Pi = (V \cup \{\#, ¢, \$\}, \mu, L_1, L_2, R_1, R_2, 2)$$

where $V = \{a\}$, $L_1 = \left\{ \begin{array}{ccccccccc} & & \# & & \# & & \# & & \\ & \# & & a & & a & & \# & \\ \# & & a & & a & & a & & \# \\ & \# & & a & & a & & \# & \\ & & \# & & \# & & \# & & \end{array} \right\}$, $\mu = [_2[_1]_1]_2$,

$R_1 = R_e \cup R_{nw} \cup R_{sw}$ with $R_{nw} = R_{sw} = \phi$



$L_2 = \phi$, $R_2 = \phi$.

It is clear that $L(\Pi) = \left\{ \begin{array}{ccccccc} & a & a & & a & a & a \\ a & a & a & , & a\ a\ a\ a & ,\ldots & \\ & a & a & & a & a & a \end{array} \right\}$ which is also in CT0LHAL [11].

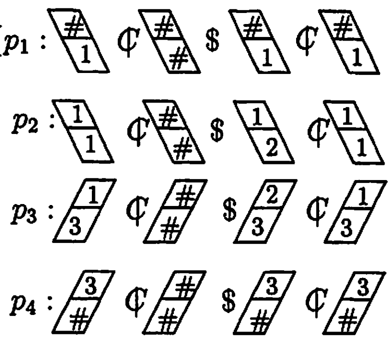
Consider the hexagonal splicing P system

$$\Pi = (V \cup \{\#, ₵, \$\}, \mu, L_1, L_2, R_1, R_2, 2)$$

where $V = \{1,2,3\}$, $L_1 = \left\{ \begin{array}{ccccccccc} & & \# & & \# & & \# & & \\ & \# & & 1 & & 1 & & \# & \\ \# & & 1 & & 2 & & 1 & & \# \\ & \# & & 3 & & 3 & & \# & \\ & & \# & & \# & & \# & & \end{array} \right\}$,

$\mu = [_2[_1]_1]_2$,
$R_1 = R_e \cup R_{nw} \cup R_{sw}$ with $R_{nw} = R_{sw} = \phi$

$R_e = \{\{p_1 :$



$\}, \{here, out\}\}$

$L_2 = \phi$, $R_2 = \phi$.

We can easily see that $L(\Pi) = \left\{ \begin{array}{c} 1\ 1 \\ 1\ 2\ 1 \\ 3\ 3 \end{array} , \begin{array}{c} 1\ 1\ 1 \\ 1\ 2\ 2\ 1 \\ 3\ 3\ 3 \end{array} ,\ldots \right\}$.

This language cannot be generated by any controlled table 0L hexagonal array grammar, since, if we allow any of the rule $11 \leftarrow 1$ or $12 \leftarrow 1$ we get a different language. Hence the proof. □

4 Conclusion

In this paper, a new P system on pictures with hexagonal arrays as objects and arrow head splicing rules on hexagonal arrays as evolution rules has been proposed. The new class of hexagonal array languages generated by the proposed system has been compared with the existing families of hexagonal array languages like local hexagonal array languages, etc.

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