On Fuzzy Regular ω-Languages

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Abstract

The concept of fuzzy local ω-language and Büchi fuzzy local ω-language are defined in [1, 2]. In this paper, we define Landweber fuzzy local ω-language and study their closure properties and also give an automata characterization for it. Finally, we conclude hierarchy among the subclasses of fuzzy regular ω-languages.

Keywords: Local automaton, Local ω -Language, Fuzzy sets, Fuzzy automaton, Fuzzy regular ω-Language.

1 Introduction

Fuzzy set was introduced by Zadeh [12] and it has application in many fields of science and engineering. To deal with imprecision due to fuzziness in system modeling, fuzzy automata and fuzzy languages have been proposed as a sound extension of classical automata and formal language theory. The mathematical formulation of fuzzy automata was first proposed by Wee [11]. The basic idea is that unlike the classical case, a fuzzy automaton can switch from one state to another with a certain possibility degree. In [11], Wee initiated the studies of fuzzy languages accepted by fuzzy automata. More recent development of algebraic theory of fuzzy

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automata and fuzzy languages can be found in book by Mordeson and Malik [8]. D.S.Malik et al. [9] and S.Gnanasekaran [6] studied the closure properties of fuzzy regular languages and fuzzy local languages on finitary case. Kamala Krithivasan [7] studied the fuzzy ω-finite state automata which accept fuzzy ω-regular languages with different acceptance criteria. Fuzzy regular language have many important applications including learning systems, pattern recognition, database theory, lexical analysis in programming language compilations and user-interface translations. Roughly speaking, in recent years their application have been further extended to include parallel processing, image generation and compression type theory for object-oriented languages, DNA computing, etc. In [1], we introduced two subclasses of fuzzy regular ω-languages that are fuzzy local ω-languages and Büchi fuzzy local ω-languages and we gave some closure properties of these classes of languages under intersection and union. In [2], we defined the notion of the deterministic fuzzy automata which accept fuzzy ω-languages with different mode of acceptance criteria and we established relationships between the various classes of fuzzy ω-languages. In this paper, we introduce Landweber fuzzy local ω-language and study their closure properties. We give automata characterization for it and we extablish the hierarchy this class with the already defined classes.

2 Preliminaries

Let Σ be a finite alphabet and Σ^* be the set of all finite words over Σ. We define the empty word by ϵ. For each $u \in \Sigma^*$, we denote by $P_1(u)$, the prefix of u of length 1 and by $F_2(u)$, the set of all factors of u of length 2. We denote by $S_1(u)$, the suffix of u of length 1. An infinite word α over Σ is a function $\alpha : N \to \Sigma$ from the set N of all positive integers to Σ. We represent the infinite word α as $\alpha = a_1a_2\cdots$ where $\alpha(i) = a_i \in \Sigma$, for all i. We denote by Σ^ω, the set of all infinite words over Σ. For $\alpha \in \Sigma^\omega$, $inf_2(\alpha)$ denotes the set of all elements of $F_2(\alpha)$, each of which repeats infinite number of times in α. An ω-language (with respect to the alphabet Σ) is any subset of Σ^ω. Fuzzy ω-languages are fuzzy sets of ω-languages. A projection map f is extended in a usual fashion to Σ^ω as follows: $f(\epsilon) = \epsilon$, $f(au) = f(a)f(u)$, for $a \in \Sigma$ and $u \in \Sigma^\omega$.

Definition 2.1 *[1] The pair $S = (\lambda_1, \lambda_2)$ is called a fuzzy local system if λ_1 is a fuzzy subset of Σ and λ_2 is a fuzzy subset of Σ^2. The fuzzy ω-language L over Σ whose membership function is defined by $L(\alpha) = \lambda_1(P_1(\alpha)) \wedge (\bigwedge_{x \in F_2(\alpha)} \lambda_2(x))$, $\forall \alpha \in \Sigma^\omega$ is called the fuzzy ω-language generated by S and we write $L = L^\omega(S)$.*

Definition 2.2 *[1] The fuzzy ω-language L over Σ is called a fuzzy local ω-language if $L = L^{\omega}_{L}(S)$ for some fuzzy local system S. The class of all fuzzy local ω-languages is denoted by $\mathcal{L}^{\omega}_{L}$.*

