Dual-Chordal and Strongly Dual-Chordal Graphs

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Abstract

A graph is chordal if and only if every cycle either has a chord or is a triangle. If an edge (or triangle) is defined to be a strength-k edge (or triangle) whenever it is in at least k maxcliques, then a graph is strongly chordal if and only if, for every $k \geq 1$, every cycle of strength-k edges either has a strength-k chord or is a strength-k triangle. Dual-chordal graphs have been defined so as to be the natural cycle/cutset duals of chordal graphs. A carefully crafted notion of dual strength allows a characterization of strongly dual-chordal graphs that is parallel to the above. This leads to a complete list of all 3-connected strongly dual-chordal graphs.

1 Background

A *maxclique* of a graph is an inclusion-maximal complete subgraph. The *clique strength* of an edge e or triangle C in a graph G—denoted by $\text{cstr}_G(e)$ and $\text{cstr}_G(C)$, respectively—is the number of maxcliques of G that contain e or C. An edge e or triangle C is a cstr_G-k *edge* or cstr_G-k *triangle*, respectively, if $\text{cstr}_G(e) \geq k$ or $\text{cstr}_G(C) \geq k$. The subscript G will be dropped from cstr_G whenever it is clear which graph G is. Thus, every edge or triangle is automatically a cstr-1 edge or a cstr-1 triangle.

A graph is *chordal* if every cycle of length 4 or more has a chord—in other words, if every cycle (of cstr-1 edges) either has a (cstr-1) chord or is a (cstr-1) triangle. As introduced in [4], a chord e of an even-length cycle C of G is a *strong chord* of C if e forms a cycle with an odd number of edges of C. A graph is *strongly chordal* if it is chordal and every cycle of even length 6 or more has a strong chord; see [2, 4, 10] for details. The results in this paper are motivated by Proposition 1 (from [5, 8], in the terminology of [9]), which restates being strongly chordal as a natural strengthening of being chordal.

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Proposition 1 *A graph is strongly chordal if and only if, for every $k \geq 1$, every cycle of* cstr-k *edges either has a* cstr-k *chord or is a* cstr-k *triangle.*

A *cutset* of a connected graph G is an inclusion-minimal set $D \subset E(G)$ such that $G - D$ is not connected. A *triad* is a size-3 cutset, and a *cyclic triad* is a triad D such each component of $G - D$ contains a cycle. An edge $e \notin D$ is a *cut-chord* of a cutset D if $\{e\}$ is a cutset of $G - D$; equivalently—and dually to chords of cycles—if D can be partitioned as $D_1 \cup D_2$ where each $D_i \cup \{e\}$ is a cutset of G. As in [6, 7], a graph is *dual-chordal* if every cutset of size 4 or more has a cut-chord—in other words, if every cutset either has a cut-chord or is a triad. (Warning: This duality should not be confused with hypergraph duality, which leads to another important notion dual to chordal: the "dually chordal graphs" of [1, 2].)

As in [7], it is useful to restrict attention to 3-connected graphs—always with at least three vertices—when studying dual-chordal graphs, especially in light of Proposition 2 from [7] (where being *cubic* means that every vertex has degree 3).

Proposition 2 *Every 3-connected dual-chordal graph G is cubic.*

Proof. If a 3-connected graph dual-chordal graph G had a vertex v of degree 4 or more, then the cutset D of all the edges incident with v would have a cut-chord e, and simultaneously deleting v and e from G would leave a disconnected graph, contradicting G being 3-connected. □

2 Strongly dual-chordal graphs

As in [7], a cut-chord e of an even-size cutset D of G is a *strong cut-chord* of D if e forms a cutset with an odd number of edges of D. A graph is *strongly dual-chordal* if it is dual-chordal and every cutset of even size 6 or more has a strong cut-chord.

Dualizing what happens with chords and cycles in chordal graphs, an easy inductive argument on $|D|$ shows that a 3-connected graph is dual-chordal if and only if every cutset D with $|D| \geq 4$ has a cut-chord e with edges $a, b \in D$ such that $\{a, b, e\}$ is a triad of G. Similarly, a dual-chordal graph is strongly dual-chordal if every cutset D with $|D| \geq 6$ has a cut-chord e with edges $a, b, c \in D$ such that $\{a, b, c, e\}$ is a cutset of G.

