Super (a, d)-edge-antimagic total labelings of generalized friendship graphs

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Abstract

An (a, d)-edge-antimagic total labeling of a graph G with p vertices and q edges is a bijection f from the set of all vertices and edges to the set of positive integers $\{1, 2, 3, ..., p + q\}$ such that all the edge-weights $w(uv) = f(u) + f(v) + f(uv); uv \in E(G)$, form an arithmetic progression starting from a and having common difference d. An (a, d)-edge-antimagic total labeling is called a super (a, d)-edge-antimagic total labeling ((a, d)-SEAMT labeling) if $f(V(G)) = \{1, 2, 3, ..., p\}$. The graph F_n consisting of n triangles with a common vertex is called the friendship graph. The generalized friendship graph $F_{m_1, m_2, ..., m_n}$ consists of n cycles of orders $m_1 \leq m_2 \leq, ..., \leq m_n$ having a common vertex. In this paper we prove that the friendship graph F_{16} does not admit a $(a, 2)$-SEAMT labeling. We also investigate the existence of (a, d)-SEAMT labeling for several classes of generalized friendship graphs.

Key Words: super (a, d)-edge antimagic total labelings, friendship graphs, generalized friendship graphs.
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1 Introduction

Throughout this paper $G = (V, E)$ is a finite undirected graph with neither loops nor multiple edges. The order and size of G are denoted by p and

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q respectively. For graph theoretic terminology we refer to Chartrand and Lesniak [6].

A labeling of a graph G is a mapping that assigns integers to the vertices or edges or both, subject to certain conditions. The labeling is called a vertex labeling or an edge labeling or a total labeling according as the domain of the mapping is $V(G)$ or $E(G)$ or $V(G)\bigcup E(G)$. If f is a total labeling then the weight of an edge uv is defined by $w(uv) = f(u) + f(v) + f(uv)$.

An (a,d)-edge-antimagic vertex labeling of a (p,q)-graph G ((a,d)-EAV labeling) is a bijection $f : V \rightarrow \{1,2,3,...,p\}$ with the property that the edge-weights $w(uv) = f(u) + f(v), uv \in E(G)$ form an arithmetic progression $a, a+d, ..., a+(q-1)d$, where $a > 0$ and $d \geq 0$ are two fixed integers.

An (a,d)-edge-antimagic total labeling of a (p,q)-graph G is a bijection $f : V \cup E \rightarrow \{1,2,3,...,p+q\}$ with the property that the edge-weights form an arithmetic progression $a, a+d, ..., a+(q-1)d$, where $a > 0$ and $d \geq 0$ are two fixed integers. If such a labeling exists then G is called an (a,d)-edge-antimagic total graph. If further the vertex labels are the integers $\{1,2,3,...,p\}$, then f is called a super (a,d)-edge-antimagic total labeling of G ((a,d)-SEAMT labeling) and a graph which admits such a labeling is called a super (a,d)-edge-antimagic total graph. This labeling was first introduced by Simanjuntak et al. [8]. Baca et al. [2] have obtained the following theorem which is useful in proving the existence of (a,d)-SEAMT labeling.

Theorem 1.1. [2, 4] *If G has an (a,d)-EAV labeling, then*

(i) G has a $(a+p+1, d+1)$-SEAMT labeling and

(ii) G has a $(a+p+q, d-1)$-SEAMT labeling.

Sudarsana et al. [9] have obtained the following theorem.

Theorem 1.2. [9] *Let G be a (a,d)-SEAMT graph with p vertices and q edges. Let f be a (a,d)-SEAMT labeling of G. Then, the labeling $\overline{f}$ defined by*

$$\overline{f}(v) = p + 1 - f(v), \forall\ v \in V \text{ and}$$
$$\overline{f}(e) = 2p + q + 1 - f(e), \forall\ e \in E$$

is a $(4p + q + 3 - a - (q-1)d, d)$-SEAMT labeling of G.

