The monophonic detour hull number of a graph

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Abstract

For vertices u and v in a connected graph $G = (V, E)$, the monophonic detour distance $d_m(u, v)$ is the length of a longest u-v monophonic path in G. An u-v monophonic path of length $d_m(u, v)$ is an u-v monophonic detour or an u-v m-detour. The set $I_{d_m}[u, v]$ consists of all those vertices lying on an u-v m-detour in G. Given a set S of vertices of G, the union of all sets $I_{d_m}[u, v]$ for $u, v \in S$, is denoted by $I_{d_m}[S]$. A set S is an m-detour convex set if $I_{d_m}[S] = S$. The m-detour convex hull $[S]_{d_m}$ of S in G is the smallest m-detour convex set containing S. A set S of vertices of G is an *m-detour set* if $I_{d_m}[S] = V$ and the minimum cardinality of an m-detour set is the *m-detour number* $md(G)$ of G. A set S of vertices of G is an *m-detour hull set* if $[S]_{d_m} = V$ and the minmimum cardinality of an m-detour hull set is the *m-detour hull number* $md_h(G)$ of G. Certain general properties of these concepts are studied. Bounds for the m-

detour hull number of a graph are obtained. It is proved that every two integers a and b with $2 \leq a \leq b$ are realizable as the m-detour hull number and the m-detour number respectively, of some graph. Graphs G of order n for which $md_h(G) = n$ or $md_h(G) = n - 1$ are characterized. It is proved that for each triple a, b and k of positive integers with $a < b$ and $k \geq 3$, there exists a connected graph G with $rad_m G = a, diam_m G = b$ and $md_h(G) = k$.

Key Words: m-detour, m-detour convex set, m-detour number, m-detour extreme vertex, m-detour hull number.
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1 Introduction

By a *graph* $G = (V, E)$, we mean a finite undirected connected graph without loops or multiple edges. The *order* and *size* of G are denoted by n and m respectively. For basic definitions and terminologies, we refer to [1, 8]. The *distance* $d(u, v)$ between two vertices u and v in a connected graph G is the length of a shortest u-v path in G. An u-v path of length $d(u, v)$ is called an *u-v geodesic*. A vertex v in G is an *extreme vertex* if the subgraph induced by its neighbors is complete. The set $I[u, v]$ consists of all vertices lying on some u-v geodesic of G; while for $S \subseteq V$, $I[S] = \bigcup_{u,v \in S} I[u, v]$. The set S is *convex* if $I[S] = S$. The *convex hull* $[S]$ is the smallest convex set containing S. A set S of vertices of G is a *hull set* of G if $[S] = V$. The *hull number* $h(G)$ of G is the minimum cardinality of a hull set and any hull set of cardinality $h(G)$ is called a *minimum hull set* of G. A set S of vertices of G is a *geodetic set* if $I[S] = V$, and a geodetic set of minimum cardinality is a *minimum geodetic set* of G. The cardinality of a minimum geodetic set of G is the *geodetic number* $g(G)$. These concepts were studied in [1, 3, 4, 5, 9].

For vertices u and v in a nontrivial connected graph G, the *detour distance* $D(u, v)$ is the length of a longest u-v path in G. An u-v path of length $D(u, v)$ is an u-v detour. It is known that the detour distance is a metric on the vertex set V of G. The *detour eccentricity* $e_D(v)$ of a vertex v in G is the maximum detour distance from v to a vertex of G. The *detour radius*, $rad_D(G)$ of G is the minimum detour eccentricity among the vertices of G, while the *detour diameter*, $diam_D(G)$ of G is the maximum detour eccentricity among the vertices of G. A vertex v in a connected graph G is a *detour extreme vertex* if it is an initial or terminal vertex of any detour in G containing the vertex v. The detour distance of a graph was studied in [2].

The set $I_D[u, v]$ consists of all vertices lying on some u-v detour of G; while for $S \subseteq V$, $I_D[S] = \bigcup_{u,v \in S} I_D[u, v]$. A set S of vertices is a *detour* *convex set* if $I_D[S] = S$. The *detour convex hull* $[S]_D$ of S in G is the smallest detour convex set containing S. A set S of vertices of G is a *detour hull set* if $[S]_D = V$ and a detour hull set of minimum cardinality is the *detour hull number* $d_h(G)$. The detour hull number of a graph was introduced and studied in [14]. A set $S \subseteq V$ is called a *detour set* if $I_D[S] = V$. The *detour number* $dn(G)$ of G is the minimum cardinality of a detour set and any detour set of cardinality $dn(G)$ is called a *minimum detour set* of G. The detour number of a graph was introduced in [6] and further studied in [12].