Proposition 2 allows us to restrict attention to cubic graphs. Results in [3] show that every 3-connected cubic graph G can be decomposed into smaller 3-connected cubic graphs $G_1, G_2, \ldots$ by repeatedly applying the procedure illustrated in Figure 1 to cyclic triads and, moreover, that the set $\{G_1, G_2, \ldots\}$ of non-decomposable graphs ultimately obtained is unique. Given the graphs $G_1, G_2, \ldots$ of the decomposition of G, let $T(G)$ denote

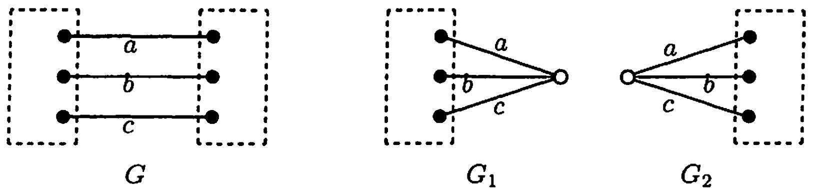

Figure 1: Decomposing G into graphs G_1 and G_2 along the triad $\{a, b, c\}$

the corresponding decomposition tree. For each node S of $T(G)$, let G_S denote the subgraph of G that is formed by the edges in S.

If each subgraph G_i in a decomposition of G is isomorphic to K_4 or $K_{3,3}$, then G is called $\{K_4, K_{3,3}\}$-*decomposable* in [3] (or see [7]), $T(G)$ is called a $\{K_4, K_{3,3}\}$-*decomposition tree*, and every node S of $T(G)$ has either $|S| = 6$ or $|S| = 9$. Figure 2 shows an example of a 3-connected, cubic, strongly dual-chordal graph G and its $\{K_4, K_{3,3}\}$-decomposition tree $T(G)$. In this example, the triad $\{7, 8, 9\}$ decomposes G into a K_4 with edge set $\{1, 2, 3, 7, 8, 9\}$ and a graph G' with edge set $\{4, 5, \ldots, 15\}$ that is obtained from G by contracting the triangle $\{1, 2, 3\}$ to a single vertex; the triad $\{7, 8, 9\}$ corresponds to the left edge shown in $T(G)$. The graph G' can then be decomposed along the triad $\{9, 10, 11\}$ into a K_4 with edge set $\{4, 5, 6, 9, 10, 11\}$ and a $K_{3,3}$ with edge set $\{7, 8, \ldots, 15\}$ that is obtained from the original G by contracting the triangles $\{1, 2, 3\}$ and $\{4, 5, 6\}$ into single vertices; the triad $\{9, 10, 11\}$ corresponds to the right edge shown in $T(G)$.

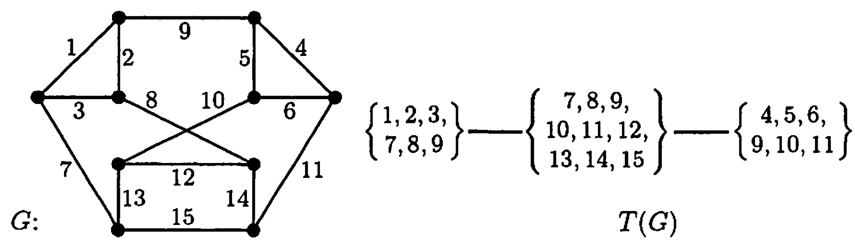

Figure 2: A strongly dual-chordal cubic graph and its decomposition tree.

Suppose G has a $\{K_4, K_{3,3}\}$-decomposition tree $T(G)$. If S is a degree-0 node of $T(G)$, then $G_S \cong K_4$ or $G_S \cong K_{3,3}$. If S is a degree-1 node of $T(G)$, then either G_S is net or a bipartite net graph, as illustrated in Figure 3.

