GENERATING THE COMPLEMENT OF A STAIRCASE STARSHAPED ORTHOGONAL POLYGON FROM STAIRCASE CONVEX CONES

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ABSTRACT. Let S be an orthogonal polygon in the plane, bounded by a simple closed curve, and let R be the smallest rectangular region containing S. Assume that S is starshaped via staircase paths. For every point p in $\mathbb{R}^2 \backslash (int\, S)$, there is a corresponding point q in $bdry\, S$ such that p lies in a maximal staircase convex cone C_q at q in $\mathbb{R}^2 \backslash (int\, S)$. Furthermore, point q may be selected to satisfy these requirements:

1) If $p \in \mathbb{R}^2 \backslash (int\, R)$, then q is an endpoint of an extreme edge of S.

2) If $p \in (int\, R) \backslash (int\, S)$, then q is a point of local nonconvexity of S and C_q is unique. Moreover, there is a neighborhood N of q such that, for s in $(bdry\, S) \cap N$ and for C_s any staircase cone at s in $\mathbb{R}^2 \backslash (int\, S), C_s \subseteq C_q$.

Thus we obtain a finite family of staircase convex cones whose union is $\mathbb{R}^2 \backslash (int\, S)$.

1. INTRODUCTION.

We begin with some definitions from [1]. Let S be a nonempty set in the plane. Point x in S is a *point of local convexity* of S if and only if there is a neighborhood N of x such that $N \cap S$ is convex. If S fails to be locally convex at q in S, then q is a *point of local nonconvexity* (lnc point) of S. Set S is called an *orthogonal polygon* if and only if S is a connected union of finitely many convex polygons (possibly degenerate) whose edges are parallel to the coordinate axes. An edge e of S is a *dent edge* if and only if both endpoints of e are lnc points of $S \cap J$, for J an appropriate closed halfplane determined by the line of e. Let λ be a simple polygonal path in the plane whose edges $[v_{i-1}, v_i], 1 \leq i \leq n$, are parallel to the coordinate axes. Such a path λ is called a *staircase path* or a $v_o - v_n$ *staircase* if and only if no two of the vectors $\overrightarrow{v_{i-1}v_i}$ have opposite direction. That is, for an appropriate labeling, for i odd the vectors $\overrightarrow{v_{i-1}v_i}$ have the same horizontal

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direction, and for i even the vectors $\overrightarrow{v_{i-1}v_i}$ have the same vertical direction. Similarly, a staircase path followed by a ray having a compatible direction is called a *staircase ray*, while a union of staircase rays originating at a common point x will be a *cone* at x. An edge $[v_{i-1}, v_i]$ or an associated ray emanating from v_{i-1} through v_i will be called *north, south, east,* or *west* according to the direction of vector $\overrightarrow{v_{i-1}v_i}$. Also, we use the terms *north, south, east, west, northeast, northwest southeast, southwest* to describe the relative position of points.

For points x and y in set S, we say *x sees y* (x is *visible* from y) *via staircase paths* if and only if there is a staircase path in S that contains both x and y. Set S is called *staircase convex* provided, for every x, y in S, x sees y via staircase paths. Set S is *starshaped via staircase paths* (*staircase starshaped*) if and only if, for some point p of S, p sees each point of S via staircase paths, and the set of all such points p is the *staircase kernel* of S, denoted $Ker\, S$.

Finally, if S is an orthogonal polygon and L is a horizontal or vertical line supporting S, any edge of S in L will be called an *extreme edge* of S.

Many results in convexity that involve the usual concept of visibility via straight line segments have interesting analogues that involve the idea of visibility via staircase paths: (See [1]-[5].) For example, the familiar Krasnosel'skii theorem [9] says that, for S a nonempty compact set in the plane, S is starshaped via segments if and only if every three points of S are visible (via segments in S) from a common point. In the staircase analogue [2], for S a simply connected orthogonal polygon in the plane, S is starshaped via staircase paths if and only if every two points of S are visible (via staircase paths in S) from a common point. Notice that, in the staircase version, the Helly number three is reduced to two.

