HOMOMORPHISM THEOREMS IN THE NEW VIEW OF FUZZY GAMMA RINGS

EBUBEKIR İNAN, HASRET YAZARLI, AND MEHMET ALI ÖZTÜRK

ABSTRACT. In this paper, we give an example of fuzzy binary operation, fuzzy group, a new fuzzy binary operation on Γ-ring M and a new fuzzy gamma ring. Also we give homomorphism theorems between two fuzzy gamma rings and investigated some related properties.

1. INTRODUCTION

In 1965, L. A. Zadeh [15] introduced the notion of a fuzzy subset of a set as a method for representing uncertainty. Then, in 1971, A. Rosenfeld used the notion of a fuzzy subset of a set to introduced the notion of a fuzzy subgroup of a group. Rosenfeld's paper inspired the development of fuzzy abstract algebra. After these studies, many mathematicians studied these subject. For more details, see [8].

In 2004, X. Yuan and E. S. Lee [14] presented a new kind of fuzzy group based on fuzzy binary operation. Recently H. Aktaş and N. Çağman [1] considered by the use of X. Yuan and E. S. Lee's definition of the fuzzy group based on fuzzy binary operation, the type of fuzzy ring.

In [10], N. Nobusawa introduced the notion of a Γ-ring, as more general than ring. W. E. Barnes [2] weakened slightly the conditions in the definition of the Γ-ring in the sense of Nobusawa. After these two papers are published, many mathematicians made good works on Γ-ring in the sense of Barnes and Nobusawa, which are parallel to the results in the ring theory.

In [4], Jun and Lee introduced concept of fuzzy Γ-ring. After this study, many mathematicians made works in this subject ([5], [6], [11]).

In this paper, we give an example of fuzzy binary operation [14], fuzzy group [14], a new fuzzy binary operation on Γ-ring M [11] and a new fuzzy gamma ring [11]. In addition, we give homomorphism theorems between two fuzzy gamma rings and investigated some related properties.

2000 *Mathematics Subject Classification.* 16D30, 16S99, 16Y99, 03E72 .

Key words and phrases. Keywords: Fuzzy binary operations; Fuzzy gamma ring; Fuzzy gamma homomorphism.

2. Preliminaries

In this section we summarize the preliminary definitions that will be required in this paper.

Definition 1. [14] *Let G be a non-empty set and R be a fuzzy subset of $G \times G \times G$. R is called a fuzzy binary operation on G, if*

(i) For all $a, b \in G$, there exists $c \in G$, such that $R(a, b, c) > \theta$,

(ii) $R(a, b, c_1) > \theta$ and $R(a, b, c_2) > \theta$ imply $c_1 = c_2$, for all $a, b, c_1, c_2 \in G$, where $\theta \in [0, 1)$ is fixed real number.

Let R be a fuzzy binary operation on G,

then we have a mapping $R : F(G) \times F(G) \to F(G)$ defined by $R(A, B)$ where

$$F(G) = \{A \mid A : G \to [0, 1] \text{ is a mapping}\}$$

and

$$R(A, B)(c) = \bigvee_{a,b \in G} (A(a) \wedge B(b) \wedge R(a, b, c)) \tag{2.1}$$

for all $c \in G$.

Let $A = \{a\}$ and $B = \{b\}$ and let $R(A, B)$ be denoted as $a \circ b$, then

$$(a \circ b)(c) = R(a, b, c), \text{ for all } c \in G \tag{2.2}$$

$$((a \circ b) \circ c)(z) = \bigvee_{d \in G} (R(a, b, d) \wedge R(d, c, z)), \text{ for all } z \in G \tag{2.3}$$

$$(a \circ (b \circ c))(z) = \bigvee_{d \in G} (R(b, c, d) \wedge R(a, d, z)), \text{ for all } z \in G \tag{2.4}$$

Definition 2. [14] *Let G be a non-empty set and R be a fuzzy binary operation on G. (G, R) is called a fuzzy group if the following conditions are true:*

$(G1)$ $((a \circ b) \circ c)(z_1) > \theta$ and $(a \circ (b \circ c))(z_2) > \theta$ imply $z_1 = z_2$ for all $a, b, c, z_1, z_2 \in G$,

$(G2)$ There exists $e_0 \in G$ such that $(e_0 \circ a)(a) > \theta$ and $(a \circ e_0)(a) > \theta$ for any $a \in G$ (e_0 is called an identity element of G),

$(G3)$ There exists $b \in G$ such that $(a \circ b)(e_0) > \theta$ and $(b \circ a)(e_0) > \theta$ for any $a \in G$ (b is called an inverse element of a and is denoted as a^{-1}).

Let M and Γ be non-empty sets, R_M a fuzzy binary operation on M and R_Γ on Γ. Thus, R_M is a fuzzy subset of $M \times M \times M$ and R_Γ a fuzzy subset of $\Gamma \times \Gamma \times \Gamma$. We assume throughout that the value of θ is the same for R_M and R_Γ.

Definition 3. [11] *Let M and Γ be two non-empty sets and let S be a fuzzy subset of $M \times \Gamma \times M \times \Gamma \times M$. S is called a fuzzy binary operation on (M,Γ) if*
(i) $\forall a,b \in M$, $\forall \alpha,\beta \in \Gamma$ and $\exists c \in M$ such that $S(a,\alpha,b,\beta,c) > \theta$,
(ii) $\forall a,b,c_1,c_2 \in M$, $\forall \gamma \in \Gamma$, $\bigvee_{\beta\in\Gamma} S(a,\gamma,b,\beta,c_1) > \theta$ and $\bigvee_{\beta\in\Gamma} S(a,\gamma,b,\beta,c_2) > \theta$ implies $c_1 = c_2$ where $\theta \in [0,1)$ is above for R_M and R_Γ.