Any graph G having exactly one cut vertex c in which every block is a cycle is called a generalized friendship graph. We denote by $F_{m_1,m_2,...,m_n}$, the generalized friendship graph having n cycles of lengths $m_1 \leq m_2 \leq ,..., \leq m_n$. If each $m_i = 3$, then we have the friendship with n 3-cycles, which we simply denote by F_n. An edge e of F_n is called a *spoke* if e is incident at c. Otherwise e is called a *rim*.

Slamin et al. [7] showed that the friendship graph F_n has a $(a,0)$-SEAMT labeling if and only if $n \in \{1,3,4,5,7\}$. Baca et al. [3] showed that for $n \in \{1,3,4,5,7\}$, the friendship graph F_n has a $(a,0)$-SEAMT labeling and $(a,2)$-SEAMT labeling. In [1], we have proved that the friendship graph F_n does not admit an $(a,2)$-SEAMT labeling, when n is even and $n \not\equiv 4\ (mod\ 12)$ and the generalized friendship graph of order $p \geq 5$ has a $(2p+2,1)$-SEAMT labeling if and only if p is odd.

In this paper we prove that F_{16} does not admit an $(a,2)$-SEAMT labeling. We also investigate the existence of $(a,2)$-SEAMT labeling for several classes of generalized friendship graphs.

2 Main Results

Theorem 2.1. *The generalized friendship graph $G = F_{3,m_2}$, where $m_2 \equiv 1\ (mod\ 4)$, has a $(\frac{3m_2+9}{2},2)$-SEAMT labeling and a $(\frac{5m_2+13}{2},0)$-SEAMT labeling.*

Proof. Let $C_1 = (c,u_1,u_2,c)$ and $C_2 = (c,v_1,v_2,\ldots,v_{m_2-1},c)$ be the two cycles in G. The order and size of G are given by $p = m_2+2$ and $q = m_2+3$. In view of Theorem 1.1, it is enough to prove that G admits a $(\frac{p+1}{2},1)$-EAV labeling. We define $f : V(F_{3,m_2}) \longrightarrow \{1,2,3,...,p\}$ by

$$\begin{aligned}
f(c) &= \frac{p+1}{4} \\
f(v_{2i}) &= \frac{p+1}{4} - i \text{ if } 1 \leq i \leq \frac{p-3}{4}, \\
f(v_{2i-1}) &= \frac{3p-1}{4} - i \text{ if } 1 \leq i \leq \frac{p-3}{4}, \\
f(v_{\frac{p-3}{2}+2i-1}) &= \frac{p+1}{2} - i \text{ if } 1 \leq i \leq \frac{p-3}{4}, \\
f(v_{\frac{p-3}{2}+2i}) &= p+1-i \text{ if } 1 \leq i \leq \frac{p-3}{4}, \\
f(u_1) &= \frac{3p-1}{4} \text{ and} \\
f(u_2) &= \frac{3p-1}{4} + 1.
\end{aligned}$$

$$\begin{aligned}
\text{Then } w(cv_1) &= p-1, \\
w(cv_{p-3}) &= p+2, \\
w(cu_1) &= p, \\
w(cu_2) &= p+1,
\end{aligned}$$

$$w(u_1u_2) = \frac{3p-1}{2} + 1,$$
$$w(v_{2i-1}v_{2i}) = p - 2i \text{ if } 1 \le i \le \frac{p-3}{4},$$
$$w(v_{2i}v_{2i+1}) = p - 2i - 1 \text{ if } 1 \le i \le \frac{p-3}{4},$$
$$w(v_{\frac{p-3}{2}+2i-1}v_{\frac{p-3}{2}+2i}) = \frac{3p+3}{2} - 2i \text{ if } 1 \le i \le \frac{p-3}{4},$$
$$w(v_{\frac{p-3}{2}+2i}v_{\frac{p-3}{2}+2i+1}) = \frac{3p+3}{2} - 2i - 1 \text{ if } 1 \le i \le \frac{p-3}{4} - 1 \text{ and}$$
$$w(v_{\frac{p-3}{2}}v_{\frac{p-3}{2}+1}) = \frac{p-3}{2} + 2.$$

Clearly the set of all edge-weights $W = \{\frac{p+1}{2}, \frac{p+3}{2}, ..., \frac{p+1}{2} + q - 1\}$ is an arithmetic progression with first term $\frac{p+1}{2}$ and common difference 1. Thus f is a $(\frac{p+1}{2}, 1)$-EAV labeling. □

Example 2.2. An $(8, 1)$-EAV labeling of $F_{3,13}$ is given in Figure 1.