Example 2.3 *Consider the fuzzy ω-language whose membership function is given by*

$$L(\alpha) = \begin{cases} 0.5 & \text{if } \alpha = a^2b^{\omega}, \\ 0 & \text{otherwise.} \end{cases}$$

Let us consider $S = (\lambda_1, \lambda_2)$, where

$$\lambda_1(x) = \begin{cases} 0.5 & \text{if } x = a, \\ 0 & \text{otherwise.} \end{cases}$$

and

$$\lambda_2(x) = \begin{cases} 0.6 & \text{if } x = ab, \\ 0.5 & \text{if } x = \{bb, aa\}, \\ 0 & \text{otherwise.} \end{cases}$$

For $\alpha \in \Sigma^{\omega}$,

$$L^{\omega}(S)(\alpha) = \lambda_1(P_1(\alpha)) \bigwedge \left(\wedge_{x \in F_2(\alpha)} \lambda_2(x)\right)$$

Now,

$$\begin{aligned} L^{\omega}(S)(a^2b^{\omega}) &= \lambda_1(a) \bigwedge \left(\lambda_2(aa) \wedge \lambda_2(ab) \wedge \lambda_2(bb)\right) \\ &= 0.5 \wedge (0.5 \wedge 0.6 \wedge 0.5) \\ &= 0.5 \wedge 0.5 \\ &= 0.5. \end{aligned}$$

Then $L = L^{\omega}_{L}(S)$ and therefore L is a fuzzy local ω-language.

Remark 2.4 *The class of all local ω-languages is a proper subset of the class of all fuzzy local ω-languages.*

Theorem 2.5 *[1] If L_1 and L_2 are fuzzy local ω-languages over Σ, then $L_1 \cap L_2$ is a fuzzy local ω-language over Σ.*

Remark 2.6 *Union of two fuzzy local ω-languages over Σ needs not be a fuzzy local ω-language.*

Theorem 2.7 *[1] If Σ_1 and Σ_2 are two disjoint subsets of the alphabet Σ whose union is Σ and if $L_1 \subseteq \Sigma_1^{\omega}$ and $L_2 \subseteq \Sigma_2^{\omega}$ are fuzzy local ω-languages, then $L_1 \cup L_2$ is also fuzzy local ω-language.*

Definition 2.8 *[1] A fuzzy ω-language L over Σ is called a Büchi fuzzy local ω-language if there exists a triple (fuzzy local system) $S = (\lambda_1, \lambda_2, \lambda_3)$ where λ_1 is a fuzzy subset of Σ, λ_2 and λ_3 are fuzzy subsets of Σ^2 such that $\lambda_3 \leq \lambda_2$ and whose membership function is $L(\alpha) = \lambda_1(P_1(\alpha)) \bigwedge(\bigwedge_{x \in F_2(\alpha)} \lambda_2 (x)) \bigwedge(\bigvee_{x \in inf_2(\alpha)} \lambda_3(x)), \forall \alpha \in \Sigma^\omega$ and we write $L = L^\omega_B(S)$. The class of all Büchi fuzzy local ω-languages is denoted by $\mathcal{L}^\omega_B$.*

Example 2.9 *Consider a fuzzy ω-language L whose membership function is given by*

$$L(\alpha) = \begin{cases} 0.5 & \text{if } \alpha \in \{a^n b^\omega : n > 0\}, \\ 0 & \text{otherwise.} \end{cases}$$

Let us consider the fuzzy local system $S = (\lambda_1, \lambda_2, \lambda_3)$, where

$$\lambda_1(x) = \begin{cases} 0.5 & \text{if } x = a, \\ 0 & \text{otherwise.} \end{cases}$$

$$\lambda_2(x) = \begin{cases} 0.6 & \text{if } x = ab, \\ 0.5 & \text{if } x \in \{aa, bb\}, \\ 0 & \text{otherwise.} \end{cases}$$

and

$$\lambda_3(x) = \begin{cases} 0.5 & \text{if } x = bb, \\ 0 & \text{otherwise.} \end{cases}$$