A *chord* of a path $P = (u_0, u_1, \dots, u_n)$ is an edge u_iu_j, with $j \geq i+2$. For vertices u and v in a connected graph G, any chordless path connecting u and v is an *u-v monophonic path* or an *m-path. The monophonic detour distance* $d_m(u,v)$ is the length of a longest u-v monophonic path in G. An u-v monophonic path of length $d_m(u,v)$ is an *u-v monophonic detour* or an *u-v m-detour*. The *m-detour eccentricity* $e_m(v)$ of a vertex v in G is the maximum m-detour distance from v to a vertex of G. The *m-detour radius*, $rad_m(G)$ of G is the minimum m-detour eccentricity among the vertices of G, while the *m-detour diameter*, $diam_m(G)$ of G is the maximum m-detour eccentricity among the vertices of G. The m-detour distance of a graph was introduced and studied in [13].

These concepts have interesting applications in Channel Assignment Problem in radio technologies [7, 10]. In [11], the distance martix and the detour matrix of a connected graph are used to discuss the applications of the graph parameters Wiener index, the detour index, the hyper-Wiener index and the hyper-detour index to a class of graphs viz. bridge and chain graphs, which in turn, capture different aspects of certain molecular graphs associated to the molecules arising in special situations of molecular problems in theoretical Chemistry. For more applications of these parameters, one may refer to [11] and the references therein.

2 The monophonic detour hull number of a graph

For vertices u and v in a connected graph G, the *monophonic detour distance* $d_m(u,v)$ is the length of a longest u-v monophonic path in G. A u-v monophonic path of length $d_m(u,v)$ is an *u-v monophonic detour* or simply an *u-v m-detour*. The set $I_{d_m}[u,v]$ consists of all vertices lying on some u-v m-detour of G, while for $S \subseteq V, I_{d_m}[S] = \cup_{u,v \in S} I_{d_m}[u,v]$. A set S of vertices is an *m-detour convex set* if $I_{d_m}[S] = S$. The *m-detour convex hull* $[S]_{d_m}$ of S in G is the smallest m-detour convex set containing S. The m-detour convex hull $[S]_{d_m}$ of S can also be formed from the sequence $\{I^k_{d_m}[S]\}(k \geq 0)$, where $I^0_{d_m}[S] = S, I^1_{d_m}[S] = I_{d_m}[S]$ and

$I^k_{d_m}[S] = I_{d_m}[I^{k-1}_{d_m}[S]]$. From some term on, this sequence must be constant. Let p be the smallest number such that $I^p_{d_m}[S] = I^{p+1}_{d_m}[S]$. Then $I^p_{d_m}[S]$ is the m-detour convex hull $[S]_{d_m}$ of S. A set S of vertices of G is an *m-detour set* of G if $I_{d_m}[S] = V$ and an m-detour set of minimum cardinality is the *m-detour number* $md(G)$ of G. A set S of vertices of G is an *m-detour hull set* if $[S]_{d_m} = V$ and an m-detour hull set of minimum cardinality is the *m-detour hull number* $md_h(G)$.

Example 2.1. For the graph G given in Figure 2.1, it is easily verified that no 2-element subset of vertices of G is an m-detour hull set of G. Now, let $S = \{u_1, u_3, u_6\}$. Then $I_{d_m}[S] = \{u_1, u_2, u_3, u_5, u_6, u_7, u_8\}$ and $I^2_{d_m}[S] = V$. Hence S is a minimum m-detour hull set of G and so $md_h(G) = 3$. It is easy to verify that no 3-element subset of vertices of G is a hull set of G. Let $S_1 = \{u_1, u_3, u_6, u_7\}$. Since $I[S_1] = V$, S_1 is a minimum hull set of G and so $h(G) = 4$. Let $S_2 = \{u_1, u_6\}$. Then $I_D[S_2] = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\}$ and $I^2_D[S_2] = V$. Hence S_2 is a minimum detour hull set and so $d_h(G) = 2$.