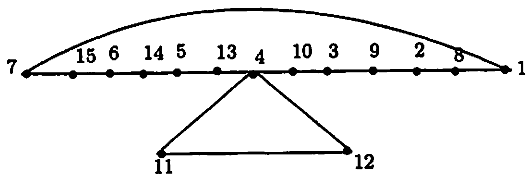

Figure 1

Theorem 2.3. *The generalized friendship graph $G = F_{4,m_2}$, where $m_2 \equiv 0 \pmod 4$, has a $(\frac{3m_2+12}{2}, 2)$-SEAMT labeling and a $(\frac{5m_2+18}{2}, 0)$-SEAMT labeling.*

Proof. Let $C_1 = (c, u_1, u_2, u_3, c)$ and $C_2 = (c, v_1, v_2, ..., v_{m_2-1}, c)$ be the two cycles in G. The order and size of G are given by $p = m_2 + 3$ and $q = m_2 + 4$. In view of Theorem 1.1, it is enough to prove that G admits a $(\frac{p+1}{2}, 1)$-EAV labeling. We define $f : V(F_{4,m_2}) \longrightarrow \{1, 2, 3, ..., p\}$ by

$$f(c) = \frac{p+1}{4},$$
$$f(v_{2i}) = \frac{p+1}{4} - i \text{ if } 1 \le i \le \frac{p-3}{4},$$
$$f(v_{2i-1}) = \frac{3p-5}{4} - i \text{ if } 1 \le i \le \frac{p-3}{4},$$
$$f(v_{\frac{p-3}{2}+2i-1}) = p + 1 - i \text{ if } 1 \le i \le \frac{p-3}{4},$$
$$f(v_{\frac{p-3}{2}+2i}) = \frac{p-1}{2} - i \text{ if } 1 \le i \le \frac{p-3}{4} - 1,$$

$$f(u_1) = \frac{3p-5}{4},$$
$$f(u_2) = \frac{3p-5}{4} + 2 \text{ and}$$
$$f(u_3) = \frac{3p-5}{4} + 1.$$

Then $w(cv_1) = p - 2,$
$$w(cv_{p-4}) = p + 2,$$
$$w(cu_1) = p - 1,$$
$$w(cu_3) = p,$$
$$w(u_1u_2) = \frac{3p-1}{2},$$
$$w(u_2u_3) = \frac{3p-1}{2} + 1,$$
$$w(v_{2i-1}v_{2i}) = p - 1 - 2i \text{ if } 1 \leq i \leq \frac{p-3}{4},$$
$$w(v_{2i}v_{2i+1}) = p - 2 - 2i \text{ if } 1 \leq i \leq \frac{p-3}{4} - 1,$$
$$w(v_{\frac{p-3}{2}+2i-1}v_{\frac{p-3}{2}+2i}) = \frac{3p+1}{2} - 2i \text{ if } 1 \leq i \leq \frac{p-3}{4} - 1,$$
$$w(v_{\frac{p-3}{2}+2i}v_{\frac{p-3}{2}+2i+1}) = \frac{3p-1}{2} - 2i \text{ if } 1 \leq i \leq \frac{p-3}{4} - 1 \text{ and}$$
$$w(v_{\frac{p-3}{2}}v_{\frac{p-3}{2}+1}) = p + 1.$$

Clearly the set of all edge-weights $W = \{\frac{p+1}{2}, \frac{p+3}{2}, \ldots, \frac{p+1}{2} + q - 1\}$ is an arithmetic progression with first term $\frac{p+1}{2}$ and common difference 1. Thus f is a $(\frac{p+1}{2}, 1)$-EAV labeling. □

Example 2.4. An $(8, 1)$-EAV labeling of $F_{4,12}$ is given in Figure 2.

