On Total Vertex Irregularity Strength of Honeycomb Derived Networks

V. Annamma
Department of Mathematics, L. N. Government College, Ponneri, India
annammasharo@yahoo.com

Abstract

Let $G(V,E)$ be a simple graph. For a labeling $\partial : V \cup E \rightarrow \{1,2,3,...,k\}$ the weight of a vertex x is defined as $wt(x) = \partial(x) + \sum_{xy \in E} \partial(xy)$. ∂ is called a vertex irregular total k-labeling if for every pair of distinct vertices x and y, $wt(x) \neq wt(y)$. The minimum k for which the graph G has a vertex irregular total k-labeling is called the total vertex irregularity strength of G and is denoted by $tvs(G)$. In this paper we obtain a bound for the total vertex irregularity strength of honeycomb and honeycomb derived networks.

1 Introduction

A graph labeling is an assignment of *labels*, represented by integers, to the vertices, edges or both of a graph. Formally, given a graph G, a vertex labeling is a function mapping vertices of G to a set of integers [8]. A graph with such a function defined is called a *vertex-labeled graph.* Likewise, an edge labeling is a function mapping edges of G to a set of labels. In this case, G is called an *edge-labeled graph.* Most *graph labelings* trace their origins to labelings presented by Alex Rosa [9]. Rosa identified three types of labelings, which he called α, β and ρ labelings. β-labelings were later renamed as *graceful* by S.W.Golomb and the name has been popular since.

Motivated by the notion of the irregularity strength of a graph introduced by Chartrand et al. [6] in 1988 and various kinds of other total labelings, Baca et al. [4] introduced the total vertex irregularity strength of a graph as follows: Let $G(V,E)$ be a simple graph. For a labeling $\partial : V \cup E \rightarrow \{1,2,3,...,k\}$ the weight of a vertex x [10] is defined as $wt(x) = \partial(x) + \sum_{xy \in E} \partial(xy)$. ∂ is called a vertex irregular total k-labeling if for every pair of distinct vertices x and y, $wt(x) \neq wt(y)$. The minimum k for which the graph G has a vertex irregular total k-labeling is called the *total vertex irregularity strength* of G and is denoted by $tvs(G)$. In Figure 1 (a), $tvs(G_1) = 2$ and in Figure 1 (b), $tvs(G_2) = 2$.

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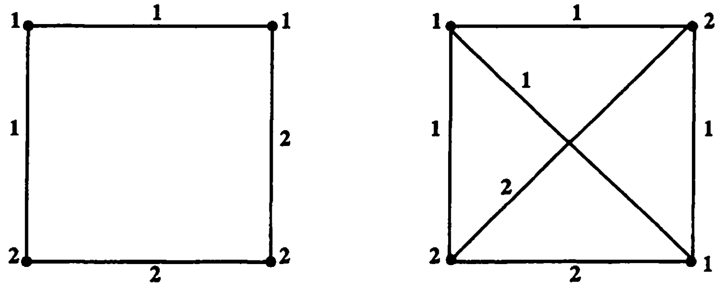

Figure 1: (a) G_1; (b) G_2

Baca, Jenrol, Miller and Ryan [4] proved that $tvs(C_n) = \left\lceil \frac{n+2}{3} \right\rceil$, $n \geq 2$; $tvs(K_n) = 2$; $tvs(K_{1,n}) = \left\lceil \frac{n+1}{2} \right\rceil$; $tvs(C_n \times P_2) = \left\lceil \frac{2n+3}{4} \right\rceil$. If T is a tree with m pendent vertices and no vertex of degree 2, they proved that $\left\lceil \frac{t+1}{2} \right\rceil \leq tvs(T) \leq m$. They also proved that if G is a (p,q) graph with minimum degree δ and maximum degree Δ, then $\left\lceil \frac{p+\delta}{\Delta+1} \right\rceil \leq tvs(G) \leq p + \Delta - 2\delta + 1$. Ahmad et. al [1, 2, 3] found the total vertex irregularity strength for Jahangir graphs, circulant graphs, convex polytope and wheel related graphs.

In this paper we investigate the total vertex irregularity strength of honeycomb networks and honeycomb derived networks and obtain a bound for the total vertex irregularity strength of these networks.

2 Honeycomb Networks

A unit honeycomb network is a hexagon denoted by $HC(1)$. Honeycomb network of size 2 denoted by $HC(2)$ can be obtained by adding six hexagons around the boundary edges of $HC(1)$. Inductively honeycomb network $HC(r)$ can be obtained from $HC(r-1)$ by adding a layer of hexagons around the boundary edges of $HC(r-1)$.

