Solving Symmetric Fully Fuzzy Linear Systems With Trapezoidal Fuzzy Number Matrices.

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ABSTRACT

In this paper, a n x n fully fuzzy linear system is solved by decomposition of positive definite symmetric coefficient matrix with trapezoidal fuzzy number matrices using Cholesky and LDLT decomposition methods. The methods are illustrated with a numerical example.

1.Introduction

Linear system of equation has applications in many areas of science, engineering, finance and economics. Fuzzy linear system whose coefficient matrix is crisp and right hand side column is an arbitrary fuzzy number was first proposed by Friedman et al.[4].Several algorithms for solving fuzzy linear systems have been introduced by many authors [1,7].

A linear system is called a fully fuzzy linear system (FFLS) if all coefficients in the system are all fuzzy numbers. Decomposition methods were used to solve FFLS [2,3,8,9,10,11].Nasseri et al [12] used linear system with trapezoidal fuzzy numbers.

Positive definite symmetric matrices are of both theoretical and computational importance in a wide variety of applications. They are used, for example, in linear least squares regression ,adjacency matrix in graph theory, finite difference methods in partial differential equations , the conductance matrix in electric circuits , Monte Carlo methods for simulating systems with multiple correlated variables.

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In this paper a n x nFFLS $\tilde{A} \otimes \tilde{x} = \tilde{b}$ where $\tilde{A}$ is a trapezoidal fuzzy matrix,$\tilde{x}$ and $\tilde{b}$ are trapezoidal fuzzy vectors is solved by decomposition of symmetric positive definite coefficient matrix using Cholesky decomposition method and LDLT decomposition method. This paper is organized as follows. Some basic definitions and results on fuzzy sets and trapezoidal fuzzy numbers are given in section 2.In section 3, Cholesky decomposition method and LDLT decomposition method to solve FFLS with trapezoidal fuzzynumbers matrices is given. Illustration with numerical examples is given in section 4. Section 5 ends this paper with conclusion.

2. Preliminaries.

Definition 2.1. A fuzzy subset $\tilde{A}$ of R is defined by its membership function by its membership function $\mu_{\tilde{A}}: R \rightarrow [0,1]$which assigns a real number $\mu_{\tilde{A}}$in the interval [0,1] to each element $x \in R$ where the value of $\mu_{\tilde{A}}$shows grade membership of x in $\tilde{A}$.

Definition 2.2. A trapezoidal fuzzy number denoted by $\tilde{A}=(m,n,\alpha,\beta)$has the membership function .

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & x \leq \alpha \\ \dfrac{x-\alpha}{m-\alpha} & \alpha \leq x \leq m \\ 1 & m \leq x \leq n \\ \dfrac{\beta - x}{\beta - n} & n \leq x \leq \beta \\ 0 & x \geq \beta \end{cases}$$

Definition 2.3. A fuzzy number $\tilde{A}$ is called positive (negative) denoted by $\tilde{A}>0$ ($\tilde{A}<0$) if its membership function $\mu_{\tilde{A}}$(x) satisfies $\mu_{\tilde{A}}= 0$, $\forall x \leq 0 (\forall x \geq 0)$ using its mean value, left and right spreads such a fuzzy number $\tilde{A}$ is symbolically written as $\tilde{A} =(m,n,\alpha,\beta)$ is positive if and only if $m-\alpha \geq 0$ and $n-\beta \geq 0$.

Definition 2.4. A Trapezoidal fuzzy number $\tilde{A} = (m,n,\alpha,\beta)$ is said to be zero trapezoidal fuzzy number if and only if
m= 0, n =0, α =0, β=0.

Definition 2.5. Two fuzzy numbers $M = (m,n,\alpha,\beta)$andN $=(x,y,\gamma,\delta)$are equal if and only if $m = x, n = y, \alpha = \gamma, \beta = \delta$.