Let S be a fuzzy binary operation on (M,Γ), then we may regard S as the mapping

$$S : F(M) \times F(\Gamma) \times F(M) \to F(M), \ (A,G,B) \mapsto S(A,G,B)$$

where

$$F(M) = \{A \mid A : M \to [0,1] \text{ is a mapping}\},$$
$$F(\Gamma) = \{G \mid G : \Gamma \to [0,1] \text{ is a mapping}\}$$

and
(2.5)
$$S(A,G,B)(c) = \bigvee_{\substack{a,b\in M\\ \alpha,\beta\in\Gamma}} \left(A(a) \wedge G(\alpha) \wedge B(b) \wedge S(a,\alpha,b,\beta,c)\right), \forall c \in M.$$

Let $A = \{a\}$, $B = \{b\}$, $G = \{\alpha\}$, and $G' = \{\alpha'\}$. Let $R_M(A,B)$, $R_\Gamma(G,G')$ and $S(A,G,B)$ be denoted as aob, $\alpha o \alpha'$ and $a{*}\alpha{*}b$, respectively. We will use the following notation to simplify the calculations:

$$(2.6) \qquad (a * \alpha * b)(c) = \bigvee_{\alpha'\in\Gamma} S(a,\alpha,b,\alpha',c) \ \ for\ all\ c \in M,$$

$$(2.7) \quad ((a * \alpha * b) * \beta * c)(z) = \bigvee_{\substack{d\in M\\ \alpha',\beta'\in\Gamma}} \left(S(a,\alpha,b,\alpha',d) \wedge S(d,\beta,c,\beta',z)\right),$$

$$(2.8) \quad (a * \alpha * (b * \beta * c))(z) = \bigvee_{\substack{d\in M\\ \alpha',\beta'\in\Gamma}} \left(S(b,\beta,c,\alpha',d) \wedge S(a,\alpha,d,\beta',z)\right),$$

$$(2.9) \qquad (a * \alpha * (b \circ c))(z) = \bigvee_{\substack{d\in M\\ \alpha'\in\Gamma}} \left(R_M(b,c,d) \wedge S(a,\alpha,d,\alpha',z)\right),$$

(2.10)
$$((a * \alpha * b) \circ (a * \alpha * c))(z) = \bigvee_{\substack{d,e\in M\\ \alpha',\beta'\in\Gamma}} \left(\begin{array}{c} S(a,\alpha,b,\alpha',d) \wedge S(a,\alpha,c,\beta',e) \\ \wedge R_M(d,e,z) \end{array} \right),$$

$$(2.11)\qquad (a*(\alpha\circ\beta)*b)(c)=\bigvee_{\gamma,\alpha'\in\Gamma}\left(R_\Gamma(\alpha,\beta,\gamma)\wedge S(a,\gamma,b,\alpha',c)\right),$$

(2.12)

$$((a*\alpha*b)\circ(a*\beta*b))(c)=\bigvee_{\substack{d,e\in M\\ \alpha',\beta'\in\Gamma}}\left(\begin{array}{c} S(a,\alpha,b,\alpha',d)\wedge S(a,\beta,b,\beta',e)\\ \wedge R_M(d,e,c)\end{array}\right),$$

$$(2.13)\qquad ((a\circ b)*\alpha*c)(z)=\bigvee_{\substack{d\in M\\ \alpha'\in\Gamma}}\left(R_M(a,b,d)\wedge S(d,\alpha,c,\alpha',z)\right),$$

(2.14)

$$((a*\alpha*c)\circ(b*\alpha*c))(z)=\bigvee_{\substack{d,e\in M\\ \alpha',\beta'\in\Gamma}}\left(\begin{array}{c} S(a,\alpha,c,\alpha',d)\wedge S(b,\alpha,c,\beta',e)\\ \wedge R_M(d,e,z)\end{array}\right).$$

Definition 4. [11] *Let M and Γ be non-empty sets, R_M, R_Γ and S fuzzy binary operations on M, Γ and $M\times\Gamma\times M\times\Gamma\times M$ (briefly (M,Γ)), respectively, all with the same value of θ. To simplify notation, R_M and R_Γ are denoted by R. Then, (M,Γ,R,S) is called a fuzzy gamma ring if the following conditions hold.*

$(M,\Gamma)_1$ *(M,R) and (Γ,R) are abelian fuzzy groups,*
$(M,\Gamma)_2$ *$\forall a,b,c,z_1,z_2\in M$, $\forall\gamma,\beta\in\Gamma$, $((a*\gamma*b)*\beta*c)(z_1)>\theta$ and $(a*\gamma*(b*\beta*c))(z_2)>\theta$ imply $z_1=z_2$,*
$(M,\Gamma)_3$ *$\forall a,b,c,z_1,z_2\in M$, $\forall\gamma,\beta\in\Gamma$,*
(i) $(a*\gamma*(b\circ c))(z_1)>\theta$ and $((a*\gamma*b)\circ(a*\gamma*c))(z_2)>\theta$ imply $z_1=z_2$,
(ii) $(a*(\gamma\circ\beta)*b)(z_1)>\theta$ and $((a*\gamma*b)\circ(a*\beta*b))(z_2)>\theta$ imply $z_1=z_2$,
(iii) $((a\circ b)*\gamma*c)(z_1)>\theta$ and $((a*\gamma*c)\circ(b*\gamma*c))(z_2)>\theta$ imply $z_1=z_2$.

The identity element of the fuzzy group (M,R) is called the zero element of (M,Γ,R,S) and denoted e_0. Let (M,Γ,R,S) be a fuzzy gamma ring and N be non-empty subset of M. Let $R_N(a,b,c)=R(a,b,c)$ and $S_N(a,\gamma,b,\beta,c)=S(a,\gamma,b,\beta,c)$ for all $a,b,c\in N$ and all $\gamma,\beta\in\Gamma$. Then, we have

$$(a\triangle b)(c)=R_N(a,b,c)=R(a,b,c)\,,\ \text{for all } a,b,c\in N$$

$$(a\diamond\gamma\diamond b)(c)=\bigvee_{\beta\in\Gamma}S_N(a,\gamma,b,\beta,c)=\bigvee_{\beta\in\Gamma}S(a,\gamma,b,\beta,c)$$

for all $a,b,c\in N$ and all $\gamma\in\Gamma$.

Definition 5. [11] *Let* (M,Γ,R,S) *be a fuzzy gamma ring and* N *be non-empty subset of* M *for which:*

(i) For all $a,b \in N$, *all* $c \in M$ *and all* $\gamma \in \Gamma$, $(a \circ b)(c) > \theta$ *implies* $c \in N$ *and* $(a * \gamma * b)(c) > \theta$ *implies* $c \in N$.

(ii) (N,Γ,R_N,S_N) *is fuzzy gamma ring.*

Then, (N,Γ,R_N,S_N) *is called a fuzzy gamma subring of* (M,Γ,R,S).

Proposition 1. [11] *Let* (M,Γ,R,S) *be a fuzzy gamma ring and* N *be non-empty subset of* M*.Then* (N,Γ,R_N,S_N) *is a fuzzy gamma subring of* M *if and only if*

(i) For all $a,b \in N$, *all* $c \in M$ *and all* $\gamma \in \Gamma$, $(a \circ b)(c) > \theta$ *implies* $c \in N$ *and* $(a * \gamma * b)(c) > \theta$ *implies* $c \in N$.

