# A Novel Digital Modulation Method for Digital Wireless Communications

E. A. Yfantis, and A. Fayed
ICIS Laboratory
Computer Science Department, College of Engineering
University of Nevada, Las Vegas
Las Vegas, NV, 89154-4019
yfantis@cs.unlv.edu

#### Abstract

Analog modulation has served us very well over the years. Digital modulation is an improvement over analog modulation because it provides better bandwidth utilization over analog modulation, less power for signal propagation, it is natural for packet transmission, forward error correction, automatic repeat request, encryption, compression, and signal transformation so that it looks like noise to the adversary. Digital wireless communication is an enormous area that is rapidly growing. Digital communication is a field in which theoretical ideas have had an unusually powerful impact on system design and practice. In this research paper we provide a digital modulation algorithm for efficient transmission based on circular probability distribution theory. Key Words; Digital modulation. Forward Error Correction, Digital Transmission, Amplitude Shift Keying, Frequency Shift Keying, Phase Shift Keying.

### 1 Introduction

The digital communication industry is an enormous industry that is rapidly growing. Digital communication is a field in which theoretical ideas have had an unusually powerful impact on system design and practice [1]. In Digital modulation the message signal is in the digital form and the carrier wave is in sinusoidal form. In this technique the Amplitude, Frequency or Phase of carrier varies according to the message (Baseband) signal. The three basic types of digital modulation are: Amplitude shift keying (ASK),

where only the amplitude of the carrier is changed in response to the information (baseband) to be transmitted, and everything else is kept fixed. Thus bit 1 is transmitted by a carrier of one particular amplitude and bit 0 is transmitted by a different amplitude but the frequency and phase remain the same regardless if bit 1 or bit 0 are being transmitted. On Off Keying (OOK) is considered to be a special case of amplitude shift keying. In the frequency shift keying the carrier frequency is different for transmitting a 0 bit than transmitting an 1 bit.

$$s(t) = \begin{cases} sin(2\pi f_1 t) & \text{for bit 0} \\ sin(2\pi f_2 t) & \text{for bit 1} \end{cases}$$
 (1)

In the phase shift keying the phase of the sinusoidal carrier changes according to the transmitted information. Thus when a 0 bit is transmitted the phase of the sinusoid is shifted by  $\pi$  radians. Thus

$$s(t) = \begin{cases} sin(2\pi ft) & \text{for bit 1} \\ sin(2\pi ft + \pi) & \text{for bit 0} \end{cases}$$
 (2)

Amplitude shift keying is combined with phase shift keying to create hybrid modulation such as the quadrature amplitude modulation (QAM). In communication systems the three important parts are the baseband (information to be transmitted), the medium (various types of wired transmission, or wireless), and the carrier signal [2-10]. The carrier is a function of the medium and the communication requirements. The medium dictates the type of carrier to be chosen. Except for cases like the Pulse Code Modulation where there is no carrier and the information and transmission are both digital, usually when we talk about a digital communication system we talk about digital information over an analog medium. Analog signals today are the music and voice transmitted via AM and FM radios, just about everything else is digital.

## 2 Background Information

Often times communication systems are placed in one of three categories, namely: power efficient, or bandwidth efficient, or cost efficient. Power efficient related to reliably send information at the lowest realistic level. Bandwidth efficiency is the ability to transmit the maximum amount of data at the minimum bandwidth possible. Cost efficiency is the ability to produce a communication system at the lowest price possible. Power efficient becomes a critical factor in the designing of hand held cellular phones,