Definition 2.6. For two fuzzy numbers $M = (m,n,\alpha,\beta)$and N $=(x,y,\gamma,\delta)$ the operations extended addition, extended opposite and extended multiplication are

$$(m,n,\alpha,\beta) \oplus (x,y,\gamma,\delta) = (m+x, n+y, \alpha+\gamma, \beta+\delta)$$
$$-M = -(m,n,\alpha,\beta) = (-m,-n,\beta,\alpha)$$

If M>0, N>0 then $(m,n,\alpha,\beta) \otimes (x,y,\gamma,\delta) = (mx, ny, m\gamma + x\alpha, n\delta + y\beta)$.....*(1)*

For scalar multiplication

$$\lambda \otimes (m,n,\alpha,\beta) = \begin{cases} \lambda m, \lambda n, \lambda\alpha, \lambda\beta & \lambda \geq 0 \\ \lambda m, \lambda n, -\lambda\alpha, -\lambda\beta & \lambda < 0 \end{cases}$$

Definition 2.7. A matrix $\tilde{A} = (\tilde{a}_{ij})$ is called a fuzzy matrix if each element of $\tilde{A}$ is a fuzzy number. A fuzzy matrix $\tilde{A}$ is positive denoted by $\tilde{A}>0$ if each element of $\tilde{A}$ is positive. Fuzzy matrix $\tilde{A} = (\tilde{a}_{ij})$ which is n x n matrix can be represented such that$\tilde{a}_{ij} = (a_{ij}, b_{ij}, m_{ij}, n_{ij})$ where $\tilde{A} = (A,B,M,N)$ where $A = (a_{ij})$ $B = (b_{ij})$ $M = (m_{ij})$ $N = (n_{ij})$are $n \times n$ crisp matrices.

Definition 2.8. A square matrix $\tilde{A} = (\tilde{a}_{ij})$ is symmetric matrix if $\tilde{a}_{ij} = \tilde{a}_{ji}$, $\forall$ i,j

Definition 2.9. A square matrix $\tilde{A} = (\tilde{a}_{ij})$is upper triangular matrix if $\tilde{a}_{ij} = \tilde{0} = (0,0,0,0)$ $\forall$ i>j and lower triangular matrix if $\tilde{a}_{ij} = \tilde{0} = (0,0,0,0)$ $\forall$ i<j.

Definition 2.10. A symmetric matrix $\tilde{A}$ is positive definite if $\tilde{x}^T \tilde{A} \tilde{x} > 0$ for all nonzero $\tilde{x}$.

Definition 2.11. Consider n x n fuzzy linear system of equations

$$(\tilde{a}_{11} \otimes \tilde{x}_1) \oplus (\tilde{a}_{12} \otimes \tilde{x}_2) \oplus \dots\dots\dots \oplus (\tilde{a}_{1n} \otimes \tilde{x}_n) = \tilde{b}_1$$
$$(\tilde{a}_{21} \otimes \tilde{x}_1) \oplus (\tilde{a}_{22} \otimes \tilde{x}_2) \oplus \dots\dots\dots \oplus (\tilde{a}_{2n} \otimes \tilde{x}_n) = \tilde{b}_2$$
$$\dots\dots\dots\dots$$
$$(\tilde{a}_{n1} \otimes \tilde{x}_1) \oplus (\tilde{a}_{n2} \otimes \tilde{x}_2) \oplus \dots\dots. \oplus (\tilde{a}_{nn} \otimes \tilde{x}_n) = \tilde{b}_n$$

The matrix of the above equation is $\tilde{A} \otimes \tilde{x} = \tilde{b}$ where coefficient matrix $\tilde{A}=(\tilde{a}_{ij})$ Where $1 \leq i, j \leq n$ is a n x n fuzzy matrix and $\tilde{x}_j, \tilde{b}_j \in F(R)$. This system is called Fully fuzzy linear system. (FFLS).

Definition 2.12. For solving n x n FFLS $\tilde{A} \otimes \tilde{x} = \tilde{b}$ where $\tilde{A} = (A,B,M,N)$, $\tilde{x} = (x, y, z, w)$ and $\tilde{b} = (b,g,h,k)$

$$(A,B,M,N) \otimes (x,y,z,w) = (b,g,h,k)$$
$$(Ax,By,Az+Mx,Bw+Ny) = (b,g,h,k)$$
$$Ax = b$$
$$By = g$$
$$Az + Mx = h$$
$$Bw + Ny = k$$

3. Methods for solving symmetric positive definite FFLS using trapezoidal fuzzy number matrices.

3.1 Cholesky decomposition method.

A symmetric positive definite matrix A can be decomposed into LL^T where L is a lower triangular matrix and L^T is a upper triangular matrix .