(ii) $a^{-1} \in N$ *for all* $a \in N$.

Definition 6. [11] *Let* (M,Γ,R,S) *be a fuzzy gamma ring . A non-empty subset* I *of* M *is called a left (right) fuzzy ideal of* M *if for all* $a,b \in I$, *all* $n,m \in M$, *and all* $\gamma \in \Gamma$, $(a \circ b)(m) > \theta$ *implies* $m \in I$, $a^{-1} \in I$, $(n * \gamma * a)(m) > \theta$ *implies* $m \in I$ *(*$(a * \gamma * n)(m) > \theta$ *implies* $m \in I$*).*

A non-empty subset I *of a fuzzy gamma ring* (M,Γ,R,S) *is called a fuzzy (two-sided) ideal of* (M,Γ,R,S) *if* I *is both a left and a right ideal of* (M,Γ,R,S).

Remark 1. [11] *From the definition of a fuzzy left (right) ideal of* (M,Γ,R,S), *then* I *is a fuzzy gamma subring of* (M,Γ,R,S).

Proposition 2. [11] *Let* I_i, $i \in \Lambda$, *be a fuzzy ideal of fuzzy gamma ring* (M,Γ,R,S), *where* Λ *is a index set. Then* $\bigcap_{i\in\Lambda} I_i$ *is a fuzzy ideal of* M.

Let I be a fuzzy ideal of fuzzy gamma ring (M,Γ,R,S) and $\Delta = \{a \circ I \mid a \in M\}$. Let define a relation over Δ by

$$a_1 \circ I \sim a_2 \circ I \Leftrightarrow \exists u \in I, \text{ such that } R\left(a_1^{-1}, a_2, u\right) > \theta$$

The fuzzy relation $\sim$ on the set Δ is a fuzzy equivalence relation by [14, Theorem 4.1]. Let $[a \circ I] = \{a' \circ I \mid a' \circ I \sim a \circ I\}$,

$\overline{a} = \{a' \mid a' \in M \text{ and } a' \circ I \sim a \circ I\}$ and $M/I = \{[a \circ I] \mid a \in M\}$. Also, (I,R) is a fuzzy subgroup of (M,R) and since (M,R) is abelian, (I,R) is a normal fuzzy group of (M,R) by [14, Theorem 3.1]. Hence, M/I denotes the set of all coset $(a \circ I)(u) = \bigvee_{x\in I} R(a,x,u)$ for all $a \in M$, then $\left(M/I, \overline{R}\right)$ is a commutative fuzzy group by [14, Theorem 4.2], where

$$\begin{aligned}([a \circ I] \oplus [b \circ I])(c \circ I) &= \overline{R}([a \circ I],[b \circ I],[c \circ I]) \\ &= \bigvee_{(a',b',c')\in \overline{a}\times\overline{b}\times\overline{c}} R(a',b',c')\end{aligned}$$

Theorem 1. [11] *Let* (M, Γ, R, S) *be a fuzzy gamma ring and* I *be a fuzzy ideal of* M. *Then the quotient fuzzy group* $\left(M/I, \bar{R}\right)$ *is a fuzzy gamma ring with*

$$([a \circ I] \otimes \gamma \otimes [b \circ I])([c \circ I]) = \bar{S}([a \circ I], \gamma, [b \circ I], \beta, [c \circ I]) = \bigvee_{(a', \gamma, b', \beta, c') \in \bar{a} \times \gamma \times \bar{b} \times \beta \times \bar{c}} S(a', \gamma, b', \beta, c')$$

Definition 7. [11] *If* (M, Γ, R, S) *be a fuzzy gamma ring and* I *be a fuzzy ideal of* M, *then the fuzzy gamma ring* $\left(M/I, \Gamma, \bar{R}, \bar{S}\right)$ *is called the fuzzy quotient gamma ring of* M *by* I.

Definition 8. [11] *Let* (M_1, Γ, R_1, S_1) *and* (M_2, Γ, R_2, S_2) *be two fuzzy gamma rings and* f *be a function from* M_1 *into* M_2. *Then* f *is called a fuzzy gamma homomorphism of* M_1 *into* M_2 *if*

(*i*) $R_1(a, b, c) > \theta$ *implies* $R_2(f(a), f(b), f(c)) > \theta$,

(*ii*) $S_1(a, \gamma, b, \beta, c) > \theta$ *implies* $S_2(f(a), \gamma, f(b), \beta, f(c)) > \theta$

for all $a, b, c \in M_1$, *and all* $\gamma, \beta \in \Gamma$.

Definition 9. [11] *A homomorphism* f *of a fuzzy gamma ring* M_1 *into a fuzzy gamma ring* M_2 *is called*

(1) *a monomorphism if* f *is one-one,*

(2) *an epimorphism if* f *is onto* M_2 *and*

(3) *an isomorphism if* f *is one-one and map* M_1 *onto* M_2.

If f *is an isomorphism of* M_1 *onto* M_2, *then two fuzzy gamma ring* M_1 *and* M_2 *are called isomorphic, denoted by* $M_1 \cong M_2$.

Theorem 2. [11] *Let* (M_1, Γ, R_1, S_1) *and* (M_2, Γ, R_2, S_2) *be two fuzzy gamma rings and let* f *be a fuzzy gamma homomorphism of* M_1 *into* M_2. *Then*

(*i*) $f(e_0) = e'_0$, *where* e'_0 *is the zero of* M_2

(*ii*) $f\left(a^{-1}\right) = f(a)^{-1}$ *for all* $a \in M_1$

(*iii*) $Imf = \{f(a) \mid a \in M_1\}$ *is a fuzzy gamma subring of* M_2.

Theorem 3. [11] *Let* (M_1, Γ, R_1, S_1) *and* (M_2, Γ, R_2, S_2) *be two fuzzy gamma rings and let* f *be a fuzzy gamma homomorphism of* M_1 *into* M_2. *Then*

(*i*) $Kerf = \{a \in M_1 \mid f(a) = e'_0\}$ *is a fuzzy ideal of* M_1,

(*ii*) *If* B *is a fuzzy ideal of* M_2, *then* $f^{-1}(B)$ *is a fuzzy ideal of* M_1,

(*iii*) *If* f *is surjective and* A *is a fuzzy ideal of* M_1, *then* $f(A)$ *is a fuzzy ideal of* M_2.

