A Construction of Rectangular Designs

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Abstract. A construction of rectangular designs from Bhaskar Rao designs is described. As special cases some series of rectangular designs are obtained.

1. Introduction

Rectangluar designs are an important class of 3-associate class partially balanced incomplete block (PBIB) designs. These designs are useful as factorial experiments. Group divisible designs are a special class of rectangluar designs. For a recent review on construction procedures of rectangular designs, see Gupta & Mukerjee (1989). Bhaskar Rao designs have been studied by Bhaskar Rao (1966, 1970), Saha & Gouri Shanker (1976), Seberry (1982, 1984), Street (1981), and Street & Rodger (1980). These authors also provide methods of construction of group divisible designs from Bhaskar Rao designs. Seberry (1982), Lam & Seberry (1984) studied generalized Bhaskar Rao designs.

Here, a method of construction of rectangular designs from Bhaskar Rao designs is described.

2. The Construction

Theorem 1.1. *The existence of a Bhakar Rao design* $(v', b', r', k', \lambda')$ *implies the existence of a rectangular design with parameters:*

$$\begin{gathered} v = 3v', \quad b = 6b', \quad r = 2b', \quad k = v' \\ \lambda_1 = 0, \quad \lambda_2 = \lambda', \quad \lambda_3 = b' - \frac{\lambda'}{2}, \quad m = v', \quad n = 3 \end{gathered} \tag{1.1}$$

Proof: Let X by the Bhaskar Rao design $(v', b', r', k', \lambda')$. First, X^I, $X^{(0,1)}$, $X^{(0,-1)}$, $X^{(1,-1)}$, $X^{(0,1,-1)}$, and $X^{(0,-1,1)}$ are obtained by permuting the elements of X with I, (0,1), (0,-1), (1,-1), (0,1,-1), (0,-1,1). Further, let X_1^p be the matrix obtained from X^p by replacing -1's by zeros, X_2^p is obtained by replacing 0's by 1, and 1 and -1 by zeros. X_3^p is obtained by replacing -1's by 1's, and 1's by zeros. Then the following incidence structure

$$N = \begin{bmatrix} X_1^I & X_1^{(0,1)} & X_1^{(0,-1)} & X_1^{(1,-1)} & X_1^{(0,1,-1)} & X_1^{(0,-1,1)} \\ X_2^I & X_2^{(0,1)} & X_2^{(0,-1)} & X_2^{(1,-1)} & X_2^{(0,1,-1)} & X_2^{(0,-1,1)} \\ X_3^I & X_3^{(0,1)} & X_3^{(0,-1)} & X_3^{(1,-1)} & X_3^{(0,1,-1)} & X_3^{(0,-1,1)} \end{bmatrix}$$

gives the rectangular design with parameters (1.1).

By the application of the above theorem and the theorems 3.1 and 3.2 given in Rao (1966, 1970) we get the following:

Corollary 1.2: There exists rectangular design with parameters

$$v' = 3v, \quad b' = 6tv(v-1), \quad r' = 2tv(v-1), \quad k' = v, \qquad (1.2)$$
$$\lambda_1' = 0, \quad \lambda_2' = 2t, \quad \lambda_3' = t(v^2 - v - 1), \quad m = v, \quad n = 3$$

Corollary 1.3: There exists rectangular design with parameters:

$$v = 3(s^2 + s + 1), \quad b = 6(s^2 + s + 1), \quad r = 2(s^2 + s + 1), \quad k = s^2 + s + 1$$
$$\lambda_1 = 0, \quad \lambda_2 = s(s-1), \quad \lambda_3 = \frac{1}{2}(s^2 + 3s + 2), \quad m = s^2 + s + 1, \quad n = 3.$$

where s is an odd prime.

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