For $\alpha \in \Sigma^\omega$,

$$L^\omega(S)(\alpha) = \lambda_1(P_1(\alpha)) \bigwedge(\bigwedge_{x \in F_2(\alpha)} \lambda_2(x)) \bigwedge(\bigvee_{x \in inf_2(\alpha)} \lambda_3(x))$$

Now,

$$\begin{aligned} L^\omega(S)(a^2 b^\omega) &= \lambda_1(a) \bigwedge (\lambda_2(aa) \wedge \lambda_2(ab) \wedge \lambda_2(bb)) \bigwedge (\bigvee \lambda_3(bb)) \\ &= 0.5 \wedge (0.5 \wedge 0.6 \wedge 0.5) \wedge (\vee 0.5) \\ &= 0.5 \wedge 0.5 \wedge 0.5 \\ &= 0.5. \end{aligned}$$

Then $L = L^\omega_B(S)$ and therefore L is a Büchi fuzzy local ω-language.

Remark 2.10 *The class of all fuzzy local ω-languages $\mathcal{L}^\omega_L$ is a subset of the class of all Büchi fuzzy local ω-languages $\mathcal{L}^\omega_B$.*

Example 2.11 *The language L in Example 2.7 is a Büchi fuzzy local ω-language. But L is not a fuzzy local ω-language, otherwise, $a^\omega \in L$. Therefore $\mathcal{L}_L^\omega \subset \mathcal{L}_B^\omega$.*

Theorem 2.12 *[1] Every fuzzy ω-regular language is a projection of a Büchi fuzzy local ω-language.*

Definition 2.13 *[2] A deterministic fuzzy automaton is a tuple $M = (Q, \Sigma, \delta, q_0, F)$ where*

- *Q is a finite non-empty set of states,*
- *Σ is a finite alphabet,*
- *$\delta : Q \times \Sigma \to Q$ is a transition function,*
- *$q_0 \in Q$ is the initial state,*
- *F is a fuzzy subset of Q.*

If $\alpha = a_1a_2a_3\cdots \in \Sigma^\omega$, the sequence $\rho = \{q_n\}_{n=0}^{\infty}$ of states in Q is called a run or path of M for α, if for $n \geq 1$, $\delta(q_{n-1}, a_n)$. We write $\rho : q_0 \xrightarrow{a_1} q_1 \xrightarrow{a_2} q_2 \to \cdots$. The range of ρ, denoted by $ran(\rho)$, is the set $\{q_0, q_1, q_2 \cdots\}$ and $inf(\rho)$ denotes the set of states which appear infinitely often in ρ. We say that for $i \in \{1, 1', 2, 2'\}$, the acceptance value of ρ on α in i-mode is $acc_i(\rho, \alpha)$ where

$$(a).\quad acc_1(\rho,\alpha) = \bigvee_{q \in ran(\rho)} F(q)$$

$$(b).\quad acc_{1'}(\rho,\alpha) = \bigwedge_{q \in ran(\rho)} F(q)$$

$$(c).\quad acc_2(\rho,\alpha) = \bigvee_{q \in inf(\rho)} F(q)$$

$$(d).\quad acc_{2'}(\rho,\alpha) = \bigwedge_{q \in inf(\rho)} F(q).$$

The fuzzy ω-language accepted by M in i-mode, is the fuzzy subset $L_i(M)$ of Σ^ω defined by $L_i(M)(\alpha) = acc_i(\rho, \alpha)$. For $i \in \{1, 1', 2, 2'\}$, we denote the class of all fuzzy ω-languages accepted by deterministic fuzzy automata in i-mode by $\mathcal{L}_i$. A fuzzy ω-language L is said to be a fuzzy regular ω-language if there exists a deterministic fuzzy automaton M such that $L = L_2^\omega(M)$.

Definition 2.14 *[2] A deterministic fuzzy automaton $M = (Q, \Sigma, \delta, q_0, F)$ is said to be local if for every $a \in \Sigma$, the set $\{\delta(q, a) : q \in Q\}$ contains at most one element.*

Theorem 2.15 *[2]*

(i). $\mathcal{L}_{1'} \subseteq \mathcal{L}_2$

(ii). $\mathcal{L}_{1'} \subseteq \mathcal{L}_{2'}$.

