Equitable Total Domination Edge Addition Stable Graphs

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Abstract

All graphs considered in our study are simple, finite and undirected. A graph is equitable total domination edge addition critical (stable) if the addition of any arbitrary edge changes (does not change) the equitable total domination number. In this paper, we introduce the following new parameters: equitable independent domination number, equitable total domination number and equitable connected domination number and study their stability upon edge addition, on special families of graphs namely cycles, paths and complete bipartite graphs. Also the relation among the above parameters is established.
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1 Introduction

Let $G = (V, E)$ be a simple, finite, undirected and connected graph. For graph theoretic terminology, we refer to Harary [4].

Domination is a rapidly developing area of research in graph theory. The concept of domination has existed for a long time and early discussions on the topic can be found in works of Ore [6]. An excellent treatment of domination is given in the book by Haynes et al. [5]. New concepts of domination emerge from practical considerations. In order to study this practical concept, Prof. E. Sampathkumar introduced various types of equitability in graphs [7].

The open neighborhood of $v \in V$ is denoted and defined by $N(v) = \{u \in V : uv \in E\}$ and the closed neighborhood of $v \in V$ is $N[v] = N(v) \cup \{v\}$. The degree of a vertex $v \in V$ is $deg(v) = |N(v)|$. A vertex of degree zero in G is called an isolated vertex and a vertex of degree one is a pendant vertex or a leaf of G. The vertex which is adjacent to a pendant vertex is called a support vertex. For any set $S \subseteq V$, the induced subgraph $< S >$ is the maximal subgraph of G with vertex set S. Thus two vertices of S are adjacent in $< S >$ if and only if they are adjacent in G. A vertex $u \in V$ dominates a vertex $v \in V$ if $uv \in E$. A vertex $v \in V$ dominates a set $S \subseteq V$ if v dominates every vertex in S.

In this study, we obtain equitable independent domination number, equitable total domination number and equitable connected domination number of graphs of some interesting graph classes and also analyze the concepts of edge addition stability of the above graph parameters on special families of graphs namely cycles, paths and complete bipartite graphs.

We give below a brief summary of definitions which are useful for the present investigations.

Definition 1. *[5] Let $G = (V, E)$ be a graph. A subset D of V is called a dominating set of G if every vertex in $V - D$ is adjacent to at least one vertex in D. A dominating set D is called a minimal dominating set if no proper subset of D is a dominating set. The domination number $\gamma(G)$ of a graph G is the minimum cardinality of a minimal dominating set in G.*

Definition 2. *[2] A dominating set D of a graph $G = (V, E)$ is a total dominating set if the induced subgraph $< D >$ has no isolated vertices. The total domination number $\gamma_t(G)$ of a graph G is the minimum cardinality of a minimal total dominating set in G.*

Definition 3. *[1] A dominating set D of a graph $G = (V, E)$ is an independent dominating set if the induced subgraph $< D >$ has no edges. The independent domination number $\gamma_i(G)$ of a graph G is the minimum cardinality of a minimal independent dominating set in G.*

Definition 4. *[7] A dominating set D of a graph $G = (V, E)$ is a connected dominating set if the induced subgraph $< D >$ is connected. The connected domination number $\gamma_c(G)$ of a graph G is the minimum cardinality of a minimal connected dominating set in G.*

Definition 5. *[7] A dominating set D in G is said to be an equitable dominating set if it possesses the 'equitable property', i.e., for every vertex $v \in V - D$ there exists an adjacent vertex $u \in D$ such that $|d(u) - d(v)| \leq 1$, where $d(u)$ denotes the degree of u. The minimum cardinality of a minimal equitable dominating set of G is the equitable domination number of G, denoted by $\gamma_e(G)$.*

Definition 6. *[3] Let P be a graph parameter. A graph G is said to be P-edge addition critical if $P(G+e) \neq P(G)$ for every edge e of $\overline{G}$, the complement of G. A graph G is said to be P-edge addition stable if $P(G+e) = P(G)$ for every edge e of $\overline{G}$.*

2 Equitable Total Domination Upon Edge Addition Stable

In this section, we extend the concept of total domination [2], to equitable total domination $\gamma_{et}(G)$ and we investigate 'equitable total domination edge addition stable' property for various classes of graphs like cycles, paths and complete bipartite graphs.