Theorem 4. [11] *Let* (M, Γ, R, S) *be a fuzzy gamma ring and* I *be a fuzzy ideal of* M. *Then, the mapping* $\Pi : M \to M/I$ *by* $\Pi(a) = [a \circ I]$ *for all*

$a \in M$ is a fuzzy gamma homomorphism, called the fuzzy canonical gamma homomorphism.

Theorem 5. [11] *Let $f : (M_1, \Gamma, R_1, S_1) \to (M_2, \Gamma, R_2, S_2)$ be a fuzzy gamma epimorphism. Then $M_1/N \cong M_2$, where $N = Kerf$.*

3. An Example of a New View of Fuzzy Gamma Rings

In [14], Yuan and Lee introduced concept of fuzzy binary operation and fuzzy group. In [11], Öztürk, Jun and Yazarlı introduced a new fuzzy binary operation on $M \times \Gamma \times M \times \Gamma \times M$ and a new fuzzy gamma ring. In this section, we give an example of fuzzy binary operation, fuzzy group, a new fuzzy binary operation on $M \times \Gamma \times M \times \Gamma \times M$ and a new fuzzy gamma ring.

Example 1. *Let $M = \{\begin{bmatrix} \bar{0} & \bar{0} & \bar{0} \end{bmatrix}, \begin{bmatrix} \bar{0} & \bar{0} & \bar{1} \end{bmatrix}\} \subset (\mathbb{Z}_2)_{1\times 3}$ and $\Gamma = \left\{ \begin{bmatrix} \bar{0} \\ \bar{0} \\ \bar{0} \end{bmatrix}, \begin{bmatrix} \bar{1} \\ \bar{0} \\ \bar{0} \end{bmatrix} \right\} \subset (\mathbb{Z}_2)_{3\times 1}$ be non-empty sets. And we will use the following notation to simplify the calculations;*

$$a = \begin{bmatrix} \bar{0} & \bar{0} & \bar{0} \end{bmatrix}, b = \begin{bmatrix} \bar{0} & \bar{0} & \bar{1} \end{bmatrix}, \alpha = \begin{bmatrix} \bar{0} \\ \bar{0} \\ \bar{0} \end{bmatrix} \text{ and } \beta = \begin{bmatrix} \bar{1} \\ \bar{0} \\ \bar{0} \end{bmatrix}.$$

Let us define R_M, R_Γ and S fuzzy binary operations on M, Γ and (M, Γ), respectively, all with the same value of $\theta = 0.7$, as follows:

$$\begin{array}{ll} R(a,a,a) = 0.9 & R(b,a,a) = 0.4 \\ R(a,a,b) = 0.2 & R(b,a,b) = 0.8 \\ R(a,b,a) = 0.4 & R(b,b,a) = 0.9 \\ R(a,b,b) = 0.9 & R(b,b,b) = 0.1 \end{array}$$

and

$$\begin{array}{ll} R(\alpha,\alpha,\alpha) = 0.9 & R(\beta,\alpha,\alpha) = 0.3 \\ R(\alpha,\alpha,\beta) = 0.1 & R(\beta,\alpha,\beta) = 0.9 \\ R(\alpha,\beta,\alpha) = 0.2 & R(\beta,\beta,\alpha) = 0.9 \\ R(\alpha,\beta,\beta) = 0.8 & R(\beta,\beta,\beta) = 0.5 \end{array}$$

(M, R_M) and (Γ, R_Γ) are fuzzy groups, since the following conditions are true:

$(G1)_M$ $((a \circ b) \circ c)(z_1) > 0.7$ and $(a \circ (b \circ c))(z_2) > 0.7$ imply $z_1 = z_2$ for all $a, b, c, z_1, z_2 \in M$,

$(G2)_M$ There exists $e_0 = a \in M$ such that $(e_0 \circ d)(d) = (a \circ d)(d) > 0.7$ and $(d \circ e_0)(d) = (d \circ a)(d) > 0.7$ for any $d \in M$ ($e_0 = a$ is called an identity element of M),

$(G3)_M$ *There exists* $b \in M$ *such that* $(a \circ b)(e_0) > 0.7$ *and* $(b \circ a)(e_0) > 0.7$ *for any* $a \in M$ *(b is called an inverse element of a and is denoted as* a^{-1}*).*

There exists $a \in M$ *such that* $(a \circ a)(e_0) > 0.7$ *and* $(a \circ a)(e_0) > 0.7$ *for any* $a \in M$. *Hence* $a^{-1} = a$.

There exists $b \in M$ *such that* $(b \circ b)(e_0) > 0.7$ *and* $(b \circ b)(e_0) > 0.7$ *for any* $b \in M$. *Hence* $b^{-1} = b$. *And,*

$(G1)_\Gamma$ $((\alpha \circ \beta) \circ \gamma)(z_1) > 0.7$ *and* $(\alpha \circ (\beta \circ \gamma))(z_2) > 0.7$ *imply* $z_1 = z_2$ *for all* $\alpha, \beta, \gamma, z_1, z_2 \in \Gamma$,

$(G2)_\Gamma$ *There exists* $e_0 = \alpha \in \Gamma$ *such that* $(e_0 \circ \beta)(\beta) = (a \circ \beta)(\beta) > 0.7$ *and* $(\beta \circ e_0)(d) = (\beta \circ \alpha)(\beta) > 0.7$ *for any* $\beta \in \Gamma$ *(*$e_0 = \alpha$ *is called an identity element of* Γ*),*

$(G3)_\Gamma$ *There exists* $\beta \in \Gamma$ *such that* $(\alpha \circ \beta)(e_0) > 0.7$ *and* $(\beta \circ \alpha)(e_0) > 0.7$ *for any* $\alpha \in \Gamma$ *(*β *is called an inverse element of* α *and is denoted as* α^{-1}*).*

There exists $\alpha \in \Gamma$ *such that* $(\alpha \circ \alpha)(e_0) > 0.7$ *and* $(\alpha \circ \alpha)(e_0) > 0.7$ *for any* $\alpha \in \Gamma$. *Hence* $\alpha^{-1} = \alpha$.

There exists $\beta \in \Gamma$ *such that* $(\beta \circ \beta)(e_0) > 0.7$ *and* $(\beta \circ \beta)(e_0) > 0.7$ *for any* $\beta \in \Gamma$. *Hence* $\beta^{-1} = \beta$.