Using the Cholesky decomposition method $A = LL^T$

$$\begin{pmatrix} a_{11} & a_{21} & . & a_{n1} \\ a_{21} & a_{22} & . & a_{n2} \\ . & . & . & . \\ a_{n1} & a_{n2} & . & a_{nn} \end{pmatrix} = \begin{pmatrix} l_{11} & 0 & 0 & 0 \\ l_{21} & l_{22} & 0 & 0 \\ . & . & . & . \\ l_{n1} & l_{n2} & . & l_{nn} \end{pmatrix} \begin{pmatrix} l_{11} & l_{21} & . & l_{n1} \\ 0 & l_{22} & . & l_{n2} \\ . & . & . & . \\ 0 & 0 & 0 & l_{nn} \end{pmatrix}$$

Entries of L can be calculated using

$$l_{jj} = \sqrt{a_{jj} - \sum_{k=1}^{j-1} l_{jk}^2}$$

$$l_{ij} = \frac{1}{l_{jj}} \left[a_{jj} - \sum_{k=1}^{j-1} l_{ik} l_{jk} \right] \text{ for } i > j$$

3.2 Solving FFLS by Cholesky decomposition method.

Let $\tilde{A}$ = (A,B,M,N) where A and B are symmetric positive definite crisp matrices.

$L \otimes U = A$ Where $L=(L_1, L_2, 0, 0)$ and $U=(L_1^T, L_2^T, U_1, U_2)$

$(L_1, L_2, 0, 0) \otimes (L_1^T, L_2^T, U_1, U_2) = (A,B,M,N)$

$(L_1L_1^T, L_2L_2^T, L_1U_1, L_2U_2) = (A,B,M,N)$

$(A,B,M,N) \otimes (x,y,z,w) = (b,g,h,k)$ becomes

$(L_1L_1^T, L_2L_2^T, L_1U_1, L_2U_2) \otimes (x,y,z,w) = (b,g,h,k)$

Using *(1)* $(L_1L_1^Tx, L_2L_2^Ty, L_1L_1^Tz + L_1U_1x, L_2L_2^Tw + L_2U_2y) = (b,g,h,k)$

$L_1L_1^Tx = b$

$L_2L_2^Ty = g$

$L_1L_1^Tz + L_1U_1x = h$

$L_2L_2^Tw + L_2U_2y = k$

we get the solution as

$$x = L_1^{T^{-1}} L_1^{-1}\, b$$

$$y = L_2^{T^{-1}} L_2^{-1}\, g$$

$$z = L_1^{T^{-1}} L_1^{-1}\, (h - L_1U_1x)$$

$$w = L_2^{T^{-1}} L_2^{-1}\, (k - L_2U_2y)$$

3.3 Algorithm

1. Compute $A = L_1L_1^T$
2. Compute $B = L_2L_2^T$
3. Compute $x = L_1^{T^{-1}} L_1^{-1}\, b$
4. Compute $y = L_2^{T^{-1}} L_2^{-1}\, g$
5. Compute $z = L_1^{T^{-1}} L_1^{-1}\, (h - L_1U_1x)$
6. Compute $w = L_2^{T^{-1}} L_2^{-1}\, (k - L_2U_2y)$

3.4 LDL^T decomposition method

A symmetric matrix A can be decomposed into LDL^T where L is a lower triangular matrix , D is a diagonal matrix and L^T is a upper triangular matrix. Using the method $A = LDL^T$

$$\begin{pmatrix} a_{11} & a_{21} & . & a_{n1} \\ a_{21} & a_{22} & . & a_{n2} \\ . & . & . & . \\ a_{n1} & a_{n2} & . & a_{nn} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ l_{21} & 1 & 0 & 0 \\ . & . & . & . \\ l_{n1} & l_{n2} & . & 1 \end{pmatrix} \begin{pmatrix} d_{11} & 0 & 0 & 0 \\ 0 & d_{22} & 0 & 0 \\ . & . & . & . \\ 0 & 0 & 0 & d_{nn} \end{pmatrix} \begin{pmatrix} 1 & l_{21} & . & l_{n1} \\ 0 & 1 & . & l_{n2} \\ . & . & . & . \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$d_{jj} = a_{jj} - \sum_{k=1}^{j-1} l_{jk}^2 . d_{kk}$$

$$l_{ij} = \frac{1}{d_{jj}} \left[a_{jj} - \sum_{k=1}^{j-1} l_{ik} l_{jk} \, d_{kk} \right] \text{for } i > j$$

3.5 Solving FFLS by LDL^T decomposition.

Consider the FFLS $\tilde{A} \otimes \tilde{x} = \tilde{b}$ where $\tilde{A}$ is symmetric matrix.