Definition 2.16 *[2] A fuzzy Muller automaton is a tuple $M = (Q, \Sigma, \delta, q_0, \mathcal{F})$ where Q, Σ, δ, q_0 are defined as in definition 2.10 and $\mathcal{F} = \{F_1, F_2, \ldots, F_n\}$ where each F_i is a fuzzy subset of Q. The fuzzy ω-language accepted by M is $L_3(M)$ defined by $L_3(M)(\alpha) = \bigvee_{i=1}^{m} \bigwedge_{q \in inf(\rho)} F_i(q)$ where ρ is a run of M for α. We say that $L_3(M)$ is accepted by M in 3-mode. We denote the class of all fuzzy ω-languages accepted by deterministic fuzzy automata in 3-mode by $\mathcal{L}_3$.*

Theorem 2.17 *[2]*

(i). $\mathcal{L}_2 \subseteq \mathcal{L}_3$

(ii). $\mathcal{L}_{2'} \subseteq \mathcal{L}_3$.

Theorem 2.18 *[2] $L \subseteq \Sigma^\omega$ is a fuzzy local ω-language if and only if L is recognized by a fuzzy local automaton in $1'$-mode.*

Theorem 2.19 *[2] $L \subseteq \Sigma^\omega$ is a Büchi fuzzy local ω-language if and only if L is accepted by a fuzzy local automaton in 2-mode.*

3 Fuzzy local automaton on Landweber fuzzy local ω-language

In this section we define the subclass of fuzzy regular ω-language that is Landweber fuzzy local ω-language and study their closure properties. We prove that if $L \subseteq \Sigma^\omega$ is a Landweber fuzzy local ω-language then L is accepted by a fuzzy local automaton in $2'$-condition. Finally we conclude hierarchy among the subclasses of fuzzy regular ω-languages.

Definition 3.1 *A fuzzy ω-language L over Σ is called a Landweber fuzzy local ω-language if there exists a triple (fuzzy local system) $S = (\lambda_1, \lambda_2, \lambda_3)$, where λ_1 is a fuzzy subset of Σ, λ_2 and λ_3 are fuzzy subsets of Σ^2 such that $\lambda_3 \leq \lambda_2$ and whose membership function is $L(\alpha) = \lambda_1(P_1(\alpha)) \bigwedge(\bigwedge_{x \in F_2(\alpha)} \lambda_2(x)) \bigwedge(\bigwedge_{x \in inf_2(\alpha)} \lambda_3(x)), \forall \alpha \in \Sigma^\omega$ and we write $L = L^\omega_{Ln}(S)$. The class of all Landweber fuzzy local ω-languages is denoted by $\mathcal{L}^\omega_{Ln}$.*

Example 3.2 *Consider the fuzzy ω-language*

$$L(\alpha) = \begin{cases} 0.5 & \text{if } \alpha \in \{a^+ b^\omega\}, \\ 0 & \text{otherwise.} \end{cases}$$

Let us consider the fuzzy local system $S = (\lambda_1, \lambda_2, \lambda_3)$, where

$$\lambda_1(x) = \begin{cases} 0.5 & \text{if } x = a, \\ 0 & \text{otherwise.} \end{cases}$$

$$\lambda_2(x) = \begin{cases} 0.6 & \text{if } x = ab, \\ 0.5 & \text{if } x = \{bb, aa\}, \\ 0 & \text{otherwise.} \end{cases}$$

and

$$\lambda_3(x) = \begin{cases} 0.5 & \text{if } x = bb, \\ 0 & \text{otherwise.} \end{cases}$$

For $\alpha \in \Sigma^\omega$,

$$L^\omega(S)(\alpha) = \lambda_1(P_1(\alpha)) \bigwedge \Big(\bigwedge_{x \in F_2(\alpha)} \lambda_2(x) \Big) \bigwedge \Big(\bigwedge_{x \in inf_2(\alpha)} \lambda_3(x) \Big)$$

Now,

$$\begin{aligned} L^\omega(S)(a^2 b^\omega) &= \lambda_1(a) \bigwedge (\lambda_2(aa) \wedge \lambda_2(ab) \wedge \lambda_2(bb)) \bigwedge (\bigwedge \lambda_3(bb)) \\ &= 0.5 \bigwedge (0.5 \wedge 0.6 \wedge 0.5) \bigwedge (\wedge 0.5) \\ &= 0.5 \wedge 0.5 \wedge 0.5 \\ &= 0.5. \end{aligned}$$

Then $L = L^\omega(S)$ and therefore L is a Landweber fuzzy local ω-language.