If S is a degree-2 or higher node of $T(G)$, then all the possibilities for G_S (up to isomorphism) are shown in Figure 4, where the vertices indicated by solid balls occur only in G_S for the one node S of $T(G)$, while the three vertices in each dashed box are distinct endpoints of edges that are in both

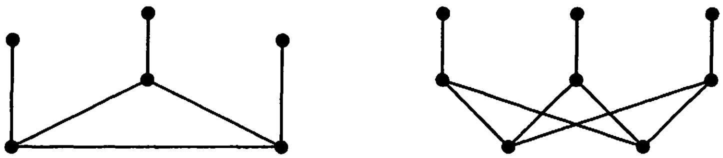

Figure 3: The *net* and *bipartite net* graphs.

G_S and $G_{S'}$ for a neighbor S' of S in $T(G)$. Contracting the subgraph inside each of the dashed boxes into a vertex would produce the K_4 or $K_{3,3}$ graph G_i in the $\{K_4, K_{3,3}\}$-decomposition that has corresponding $E(G_S)$ and $E(G_i)$. For instance, in the degree-4 center column graph in Figure 4, $G_S \cong 2K_{1,3} \cup 3K_2$ (where $3K_2$ denotes the order-6 graph with three edges) with the corresponding $G_i \cong K_{3,3}$.

The results in Lemma 3 are all from [7] (where the *symmetric difference* of sets $S_1, \ldots, S_n$ is the set of elements that occur in an odd number of $S_1, \ldots, S_n$).

Lemma 3 ([7]) *The following hold for all 3-connected cubic graphs G:*

(1) *A triad D of G is cyclic if and only if $T(G)$ has an edge SS' with $D = S \cap S'$.*

(2) *A triad D of G is cyclic if and only if the three edges in D have a total of six distinct endpoints in G.*

(3) *G is dual-chordal if and only if every cutset D of G is the symmetric difference of triads $D_1, \ldots, D_{|D|-2}$ where $D_1 \cap D, \ldots, D_{|D|-2} \cap D$ partition D.*

(4) *G is dual-chordal if and only if G is $\{K_4, K_{3,3}\}$-decomposable.*

3 Defining dual strength

Suppose G is a 3-connected cubic graph with a decomposition tree $T(G)$. Define the *dual strength of an edge* e of G—denoted by $\text{str}^*_G(e)$—to be the number of nodes of $T(G)$ that contain e. In the graph G in Figure 2, for instance, $\text{str}^*_G(1) = 1$ and $\text{str}^*_G(9) = 3$. Define the *dual strength of a triad* D of G—denoted by $\text{str}^*_G(D)$—to be the number of nodes S of $T(G)$ that contain D such that the following holds:

> If $|S| = 9$, then S is the unique size-9 node, G_S is neither of the top two graphs in the right column of Figure 4, and $T(G)$ is a star with center S.

In the graph G in Figure 2, for instance, the center node S of the star $T(G)$ corresponds to the graph G_S at the top of the center column of Figure 4,