Taking trapezoidal fuzzy number matrices

$$(A, B, M, N) \otimes (x, y, z, w) = (b, g, h, k)$$

Let $A = L_1 D_1 L_1^T$ and $B = L_2 D_2 L_2^T$

$$(L_1 D_1 L_1^T, L_2 D_2 L_2^T, M, N) \otimes (x, y, z, w) = (b, g, h, k)$$

$$(L_1 D_1 L_1^T x, L_2 D_2 L_2^T y, L_1 D_1 L_1^T z + Mx, L_2 D_2 L_2^T w + Ny) = (b, g, h, k)$$

$$L_1 D_1 L_1^T x = b \Rightarrow x = L_1^{T^{-1}} D_1^{-1} L_1^{-1} b$$

$$L_2 D_2 L_2^T y = g \Rightarrow y = L_2^{T^{-1}} D_2^{-1} L_2^{-1} g$$

$$L_1 D_1 L_1^T z + Mx = h \Rightarrow z = L_1^{T^{-1}} D_1^{-1} L_1^{-1} (h - Mx)$$

$$L_2 D_2 L_2^T w + Ny = k \Rightarrow w = L_2^{T^{-1}} D_2^{-1} L_2^{-1} (k - Ny)$$

3.6 Algorithm

1. Compute $A = L_1 D_1 L_1^T$

2. $x = L_1^{T^{-1}} D_1^{-1} L_1^{-1} b$

3. Compute $B = L_2 D_2 L_2^T$

4. $y = L_2^{T^{-1}} D_2^{-1} L_2^{-1} g$

5. $z = L_1^{T^{-1}} D_1^{-1} L_1^{-1} (h - Mx)$

6. $w = L_2^{T^{-1}} D_2^{-1} L_2^{-1} (\mathrm{k} - Ny)$

4. Numerical examples

In this section the algorithmsare applied for solving FFLS with fuzzy trapezoidal number matrices.

Example(4.1) Solve by Cholesky decomposition method

$$\begin{bmatrix} (1,4,4,4) & (2,6,6,6) & (3,6,1,2) & (2,4,3,6) \\ (2,6,6,6) & (8,18,4,3) & (8,12,4,3) & (6,9,4,6) \\ (3,6,1,2) & (8,12,4,3) & (14,11,4,2) & (11,9,4,8) \\ (2,4,3,6) & (6,9,4,6) & (11,9,4,8) & (10,13,2,3) \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} (10,36,172,160) \\ (20,42,216,186) \\ (30,40,56,100) \\ (40,36,82,82) \end{bmatrix}$$

Solution:

$$A = \begin{pmatrix} 1 & 2 & 3 & 2 \\ 2 & 8 & 8 & 6 \\ 3 & 8 & 14 & 11 \\ 2 & 6 & 11 & 10 \end{pmatrix} = L_1 L_1^T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ 3 & 1 & 2 & 0 \\ 2 & 1 & 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B = \begin{pmatrix} 4 & 6 & 6 & 4 \\ 6 & 18 & 12 & 9 \\ 6 & 12 & 11 & 9 \\ 4 & 9 & 9 & 13 \end{pmatrix} = L_2 L_2^T = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 3 & 3 & 0 & 0 \\ 3 & 1 & 1 & 0 \\ 2 & 1 & 2 & 2 \end{pmatrix} \begin{pmatrix} 2 & 3 & 3 & 2 \\ 0 & 3 & 1 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

$$x = L_1^{T^{-1}} L_1^{-1} b =$$
$$\begin{pmatrix} 1 & -1 & -1 & 1 \\ 0 & 1/2 & -1/4 & 0 \\ 0 & 0 & 1/2 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ -1 & 1/2 & 0 & 0 \\ -1 & -1/4 & 1/2 & 0 \\ 1 & 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} 10 \\ 20 \\ 30 \\ 40 \end{pmatrix} = \begin{pmatrix} 30 \\ 0 \\ -20 \\ 20 \end{pmatrix}$$
$$y = L_2^{T^{-1}} L_2^{-1} g$$
$$= \begin{pmatrix} 1/2 & -1/2 & -1 & 3/4 \\ 0 & 1/3 & -1/3 & 1/6 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1/2 \end{pmatrix} \begin{pmatrix} 1/2 & 0 & 0 & 0 \\ -1/2 & 1/3 & 0 & 0 \\ -1 & -1/3 & 1 & 0 \\ 3/4 & 1/6 & -1 & 1/2 \end{pmatrix} \begin{pmatrix} 36 \\ 42 \\ 40 \\ 36 \end{pmatrix}$$
$$= \begin{pmatrix} 30 \\ 4 \\ -22 \\ 6 \end{pmatrix}$$
$$z = L_1^{T^{-1}} L_1^{-1}(h - L_1 U_1 x) = \begin{pmatrix} 32 \\ 5 \\ -22 \\ 18 \end{pmatrix} \quad , \quad w = L_2^{T^{-1}} L_2^{-1}(k - L_2 U_2 y) =$$
$$\begin{pmatrix} 28 \\ 4 \\ -24 \\ 8 \end{pmatrix}$$
$$x = \begin{pmatrix} 30 \\ 0 \\ -20 \\ 20 \end{pmatrix}, y = \begin{pmatrix} 30 \\ 4 \\ -22 \\ 6 \end{pmatrix}, z = \begin{pmatrix} 32 \\ 5 \\ -22 \\ 18 \end{pmatrix}, w = \begin{pmatrix} 28 \\ 4 \\ -24 \\ 8 \end{pmatrix}$$