Remark 3.3 *The class of all fuzzy local ω-languages $\mathcal{L}_L^\omega$ is a subset of the class of all Landweber fuzzy local ω-languages $\mathcal{L}_{Ln}^\omega$.*

Example 3.4 *The language L in Example 3.2 is a Landweber fuzzy local ω-language. But L is not a fuzzy local ω-language, otherwise, $a^\omega \in L$. Therefore $\mathcal{L}_L^\omega \subset \mathcal{L}_{Ln}^\omega$.*

Theorem 3.5 *If L_1 and L_2 are Landweber fuzzy local ω-languages over Σ, then $L_1 \cap L_2$ is a Landweber fuzzy local ω-language over Σ.*

Proof: If L_1 and L_2 are fuzzy local ω-languages, then $L_1 = L^\omega(S_1)$ for some fuzzy local system $S_1 = (\lambda_1', \lambda_2')$ and $L_2 = L^\omega(S_2)$ for some fuzzy local system $S_2 = (\lambda_1'', \lambda_2'')$. Consider the fuzzy local system $S = (\lambda_1, \lambda_2, \lambda_3)$ where $\lambda_1 = \lambda_1' \wedge \lambda_1''$, $\lambda_2 = \lambda_2' \wedge \lambda_2''$ and $\lambda_3 = \lambda_3' \wedge \lambda_3''$. We show that $L^\omega_{Ln}(S) = L^\omega_{Ln}(S_1) \cap L^\omega_{Ln}(S_2) = L_1 \cap L_2$. For $\alpha \in \Sigma^\omega$,

$$\begin{aligned}
L^\omega(S)(\alpha) &= \lambda_1(P_1(\alpha)) \bigwedge (\bigwedge_{x \in F_2(\alpha)} \lambda_2(x)) \bigwedge (\bigwedge_{x \in inf_2(\alpha)} \lambda_3(x)) \\
&= \left(\lambda_1'(P_1(\alpha)) \wedge \lambda_1''(P_1(\alpha))\right) \bigwedge \left(\wedge_{x \in F_2(\alpha)} (\lambda_2'(x) \wedge \lambda_2''(x))\right) \\
&\quad \bigwedge \left(\wedge_{x \in inf_2(\alpha)} (\lambda_3'(x) \wedge \lambda_3''(x))\right) \\
&= \left((\lambda_1' \wedge \lambda_1'')(P_1(\alpha))\right) \bigwedge \left(\wedge_{x \in F_2(\alpha)} (\lambda_2' \wedge \lambda_2'')(x)\right) \bigwedge \Big(\\
&\qquad \wedge_{x \in inf_2(\alpha)} (\lambda_3' \wedge \lambda_3'')(x)\Big) \\
&= \left(\lambda_1'(P_1(\alpha)) \wedge (\wedge_{x \in F_2(\alpha)} \lambda_2'(x)) \wedge (\wedge_{x \in inf_2(\alpha)} \lambda_3'(x))\right) \\
&\quad \bigwedge \left(\lambda_2''(P_1(\alpha)) \wedge (\wedge_{x \in F_2(\alpha)} \lambda_2''(x)) \wedge (\wedge_{x \in inf_2(\alpha)} \lambda_3''(x))\right) \\
&= L^\omega(S_1)(\alpha) \wedge L^\omega(S_2)(\alpha) \\
&= L_1(\alpha) \wedge L_2(\alpha) \\
&= (L_1 \cap L_2)(\alpha)
\end{aligned}$$

Thus $L^\omega_{Ln}(S) = L_1 \cap L_2$.