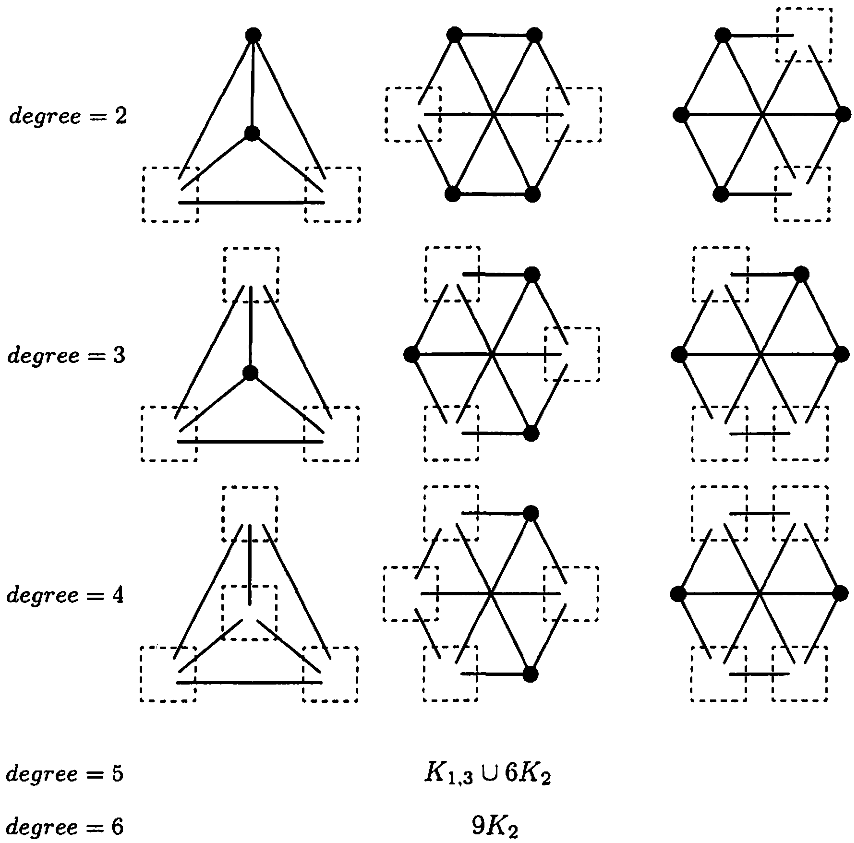

Figure 4: The possible G_S subgraphs for degree-2 or higher nodes S of $T(G)$.

the two cyclic triads $\{7, 8, 9\}$ and $\{9, 10, 11\}$ have dual strength 2, and each of the ten noncyclic triads D has $\text{str}^*(D) = 1$ (dropping the subscript G from str^*_G since it is clear which graph G is). Note that triads D always satisfy $\text{str}^*(D) \leq \min\{\text{str}^*(e) : e \in D\}$.

The definition of the dual strength of a triad may well look contrived, but notice that the $|S| = 9$ restriction only occurs for nonplanar graphs: $K_{3,3}$ will only occur in Lemma 3(4) when the graph G is nonplanar—if G is planar, then every S has $|S| = 6$. Also, Theorem 4 will show that the definition of $\text{str}^*(D)$ can be defended by traditional graph-theoretic duality whenever G is planar (interchanging vertices with faces in a plane embedding).

Theorem 4 *If a 3-connected cubic planar graph G has dual graph G^*, then every edge e of G has* $\mathrm{str}^*_G(e) = \mathrm{cstr}_{G^*}(e)$ *and every triad D of G has* $\mathrm{str}^*_G(D) = \mathrm{cstr}_{G^*}(C)$ *where C is the triangle in G^* that is dual to D.*

Proof. Suppose a 3-connected cubic planar graph G is embedded in the plane and G^* is the vertex-face planar dual of G; thus the cutsets (and their dual-chords) of G correspond exactly to the cycles (and their chords) of G^*, and so G^* will be chordal [7]. By Lemma 3(4), every node of $T(G)$ will have size 6. For each leaf node S of $T(G)$, the graph G_S will be a net; say $S = \{a, b, c, d, e, f\}$ where a, b, c form a triangle in G and d, e, f form a triad of G. (Thus a, b, c will form a triad of the dual graph G^*—indeed, a, b, c will have a common degree-3 endpoint in G^*.) Let G_1 be the graph with edge set $E(G) - \{a, b, c\}$ in the decomposition of G along the triad $\{d, e, f\}$, and repeat the above for the 3-connected cubic planar graph G_1. In this way, repeatedly removing leaves from the trees $T(G), T(G_1), T(G_2), \ldots$ for this decomposition of G will correspond to a simplicial elimination scheme for G^*, see [2, 10], with the nodes of $T(G)$ corresponding to the edge sets of the maxcliques of G^*. Therefore, every edge e of G has $\mathrm{str}^*_G(e) = \mathrm{cstr}_{G^*}(e)$ and every triad D of G has $\mathrm{str}^*_G(D) = \mathrm{cstr}_{G^*}(C)$ where C is the triangle in G^* that is dual to D. □

Call an edge e or triad D a str*-k *edge* or str*-k *triad*, respectively, if $\mathrm{str}^*(e) \geq k$ or $\mathrm{str}^*(D) \geq k$. Thus, every edge or triad is automatically a str*-1 edge or a str*-1 triad.