Definition 7. *A total dominating set D of a graph $G = (V, E)$ is called an equitable total dominating set if it also possesses the equitable property. The minimum cardinality of a minimal equitable total dominating set of G is the equitable total domination number of G, denoted by $\gamma_{et}(\overline{G})$.*

Remark 8. *Since any equitable total dominating set is also a total dominating set, $\gamma_t(G) \leq \gamma_{et}(G)$ for any graph G.*

Theorem 9. *If G is regular or $(k, k+1)$ bi-regular, for some k, then $\gamma_{et}(G) = \gamma_t(G)$.*

Proof. *Suppose G is a regular or bi-regular graph. Then every vertex of G has degree either k or $k+1$. Let D be a minimum total dominating set of G. Then $|D| = \gamma_t(G)$. Let $u \in V - D$. Then as D is a total dominating set, there exists $v \in D$ and $uv \in E(G)$. Also $deg(u) = deg(v) = k$. Therefore $|deg(u) - deg(v)| = 0$ or $1 \leq 1$. Therefore D is a equitable total dominating set of G, so that $\gamma_{et}(G) \leq |D| = \gamma_t(G)$. But $\gamma_t(G) \leq \gamma_{et}(G)$. Therefore $\gamma_{et}(G) = \gamma_t(G)$.*

In the following proposition, equitable total domination number γ_{et} for various classes of graphs are given.

Proposition 10. *1. For the cycle C_n of order $n \geq 3$,*

$$\gamma_{et}(C_n) = \begin{cases} \lceil n/2 \rceil + 1 & \text{when } n = 4k+2, k \geq 1 \\ \lceil n/2 \rceil & \text{otherwise.} \end{cases}$$

2. For the path P_n of order $n \geq 2$,

$$\gamma_{et}(P_n) = \begin{cases} \lceil n/2 \rceil + 1 & \text{when } n = 4k+2, k \geq 1 \\ \lceil n/2 \rceil & \text{otherwise.} \end{cases}$$

3. For the wheel graph W_n of order $n \geq 4$,

$$\gamma_{et}(W_{1,n}) = \begin{cases} \lceil n/3 \rceil & \text{when } n = 4 \\ \lceil n/3 \rceil + 1 & \text{otherwise.} \end{cases}$$

4. For the complete graph K_n of order $n \geq 2$, $\gamma_{et}(K_n) = 2$.

5. For the star graph $K_{1,n}$ of order $n \geq 3$, $\gamma_{et}(K_{1,n}) = n + 1$.

6. For the complete bipartite graph $K_{m,n}$, $m, n \geq 2$, $\gamma_{et}(K_{m,n}) = 2$, when $m = n$ or $m = n + 1$.

Definition 11. *A graph G is said to be an equitable total domination edge addition stable or in short γ_{et}^{+}-stable, if addition of any edge to G does not change the equitable total domination number. In other words, a graph G is γ_{et}^{+}-stable if $\gamma_{et}(G + e) = \gamma_{et}(G)$ for every edge $e \in E(\overline{G})$.*

Theorem 12. *Cycle graphs C_n are γ_{et}^{+}-stable if $n = 4k$, $k \geq 1$.*

Proof. *Let $C_n : v_1, v_2, \ldots, v_n$ be a cycle of length $n \geq 4$. An equitable total dominating set of C_n can be obtained by taking $D = \{v_{4i+1}, v_{4i+2} / i = 0, 1, 2, \ldots, k-1\}$. Adding an edge between non-adjacent vertices v_i and v_j in C_n increases the degrees of v_i and v_j by 1. Therefore, the equitability property is not affected up on edge addition and also equitable total dominating set remains the same and hence the equitable total domination number is not changed. Hence C_n is γ_{et}^{+}-stable, when $n = 4k$, $k \geq 1$.*

Corollary 13. *C_n is not γ_{et}^{+}-stable if $n \neq 4k$, $k \geq 1$.*

Proof. *By using the labeling of C_n as in the above theorem, an equitable total dominating set of C_n, $n = 4k + 1$ is given by $D = \{v_{4i+1}, v_{4i+2} / i = 0, 1, 2, \ldots, k - 1\} \cup \{v_n\}$ containing k vertices and $D = \{v_{4i+1}, v_{4i+2} / i = 0, 1, 2, \ldots, k-1\} \cup \{v_{n-1}, v_n\}$, when $n = 4k+2$ or $4k+3$. Up on adding an edge between v_1 and v_{n-1}, the equitable total domination number is reduced by 1. Hence C_n is not γ_{et}^{+}-stable, when $n \neq 4k$, $k \geq 1$.*

Theorem 14. *For $n \geq 5$, path graphs P_n are not γ_{et}^{+}-stable.*

Proof. *Let $P_n : v_1, v_2, \ldots, v_n$ be the path graph of order $n \geq 5$. Addition of an edge between the support vertices v_2 and v_{n-1} forces us to include both v_1 and v_n into the equitable total dominating set in order to maintain equitability. Hence adding an edge between support vertices increases the equitable total domination number implying that P_n, $n \geq 5$ are not γ_{et}^{+}-stable.*

Note. The path graphs P_3 and P_4 are γ_{et}^{+}-stable.