Therefore $L_1 \cap L_2$ is a Landweber fuzzy local ω-language.

Remark 3.6 *Union of two Landweber fuzzy local ω-languages over Σ needs not be a Landweber fuzzy local ω-language.*

Example 3.7 *Consider the Landweber fuzzy local ω-languages L_1 and L_2 over $\Sigma = \{a, b, c\}$ with membership function,*

$$L_1(\alpha) = \begin{cases} 0.3 & \textit{if } \alpha = a(bc)^\omega, \\ 0 & \textit{otherwise.} \end{cases}$$

and

$$L_2(\alpha) = \begin{cases} 0.4 & \textit{if } \alpha = a^\omega, \\ 0 & \textit{otherwise.} \end{cases}$$

Therefore

$$(L_1 \cup L_2)(\alpha) = \begin{cases} 0.4 & \textit{if } \alpha = a^\omega, \\ 0.3 & \textit{if } \alpha = a(bc)^\omega, \\ 0 & \textit{otherwise.} \end{cases}$$

If $L_1 \cup L_2$ is Landweber fuzzy local ω-language, then there exists a fuzzy local system $S = (\lambda_1, \lambda_2, \lambda_3)$ such that $L_1 \cup L_2 = L^{\omega}_{Ln}(S)$. Here $\lambda_1(a)$, $\lambda_2(aa)$, $\lambda_2(ab)$, $\lambda_2(bc)$, $\lambda_2(cb)$, $\lambda_3(bc)$ and $\lambda_3(aa)$ are all greater than zero and therefore $L_1 \cup L_2(a^n(bc)^{\omega}) \neq 0$, $n \geq 1$. But $L_1(a^n(bc)^{\omega}) = 0$ and $L_2(a^n(bc)^{\omega}) = 0$ which is a contradiction.

Theorem 3.8 *If $L \subseteq \Sigma^{\omega}$ is a Landweber fuzzy local ω-language then L is accepted by a fuzzy local automaton in $2'$-mode.*

Proof: Let L be a Landweber fuzzy local ω-language. Then there exists a triple $S = (\lambda_1, \lambda_2, \lambda_3)$ where λ_1 is a fuzzy subset of Σ and λ_2, λ_3 are fuzzy subset of Σ^2 such that $\lambda_3 \leq \lambda_2$ and $L(\alpha) = \lambda_1(P_1(\alpha)) \wedge (\bigwedge_{x \in F_2(\alpha)} \lambda_2(x)) \wedge (\bigwedge_{x \in inf_2(\alpha)} \lambda_3(x)), \forall \alpha \in \Sigma^{\omega}$. Consider the deterministic fuzzy automaton $M = (Q, \Sigma, \delta, q_0, F)$ where

- $Q = \{\{[\epsilon]\} \cup \{[a] : \lambda_1(a) \neq 0\} \cup \{[u] : \lambda_2(u) \neq 0\}\}$,
- $q_0 = \{[\epsilon]\}$,
- δ is defined as follows:

For all $a, b \in \Sigma$,

$$\delta([\epsilon], a) = [a] \quad if \quad \lambda_1(a) \neq 0,$$
$$\delta([a], b) = [ab] \quad if \quad \lambda_2(ab) \neq 0,$$

For all $u = ab \in Q$ and $c \in \Sigma$,

$$\delta([ab], c) = [bc] \quad if \quad \lambda_2(bc) \neq 0 \text{ and}$$

- F is the fuzzy final state, defined by

$$F([u]) = \begin{cases} \lambda_3(u) & \text{if } u \in \Sigma^2, \\ 0 & \text{otherwise.} \end{cases}$$

Then M is local and therefore $L = L^{\omega}_{2'}(M)$.

4 Conclusion

The following Figure 1, represents hierarchy among the subclasses of fuzzy regular ω-languages.

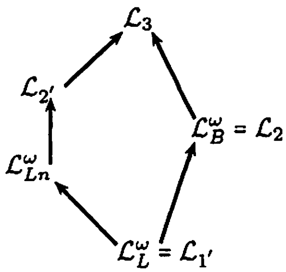

Figure 1

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