Corollary 5 *A planar 3-connected cubic graph is strongly dual-chordal if and only if, for every $k \geq 1$, every cutset of* str*-k *edges either has a* str*-k *cut-chord or is a* str*-k *triad.*

Proof. For every 3-connected cubic planar graph G with dual graph G^*, the corollary follows from Proposition 1, Theorem 4, the identification of the cutsets (and their cut-chords) of G with the cycles (and their chords) of G^*, and the definitions of G being strongly dual-chordal and G^* being strongly chordal. □

4 Strongly dual-chordal characterization

This section will generalize the result of Corollary 5 for planar 3-connected cubic graphs to arbitrary 3-connected cubic graphs. Theorem 7 will be the promised dual version of Proposition 1 (remembering that Proposition 2 allows the restriction to cubic graphs). Before that, several examples of dual-chordal graphs that are not strongly dual-chordal will be helpful.

Figure 5 shows a planar 3-connected graph that is dual-chordal, but not strongly dual-chordal: cutset $D = \{2,3,5,6,13,14\}$ has cut-chords 8, 10, and 12, but no strong cut-chords; also, $D' = \{8,10,12\}$ is a triad of str*-2 edges with $\text{str}^*(D') = 1$.

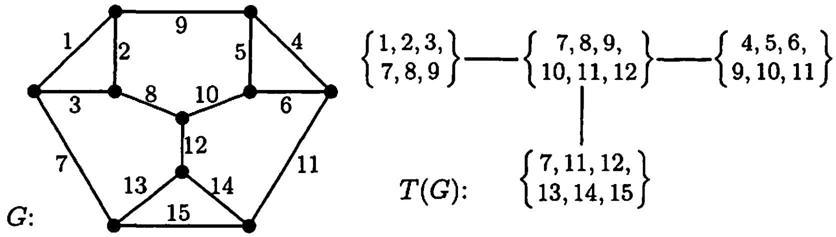

Figure 5: A planar 3-connected dual-chordal but not strongly dual-chordal graph and its decomposition tree.

Figure 6 shows two additional dual-chordal graphs that are not strongly dual-chordal; in each, the size-9 node of $T(G)$ does not satisfy the $|S| = 9$ condition in the definition of $\text{str}^*(D)$. In the top example, G_S is the top-right graph in Figure 4; thus $\text{str}^*(\{4,5,6\}) = 1$, which is less than $\min\{\text{str}^*(4), \text{str}^*(5), \text{str}^*(6)\} = 2$. In the bottom example, G_S is a bipartite net, but S is not the center node of a star; thus $\text{str}^*(\{7,8,9\}) = 1$, which is less than $\min\{\text{str}^*(7), \text{str}^*(8), \text{str}^*(9)\} = 2$.

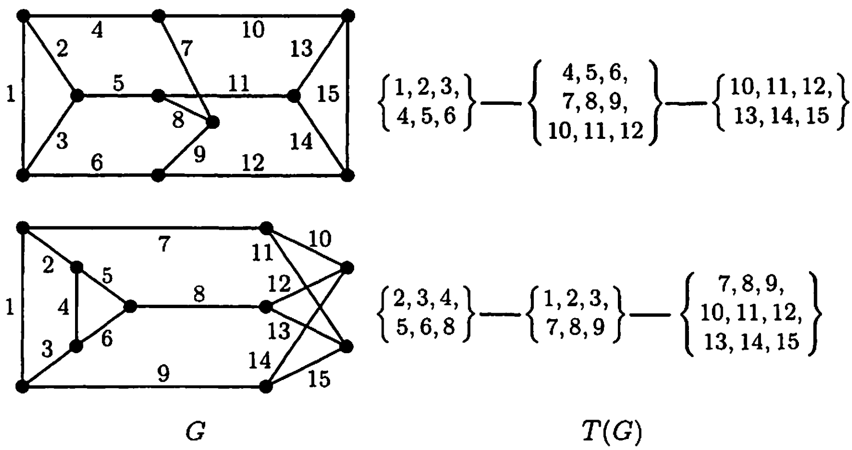

Figure 6: Two dual-chordal, but not strongly dual-chordal, graphs that have size-9 nodes in their decomposition trees.