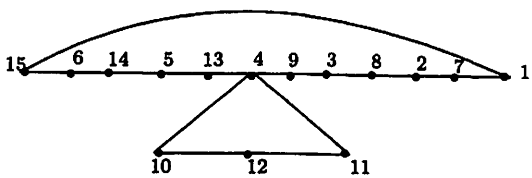

Figure 2

Theorem 2.5. *The generalized friendship graph $G = F_{5,m_2}$, where $m_2 \equiv 3 \pmod 4$, has a $(\frac{3m_2+15}{2}, 2)$-SEAMT labeling and a $(\frac{5m_2+23}{2}, 0)$-SEAMT labeling.*

Proof. Let $C_1 = (c, u_1, u_2, u_3, u_4, c)$ and $C_2 = (c, v_1, v_2, ..., v_{m_2-1}, c)$ be the two cycles in G. The order and size of G are given by $p = m_2 + 4$ and $q = m_2 + 5$. In view of Theorem 1.1, it is enough to prove that G admits a $(\frac{p+1}{2}, 1)$-EAV labeling. We define $f : V(F_{5,m_2}) \longrightarrow \{1, 2, 3, ..., p\}$ by

$$\begin{aligned}
f(c) &= \frac{p+1}{4},\\
f(v_{2i}) &= \frac{p+1}{4} - i \text{ if } 1 \leq i \leq \frac{p-7}{4},\\
f(v_{2i-1}) &= \frac{3p-9}{4} - i \text{ if } 1 \leq i \leq \frac{p-3}{4},\\
f(v_{\frac{p-1}{2}}) &= 1,\\
f(v_{\frac{p-3}{2}}) &= p,\\
f(v_{\frac{p-3}{2}+2i}) &= p - i \text{ if } 1 \leq i \leq \frac{p-3}{4} - 1,\\
f(v_{\frac{p-3}{2}+2i+1}) &= \frac{p-3}{2} - i \text{ if } 1 \leq i \leq \frac{p-11}{4},\\
f(u_1) &= \frac{3p-9}{4},\\
f(u_2) &= \frac{3p-9}{4} + 2,\\
f(u_3) &= \frac{3p-9}{4} + 3 \text{ and}\\
f(u_4) &= \frac{3p-9}{4} + 1.
\end{aligned}$$

Then
$$\begin{aligned}
w(cv_1) &= p - 3,\\
w(cv_{p-5}) &= p + 2,\\
w(cu_1) &= p - 2,\\
w(cu_4) &= p - 1,\\
w(u_1u_2) &= 3\left(\frac{p-3}{2}\right) + 2,\\
w(u_2u_3) &= 3\left(\frac{p-3}{2}\right) + 5,\\
w(u_3u_4) &= 3\left(\frac{p-3}{2}\right) + 4,\\
w(v_{2i-1}v_{2i}) &= p - 2 - 2i \text{ if } 1 \leq i \leq \frac{p-3}{4} - 1,\\
w(v_{2i}v_{2i+1}) &= p - 3 - 2i \text{ if } 1 \leq i \leq \frac{p-3}{4} - 1,
\end{aligned}$$

$$w(v_{\frac{p-1}{2}}v_{\frac{p-1}{2}+1}) = p,$$
$$w(v_{\frac{p-1}{2}}v_{\frac{p-1}{2}-1}) = p+1,$$
$$w(v_{\frac{p-1}{2}-1}v_{\frac{p-1}{2}-2}) = \frac{3p-3}{2},$$
$$w(v_{\frac{p-3}{2}+2i}v_{\frac{p-3}{2}+2i+1}) = 3\left(\frac{p-1}{2}\right) - 2i \text{ if } 1 \le i \le \frac{p-3}{4} - 2 \text{ and}$$
$$w(v_{\frac{p-3}{2}+2i+1}v_{\frac{p-3}{2}+2i+2}) = 3\left(\frac{p-1}{2}\right) - 1 - 2i \text{ if } 1 \le i \le \frac{p-3}{4} - 2.$$

Clearly the set of all edge-weights $W = \{\frac{p+1}{2}, \frac{p+3}{2}, ..., \frac{p+1}{2} + q - 1\}$ is an arithmetic progression with first term $\frac{p+1}{2}$ and common difference 1. Thus f is a $(\frac{p+1}{2}, 1)$-EAV labeling. □

Example 2.6. An $(8, 1)$-EAV labeling of $F_{5,11}$ is given in Figure 3.