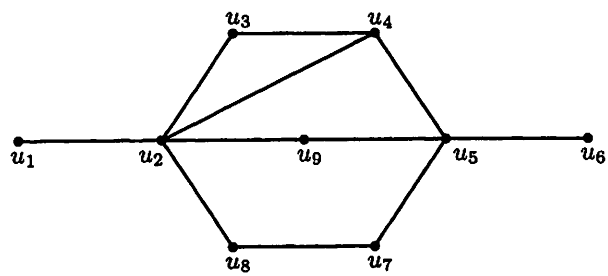

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Figure 2.1

Definition 2.2. A vertex v in a connected graph G is an *m-detour extreme vertex* if it is an initial vertex or a terminal vertex of a m-detour in G containing the vertex v.

Example 2.3. For the graph G in Figure 2.1, the vertices u_1, u_3 and u_6 are m-detour extreme vertices.

Remark 2.4. Each extreme vertex is an m-detour extreme vertex. However, an m-detour vertex need not be an extreme vertex. Let $W_{1,5} = K_1 + C_5$, where $K_1 = \{v_1\}$ and C_5 is the cycle $C_5 = (v_2, v_3, v_4, v_5, v_6, v_2)$. Then $d_m(u, v) = 3$ for any two non-adjacent vertices u, v in $W_{1,5}$ and it is clear that the vertex v_1 does not lie on any u-v m-detour for any non-adjacent vertices u, v in $W_{1,5}$. Since any u-v m-detour containing v_1 is of the form $v_1v_i (i = 2, 3, \ldots, 6)$ or $v_iv_1 (i = 2, 3, \ldots, 6)$, it follows that v_1 is a

m-detour extreme vertex. Note that v_1 is not a extreme vertex. Also v_1 is not a detour extreme vertex.

Observation 2.5. A vertex v in a connected graph G is an m-detour extreme vertex if and only if $V(G) - \{v\}$ is an m-detour convex set.

Proposition 2.6. *Each m-detour extreme vertex of a nontrivial connected graph G belongs to every m-detour hull set of G. Moreover, if the set of all m-detour extreme vertices S of G is an m-detour hull set of G, then S is the unique minimum m-detour hull set of G.*

Proof. Let v be an m-detour extreme vertex of G. Then v is either an initial vertex or a terminal vertex of any m-detour containing the vertex v. Hence it follows that v belongs to every m-detour hull set of G. □

Definition 2.7. Let $k \geq 2$ be an integer. A connected graph G is said to be *k-monophonic regular graph* if $d_m(u, v) \geq k$ for each pair of non-adjacent vertices u and v in G.

Note 2.8. For $k \geq 3$ every k-monophonic regular graph is $(k-1)$-monophonic regular.

Example 2.9. The cycle $C_n (n \geq 4)$ and the wheel $W_{1,n} (n \geq 4)$ are $\lfloor n/2 \rfloor$-monophonic regular graphs.

Proposition 2.10. *Let G be a connected graph of order n and let u, v and w be three distinct vertices such that $w \in I_{d_m}[u, v]$. Then $deg(w) \leq n - d_m(u, v) + 1$.*

Proof. Let P be a u-v monophonic path of length $d_m(u, v)$ containing w. Since P is an induced path, w is nonadjacent to $d_m(u, v) - 2$ vertices of P and hence the result follows. □

Corollary 2.11. *Let G be a k-monophonic regular graph and let w be any vertex in G such that $deg(w) \geq n - k + 2$. Then w is an m-detour extreme vertex of G.*

Proof. This follows from Proposition 2.10. □

Corollary 2.12. *Let G be a 3-monophonic regular graph. Then every full degree vertex is an m-detour extreme vertex of G.*

Proof. This follows from Corollary 2.11. □

Remark 2.13. The converse of Corollary 2.11 is not true. For the graph G given in Figure 2.2, the vertex v is an m-detour extreme vertex. Also, since $d_m(x, y) \geq 3$ for all non-adjacent vertices x and y in G,we have G is a 3-monophonic regular graph. However, $deg(v) = 2 < 7 = n - k + 1$.