The number of vertices and edges of $HC(r)$ are $6r^2$ and $9r^2 - 3r$ respectively. Cellular phone station placement, the representation of benzenoid hydrocarbons, computer graphics and image processing are mainly based

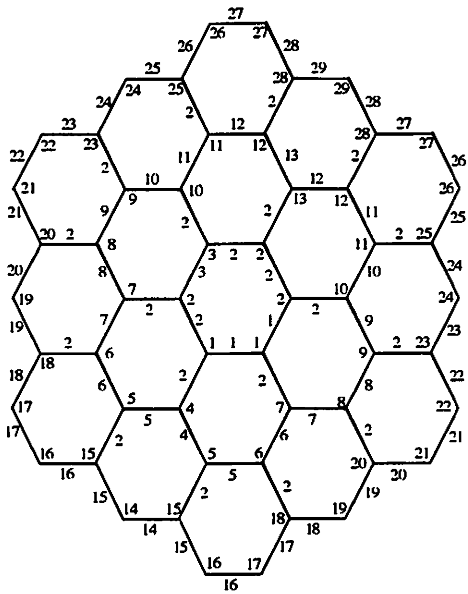

Figure 2: $tvs(H(3)) \leq 29$

on hexagonal tesselations. Honeycomb architectures begin with hexagonal tessellations but use cells (instead of vertices) as processors [7].

Theorem 1 $tvs(HC(r)) \leq 3r^2 + r - 1$, $r > 1$.

Procedure Upper Bound for $tvs(HC(r))$
Input: $HC(r), r > 1$
Algorithm:
Case 1: r **even**

Step 1 Vertices and edges of $HC(1)$ in the anticlockwise direction starting from $v_1$receive label as shown in Figure2.

Having labeled the vertices and edges of $HC(r-1)$, the vertices and edges of $HC(r), r > 1$ are labeled inductively as follows.

Step 2 Consecutive vertices of $HC(r)$ in the outer cycle receive labels using consecutive integers from $3(r-1)^2 + r - 1$ to $3r^2 + r - 1$ in the anticlockwise direction, starting with the vertex adjacent to the first vertex labeled in $HC(r-1)$.

Step 3 Remaining adjacent vertices of $HC(r)$ in the same cycle in the anticlockwise direction receive labels using integers from $3r^2+r-2$ to $3(r-1)^2+r$ in the descending order.

Step 4 Consecutive edges of $HC(r)$ in the outer cycle in the anticlockwise direction receive labels in the same way starting with the edge incident at the vertex labeled first in $HC(r-1)$.

Step 5 All other edges receive label 2.

Case 2: r odd

Step 1 Vertices and edges of $HC(1)$ in the anticlockwise direction starting from $v_1$receive labels as shown in Figure 2.

Having labeled the vertices and edges of $HC(r-1)$, the vertices and edges of $HC(r)$, $r>1$ are labeled inductively as follows.

Step 2 Consecutive vertices of $HC(r)$ in the outer cycle in the anticlockwise direction receive labels using consecutive integers from $3(r-1)^2+r-1$ to $3r^2+r-1$ starting with the vertex diagonally opposite to the first vertex labeled in $HC(r-1)$.

Step 3 Remaining consecutive vertices of $HC(r)$ in the same cycle in the anticlockwise direction receive labels using integers from $3r^2+r-2$ to $3(r-1)^2+r$ in the descending order.

Step 4 Consecutive edges of $HC(r)$ in the outer cycle in the anticlockwise direction receive labels in the same way starting with the edge incident at the vertex labeled first in $HC(r)$.

Step 5 All other edges receive label 2. See Figure 2.

Output: $tvs(HC(r)) \leq 3r^2+r-1, r>1$.

End Procedure Upper Bound for $tvs(HC(r))$.

Proof. We prove that the weights of the vertices of $HC(r)$ are distinct using induction method. The weights of the vertices of $HC(1)$ constitute the set $\{5,6,7,8,9,10,14,15,16,17,19,22,23,24,25,26,28,31,32,33,34,35,37,40\}$. Now we assume that the weights of the vertices of $HC(r-1)$ are distinct.We claim that the weights of the vertices of $HC(r)$ are distinct. The weights of the vertices of $HC(r)$ adjacent to the vertices of $HC(r-1)$

are obtained using 4 integers and each integer is greater than the labels of the corresponding vertex of $HC(r-1)$ and hence all the weights of $HC(r)$ are distinct. The weights are obtained using the labels from the set $\{1, 2, 3, ..., 3r^2 + r - 1\}$. Hence $tvs(HC(r)) \leq 3r^2 + r - 1$, $r > 1$.