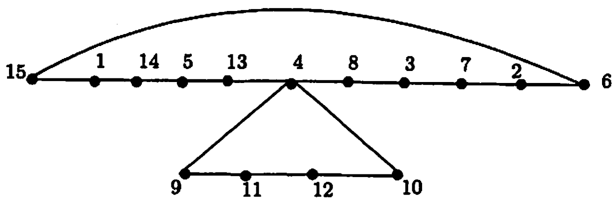

Figure 3

3 Friendship graphs

Let $f : V \cup E \to \{1, 2, 3, ..., p+q\}$ be an (a, d)-edge-antimagic total labeling of a graph $G = (V, E)$. Then $W = \{w(uv) : w(uv) = f(u) + f(v) + f(uv), uv \in E(G)\} = \{a, a+d, ..., a+(q-1)d\}$. In the computation of the edge-weights of G each edge label is used once and each vertex label $v \in V(G)$ is used deg v times. Thus the following equation holds.

$$\sum_{v \in V(G)} deg(v)f(v) + \sum_{e \in E(G)} f(e) = \sum_{e \in E(G)} w(e) \qquad (1)$$

This equation was first observed by Baca and Yousef [5], which we repeatedly use.

In [1] we have proved that the friendship graph F_n does not admit an $(a, 2)$-SEAMT labeling when n is even and $n \not\equiv 4$ $(mod\ 12)$. The problem still remains open for $n \equiv 4$ $(mod\ 12)$. In the following theorem we prove that F_{16} has no $(a, 2)$-SEAMT labeling.

Theorem 3.1. *The friendship graph F_{16} has no $(a,2)$-SEAMT labeling.*

Proof. The order and size of F_{16} are given by $p = 33$ and $q = 48$. Let c denote the central vertex of F_{16}. Suppose there exists an $(a,2)$-SEAMT labeling $f : V \cup E \to \{1,2,3,...,81\}$ for F_{16} and let $f(c) = i$. Since $deg\ v_i = 2$, for every $v_i \neq c$ and $deg\ c = 32$, from equation (1), we get

$$a = \frac{271+5i}{8}.$$

Now the minimum possible edge-weight is 37 and hence $\frac{271+5i}{8} = a \geq 37$ which gives $i \geq 5$. Thus $5 \leq i \leq 33$. Let $i = 5+k$, where $0 \leq k \leq 28$. Then $a = 37 + \frac{5k}{8}$ and hence it follows that $k = 0, 8, 16$ or 24. Thus $i = 5, 13, 21$ or 29.

Case(i): $i = 5$.

Then $a = 37$. Obviously any edge incident with the central vertex c cannot have weight 37 or 39. Let $e_1 = u_1v_1$ be the edge with $w(e_1) = 37$. Then $f(e_1) = 34$, $f(u_1) = 1$ and $f(v_1) = 2$. Clearly there is no edge with weight 39, a contradiction.

Case(ii): $i = 29$.

Then $a = 52$. Now, if f is a $(52,2)$-SEAMT labeling of F_{16}, then by Theorem 1.2, $\overline{f}$ is a $(37,2)$-SEAMT labeling, which does not exist by case(i).

Case(iii): $i = 13$.

Then $a = 42$ and the minimum possible edge-weight for an edge incident at c is 48. Hence the edges with weights 42, 44 and 46 are rim edges. We now consider two subcases.

Sub case(iii a): The edge e with $w(e) = 48$ is incident at c.

