Corrections to "The Ramsey Numbers for a Quadrilateral vs. All Graphs on Six Vertices"

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Keywords

Graph Theory, extremal problem, Ramsey number, small graph, cycle.

Abstract

The values of the Ramsey numbers $R(C_4, H)$, for any graph H on 6 vertices, are shown in [3]. An erratum is corrected in [4, 6], giving $R(C_4, K_{3,3}) = 11$.

In this paper, we correct other three errata of [3], proving that $R(C_4, K_1 + (K_{2,3} - e)) = 9$, $R(C_4, \overline{K_3 \cup P_3}) = 11$ and $R(C_4, \overline{2P_3}) = 11$, instead of 10.

We follow the notation used in [5] and Figure 1. Let H be a graph. Define $H[A]$ to be the subgraph of H induced by a set of vertices $A \subseteq V(H)$.

Our results are the following:

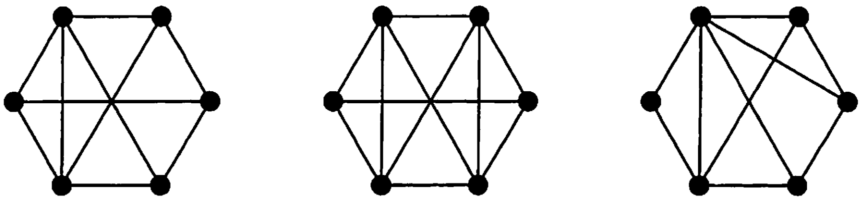
Theorem 1
$R(C_4, \overline{K_3 \cup P_3}) = 11$, $R(C_4, \overline{2P_3}) = 11$ and $R(C_4, K_1 + (K_{2,3} - e)) = 9$.

Proof. [4, 6] $11 = R(C_4, K_{3,3}) \leq R(C_4, \overline{K_3 \cup P_3}) \leq R(C_4, \overline{2P_3}) \leq R(C_4, K_1 + \overline{P_3 \cup K_2}) = 11$ [3].

$R(C_4, K_1 + (K_{2,3} - e)) \geq R(C_4, K_1 + (C_4 \cup K_1)) = 9$ [3]. Let G be a C_4-free graph on 9 vertices. G has at most 13 edges [2], thus there exists a vertex v of G with degree at most 2. Let W be the set of vertices non-adjacent to v. As W has at least 6 vertices and $R(C_4, K_{2,3} - e) = 6$ [1], $\overline{G}[W]$ contains $K_{2,3} - e$ and $\overline{G}[W \cup \{v\}]$ contains $K_1 + (K_{2,3} - e)$. Therefore, $R(C_4, K_1 + (K_{2,3} - e)) = 9$. ■

Remark

We have obtained with the help of computer algorithms all values of $R(C_4, H)$, being H of order 6, checking the corrections of [4, 6] and this paper and the remaining 152 values of [3].



$\overline{K_3 \cup P_3}$ $\overline{2P_3}$ $K_1 + (K_{2,3} - e)$

Figure 1:

References

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