Example (4.2) Solve by LDL^T decomposition method.

$$\begin{bmatrix} (8,5,6,2) & (3,1,2,1) \\ (3,1,2,1) & (8,5,6,2) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} (30,30,60,50) \\ (25,30,46,46) \end{bmatrix}$$

Solution: $A = \begin{bmatrix} 8 & 3 \\ 3 & 8 \end{bmatrix}, B = \begin{bmatrix} 5 & 1 \\ 1 & 5 \end{bmatrix}, M = \begin{bmatrix} 6 & 2 \\ 2 & 6 \end{bmatrix}, N = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

$b = \begin{bmatrix} 30 \\ 25 \end{bmatrix}, g = \begin{bmatrix} 30 \\ 30 \end{bmatrix}, h = \begin{bmatrix} 60 \\ 46 \end{bmatrix}, k = \begin{bmatrix} 50 \\ 46 \end{bmatrix}$

$$A = L_1 D_1 L_1^T$$

$$\begin{pmatrix} 8 & 3 \\ 3 & 8 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 3/8 & 1 \end{pmatrix} \begin{pmatrix} 8 & 0 \\ 0 & 55/8 \end{pmatrix} \begin{pmatrix} 1 & 3/8 \\ 0 & 1 \end{pmatrix}$$

$x = L_1^{T^{-1}} D_1^{-1} L_1^{-1} b$

$$x = \begin{pmatrix} 1 & -3/8 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1/8 & 0 \\ 0 & 8/55 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -3/8 & 1 \end{pmatrix} \begin{pmatrix} 30 \\ 20 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

$B = L_2 D_2 L_2^T$

$$\begin{pmatrix} 5 & 1 \\ 1 & 5 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1/5 & 1 \end{pmatrix} \begin{pmatrix} 5 & 0 \\ 0 & 24/5 \end{pmatrix} \begin{pmatrix} 1 & 1/5 \\ 0 & 1 \end{pmatrix}$$

$$y = L_2^{T^{-1}} D_2^{-1} L_2^{-1} g$$

$$y = \begin{pmatrix} 1 & -1/5 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1/5 & 0 \\ 0 & 5/24 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/5 & 1 \end{pmatrix} \begin{pmatrix} 30 \\ 30 \end{pmatrix} = \begin{pmatrix} 5 \\ 5 \end{pmatrix}$$

$$z = L_1^{T^{-1}} D_1^{-1} L_1^{-1} (h - Mx) = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

$$w = L_2^{T^{-1}} D_2^{-1} L_2^{-1} (k - Ny) = \begin{pmatrix} 6 \\ 5 \end{pmatrix}$$

Hence the solution of the fully fuzzy linear system is

$$x = \begin{pmatrix} 3 \\ 2 \end{pmatrix} y = \begin{pmatrix} 5 \\ 5 \end{pmatrix} z = \begin{pmatrix} 4 \\ 2 \end{pmatrix} w = \begin{pmatrix} 6 \\ 5 \end{pmatrix}$$

5. Conclusion

In this paper a solution of FFLS is obtained by Cholesky and LDL^T decomposition methods by a new methodology in the form of trapezoidal fuzzy number matrices. Cholesky's method is efficient for symmetric positive definite system as it speeds up the execution time and minimizes the memory usage by using symmetry of coefficient matrix compared with Crammer's rule and LU.decomposition methods. LDL^Tdecomposition methodavoids calculation of square roots and can be implemented if Cholesky method fails.

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