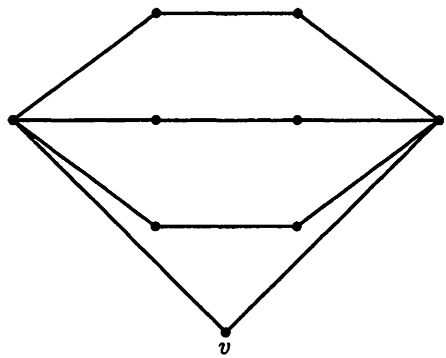

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Figure 2.2

Proposition 2.14. *Let S be a minimum m-detour hull set of a connected graph G and let $x, y \in S$. If z is a vertex distinct from both x and y such that $z \in I_{d_m}[x, y]$, then $z \notin S$.*

Proof. Assume to the contrary that $z \in S$. Since $z \in I_{d_m}[x, y] \subseteq I_{d_m}[S - \{z\}]$, we have $S \subseteq I_{d_m}[S - \{z\}]$. This shows that $S - \{z\}$ is an m-detour hull set of G, which is a contradiction. □

Theorem 2.15. *Let S be any m-detour hull set of a connected graph G. Then*

(i) If v is a cutvertex of G and C is a component of $G-v$, then $S \cap V(C) \neq \emptyset$.

(ii) No cutvertex of G belongs to any minimum m-detour hull set of G.

Proof. (i) If $S \cap V(C) = \emptyset$ for some component C of $G - v$, then $I^n_{d_m}[S] \subseteq V(G) - V(C)$ for each $n = 1, 2, 3, \ldots$ and so $[S]_{d_m} \subseteq V(G) - V(C)$. Thus $[S]_{d_m} \neq V(G)$ and so S is not an m-detour hull set of G, which is a contradiction. Hence it follows that $S \cap V(C) \neq \emptyset$.

(ii) Let S be a minimum m-detour hull set of G. Let v be a cutvertex of G and let $C_1, C_2, \ldots, C_k (k \geq 2)$ the components of $G - v$. Then by (i) $S \cap V(C_i) \neq \emptyset$ and $S \cap V(C_j) \neq \emptyset (i \neq j)$. Let $x \in S \cap V(C_i)$ and $y \in S \cap V(C_j) (i \neq j)$. Now, since v is a cutvertex of G, it follows that $v \in I_{d_m}[x, y]$ and so by Proposition 2.14, $v \notin S$. □

3 Graphs G of order n with m-detour hull numbers n or $n-1$

Theorem 3.1. *Let G be a connected graph of order n with m-detour diameter d_m. If k is the number of m-detour extreme vertices of G, then $k \leq md_h(G) \leq n - d_m + 1$.*

Proof. The left inequality follows from Proposition 2.6. Let u and v be vertices such that $d_m(u, v) = d_m$. Let P be any u-v m-detour of G. Then it is obvious that $(V(G) - V(P)) \cup \{u, v\}$ is a m-detour hull set of G and hence it follows that $md_h(G) \leq n - d_m + 1$. □

Theorem 3.2. *Let G be a connected graph of order n. Then $md_h(G) = n$ if and only if $G = K_n$.*

Proof. If $G = K_n$, then it follows from Proposition 2.6, that $md_h(G) = n$. Conversely, assume that $md_h(G) = n$. Then it follows from Theorem 3.1 that $d_m = 1$ and so $G = K_n$. □

Theorem 3.3. *Let G be a connected graph of order n with $n \geq 3$. Then $md_h(G) = n - 1$ if and only if $G = K_1 + \cup m_j K_j$ where $\sum_j m_j \geq 2$.*

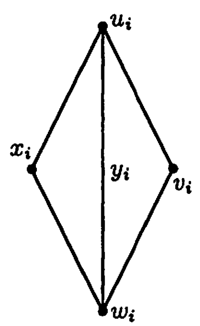
Proof. Suppose that $G = K_1 + \cup m_j K_j$ with $\sum_j m_j \geq 2$. Then G has exactly one cutvertex, say v and all other vertices are extreme and hence by Proposition 2.6 and Theorem 2.15, we have $md_h(G) = n - 1$.