And lastly,

$$
\begin{array}{ll}
S(a,\alpha,a,\alpha,a) = 0.9 & S(b,\alpha,a,\alpha,a) = 0.2 \\
S(a,\alpha,a,\alpha,b) = 0.1 & S(b,\alpha,a,\alpha,b) = 0.9 \\
S(a,\alpha,b,\alpha,a) = 0.3 & S(b,\alpha,b,\alpha,a) = 0.8 \\
S(a,\alpha,b,\alpha,b) = 0.8 & S(b,\alpha,b,\alpha,b) = 0.1 \\
S(a,\alpha,a,\beta,a) = 0.8 & S(b,\alpha,a,\beta,a) = 0.1 \\
S(a,\alpha,a,\beta,b) = 0.2 & S(b,\alpha,a,\beta,b) = 0.8 \\
S(a,\alpha,b,\beta,a) = 0.4 & S(b,\alpha,b,\beta,a) = 0.8 \\
S(a,\alpha,b,\beta,b) = 0.9 & S(b,\alpha,b,\beta,b) = 0.5 \\
S(a,\beta,a,\alpha,a) = 0.8 & S(b,\beta,a,\alpha,a) = 0.4 \\
S(a,\beta,a,\alpha,b) = 0.3 & S(b,\beta,a,\alpha,b) = 0.8 \\
S(a,\beta,b,\alpha,a) = 0.5 & S(b,\beta,b,\alpha,a) = 0.9 \\
S(a,\beta,b,\alpha,b) = 0.8 & S(b,\beta,b,\alpha,b) = 0.3 \\
S(a,\beta,a,\beta,a) = 0.9 & S(b,\beta,a,\beta,a) = 0.2 \\
S(a,\beta,a,\beta,b) = 0.1 & S(b,\beta,a,\beta,b) = 0.9 \\
S(a,\beta,b,\beta,a) = 0.2 & S(b,\beta,b,\beta,a) = 0.9 \\
S(a,\beta,b,\beta,b) = 0.8 & S(b,\beta,b,\beta,b) = 0.6
\end{array}
$$

Therefore, since

$(M,\Gamma)_1$ (M, R_M) *and* (Γ, R_Γ) *are abelian fuzzy groups,*

$(M,\Gamma)_2$ $\forall a, b, c, z_1, z_2 \in M$, $\forall \gamma, \beta \in \Gamma$, $((a * \gamma * b) * \beta * c)(z_1) > 0.7$ *and*

$(a * \gamma * (b * \beta * c))(z_2) > 0.7$ *implies* $z_1 = z_2$,
$(M,\Gamma)_3$ $\forall a, b, c, z_1, z_2 \in M$, $\forall \gamma, \beta \in \Gamma$,
(i) $(a * \gamma * (b \circ c))(z_1) > 0.7$ *and* $((a * \gamma * b) \circ (a * \gamma * c))(z_2) > 0.7$ *imply* $z_1 = z_2$,
(ii) $(a * (\gamma \circ \beta) * b)(z_1) > 0.7$ *and* $((a * \gamma * b) \circ (a * \beta * b))(z_2) > 0.7$ *imply* $z_1 = z_2$,
(iii) $((a \circ b) * \gamma * c)(z_1) > 0.7$ *and* $((a * \gamma * c) \circ (b * \gamma * c))(z_2) > 0.7$ *imply* $z_1 = z_2$.
hold, (M, Γ, R, S) *is a fuzzy gamma ring.*

4. Homomorphism Theorems

Theorem 6. (M_1, Γ, R_1, S_1) *and* (M_2, Γ, R_2, S_2) *be fuzzy gamma rings and let* f *be a fuzzy gamma homomorphism of* M_1 *into* M_2. *Then,*

(i) *If* J *is a fuzzy ideal of* M_2, $f^{-1}(J)$ *is a fuzzy ideal of* M_1 *containing* $Kerf$,

(ii) *If* I *is a fuzzy ideal of* M_1 *containing* $Kerf$, *then* $f^{-1}(f(I)) = I$,

(iii) f *induces a one-one inclusion preserving correspondence between the fuzzy ideals of* M_1 *containing* $Kerf$, *and the fuzzy ideals of* M_2 *in such a way that if* I *is a fuzzy ideal of* M_1 *containing* $Kerf$, *then* $f(I)$ *is the corresponding fuzzy ideal of* M_2 *and if* J *is a fuzzy ideal of* M_2, *then* $f^{-1}(J)$ *is the corresponding fuzzy ideal of* M_1.

Proof. (i) Let be a fuzzy ideal of M_2. From Theorem 3, $f^{-1}(J)$ is a fuzzy ideal of M_1. Let $x \in Kerf$. Since J is a fuzzy ideal of M_2, $f(x) = e_0' \in J$ and so $x \in f^{-1}(J)$. Hence $Kerf \subseteq f^{-1}(J)$.

(ii) Since $f(x) \in f(I)$ for all $x \in I$, we have $x \in f^{-1}(f(I))$. That is, $I \subseteq f^{-1}(f(I))$.

Let $y \in f^{-1}(f(I))$. Therefore $f(y) \in f(I)$ and so there exists $a \in I$ such that $f(y) = f(a)$. Since $f(y) = f(a)$, we get $R_2\left(f(a), f(y)^{-1}, e_0'\right) > \theta$. Since M_1 is a fuzzy gamma ring and a, $y \in M_1$, there exists $c \in M_1$ such that $R_1\left(a, y^{-1}, c\right) > \theta$. Since f is a fuzzy gamma homomorphism, we get $R_2\left(f(a), f\left(y^{-1}\right), f(c)\right) = R_2\left(f(a), f(y)^{-1}, f(c)\right) > \theta$. Hence $f(c) = e_0'$. That is, $c \in Kerf$. Since $Kerf \subseteq I$, we have $c \in I$. Since I is a fuzzy ideal of M_1 and $R_1\left(a, y^{-1}, c\right) > \theta$, we get $y \in I$. Hence $f^{-1}(f(I)) \subseteq I$.