In recent work by Guillermo Hansen and Horst Martini [8], the authors obtain a family of convex cones to generate the complement of a closed starshaped set in $\mathbb{R}^d$. Here we provide an analogue for the complement of an orthogonal polygon S in the plane, obtaining a finite family of staircase convex cones whose union is $\mathbb{R}^2 \backslash (int\, S)$. As in previous work (in [3] and in [4], for example), points of local nonconvexity of S play an important role in the result.

Throughout the paper, $int\, S, bdry\, S$, and $cl\, S$ will denote the interior, the boundary, and the closure, respectively, of set S. If λ is a simple path containing points x and y, then $\lambda(x, y)$ will represent the subpath of λ from x to y (ordered from x to y), containing points x and y. When x and y are distinct points, $L(x, y)$ will represent their corresponding line. Readers may refer to Valentine [13], to Lay [10], to Danzer, Grünbaum, Klee [6], to Eckhoff [7], and to numerous papers, including Martini and Soltan

[11], Martini and Wenzel [12], and Hansen and Martini [8], for discussions concerning visibility via straight line segments and starshaped sets.

2. The results.

Some preliminary observations from [1] will be helpful.

Preliminary Observations. Let S be an orthogonal polygon in the plane, bounded by a simple closed curve. Let T be the union of all points of S, together with all segments joining pairs of horizontal or vertical points in S. Then by [1, Lemma 1], T is a staircase convex polygon and is the minimal staircase convex set containing S.

Certainly every point of T lies on a horizontal or vertical segment whose endpoints are in *bdry* S. Moreover, if $T \neq S$, then $T \backslash S$ consists of finitely many components $A_1, \ldots, A_m, m \geq 1$. Each set A_i has as its boundary a simple closed curve $\lambda(x,y) \cup \delta(y,x)$, where $\delta(x,y)$ is a connected subset of *bdry* S and where $\lambda(x,y)$ is a staircase path in *bdry* T meeting S only in x and y. Each point of A_i necessarily lies on a segment joining points of $\delta(x,y)$, and thus each point of $\lambda(x,y)$ lies on a segment joining x or y to another point of $\delta(x,y)$. Hence the staircase $\lambda(x,y)$ consists of either one or two segments. (See Figure 1.)

Clearly *bdry* T is a simple closed curve. Order *bdry* T in a clockwise direction and classify each edge of *bdry* T as north, south, east, or west relative to this ordering. Here are the possible classifications for edges of $\lambda(x,y)$: If $\lambda(x,y)$ is a segment, its direction (from x to y) may be east, south, west, or north. If $\lambda(x,y)$ consists of two segments, their directions (from x to y) may be east followed by north, south followed by east, west followed by south, or north followed by west.

If we order *bdry* S in a clockwise direction, then each direction listed above for $\lambda(x,y)$ is associated with a dent edge of S having the same direction and contained in $\delta(x,y)$: If $[x,r]$ is the first edge of $\lambda(x,y)$ and is east, let line L be parallel to $L(x,r)$ and supporting A_i, with L strictly south of $L(x,r)$. Then each component of $L \cap cl\, A_i$ is a dent edge of S and is east. If $r \neq y$, a parallel argument holds for the north edge $[r,y]$ of $\lambda(x,y)$.

Finally, let R represent the smallest rectangular region determined by S (and by T). Let $L_i, 1 \leq i \leq 4$, denote the lines determined by the four edges of R. Certainly each line L_i contains at least one edge of *bdry* S, and such an edge is an extreme edge of S. Observe that there are at most four components of $R \backslash T$, one at each vertex of R. Moreover, if B is a component of $R \backslash T$, then *bdry* B is a simple closed curve $\lambda(x,y) \cup \delta(y,x)$, where $\delta(x,y)$ is a staircase path in *bdry* T and $\lambda(x,y)$ is a staircase path of two segments in *bdry* R meeting T only in x and y.

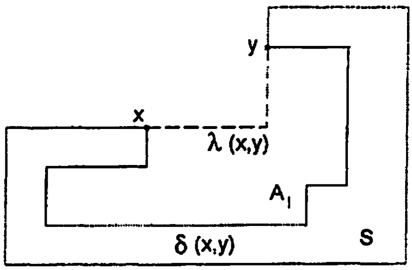

Figure 1.