Proposition 15. *For $n \geq 5$, $\gamma_{et}(P_n + e) = \gamma_{et}(P_n)$ if an edge e is added between two pendant vertices or pendant and support vertices.*

Proposition 16. *For $n \geq 5$,*

$$\gamma_{et}(P_n + e) = \begin{cases} \gamma_{et}(P_n) & \text{when } n = 4k+2 \\ \gamma_{et}(P_n)+1 & \text{when } n = 4k+1 \text{ or } 4k+3 \\ \gamma_{et}(P_n)+2 & \text{when } n = 4k \end{cases}$$

if an edge e is added between two support vertices.

Theorem 17. *For $n \geq 2$, complete bipartite graphs $K_{n,n}$ are γ_{et}^{+}-stable.*

Proof. *Let the vertex partition of $K_{n,n}$, $n \geq 2$ be $V_1 = \{u_1, u_2, \ldots, u_n\}$ and $V_2 = \{v_1, v_2, \ldots, v_n\}$. The equitable total dominating set of $K_{n,n}$ is $D = \{u_1, v_1\}$. Therefore, $\gamma_{et}(K_{n,n}) = 2$. By adding an edge e between u_i and u_j or v_i and v_j, $1 \leq i, j \leq n$ in V_1 or V_2 increases the degrees of these vertices by 1. Therefore, the equitability property is not affected up on edge addition and also equitable total domination number is not changed. Therefore complete bipartite graphs $K_{n,n}$, $n \geq 2$ are γ_{et}^{+}-stable.*

Theorem 18. *For $n \geq 2$, complete bipartite graphs $K_{n+1,n}$ are not γ_{et}^{+}-stable.*

Proof. *Let the vertex partition of $K_{n+1,n}$, $n \geq 2$ be $V_1 = \{u_1, u_2, \ldots, u_{n+1}\}$ and $V_2 = \{v_1, v_2, \ldots, v_n\}$ with $|V_1| = n+1$ and $|V_2| = n$. We see that $\gamma_{et}(K_{n+1,n}) = 2$. If an edge $e = (v_i, v_j)$, $1 \leq i \neq j \leq n$, is added in V_1, then equitable total domination remains the same. When $n \geq 3$, by adding any arbitrary edge between vertices in V_2, the equitable total domination number is increased by one. Therefore the equitable total domination number is changed. Therefore, $K_{n+1,n}$, $n \geq 2$ is not γ_{et}^{+}-stable.*

Result 19. *(i) For $n \geq 2$, $\gamma_{et}(K_{n+1,n} + e) = \gamma_{et}(K_{n+1,n})$, where $|V_1| = n+1$, $|V_2| = n$, $e = (v_i, v_j)$, $1 \leq i \neq j \leq n$ and $v_i, v_j \in V_1$.*

(ii) For $n \geq 2$, $\gamma_{et}(K_{n+1,n} + e) = \gamma_{et}(K_{n+1,n}) + 1$, where $e = (v_i, v_j)$, $1 \leq i \neq j \leq n$ and $v_i, v_j \in V_2$.

3 Equitable Independent Domination Upon Edge Addition Stable

In this section, we extend the notion of independent domination [1], to equitable independent domination $\gamma_{ei}(G)$ and we investigate 'equitable independent domination edge addition stable' property for various classes of graphs like cycles, paths and complete bipartite graphs.

Definition 20. *An independent dominating set D of a graph $G = (V, E)$ is called an equitable independent dominating set if it also possesses the equitable property. The minimum cardinality of a minimal equitable independent dominating set of G is the equitable independent domination number of G, denoted by $\gamma_{ei}(G)$.*

Remark 21. *Since any equitable independent dominating set is also an independent dominating set, $\gamma_i(G) \leq \gamma_{ei}(G)$ for any graph G.*

Theorem 22. *If G is regular or $(k, k+1)$ bi-regular, for some k, then $\gamma_{ei}(G) = \gamma_i(G)$.*

The proof is similar to the one given in Theorem 9.