(iii) Let N_1 be a set of all fuzzy ideals of M_1 containing $Kerf$ and N_2 be a set of all fuzzy ideals of M_2. Let φ be a mapping N_1 into N_2 defined by $\varphi(I) = f(I)$. Since f is well-defined, φ is well-defined. Let $f(I_1) = f(I_2)$, for I_1, $I_2 \in N_1$. Since $f^{-1}(f(I_1)) = f^{-1}(f(I_2))$, we get $I_1 = I_2$ from (ii). Therefore φ is one-one. Let $J \in N_2$. From (ii), $f^{-1}(J) \in N_1$. Let $y \in f\left(f^{-1}(J)\right)$. There exists $x \in f^{-1}(J)$ such that $y = f(x)$. Since

$y = f(x)$, $f(x) \in J$, we get $f\left(f^{-1}(J)\right) \subseteq J$. On the other hand, let $x \in J$. Since $x \in J \subseteq M_2 = f(M_1)$, there exists $a \in M_1$ such that $f(a) = x$. Therefore $f(a) \in J$ and so $a \in f^{-1}(J)$. Hence $x = f(a) \in f\left(f^{-1}(J)\right)$ and so $J = f\left(f^{-1}(J)\right)$. That is, $\varphi\left(f^{-1}(J)\right) = f\left(f^{-1}(J)\right) = J$. Hence φ is surjective.

Let I_1 and I_2 be fuzzy ideals of M_1 containing such that $I_1 \subset I_2$. Then, $f(I_1) \subseteq f(I_2)$. If $f(I_1) = f(I_2)$, then $I_1 = I_2$ since the mapping $\varphi : N_1 \to N_2$, $\varphi(I) = f(I)$ is one-one. This is a contradiction. Hence $f(I_1) \neq f(I_2)$. Therefore, $f(I_1) \subset f(I_2)$.

Conversely, let $f(I_1) \subset f(I_2)$. Therefore, $f^{-1}(f(I_1)) \subseteq f^{-1}(f(I_2))$. From (ii), $I_1 = f^{-1}(f(I_1)) \subseteq f^{-1}(f(I_2)) = I_2$. Since $f(I_1) \subset f(I_2)$, we get $I_1 \neq I_2$. Hence $I_1 \subset I_2$. □

Let (M, Γ, R, S) be a fuzzy gamma ring, I_1 and I_2 be non-empty subsets of M. Then,

$$I_1 \circ I_2 := \{c \in M \mid (a_1 \circ a_2)(c) > \theta, \forall a_1 \in I_1 \text{ and } \forall a_2 \in I_2\}.$$

Lemma 1. *Let (M, Γ, R, S) be a fuzzy gamma ring, I_1 and I_2 be fuzzy ideal of M. Then, $I_1 \circ I_2$ is a fuzzy ideal of M.*

Proof. Since I_1 and I_2 are fuzzy ideals of M, $e_0 \in I_1$and $e_0 \in I_2$. Since (M, R) is fuzzy group, $(e_0 \circ e_0)(e_0) > \theta$. Thus $e_0 \in I_1 \circ I_2$ and so $I_1 \circ I_2 \neq \emptyset$.

Let $c_1, c_2 \in I_1 \circ I_2$, $(c_1 \circ c_2)(c) > \theta$, for $c \in M$. Since $c_1, c_2 \in I_1 \circ I_2$, then there exist $a_1, b_1 \in I_1$ and $a_2, b_2 \in I_2$ such that $(a_1 \circ a_2)(c_1) > \theta$ and $(b_1 \circ b_2)(c_2) > \theta$. Hence $R(a_1, a_2, c_1) > \theta$ and $R(b_1, b_2, c_2) > \theta$.

Since I_1 and I_2 are fuzzy ideals of M, there exist $d_1 \in I_1$ and $d_2 \in I_2$ such that $R(a_1, b_1, d_1) > \theta$ and $R(a_2, b_2, d_2) > \theta$. Since $d_1, d_2 \in M$ and (M, R) is fuzzy group, there exists $t \in M$ such that $R(d_1, d_2, t) > \theta$. Since $d_1 \in I_1$ and $d_2 \in I_2$, $t \in I_1 \circ I_2$.

Since $R(a_1, b_1, d_1) > \theta$ and (M, R) is abelian fuzzy group, we have $R\left(d_1, b_1^{-1}, a_1\right) > \theta$. Let $t_1, t_2 \in M$ such that $R(a_2, d_1, t_1) > \theta$ and $R\left(t_1, b_1^{-1}, t_2\right) > \theta$. Since (M, R) is abelian fuzzy group, $R(a_1, a_2, c_1) > \theta$ implies $R(a_2, a_1, c_1) > \theta$. Then we get

$$\left(a_2 \circ \left(d_1 \circ b_1^{-1}\right)\right)(c_1) \geq R\left(d_1, b_1^{-1}, a_1\right) \wedge R(a_2, a_1, c_1) > \theta$$

and

$$\left((a_2 \circ d_1) \circ b_1^{-1}\right)(t_2) \geq R(a_2, d_1, t_1) \wedge R\left(t_1, b_1^{-1}, t_2\right) > \theta.$$

Since (M, R) is fuzzy group, we get $c_1 = t_2$ and so $R\left(t_1, b_1^{-1}, c_1\right) > \theta$. Since (M, R) is abelian fuzzy group, $R(c_1, b_1, t_1) > \theta$.

Since $t_1, b_2 \in M$ and (M, R) is fuzzy group, there exists $c_3 \in M$ such that $R(t_1, b_2, c_3) > \theta$. Then we get

$$(c_1 \circ (b_1 \circ b_2))(c) \geq R(b_1, b_2, c_2) \wedge R(c_1, c_2, c) > \theta$$

and

$$((c_1 \circ b_1) \circ b_2)(c_3) \geq R(c_1, b_1, t_1) \wedge R(t_1, b_2, c_3) > \theta.$$

Since (M, R) is fuzzy group, we get $c = c_3$ and so $R(t_1, b_2, c) > \theta$.

Since $R(a_2, b_2, d_2) > \theta$, $R(t_1, b_2, c) > \theta$, $R(d_1, d_2, t) > \theta$ and (M, R) is abelian fuzzy group, $R(b_2, a_2, d_2) > \theta$, $R(b_2, t_1, c) > \theta$ and $R(d_2, d_1, t) > \theta$. Also, since

$$(b_2 \circ (a_2 \circ d_1))(c) \geq R(a_2, d_1, t_1) \wedge R(b_2, t_1, c) > \theta$$

and

$$((b_2 \circ a_2) \circ d_1)(t) \geq R(b_2, a_2, d_2) \wedge R(d_2, d_1, t) > \theta,$$

we get $t = c$.

Hence $c_1, c_2 \in I_1 \circ I_2$ and $(c_1 \circ c_2)(c) > \theta$ imply $c \in I_1 \circ I_2$, since $t \in I_1 \circ I_2$.