We begin with the following lemma.

Lemma 1. *Let S be an orthogonal polygon in the plane, bounded by a simple closed curve, and let q be a point of local nonconvexity of S. If there exists a staircase ray at q in $\mathbb{R}^2\backslash(int\, S)$, then there exists a maximal staircase convex cone C_q at q in $\mathbb{R}^2\backslash(int\, S)$. Moreover, the cone C_q is unique and contains all such staircase rays at q.*

Proof. Define the set C_q to be the union of all the staircase rays at q in $\mathbb{R}^2\backslash(int\, S)$. Certainly $C_q \neq \emptyset, C_q$ defines a closed cone at q, and C_q contains every staircase convex cone at q in $\mathbb{R}^2\backslash(int\, S)$. We will show that C_q is staircase convex.

Let $a, b \in C_q$ to find a staircase path from a to b in C_q. The points a and b lie on corresponding staircase rays α and β, respectively, at q in $\mathbb{R}^2\backslash(int\, S)$. Without loss of generality, assume that q is the origin. If α and β lie in nonconsecutive quadrants, then $\alpha \cup \beta$ contains an $a-b$ staircase path in C_q, the desired result. Therefore, we may restrict our attention to the case in which α and β lie either in consecutive quadrants or in the same quadrant. For convenience, assume that α and β lie in the closed halfplane $cl\, H_1$ on and above the x axis. Let D denote the connected subset of $cl\, H_1$ whose boundary is $\alpha \cup \beta$. (See Figure 2.)

We assert that D is a subset of $\mathbb{R}^2\backslash(int\, S)$. Suppose, on the contrary, that D contains a point s of $int\, S$ to reach a contradiction. Then s lies on a segment $[a', b']$, where $a' \in \alpha$ and $b' \in \beta$. This forces the connected set $int\, S$ to lie in region D, in turn forcing the edges of $bdry\, S$ at q to lie in $cl\, H_1$. However, then q cannot be an lnc point of S, contradicting our hypothesis. Our supposition must be false, and $D \subseteq \mathbb{R}^2\backslash(int\, S)$ establishing the assertion.

It is easy to find a staircase path from a to b in D: Let M be any closed rectangular region of the plane containing a, b, and q. The boundary of $M \cap D$ is $\alpha(q, a'') \cup \beta(q, b'') \cup [a'', b'']$, where $[a'', b'']$ is an appropriately chosen horizontal segment with $a'' \in \alpha$ and $b'' \in \beta$. Since the boundary of $M \cap D$ is a union of three staircase paths, we may apply [5, Lemma 2] to conclude that $M \cap D$ is staircase convex. Hence $M \cap D$ (and therefore D) contains an $a - b$ staircase. Let $\mu(a, b)$ represent such a staircase path.

It remains to show that $\mu(a, b) \subseteq C_q$. Choose point t on $\mu(a, b)$. Since q and t belong to the staircase convex set $M \cap D$, $M \cap D$ contains a staircase path from q to t, say $\gamma(q, t)$. In case the north ray n at t lies in D, then $\gamma(q, t)$ followed by n produces a staircase ray in $D \subseteq \mathbb{R}^2 \backslash (int\, S)$, so $t \in C_q$. In case the north ray at t does not lie in D, then rays α, β must lie in the same quadrant in $cl\, H_1$. (Again see Figure 2.) Furthermore, α, β, and $\mu(q, t)$ necessarily use compatible directions. The north ray at t meets $\alpha \cup \beta$ at a first point u. Without loss of generality, assume that $u \in \beta$. Then $\gamma(q, t) \cup [t, u]$, followed by $\beta \backslash \beta(q, u)$, determines a staircase ray in $D \subseteq \mathbb{R}^2 \backslash (int\, S)$, and again $t \in C_q$. We conclude that $\mu(a, b) \subseteq C_q$, so C_q is staircase convex. Finally, since C_q contains every staircase ray at q in $\mathbb{R}^2 \backslash (int\, S)$, certainly C_q is maximal and is unique, finishing the proof of the lemma. □

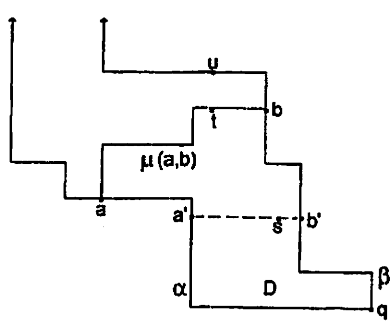

Figure 2.