Let $e = cu$. Then $f(e) = 34$ and $f(u) = 1$. Since the rim edges are mutually nonadjacent for the edge-weights 42, 44, 46 on the rim, we need three edge labels from the set $A = \{35,36,37,38,39,40\}$ and six vertex labels from the set $B = \{1,2,3,...,9\}$ such that the sum of these labels is $42+44+46 = 132$. There exist ten possible such sets of nine elements, which are given below.

$$\begin{array}{llllllllll}
A_1 = \{1 & 2 & 3 & 4 & 5 & 6 & 35 & 36 & 40\} \\
A_2 = \{1 & 2 & 3 & 4 & 5 & 6 & 35 & 37 & 39\} \\
A_3 = \{1 & 2 & 3 & 4 & 5 & 7 & 35 & 36 & 39\} \\
A_4 = \{1 & 2 & 3 & 4 & 5 & 9 & 35 & 36 & 37\} \\
A_5 = \{1 & 2 & 3 & 4 & 5 & 6 & 36 & 37 & 38\} \\
A_6 = \{1 & 2 & 3 & 4 & 5 & 7 & 35 & 37 & 38\} \\
A_7 = \{1 & 2 & 3 & 4 & 5 & 8 & 35 & 36 & 38\} \\
A_8 = \{1 & 2 & 3 & 4 & 6 & 7 & 35 & 36 & 38\} \\
A_9 = \{1 & 2 & 3 & 4 & 6 & 8 & 35 & 36 & 37\} \\
A_{10} = \{1 & 2 & 3 & 5 & 6 & 7 & 35 & 36 & 37\}
\end{array}$$

If A_1, A_2, A_3 or A_4 is used for getting the edge-weights $42, 44$ and 46, then there is no edge with weight 50. If A_5 is used for getting the edge-weights $42, 44$ and 46, then there is exactly one way for getting edge-weight 50, namely, $7, 8, 35$ on the rim or $13, 2, 35$ on the spoke and in this case there is no edge with weight 52 on the rim or spoke. A similar contradiction arises, if A_i, $6 \leq i \leq 10$, is used for getting the edge-weights $42, 44$ and 46.

Sub case(iii b): The edge e with $w(e) = 48$ is on the rim.

In this case for getting the edge-weights $42, 44, 46$ and 48, we need four edge labels from the set $A = \{34, 35, 36, 37, 38, 39\}$ and eight vertex labels from the set $B = \{1, 2, 3, ..., 10\}$ such that the sum of these labels is $42 + 44 + 46 + 48 = 180$. There exist five possible such sets of twelve elements, which are given below.

$$
\begin{aligned}
A_1 &= \{1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 34\ 35\ 36\ 39\} \\
A_2 &= \{1\ 2\ 3\ 4\ 5\ 6\ 7\ 9\ 34\ 35\ 36\ 38\} \\
A_3 &= \{1\ 2\ 3\ 4\ 5\ 6\ 7\ 10\ 34\ 35\ 36\ 37\} \\
A_4 &= \{1\ 2\ 3\ 4\ 5\ 6\ 8\ 9\ 34\ 35\ 36\ 37\} \\
A_5 &= \{1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 34\ 35\ 37\ 38\}
\end{aligned}
$$

If $A_i, 1 \leq i \leq 4$ is used for getting the edge-weights $42, 44, 46$ and 48, then there is no edge with weight 50. If A_5 is used for getting the edge-weights $42, 44, 46$ and 48, then there is exactly one way for getting edge-weight 50, namely, $13, 1, 36$ and in this case there is no edge with weight 52 on the rim or spoke.

Case(iv): $i = 21$.

Then $a = 47$. Now, if f is a $(47, 2)$-SEAMT labeling of F_{16}, then by Theorem 1.2, $\overline{f}$ is a $(42, 2)$-SEAMT labeling, which does not exist by case(iii). □

4 Conclusion and scope

We have proved that the generalized friendship graph F_{m_1,m_2}, when $m_1 = 3, 4, 5$ and $m_1 \leq m_2$ admits a $(a, 2)$-SEAMT and a $(b, 0)$-SEAMT labeling. The problem remains open for other values of m_1 and m_2 and for generalized friendship graphs with more than two cycles. We have also proved that, the friendship graph F_{16} does not admit a $(a, 2)$-SEAMT labeling. The problem remains open for other values of n with $n \equiv 4(mod\ 12)$.

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