This concludes the proof. □

3 Honeycomb Derived Networks

In this section we determine an upper bound for the total vertex irregularity strength of honeycomb derived networks.

A honeycomb derived network [5] is obtained from $HC(r)$ by joining pairs of vertices in each hexagon which are diametrically opposite to each other and is denoted by $HC_1(r)$.

Theorem 2 $tvs(HC_1(r)) \leq 3r^2 + r - 1$, $r > 1$.

Procedure Upper Bound for $tvs(HC_1(r))$, $r > 1$
Input: $HC_1(r)$, $r > 1$
Algorithm: Upper Bound for $tvs(HC_1(r))$, $r > 1$

Step 1 Vertices and edges of $HC_1(1)$ in the anticlockwise direction starting from v_1 receive labels as shown in Figure 3.

Having labeled the vertices and edges of $HC_1(r-1)$, the vertices and edges of $HC_1(r)$, $r > 1$ receive labels inductively as follows.

Step 2 Consecutive vertices of $HC_1(r)$ in the outer cycle receive labels using consecutive integers from $3(r-1)^2 + r - 1$ to $3r^2 + r - 1$ in the anticlockwise direction, starting with the vertex adjacent to the first vertex labeled in $HC_1(r)$.

Step 3 Remaining adjacent vertices of $HC_1(r)$ in the same cycle receive labels in the anticlockwise direction using integers from $3r^2 + r - 2$ to $3(r-1)^2 + r$ in the descending order.

Step 4 Consecutive edges of $HC_1(r)$ in the outer cycle in the anticlockwise direction receive labels in the same way starting with the edge incident at the vertex labeled first in $HC_1(r)$.

Step 5 All other edges receive label 1. See Figure 3.

Output: $tvs(HC_1(r)) \leq 3r^2 + r - 1$, $r > 1$

End Procedure Upper Bound for $tvs(HC_1(r))$.

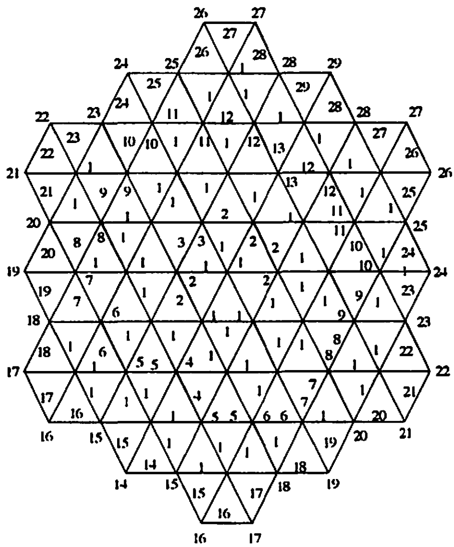

Figure 3: $tvs(HC_1(3)) \leq 29$

Proof. We prove that the weights of the vertices of $HC_1(r)$ are distinct using induction method. The weights of the vertices of $HC_1(1)$ constitute the set $\{4,5,6,7,8,9\}$ and the weights of the vertices of $HC_1(2)$ constitute the set $\{7,8,9,10,11,12,15,16,17,18,21,23,24,25,26,27,29,32,33,34,35,36,38,41\}$. Now we assume that the weights of the vertices of $HC_1(r-1)$ are distinct. We claim that the weights of the vertices of $HC_1(r)$ are distinct. The weights of the vertices of $HC_1(r)$ adjacent to the vertices of $HC_1(r-1)$ are obtained using 4 integers and each integer is greater than the labels of the corresponding vertex of $HC_1(r-1)$ and hence all the weights of $HC_1(r)$ are distinct. The weights are obtained using the labels from the set $\{1,2,3,...,3r^2+r-1\}$. Hence $tvs(HC_1(r)) \leq 3r^2+r-1$, $r > 1$.This concludes the proof. □

4 Conclusion

In this paper we have obtained a bound for the total vertex irregularity strength of honeycomb and honeycomb derived networks. Total vertex irregular k-labeling for networks like hexagonal network, butterfly network and benes network is under investigation.

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