Let $c \in I_1 \circ I_2$, let us show that $c^{-1} \in I_1 \circ I_2$. Since $c \in I_1 \circ I_2$, there exist $a_1 \in I_1$ and $a_2 \in I_2$ such that $(a_1 \circ a_2)(c) > \theta$. Since I_1 and I_2 are fuzzy ideals of M, we have $a_1^{-1} \in I_1$ and $a_2^{-1} \in I_2$. Since (M, R) is abelian fuzzy group, $R(a_1, a_2, c) > \theta$ implies $R(a_1^{-1}, a_2^{-1}, c^{-1}) > \theta$. Thus we get $c^{-1} \in I_1 \circ I_2$.

Let us show that for all $c \in I_1 \circ I_2$, $m \in M$ and $\gamma \in \Gamma$, $(c * \gamma * m)(n) > \theta$, $n \in M$ implies $n \in I_1 \circ I_2$. Since $(c * \gamma * m)(n) > \theta$,

we have $S(c, \gamma, m, \tau, n) > \theta$ for all $\tau \in \Gamma$.

Since $c \in I_1 \circ I_2$, there exist $a_1 \in I_1$ and $a_2 \in I_2$ such that $R(a_1, a_2, c) > \theta$. Since I_1 and I_2 are fuzzy ideals of M, there exist $n_1 \in I_1$ and $n_2 \in I_2$ such that $S(a_1, \gamma, m, \beta, n_1) > \theta$ and $S(a_2, \gamma, m, \alpha, n_2) > \theta$ for all $m \in M$ and $\gamma, \alpha, \beta \in \Gamma$. Since (M, R) is fuzzy group, there exists $t \in M$ such that $R(n_1, n_2, t) > \theta$. Since $n_1 \in I_1$ and $n_2 \in I_2$, we have $t \in I_1 \circ I_2$. Then,

$$((a_1 \circ a_2) * \gamma * m)(n) \geq R(a_1, a_2, c) \wedge S(c, \gamma, m, \tau, n) > \theta$$

and

$$((a_1 * \gamma * m) \circ (a_2 * \gamma * m))(t) > S(a_1, \gamma, m, \beta, n_1) \wedge S(a_2, \gamma, m, \alpha, n_2) \wedge R(n_1, n_2, t) > \theta.$$

Since (M, Γ, R, S) is fuzzy gamma ring, we get $t = n$ and so $n \in I_1 \circ I_2$. Therefore, $I_1 \circ I_2$ is a right fuzzy ideal of M. It can be shown that $I_1 \circ I_2$ is a left fuzzy ideal of M. Hence, $I_1 \circ I_2$ is a fuzzy ideal of M. □

Definition 10. *Let (M, Γ, R, S) be a fuzzy gamma ring, I_1 and I_2 be fuzzy ideal of M. Then,*

$$I_1 \circ I_2 = \{c \in M \mid (a_1 \circ a_2)(c) > \theta, \forall a_1 \in I_1 \text{ and } \forall a_2 \in I_2\}$$

is called a fuzzy addition of two fuzzy ideals of M.

Theorem 7. *Let (M, Γ, R, S) be a fuzzy gamma ring, I_1 and I_2 be fuzzy ideal of M. Then $(I_1 \circ I_2)/I_2 \cong I_1/(I_1 \cap I_2)$.*

Proof. Let $a \in I_2$. Since I_1 is fuzzy ideal of M, $e_0 \in I_1$. Since (M, R) is fuzzy group, $R(e_0, a, a) > \theta$. Thus $a \in I_1 \circ I_2$. That is, $I_2 \subseteq I_1 \circ I_2$. Since I_2 is fuzzy ideal of M, for all $a, b \in I_2$ and $m \in M$, $(a \circ b)(m) > \theta$ implies $m \in I_2$, $a^{-1} \in I_2$. Let $n \in I_1 \circ I_2$, $a \in I_2$, $\gamma \in \Gamma$. Since $I_1 \circ I_2 \subseteq M$ and I_2 is ideal of M, for all $n \in I_1 \circ I_2$, $a \in I_2$ and $\gamma \in \Gamma$, $(n * \gamma * a)(m) > \theta$, $m \in I_1 \circ I_2$ $((a * \gamma * n)(m) > \theta,\ m \in I_1 \circ I_2)$ implies $m \in I_2$. Therefore I_2 is a fuzzy ideal of $I_1 \circ I_2$.

Let $\varphi : I_1 \to I_1 \circ I_2 / I_2$, $\varphi(a) = [a \circ I_2]$. It is clear that φ is surjective.

(1) Let $a = b$, for $a, b \in I_1$. Since $R(a, b^{-1}, e_0) > \theta$ and $e_0 \in I_2$, we get $a \circ I_2 \sim b \circ I_2$ and so $[a \circ I_2] = [b \circ I_2]$. Thus φ is well-defined.

(2) Let $a, b, c \in I_1$ such that $R(a, b, c) > \theta$. Then, we get

$$\begin{aligned}\overline{R}([a \circ I_2], [b \circ I_2], [c \circ I_2]) &= \bigvee_{(a', b', c') \in \overline{a} \times \overline{b} \times \overline{c}} R(a', b', c') \\ &\geq R(a, b, c) > \theta.\end{aligned}$$

Thus $\overline{R}(\varphi(a), \varphi(b), \varphi(c)) > \theta$.

(3) Let $a, b, c \in I_1$ and $\gamma, \beta \in \Gamma$ such that $S(a, \gamma, b, \beta, c) > \theta$. Then, we get

$$\begin{aligned}\overline{S}([a \circ I_2], \gamma, [b \circ I_2], \beta, [c \circ I_2]) &= \bigvee_{(a', \gamma, b', \beta, c') \in \overline{a} \times \gamma \times \overline{b} \times \beta \times \overline{c}} S(a', \gamma, b', \beta, c') \\ &\geq S(a, \gamma, b, \beta, c) > \theta.\end{aligned}$$

Thus $\overline{S}(\varphi(a), \varphi(b), \varphi(c)) > \theta$.