Conversely, assume that $md_h(G) = n - 1$. Then by Theorem 3.1, $d_m \leq 2$. If $d_m = 1$, then $G = K_n$ and so by Theorem 3.2,we have $md_h(G) = n$, which is impossible. Hence $d_m = 2$. Let S be a minimum m-detour hull set of G. Then $|S| = n - 1$. Let $v \notin S$. We claim that v is cutvertex of G. Otherwise, $G - v$ has just one component. By Proposition 2.6, v is not an extreme vertex of G. Hence there exist vertices x and y such that x and y are adjacent to v, and x and y are not adjacent. Let P be an $x - y$ m-detour in $G - v$ such that $d_m(x, y) \geq 2$ in $G - v$. Choose a vertex $z \in V(P)$ such that $z \neq x, y$. Then $z \neq v$. Let $S' = V(G) - \{v, z\}$. Then, since $d_m = 2$, the $x - y$ m-detour P in $G - v$ is also an $x - y$ m-detour in G, and in fact, $d_m(x, y) = 2$. Also $Q : x, v, y$ is another $x - y$ m-detour in G. Hence $I[S'] = V$ and so S' is an m-detour hull set of G. Thus $md_h(G) \leq n - 2$, which is a contradiction. Hence by Theorem 2.15, v is the only cutvertex of G. Now let $G_1, G_2, \ldots, G_r (r \geq 2)$ be the components of $G - v$. We claim that each $G_i (1 \leq i \leq r)$ is complete. Suppose that some component say G_1, is not complete. Then there exist two vertices x and y in G_1 such that x and y are not adjacent. Choose a vertex z in an $x - y$ m-detour such that $z \neq x, y$. Then $S'' = V(G) - \{v, z\}$ is an m-detour hull set of G so that $md_h(G) \leq n - 2$, which is a contradiction. Hence each G_i is complete.

Now, it remains prove that v is adjacent to every vertex of G_i for each $i(1 \leq i \leq r)$. Otherwise, there exists a component, say G_i such that v is not adjacent to at least one vertex u of G_i. Choose a vertex w in G_i such that v and w are adjacent. Choose a vertex t in $G_j(i \neq j)$ such that v and t are adjacent in G. Then $Q = (u, w, v, t)$ is a monophonic path of length 3 so that $d_m(u,t) = 3$, which is a contradiction to $d_m = 2$. Hence the proof is complete. □

Theorem 3.4. *For each pair of integers a, b with $2 \leq a \leq b$, there exists a connected graph G with $md_h(G) = a, md(G) = b$.*

Proof. For $a = b$, take $G = K_{1,a}$. Then by Proposition 2.6,we have $md_h(G) = md(G) = a$. Assume that $a < b$. Let $G_i(i = 1, 2, \ldots, b-a)$ be the graph in Figure 3.1 with vertex set $V(G_i) = \{x_i, y_i, u_i, v_i, w_i\}$. Now, let H be the graph obtained from these $G_i(1 \leq i \leq b-a)$ by joining the edges v_ix_{i+1} for $i = 1, 2, \ldots, b-a-1$. Let G be the graph obtained from H by adding the new vertices $t_1, t_2, \ldots, t_a$ and joining t_1 to v_{b-a} and $t_i(2 \leq i \leq a)$ to x_1. The graph G is shown in Figure 3.2. by Proposition 2.6, the set $S = \{t_1, t_2, \ldots, t_a\}$ is contained in every m-detour hull set as well as every m-detour set of G. It is clear that the vertex $y_i(1 \leq i \leq b-a)$ lies only on the m-detour path $P_i = (u_i, y_i, w_i)$. Hence it follows that the vertices $y_1, y_2, \ldots, y_{b-a}$ belong to every m-detour set of G. Now, since $S \cup \{y_1, y_2, \ldots, y_{b-a}\}$ is a m-detour set of G, we have $md(G) = b$. Also, $I_D[S] = V(G) - \{y_1, y_2, \ldots, y_{b-a}\}$and $I_D^2[S] = V(G)$. Hence S is a m-detour hull set of G and so by Proposition 2.6, $md_h(G) = |S| = a$. □