Lemma 6 *If G is a 3-connected dual-chordal graph such that, for every $k \geq 1$, every cutset of str*-k edges either has a str*-k cut-chord or is a str*-k triad, then every nonleaf node S of the decomposition tree $T(G)$ must have G_S be the left or middle graph in the top row of Figure 4—thus S must have degree 2 in $T(G)$, and so $T(G)$ must be a path.*

Proof. Suppose G is a 3-connected dual-chordal graph such that, for every $k \geq 1$, every cutset of str*-k edges either has a str*-k cut-chord or is a str*-k triad. Suppose S is a nonleaf node of $T(G)$. The proof will show contradictions for all the possibilities for G_S in Figure 4 other than the left and middle graphs in the top row.

Suppose for the moment that G_S is one of the top two graphs in the right column of Figure 4. If $D = \{a, b, c\} \subset S$ is a triad of G such that a, b, c all have endpoints in the same dashed box of G_S and if D is in k nodes of $T(G)$, then D would be a triad of str*-k edges with $\text{str}^*(D) < k$, since S would not be counted in the definition of $\text{str}^*(D)$ [contradicting that D would have to be a str*-k triad].

Next suppose that G_S is the bottom graph in the right column of Figure 4. Let v and w be the two vertices that only occur in G_S (indicated by solid balls) with edges a, b, c having endpoint v and c, d, e having endpoint w. Edge c only occurs in the one node S of $T(G)$, and so $\text{str}^*(c) = 1$, while $\{a, b, d, e\}$ is a cutset of str*-2 edges. Furthermore, c is the only cut-chord of $\{a, b, d, e\}$ [contradicting that every cutset of four str*-2 edges would have to have a str*-2 cut-chord].

Finally, suppose G_S is one of the six graphs that are not in the top row or the right column in Figure 4. In each case, there is a triad $D = \{e_1, e_2, e_3\}$ where each e_i has an endpoint in a dashed box that does not contain an endpoint of either e_j with $j \neq i$; thus each e_i is in a node of $T(G)$ that does not contain all of D. If $k = \min\{\text{str}^*(e_1), \text{str}^*(e_2), \text{str}^*(e_3)\}$, then D would be a triad of str*-k edges with $\text{str}^*(D) < k$ [contradicting that D must be a str*-k triad]. □

Theorem 7 *A 3-connected cubic graph is strongly dual-chordal if and only if, for every $k \geq 1$, every cutset of* str*-k *edges either has a* str*-k *cut-chord or is a* str*-k *triad.*

Proof. First suppose G is a 3-connected strongly dual-chordal graph. Argue by induction on $k \geq 1$, where the $k = 1$ case follows from the definition of dual-chordal and every edge and triad being a str*-1 edge or a str*-1 triad. Suppose $k \geq 2$ and every cutset of str*-$(k-1)$ edges either has a str*-$(k-1)$ cut-chord or is a str*-$(k-1)$ triad. Suppose D is a cutset of str*-k edges.

Suppose for the moment that $|D| \geq 4$. The induction hypothesis implies that D has a str*-$(k-1)$ cut-chord e, and so G has a cutset D_2 of str*-$(k-1)$ edges with $e \in D_2 \subset D \cup \{e\}$ and $|D_2| < |D|$. Repeat this if $|D_2| \geq 4$, eventually reaching a str*-$(k-1)$ cut-chord e of D and edges $a, b \in D$ such that $D_i = \{a, b, e\}$ is a triad of str*-$(k-1)$ edges of G. The induction hypothesis implies that D_i is a str*-$(k-1)$ triad, and so $e \in D_i$ is in at least $k-1$ nodes of $T(G)$ that contain D_i. Since e is also in a triad that is contained in $D \cup \{e\} - \{a, b\}$, the edge e is in another node that does not contain D_i. Therefore, str*$(e) \geq k$, and so e is a str*-k cut-chord of D.