The equitable independent domination number of various families of graphs are given in the following proposition.

Proposition 23.

1. *For the cycle C_n of order $n \geq 3$, $\gamma_{ei}(C_n) = \lceil n/3 \rceil$.*
2. *For the path P_n of order $n \geq 2$, $\gamma_{ei}(P_n) = \lceil n/3 \rceil$.*
3. *For the complete graph K_n of order $n \geq 2$, $\gamma_{ei}(K_n) = 1$.*
4. *For the complete bipartite graph $K_{m,n}$, $m, n \geq 2$, $\gamma_{ei}(K_{m,n}) = n$, when $m = n$ or $m = n + 1$.*

Definition 24. *A graph G is said to be an equitable independent domination edge addition stable or in short γ_{ei}^{+}-stable, if addition of any edge to G does not change the equitable independent domination number. In other words, a graph G is γ_{ei}^{+}-stable if $\gamma_{ei}(G + e) = \gamma_{ei}(G)$ for every edge $e \in E(\overline{G})$.*

Theorem 25. *Cycle graphs C_n are γ_{ei}^{+}-stable if $n \neq 3k + 1$, $k \geq 1$.*

Proof. *Let $C_n : v_1, v_2, \ldots, v_n$ be a cycle of length $n \geq 4$. Here n takes one of the two forms $3k$ or $3k + 2$, $k \geq 1$ (as $n \neq 3k + 1$). An equitable independent dominating set of C_n can be obtained by taking $D = \{v_{3i-1}/i = 1, 2, \ldots, k\}$ if $n = 3k$ or $D = \{v_{3i+2}/i = 0, 1, 2, \ldots, k\}$ if $n = 3k+2$. Adding an edge between non-adjacent vertices v_i and v_j in C_n increases the degrees of v_i and v_j by 1. Therefore, the equitability property is not affected up on edge addition and also equitable independent dominating set remains the same and hence the equitable independent domination number is not changed. Hence C_n is γ_{ei}^{+}-stable when $n \neq 3k + 1$, $k \geq 1$.*

Corollary 26. *C_n is not γ_{ei}^{+}-stable if $n = 3k + 1$, $k \geq 1$.*

Proof. *By using the labeling of* C_n *as in the above theorem, an equitable independent dominating set of* C_n, $n = 3k+1$ *is given by* $D = \{v_{3i-1}/i = 1, 2, \ldots, k\} \cup \{v_n\}$ *containing* $k+1$ *vertices. Up on adding an edge between* v_2 *and* v_n, *the equitable independent domination number is reduced by 1. Hence* C_n *is not* γ_{ei}^+*-stable, when* $n = 3k+1$, $k \geq 1$.

Theorem 27. *For* $n \geq 5$, *path graphs* P_n *are not* γ_{ei}^+*-stable.*

Proof. *Let* $P_n : v_1, v_2, \ldots, v_n$ *be the path graph of order* $n \geq 5$. *Addition of an edge between the support vertices* v_2 *and* v_{n-1} *forces us to include both* v_1 *and* v_n *into the equitable independent dominating set in order to maintain equitability (except* P_6, *for which addition of such an edge results in graph for which there is no equitable independent dominating set). Hence adding an edge between support vertices increases the equitable independent domination number implying that* P_n, $n \geq 5$ *are not* γ_{ei}^+*-stable.*

Note. The path graphs P_3 and P_4 are γ_{ei}^+-stable.

Proposition 28. *For* $n \geq 5$, $\gamma_{ei}(P_n + e) = \gamma_{ei}(P_n)$ *if an edge* e *is added between two non-support vertices.*

Proposition 29. *For* $n = 3k$, $k > 2$, $\gamma_{ei}(P_n + e) = \gamma_{ei}(P_n) + 2$ *if an edge* e *is added between two support vertices.*

Proposition 30. *For* $n \geq 5$, $n \neq 3k$, $k > 1$, $\gamma_{ei}(P_n + e) = \gamma_{ei}(P_n) + 1$ *if an edge* e *is added between two support vertices.*

Theorem 31. *For* $n \geq 2$, *complete bipartite graphs* $K_{n,n}$ *are not* γ_{ei}^+*-stable.*

Proof. *Let the vertex partition of* $K_{n,n}$, $n \geq 2$ *be* $V_1 = \{u_1, u_2, \ldots, u_n\}$ *and* $V_2 = \{v_1, v_2, \ldots, v_n\}$. *We observe that* $\gamma_{ei}(K_{n,n}) = n = |V_1|$ *or* $|V_2|$. *Without loss of generality let us assume that an edge* e *is added between* u_i *and* u_j, $1 \leq i \neq j \leq n$ *in* V_1. *This reduces the equitable independent domination number by one as it is evident that only one of* u_i *or* u_j *needs to be in the equitable independent dominating set. Therefore complete bipartite graphs* $K_{n,n}$, $n \geq 2$ *are not* γ_{ei}^+*-stable.*

The following proposition is an immediate consequence of the argument in the previous theorem.