G_i
Figure 3.1

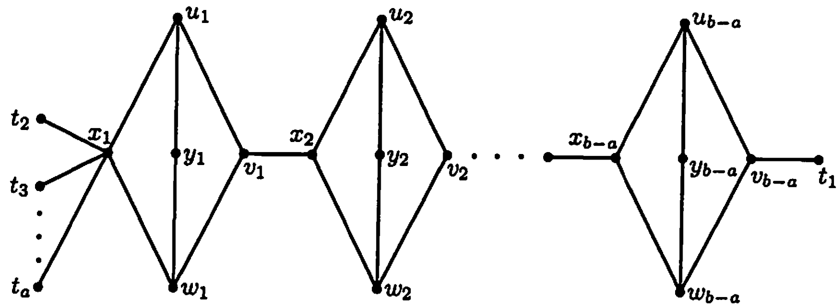


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Figure 3.2

Theorem 3.5. *For each triple a, b and k of positive integers with $a < b$ and $k \geq 3$, there exists a connected graph G with $rad_m G = a, diam_m G = b$ and $md_h(G) = k$.*

Proof. For $a < b \leq 2a$, let $C_1 = (u_1, u_2, \ldots, u_{a+2}, u_1)$ be a cycle of order $a+2$ and $C_2 = (v_1, v_2, \ldots, v_{b-a+2}, v_1)$ be a cycle of order $b - a + 2$. Let G_1 be the graph obtained from $C_1$by identifying the vertex u_1 of C_1 and v_1 of C_2. Let G be the graph obtained from G_1 by adding $k - 2$ new vertices $w_1, w_2, \ldots, w_{k-2}$ and joining each $w_i (1 \leq i \leq k-2)$ to u_1. It can be easily verified that $rad_m(G) = a$ and $diam_m(G) = b$. Now, it follows from Proposition 2.6 and Theorem 2.15 that any m-detour hull set contains $w_1, w_2, \ldots, w_{k-2}$ and at least one vertex from C_1, and at least one vertex from C_2. Since $S = \{w_1, w_2, \ldots, w_{k-2}, u_2, v_3\}$ is a m-detour hull set of G, we have $md_h(G) = |S| = k$.

For $b > 2a$, let G be the graph of order $b + 2a + 2$ obtained by identifying the central vertex of the wheel $W = K_1 + C_{b+2} (b \geq 2)$ and an end vertex v_1 of the path P_{2a}. Let $K_1 = \{v_1\}$. Let the cycle C_{b+2} be $C_{b+2} = (u_1, u_2, \ldots, u_{b+2}, u_1)$ and let $P_{2a} = (v_1, v_2, \ldots, v_{2a})$ be a path of order $2a$. Let G be a graph obtained by adding $k - 3$ new vertices $w_1, w_2, \ldots, w_{k-3}$ and joining each $w_i (1 \leq i \leq k-3)$ to v_{2a-1}. The graph G is shown in Figure 3.3. Since $b > 2a$, $e_m(x) = b$ for any vertex $x \in V(C_{b+2})$. Also, $a \leq e_m(x) \leq 2a$ for any vertex $x \in V(P_{2a})$ and $e_m(v_a) = a$. Hence $rad_m(G) = a$ and $diam_m(G) = b$. Now, by Proposition 2.6, the vertices $w_1, w_2, \ldots, w_{k-3}$, v_{2a} must belong to every m-detour hull set of G. Also, by Theorem 2.15, every m-detour hull set contains at least one vertex from the cycle C_{b+2}. Now, for any $i = 1, 2, \ldots, b+2$, the set $S = \{w_1, w_2, \ldots, w_{k-3}, v_{2a}, u_i\}$ is not a m-detour hull set of G. Since

$S_1 = \{w_1, w_2, \ldots, w_{k-3}, v_{2a}, u_1, u_3\}$ is a m-detour hull set, it follows that $md_h(G) = |S| + 1 = k$.

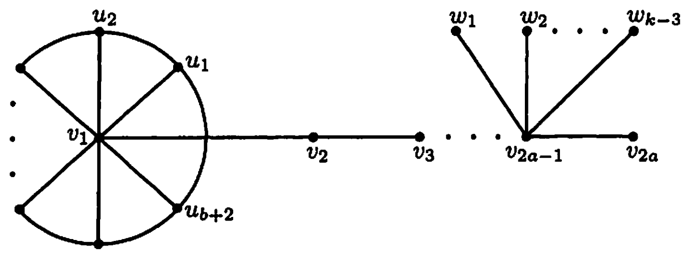

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Figure 3.3

□

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