Henceforth assume instead that $|D| = 3$. Since D is a triad of str*-k edges with $k \geq 2$, there must be two adjacent nodes of $T(G)$ that contain D. Also, the inductive hypothesis implies that D is a str*-$(k-1)$ triad. Suppose str*$(D) = k-1$, which means D is not a str*-k triad [arguing by contradiction]. Since that makes str*$(D) < k \leq \min\{$str*$(e) : e \in D\}$, the definition of str*(D) implies that at least one of the following must hold:

(*i*) D is contained in a size-9 node S of $T(G)$ where G_S is one of the top two graphs in the right column of Figure 4.

(*ii*) Each edge of D is in a node of $T(G)$ that does not contain D.

(*iii*) D is contained in a size-9 node S of $T(G)$ that has a neighbor S' that either is not a leaf of $T(G)$ or has $|S'| = 9$.

In case (*i*): Let $D = \{a, b, c\}$. Inspection of the two possibilities for G_S shows that there are six distinct edges a_1, a_2 and b_1, b_2 and c_1, c_2 in G that share common endpoints with a, with b, and with c, respectively. Those edges form a size-6 cutset D' of G that has cut-chords a, b, c. In fact, a, b, c are the only cut-chords of D' (if G_1 and G_2 are the graphs obtained from decomposing G along the triad D' as in Figure 1, then a, b, c have a common endpoint v in both G_1 and G_2, with $G_1 - v$ and $G_2 - v$ both 2-connected). Furthermore, none of a, b, c is a strong cut-chord of D' (since $\{a, a_1, a_2\}$ and $\{b, b_1, b_2\}$ and $\{c, c_1, c_2\}$ are the only cutsets of G in $D' \cup \{a, b, c\}$ that contain exactly one of a, b, c). Therefore, D' cannot have a strong cut-chord [contradicting that G is strongly chordal].

In case (*ii*), assuming that (*i*) does not hold: As mentioned before, D is in adjacent nodes of $T(G)$. In particular, D is in a node S that has degree $\delta \geq 3$ in $T(G)$ (using (*ii*), since no two triads can have two edges in common: if $|D_1 \cap D_2| = 2$, then the symmetric difference $D_1 \oplus D_2$ would be a size-2 cutset of G, contradicting that G is 3-connected). Let $S_1, \ldots, S_\delta$ be the neighbors of S in $T(G)$. If $\delta \geq 4$, then G can be decomposed along the triads $S \cap S_4$ through $S \cap S_\delta$ (as in Figure 1) so as to obtain a graph G_i that contains $S_1 \cup S_2 \cup S_3$—and this would reduce the problem to the $\delta = 3$ case. Henceforth assume $\delta = 3$, and so G_S is the left or middle graph in the degree-3 row of Figure 4. Let $D = \{a, b, c\}$. Inspection of those two graphs shows that each of the edges a, b, c has an endpoint in one common dashed box in Figure 4 and a second endpoint outside of that box. Let a_1, a_2 and b_1, b_2 and c_1, c_2 be the edges of G that share common endpoints with a, with b, and with c, respectively, and lie outside of that box. The edges $a_1, a_2, b_1, b_2, c_1, c_2$ form a size-6 cutset D' of G that has cut-chords a, b, c. By exactly the same argument as in case (i), D' cannot have a strong cut-chord [contradicting that G is strongly chordal].