It is interesting to observe that the result in Lemma 1 fails without the requirement that $bdry\, S$ be a simple closed curve. Consider the following example.

Example 1. Let S be the simply connected orthogonal polygon in Figure 3, with q the only lnc point of S. Then there are three maximal staircase convex cones at q in $\mathbb{R}^2 \backslash (int\, S)$: Cone 1 (illustrated) contains all points northeast of q but no points southwest of q, cone 2 contains all points southwest of q but no points northeast of q, and cone 3 consists of all points northeast of q and all points southwest of q.

Of course, $int\, S$ is not connected in this example.

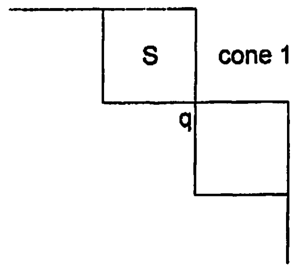

Figure 3.

We are ready for the main result.

Theorem 1. *Let S be an orthogonal polygon in the plane, bounded by a simple closed curve, and let R be the smallest rectangular region containing S. Assume that S is starshaped via staircase paths. For every point p in $\mathbb{R}^2 \backslash (int\, S)$, there is a corresponding point q in $bdry\, S$ such that p lies in a maximal staircase convex cone C_q at q in $\mathbb{R}^2 \backslash (int\, S)$. Furthermore, point q may be selected to satisfy these requirements:*

1) If $p \in \mathbb{R}^2 \backslash (int\, R)$, then q is an endpoint of an extreme edge of S.

2) If $p \in (int\, R) \backslash (int\, S)$, then q is an lnc point of S and C_q is unique. Moreover, there is a neighborhood N of q such that, for s in $(bdry\, S) \cap N$ and for C_s any staircase cone at s in $\mathbb{R}^2 \backslash (int\, S), C_s \subseteq C_q$.

Proof. As in our preliminary remarks, let T denote the minimal orthogonally convex set containing S. Certainly $S \subseteq T \subseteq R$.

There are three cases to consider.

Case 1. Assume that $p \in \mathbb{R}^2 \backslash (int\, R)$. Then p lies beyond (or on) at least one and at most two of the four lines supporting R along its edges. Without loss of generality, assume that p lies beyond (or on) the line L supporting the northern edge of R, and let q be an endpoint of a component of $L \cap S$. Then q is an endpoint of an extreme edge of S. Any $q-p$ staircase, followed by a north ray at p, yields a staircase convex cone at p in $\mathbb{R}^2 \backslash (int\, S)$. Certainly we may extend this cone to a maximal staircase cone C_q at q in $\mathbb{R}^2 \backslash (int\, S)$ to satisfy the theorem.

Case 2. Assume that $p \in (int\, R) \backslash (int\, T)$. There are at most four components of $(int\, R) \backslash (int\, T)$, one at each vertex of R, and p belongs to one of these components. Without loss of generality, assume that p belongs to the component at the northwest vertex of R. Using a clockwise orientation for $bdry\, S$, point p lies directly north of an east edge e of $bdry\, S$ and directly west of a north edge n of $bdry\, S$. (See Figure 4a.) From point p, consider the maximal length segment in $\mathbb{R}^2\backslash(int\, S)$ south to edge e, then follow maximal length segments east and south in $bdry\, S$ as far as possible. We arrive at an $ln\, c$ point q of S. Reverse the path from q to p to obtain a northwest staircase from q to p in $\mathbb{R}^2\backslash(int\, S)$. Follow this by the north (or west) ray at p to obtain a staircase ray at q containing p. Using Lemma 1, extend this to the unique maximal staircase convex cone C_q at q in $\mathbb{R}^2\backslash(int\, S)$.