Proposition 32. *For* $n \geq 2$, $\gamma_{ei}(K_{n,n} + e) = \gamma_{ei}(K_{n,n}) - 1$, *where* $e \in E(\overline{K_{n,n}})$.

Theorem 33. *For* $n \geq 2$, *complete bipartite graphs* $K_{n+1,n}$ *are not* γ_{ei}^+*-stable.*

Proof. *Let the vertex partition of $K_{n+1,n}$, $n \geq 2$ be $V_1 = \{u_1, u_2, \ldots, u_{n+1}\}$ and $V_2 = \{v_1, v_2, \ldots, v_n\}$ with $|V_1| = n+1$ and $|V_2| = n$. We see that $\gamma_{ei}(K_{n+1,n}) = n = |V_2|$. If an edge $e = (v_i, v_j)$, $1 \leq i \neq j \leq n$, is added to $K_{n+1,n}$, then $\gamma_{ei}(K_{n+1,n} + e) = n - 1$. Therefore, $K_{n+1,n}$, $n \geq 2$ is not γ_{ei}^+-stable.*

Result 34. *(i) For $n \geq 2$, $\gamma_{ei}(K_{n+1,n} + e) = \gamma_{ei}(K_{n+1,n})$, where $|V_1| = n+1$, $|V_2| = n$, $e = (v_i, v_j)$, $1 \leq i \neq j \leq n$ and $v_i, v_j \in V_1$.*

(ii) For $n \geq 2$, $\gamma_{ei}(K_{n+1,n} + e) = \gamma_{ei}(K_{n+1,n}) - 1$, where $e = (v_i, v_j)$, $1 \leq i \neq j \leq n$ and $v_i, v_j \in V_2$.

4 Equitable Connected Domination Upon Edge Addition Stable

In this section, connected domination [7] is extended to equitable connected domination number $\gamma_{ec}(G)$ and we do a similar study for equitable connected domination for various classes of graphs like cycles, paths and complete bipartite graphs.

Definition 35. *A connected dominating set D of a graph $G = (V, E)$ is said to be an equitable connected dominating set if it also possess the equitable property. The minimum cardinality of a minimal equitable connected dominating set of G is the equitable connected domination number of G, denoted by $\gamma_{ec}(G)$.*

Remark 36. *Since any equitable connected dominating set is also a connected dominating set, $\gamma_c(G) \leq \gamma_{ec}(G)$ for any graph G.*

Theorem 37. *If G is regular or $(k, k+1)$ bi-regular, for some k, then $\gamma_{ec}(G) = \gamma_c(G)$.*

The proof is similar to that of Theorem 9.

We list in the following proposition the equitable connected domination number of some classes of graphs.

Proposition 38. *1. For the cycle C_n of order $n \geq 4$, $\gamma_{ec}(C_n) = n - 2$.*

2. For the path P_n of order $n \geq 4$, $\gamma_{ec}(P_n) = n - 2$.

3. For the wheel graph W_n of order $n \geq 4$,

$$\gamma_{ec}(W_{1,n}) = \begin{cases} \lceil n/3 \rceil & \text{when } n = 4 \\ \lceil n/3 \rceil + 1 & \text{otherwise.} \end{cases}$$

4. *For the complete graph K_n of order $n \geq 2$, $\gamma_{ec}(K_n) = 2$.*

5. *For the star graph $K_{1,n}$ of order $n \geq 3$, $\gamma_{ec}(K_{1,n}) = n + 1$.*

6. *For the complete bipartite graphs $K_{m,n}$, $m, n \geq 2$,*

$$\gamma_{ec}(K_{m,n}) = \begin{cases} 2 & m = n \\ n & \text{when } m = n + 1. \end{cases}$$

Definition 39. *A graph G is said to be an equitable connected domination edge addition stable or in short γ_{ec}^{+}-stable, if addition of any edge to G does not change the equitable connected domination number. In other words, a graph G is γ_{ec}^{+}-stable if $\gamma_{ec}(G + e) = \gamma_{ec}(G)$ for every edge $e \in E(\overline{G})$.*