(4)

$$\begin{aligned}Ker\varphi &= \{a \in I_1 \mid \varphi(a) = [e_0 \circ I_2]\} \\ &= \{a \in I_1 \mid [a \circ I_2] = [e_0 \circ I_2]\} \\ &= \{a \in I_1 \mid a \circ I_2 \sim e_0 \circ I_2\} \\ &= \left\{a \in I_1 \mid R\left(a^{-1}, e_0, h\right) > \theta, \exists h \in I_2\right\} \\ &= \{a \in I_1 \mid a \in I_2\} = I_1 \cap I_2.\end{aligned}$$

Thus, we get $I_1 \circ I_2 / I_2 \cong I_1 / I_1 \cap I_2$ from Theorem 5. □

Theorem 8. *Let (M, Γ, R, S) be a fuzzy gamma ring, I_1 and I_2 be fuzzy ideal of M such that $I_1 \subseteq I_2$. Then $(M/I_1)/(I_2/I_1) \cong M/I_2$.*

Proof. Let $\varphi : M/I_1 \to M/I_2$, $\varphi([a \circ I_1]) = [a \circ I_2]$. It is clear that φ is surjective.

(1) Let $[a \circ I_1] = [b \circ I_1]$, for $a, b \in M$. There exists $h \in I_1$ such that $R\left(a^{-1}, b, h\right) > \theta$, since $a \circ I_1 \sim b \circ I_1$. Since $I_1 \subseteq I_2$, $h \in I_2$. Hence $a \circ I_2 \sim b \circ I_2$ and so $[a \circ I_2] = [b \circ I_2]$.

(2) $\overline{R}([a\circ I_1],[b\circ I_1],[c\circ I_1]) > \theta$. We have that there exist $a_1 \in \overline{a}$, $b_1 \in \overline{b}$, $c_1 \in \overline{c}$ such that $R(a_1,b_1,c_1) > \theta$. Since $a_1 \circ I_1 \sim a \circ I_1$, $b_1 \circ I_1 \sim b \circ I_1$ and $c_1 \circ I_1 \sim c \circ I_1$, and so there exist $h_1, h_2, h_3 \in I_1$ such that $R(a_1,h_1,a) > \theta$, $R(b_1,h_2,b) > \theta$, $R(c_1,h_3,c) > \theta$. Also, $h_1,h_2,h_3 \in I_2$, since $I_1 \subseteq I_2$.

Let $u \in M$ such that $R(a,b,u) > \theta$. Similar to the proof of [14, Theorem 4.2], we have $\exists h' \in I_2$ such that $R(c_1,h,u) > \theta$. Then $c\circ I_2 \sim u \circ I_2$, consequently, $\overline{R}([a\circ I_2],[b\circ I_2],[u\circ I_2]) = \overline{R}([a\circ I_2],[b\circ I_2],[c\circ I_2]) > \theta$.

(3) Let $\overline{S}([a\circ I_1],\gamma,[b\circ I_1],\beta,[c\circ I_1]) > \theta$, for all $\gamma,\beta \in \Gamma$. Similar to the proof of (ii), we have $\overline{S}([a\circ I_2],\gamma,[b\circ I_2],\beta,[c\circ I_2]) > \theta$.

(4)

$$\begin{aligned} Ker\varphi &= \{[a\circ I_1] \in M/I_1 \mid \varphi([a\circ I_1]) = [e_0 \circ I_2]\} \\ &= \{[a\circ I_1] \in M/I_1 \mid [a\circ I_2] = [e_0\circ I_2]\} \\ &= \{[a\circ I_1] \in M/I_1 \mid a\circ I_2 \sim e_0 \circ I_2\} \\ &= \left\{[a\circ I_1] \in M/I_1 \mid R\left(a^{-1}, e_0, h\right) > \theta,\ \exists h \in I_2\right\} \\ &= \{[a\circ I_1] \in M/I_1 \mid a \in I_2\} = I_2/I_1. \end{aligned}$$

Thus, we get $(M/I_1)/(I_2/I_1) \cong M/I_2$ from Theorem 5. □

References

[1] Aktaş H. and Çağman N., A type of fuzzy ring, Arch. Math. Logic 46(3-4) (2007), 117-165.

[2] Barnes W. E., On the Γ-rings of Nobusawa, Pasific J. Math. 18 (1966), 411-422.

[3] Hong S. M., Jun Y. B., A note on fuzzy ideals in gamma rings, Bull. Honam Math. Soc. 12 (1995), 39-48.

[4] Jun Y.B. and Lee C.Y., Fuzzy Γ-rings, Pusan Kyongnam Math. J. (Presently, East Asian Math. J.) 8 (1992), 163-170.

[5] Jun Y. B. and Lee C.Y., Fuzzy prime ideals in Γ-rings, Pusan Kyongnam Math. J. (Presently, East Asian Math. J.) 9 (1) (1993), 105-111.

[6] Jun Y. B., Uçkun M. and Öztürk M. A., Fuzzy quotient gamma rings, J. Fuzzy Math. 15(2)(2007), 363-374.

[7] Kyuno S., On prime gamma rings, Pasific J. Math. 75 (1) (1978), 185-190.

[8] Morderson J. M. and Malik D. S., Fuzzy commutative algebra, World Scientitific, Singapore, 1998.

[9] Mukherjee T. K. and Sen M. K., On fuzzy ideals of a ring I, Fuzzy Sets and Systems 21 (1)(1987), 99-104.

[10] Nobusawa N., On generalization of the ring theory, Osaka J. Math. 1 (1978), 185-190.

[11] Öztürk M. A., Jun Y. B. and Yazarlı H., A new view of fuzzy gamma rings, Hacettepe Journal of Mathematics and Statistics 39(3) (2010), 365-378.

[12] Rosenfeld A., Fuzzy groups, J. Math. Anal. Appl. 35 (1971), 512-517.

[13] Yong-Chai Y., Fuzzy ideal and fuzzy quotient rings, J. Fuzzy Math. 12 (1985), 19-26.

[14] Yuan X. and Lee E. S., Fuzzy group based on fuzzy binary operation, Comput. Math. Appl. 47 (2004), 631-641.
[15] Zadeh L. A., Fuzzy sets, Inform. Control 8 (1965), 338-353.

ADIYAMAN UNIVERSITY, FACULTY OF ARTS AND SCIENCES, DEPARTMENT OF MATHEMATICS, 02040-ADIYAMAN, TURKEY
E-mail address: einan@adiyaman.edu.tr

CUMHURIYET UNIVERSITY, FACULTY OF ARTS AND SCIENCES, DEPARTMENT OF MATHEMATICS, 58140 SIVAS, TURKEY
E-mail address: hyazarli@cumhuriyet.edu.tr

ADIYAMAN UNIVERSITY, FACULTY OF ARTS AND SCIENCES, DEPARTMENT OF MATHEMATICS, 02040-ADIYAMAN, TURKEY
E-mail address: maozturk@adiyaman.edu.tr