To see that q satisfies the theorem, observe that for s near q in $bdry\, S$, s is either directly north of q or directly west of q, and s lies on a staircase ray at q lying in C_q. Thus staircase rays at s exist in $\mathbb{R}^2\backslash(int\, S)$. Let C_s be the union of all these rays. Using an argument from the proof of Lemma 1, for s sufficiently close to q, C_s is staircase convex.

We will show that for s sufficiently near $q, C_s \subseteq C_q$. Certainly $q \in T$. There are two possibilities to consider: If $q \in bdry\, T$, then C_q contains staircase rays at q in each of the directions northeast, northwest, and southwest. (See Figure 4b.) Moreover, for s near q in $bdry\, S, C_s = C_q$. If $q \notin bdry\, T$, then $q \in int\, T$, and all staircase rays at q in C_q have direction northwest. (Again see Figure 4a.) For s near q in $bdry\, S$, all staircase rays at s in C_s are northwest as well. Moreover, every such ray at s in C_s lies in a corresponding ray at q in C_q, and $C_s \subseteq C_q$. We conclude that q satisfies the theorem, finishing Case 2.

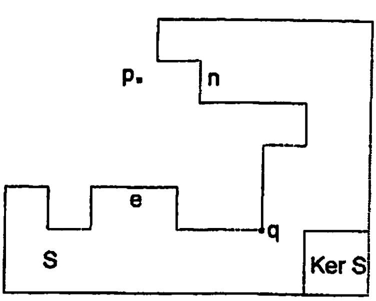

Figure 4a.

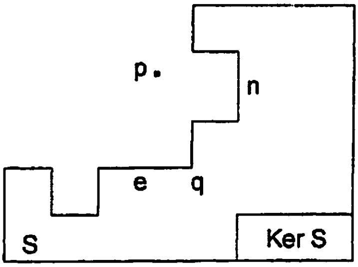

Figure 4b.

Case 3. Assume that $p \in (int\,T)\backslash(int\,S)$. Then p belongs to $cl\,A_i$ for some component A_i of $T\backslash S$. For convenience of notation, let $A_i = A$. By our preliminary observations, $bdry\,A$ is a simple closed curve $\lambda(x,y) \cup \delta(y,x)$, where $\delta(x,y)$ is a connected subset of $bdry\,S$ (ordered in a clockwise direction along $bdry\,S$) and where $\lambda(x,y)$ is a staircase of one or two segments in $bdry\,T$ (ordered in a clockwise direction along $bdry\,T$), meeting S only in x and y. By our earlier comments, there are eight possible classifications for $\lambda(x,y)$. Without loss of generality, assume that $\lambda(x,y)$ either consists of exactly one edge whose direction is east or consists of exactly two edges, one that is east followed by one that is north. In the first case, for every point z of $\lambda(x,y)$, there are points of $cl\,A$ directly south of z. In the second case, for every point z on the east edge of $\lambda(x,y)$, there are points of $cl\,A$ strictly south of z, while for every z on the north edge of $\lambda(x,y)$, there are points of $cl\,A$ strictly east of z.

Let $[x,r]$ be the east edge of $\lambda(x,y)$, and let line L be parallel to $L(x,r)$ and supporting $cl\,A$, with L strictly south of $L(x,r)$. By our preliminary comments, L contains an east dent edge of S. Moreover, the existence of this dent edge forces $Ker\,S$ to lie in the closed halfplane determined by L and south of L. There are two possibilities to consider.

First suppose that $\lambda(x,y)$ consists of just one edge $[x,y] = [x,r]$. (See Figure 5.) There can be no west dent edge of S in $cl\,A$, for otherwise $Ker\,S$ would lie north of the associated line, impossible since $Ker\,S$ is south of line L. Thus all points of $cl\,A$ must lie on or strictly south of the line $L(x,y)$. Similarly, since $Ker\,S \neq \emptyset, cl\,A$ cannot contain both north and south dent edges of S. Without loss generality, assume that $cl\,A$ contains no south dent edge of S. Using our orientation along $\delta(x,y)$ from x to y, we conclude that, for any dent edge d of S in $cl\,A$, either d is an east edge of $\delta(x,y)$ (with points of $cl\,A$ north of d) or d is a north edge of $\delta(x,y)$ (with points of $cl\,A$ west of d). Furthermore, all points of $cl\,A$ are southeast of point x.