Theorem 40. *For $n \geq 4$, cycle graphs C_n are γ_{ec}^{+}-critical.*

Proof. *Let $C_n : v_1, v_2, \ldots, v_n$ be the cycle graph of order n, $n \geq 4$. An equitable connected dominating set with minimum cardinality can be obtained by taking $D = \{v_1, v_2, \ldots, v_{n-2}\}$ and therefore, $\gamma_{ec}(C_n) = n-2$. By adding an edge between vertices at distance 2, equitable connected domination number is reduced by 1. But adding an edge e between vertices v_i and v_j, $i < j$ at distance k, $3 \leq k \leq \lfloor n/2 \rfloor$, results in a smaller equitable connected dominating set with vertices $\{v_i, v_j, v_{j-1}, v_{j-2}, \ldots, v_{i+3}, v_{j+1}, v_{j+2}, \ldots, v_{i-3}\}$, where the addition in the subscripts are such that, when they exceed n, subtract n; reduces the equitable connected domination number by 2. Therefore cycle graphs are critical.*

Theorem 41. *For $n \geq 5$, path graphs P_n are not γ_{ec}^{+}-stable.*

Proof. *Let $P_n : v_1, v_2, \ldots, v_n$ be the path graph of order $n \geq 5$. The set of inner vertices, namely $\{v_2, v_3, \ldots, v_{n-1}\}$ forms the minimal equitable connected dominating set. If an edge is added between v_1 and a center vertex, the equitable connected domination number is decreased by one. Therefore, path graphs are not γ_{ec}^{+}-stable.*

Proposition 42. *(i) For $n \geq 6$, $\gamma_{ec}(P_n + e) = \gamma_{ec}(P_n)$ if an edge e is added between two non-center vertices.*

(ii) For $n \geq 8$, $\gamma_{ec}(P_n + e) = \gamma_{ec}(P_n) - 1$ if an edge e is added between v_i and v_j and either v_i or v_j is a center vertex.

Theorem 43. *For $n \geq 3$, complete bipartite graphs $K_{n,n}$ are γ_{ec}^{+}-stable.*

Proof. *Let $K_{n,n}$ be a complete bipartite graph with $n \geq 3$ and let the vertex partition be $V = V_1 \cup V_2$, where $V_1 = \{u_1, u_2, \ldots, u_n\}$ and $V_2 = \{v_1, v_2, \ldots, v_n\}$. An equitable connected dominating set is $D = \{u_1, v_1\}$ and hence $\gamma_{ec}(K_{n,n}) = 2$. By adding any arbitrary edge between vertices in V_1 or V_2, the equitable connected domination number is not affected. Therefore, complete bipartite graphs $K_{n,n}$ are γ_{ec}^{+}-stable, for $n \geq 3$.*

Note. $K_{2,2}$ is not γ_{ec}^{+}-stable.

Theorem 44. *For $n \geq 2$, complete bipartite graphs $K_{n+1,n}$ are not γ_{ec}^{+}-stable.*

Proof. *Let $K_{n+1,n}$ be a complete bipartite graph with $n \geq 2$ and let the vertex partition be $V = V_1 \cup V_2$, where $V_1 = \{u_1, u_2, \ldots, u_{n+1}\}$ and $V_2 = \{v_1, v_2, \ldots, v_n\}$. An equitable connected dominating set is $D = \{u_1, v_1\}$ and hence $\gamma_{ec}(K_{n+1,n}) = 2$. When $n \geq 3$, by adding any arbitrary edge between vertices in V_2, the equitable connected domination number is increased by one. When $n = 2$, $\gamma_{ec}(K_{n+1,n} + e) = 4$, where $e = (v_1, v_2)$. Therefore, complete bipartite graphs $K_{n+1,n}$ are not γ_{ec}^{+}-stable, for $n \geq 2$.*

Proposition 45. *For $n \geq 2$, $\gamma_{ec}(K_{n+1,n} + e) = \gamma_{ec}(K_{n+1,n})$ if an edge e is added between vertices in V_1.*

Result 46. *For any graph G of order $n \geq 4$, $\gamma_{ei}(G) \leq \gamma_{et}(G) \leq \gamma_{ec}(G)$.*

Conclusion

In our future work, we propose to analyse edge addition stability for other families of graphs. This study can further be extended by identifying edge addition stability of graph families for which these parameters are equal to other kinds of domination upon edge addition stability.

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