In case (iii): Let G_i and G_j be the graphs in the $\{K_4, K_{3,3}\}$-decomposition of G that have $E(G_i) = S$ and $E(G_j) = S'$. Let the triad $S \cap S'$ equal $\{a, b, c\}$; let a_1, a_2 and b_1, b_2 be the edges of G_i in $S - S'$ that share an endpoint with a and with b, respectively; and let c_1, c_2 be the edges of G_j in $S' - S$ that share an endpoint with c. The edges $a_1, a_2, b_1, b_2, c_1, c_2$ form a size-6 cutset D' of G that has cut-chords a, b, c. None of a, b, c is a strong cut-chord of D' (since $\{a, a_1, a_2\}$ and $\{b, b_1, b_2\}$ and $\{c, c_1, c_2\}$ are the only cutsets of G in $D' \cup \{a, b, c\}$ that contain exactly one of a, b, c). Since $\{a, b, c, a_1, a_2, b_1, b_2\} \subset S$ and $|S| = 9$, there are two additional edges d_1 and d_2 in S where, without loss of generality and using $G_i \cong K_{3,3}$, edge d_1 shares one endpoint with a_1 and b_1 (and the other endpoint with c and d_2), while d_2 shares one endpoint with a_2 and b_2 (and the other endpoint with c and d_1). Thus G_i is a bipartite net, and d_1 and d_2 are the only other possible cut-chords of D' (since S' not being a size-6 leaf of $T(G)$ implies that $G_{S'}$ is not a net). But neither d_1 nor d_2 is a strong cut-chord of D' (since $\{a_1, b_1, d_1\}$ and $\{a_2, b_2, d_2\}$ are the only cutsets of G in $D' \cup \{d_1, d_2\}$ that contain exactly one of d_1, d_2). Therefore, D' cannot have a strong cut-chord [contradicting that G is strongly chordal].

Conversely, suppose G is a 3-connected dual-chordal graph such that, for every $k \geq 1$, every cutset of str*-k edges either has a str*-k cut-chord or is a str*-k triad. By Lemma 6, $T(G)$ is a path and each internal node S of $T(G)$ has G_S in the left or middle position on the top row of Figure 4. If every internal node S has G_S the left top graph, then G is planar and so is strongly dual-chordal by Corollary 5.

Henceforth assume that S is a size-9 node of $T(G)$ and G_S is the middle graph of the top row of Figure 4. If $T(G)$ is not a star whose center S is the unique size-9 node, then S would not be counted in the definition of $\text{str}^*(D)$ for any triad $D \subset S$ [contradicting that $\text{str}^*(D) \geq \min\{\text{str}^*(e) : e \in D\}$]. Therefore, the path $T(G)$ must be a star whose center S is the unique size-9 node, and so G is one of the three strongly dual-chordal graphs shown in Figure 7 (in which, from left to right, $T(G)$ has order 3, 2, or 1). □

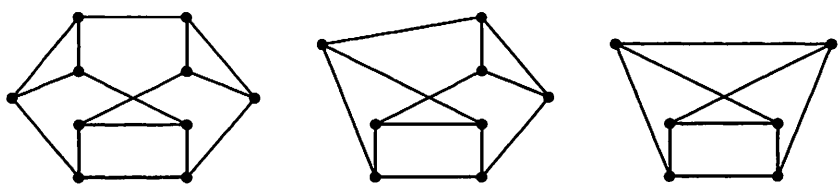

Figure 7: Three nonplanar 3-connected strongly dual-chordal graphs.

Corollary 8 *Every 3-connected strongly dual-chordal graph is one of the three nonplanar graphs shown in Figure 7 or is K_4 or a triangular prism with any number of parallel struts* (as illustrated in Figure 8).

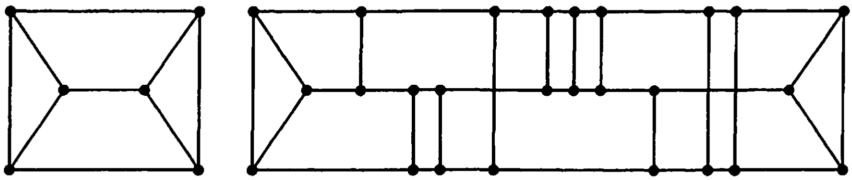

Figure 8: Triangular prisms with zero and ten parallel struts.

Proof. The three nonplanar cases follow by the final two paragraphs of the proof of Theorem 7. The planar cases of K_4 and triangular prisms with parallel struts are from [7, Thm. 11]. □

References

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