For p in $cl\,A$, we will find a northwest staircase in $cl\,A$ from p to $\lambda(x,y)$: From p, travel north in $cl\,A$ as far as possible, say to p_1. If $p_1 \in [x,y]$, then $[p,p_1]$ is an appropriate staircase. If $p_1 \notin [x,y]$, travel west of p_1 in $cl\,A$ as far as possible, say to p_2. Observe that there is a nondegenerate segment in $cl\,A$ at p_2 and north of p_2. If not, this would imply the existence of a forbidden dent edge in S. Continue the procedure until we reach $\lambda(x,y)$ via a northwest staircase. As in Case 2, follow a southeast path from p in $\mathbb{R}^2\backslash(int\,S)$ (and in $cl\,A$) to arrive at an lnc point q of S. Reverse the path to obtain a northwest staircase from q to p in $cl\,A$. Follow this by the northwest staircase in $cl\,A$ from p to $\lambda(x,y)$ found earlier, then take a north (or west) ray to obtain a northwest staircase ray at q. Finally, using Lemma 1, extend this to the maximal staircase cone C_q at q in $\mathbb{R}^2\backslash(int\,S)$.

Now suppose that $\lambda(x,y)$ is a union of two edges, east edge $[x,r]$ followed by north edge $[r,y]$. Then every point of $cl\,A$ is either south of $L(x,y)$ or east of $L(r,y)$ or both. Moreover, for any dent edge d of S in $cl\,A$, either d is an east edge of $\delta(x,y)$ or d is a north edge of $\delta(x,y)$. If p is south of $L(x,r)$, then an argument similar to the one above produces a maximal staircase cone C_q lying in $\mathbb{R}^2\backslash(int\,S)$ and containing p. If p is east of $L(r,y)$, an analogous argument yields a parallel result.

Finally, observe that $q \in int\,T$. The associated argument from Case 2 may be used to show that, for s near q in $bdry\,S$, the corresponding staircase convex cone C_s lies in C_q. Thus the point q satisfies the theorem, finishing Case 3 and completing the proof. ☐

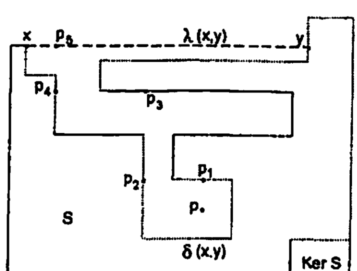

Figure 5.

Corollary. *Let S be an orthogonal polygon in the plane, and assume that S is starshaped via staircase paths. There is a finite collection of staircase convex cones whose union is $\mathbb{R}^2\backslash(int\,S)$.*

Proof. If S is bounded by a simple closed curve, we may apply Theorem 1 to select our cones. Using the notation in that theorem, certainly an appropriate choice of two cones satisfying requirement 1) will cover $\mathbb{R}^2\backslash(int\, R)$. We choose the remaining cones to satisfy requirement 2). Each of these will be uniquely determined by an lnc point of S, yielding a finite family of cones that cover $(int\, R)\backslash(int\, S)$ and a finite family in all.

If S fails to be bounded by a simple closed curve, then appropriate modifications in the proof of Theorem 1 yield the result. Although a maximal staircase convex cone C_q determined by an lnc point q of S need not be unique (as Example 1 has demonstrated), still at most finitely many (in fact, at most three) such cones can exist at q. Again we obtain a finite family of cones whose union is $\mathbb{R}^2\backslash(int\, S)$. □

We close with a series of observations and examples.

Using the setting and terminology in Theorem 1, it is easy to see that a minimal family of cones $C_q \subseteq \mathbb{R}^2\backslash(int\, S)$ covering $\mathbb{R}^2\backslash(int\, S)$ need not be unique. More interesting is the observation that a minimal family of these cones covering $(int\, R)\backslash(int\, S)$ need not be unique either. Consider the following example.

Example 2. Let S be the staircase starshaped orthogonal polygon in Figure 6, with R the smallest rectangular region containing S. For labeled lnc points $q_i, 1 \leq i \leq 5$, let C_i denote the corresponding maximal cone at q_i in $\mathbb{R}^2\backslash(int\, S)$, defined in Lemma 1. The family of cones $\{C_1, C_3, C_4\}$ is a minimal covering family for $(int\, R)\backslash(int\, S)$. (That is, no proper subfamily will cover $(int\, R)\backslash(int\, S$.) Similarly, $\{C_2, C_3, C_4\}$ is a minimal covering family, as is $\{C_3, C_4, C_5\}$. Notice that $C_5 \subsetneq C_1 \subsetneq C_2$.

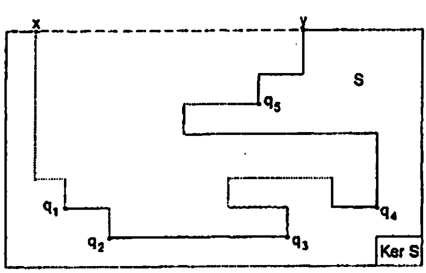

Figure 6.

However, in Theorem 1, if we select the C_q cones that are maximal (in the sense that no cone selected is a proper subset of any other such cone), then we obtain a minimal covering family for $(int\, R)\backslash(int\, S)$. To see this, consider various cases possible for p in $(intR)\backslash(int\, S)$. Because the arguments for the cases are similar, we restrict out attention to the case in which p belongs to some component A of $T \backslash S$. Assume that A is bounded by $\lambda(x,y) \cup \delta(x,y)$, where $\delta(x,y)$ is a connected subset of *bdry* S and where $\lambda(x,y)$ is an east segment in *bdry* T. Assume also that every dent edge of S in *cl* A is either east or north. Using an argument in Theorem 1, an appropriate cone C_q selected for p is determined by an *lnc* point q of S in $\delta(x,y)$. Furthermore, q lies at the intersection of an east edge and a north edge of $\delta(x,y)$. Any such cone C_q that fails to be maximal will be a proper subset of a cone C_r determined by some *lnc* point r, where r lies at the intersection of a south edge and an east edge of $\delta(x,y)$. There is at most one such r for which C_r is maximal (in the sense above). Then C_r (if it exists), together with the maximal C_q cones already chosen, will cover *cl* A, and no proper subcollection covers *cl* A. Finally, these are the only maximal cones determined by *lnc* points of S in $\delta(x,y)$.

Of course, it is the cones that are unique for A and not the *lnc* points generating them, since distinct *lnc* points may generate the same cone. Consider Example 3 below.

Example 3. Let S be the orthogonal polygon in Figure 7. Then for *lnc* points q_1 and q_2, their associated cones C_1 and C_2 contain the same points.

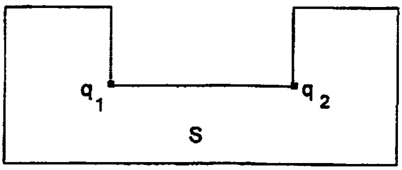

Figure 7.

Finally, it is interesting to note that Theorem 1 fails without the requirement that the orthogonal polygon S be staircase starshaped, as Example 4 demonstrates.

Example 4. Let S be he orthogonal polygon in Figure 8, with q_1 and q_2 two of the *lnc* points of S. Observe that $Ker\, S = \emptyset$. Moreover, for every point p of $\mathbb{R}^2 \backslash (int\, S)$ near edge $[q_1, q_2]$, there is no staircase convex cone in $\mathbb{R}^2 \backslash (int\, S)$ that contains point p.

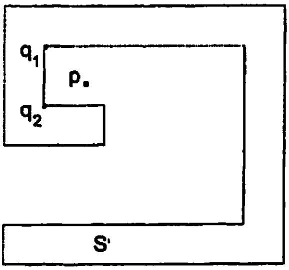

Figure 8.

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