and tablets, because they have to run on batteries. Cost is also of high priority so that more users can afford to buy them. These systems are not bandwidth efficient. In the case of digital microwaves radios for example, the highest priority is low bit error rate and bandwidth efficiency. Power is not a problem since they have plenty of power available, receiver cost is also not of great concern since they do not need to built a relatively large number of them, With the increase of communication systems competing for using the radio frequency spectrum, the radio spectrum has become very valuable and operators who do not use the spectrum efficiently could lose their existing licenses or lose out in the competition for new ones. Digital modulation has the advantage over analog modulation because it provides quick system availability, better communication quality, since it includes forward error correction and automatic repeat request, better bandwidth utilization and therefore more information capacity, higher data security, and authentication. In general analog transmitters and receivers are simpler than digital transmitters and receivers. Since digital transmitters are more complex than the analog ones they are more difficult to design, build, and test. Digital modulation has recently dominated communications. with the most popular modulation methods QPSK (Quadrature Phase Shift Keying), FSK (Frequency Shift Keying), MSK (Minimum Shift Keying), and QAM (Quadrature Amplitude Modulation). Multiplexing is a feature in many new digital systems. The most dominant multiplexing methods are TDMA (Time Division Multiple Access), and CDMA (Code Division Multiple Access), which allow to different signals to be separated from one another. The signal bandwidth for the communication channel depends on the symbol rate rather than the bit rate. In the BPSK (Binary Phase Shift Keying) which is used in the deep space telemetry, cable modems, and paging a symbol consists of one bit, the transmitted bit is 1 if the phase is 0, and 0 if the phase is  $\pi$ . In the QPSK (Quad phase shift keying), a symbol consists of two bits 00 at  $\frac{\pi}{4}$ , 10 at  $\frac{\pi}{4} + \frac{\pi}{2}$ , 11 at  $\frac{\pi}{4} + \pi$  01 at  $\frac{3\pi}{4} + \pi$ . These systems are used by Satellite, CDMA, NADC, TETRA, PHS, LMDS, DVB-S, cable modems, and TFTS. used by 8PSK satellite and aircraft.16 QAM used by microwave digital radio, modems, DVB-C, DVB-T, 32 QAM used by terrestrial microwave, 64 QAM used by DVB-C, modems, broadband setup boxes, MMDS, and 256 QAM used by modems, DVB-C, and Digital Video. Wireless communications is very widely used. From the infrared wireless transmission used as a remote control to change the television channels, to television, radio, wireless telephony, wireless networks, robot communication, satellite communication, and so many other applications, military and civilian. To accommodate all these digital wireless communication application there is a large number of modulation methods and wireless communication protocols. The IEEE 802.11 wireless transmission protocol includes several standards. Some of these standards are 802.11n

variance

$$\sigma^2 = E(\theta - \theta_0)^2 = \frac{1}{2a^2} \left[ \frac{\pi^2}{6} - 1 \right]$$
 (5)

and points of inflection:

$$\theta_0 + \frac{\pi}{4a}, \theta_0 - \frac{\pi}{4a} \tag{6}$$

Proof

First we will prove that  $f(\theta) = \frac{2a}{\pi} cos^2 (a(\theta - \theta_0))$ , where

$$\theta_0 - \frac{\pi}{2a} < \theta < \theta_0 + \frac{\pi}{2a} \tag{7}$$

is a probability function. Indeed  $f(\theta) > 0$  for every

$$\theta_0 - \frac{\pi}{2a} < \theta < \theta_0 + \frac{\pi}{2a} \tag{8}$$

and the

$$\int_{\theta_0 - \frac{\pi}{2a}}^{\theta_0 + \frac{\pi}{2a}} f(\theta) d\theta = \frac{2a}{\pi} \int_{\theta_0 - \frac{\pi}{2a}}^{\theta_0 + \frac{\pi}{2a}} \cos^2(a(\theta - \theta_0)) d\theta$$
$$= \frac{2a}{\pi} \int_{\theta_0 - \frac{\pi}{2a}}^{\theta_0 + \frac{\pi}{2a}} \frac{1 + \cos(2a(\theta - \theta_0))}{2} d\theta$$

$$= \frac{a}{\pi}(\theta_0 + \frac{\pi}{2a} - \theta_0 + \frac{\pi}{2a}) = 1(9)$$

The function  $f(\theta)$  is positive in its domain, and integrates to 1, therefore it is a probability density function. The first moment is:

$$E(\theta) = \frac{2a}{\pi} \int_{\theta_0 - \frac{\pi}{2a}}^{\theta_0 + \frac{\pi}{2a}} \theta * \cos^2(a(\theta - \theta_0)) d\theta = \frac{2a}{\pi} \int_{\theta_0 - \frac{\pi}{2a}}^{\theta_0 + \frac{\pi}{2a}} \theta * \frac{1 + \cos(2a(\theta - \theta_0))}{2} d\theta$$
$$= \frac{a}{2\pi} [(\theta_0 + \frac{\pi}{2a})^2 - (\theta_0 - \frac{\pi}{2a})^2] + 0 = \theta_0$$

The second moment is:

$$\begin{split} E(\theta^2) &= \frac{2a}{\pi} \int_{\theta_0 - \frac{\pi}{2a}}^{\theta_0 + \frac{\pi}{2a}} \theta^2 * cos^2(a(\theta - \theta_0)) d\theta \\ &= \frac{2a}{\pi} \int_{\theta_0 - \frac{\pi}{2a}}^{\theta_0 + \frac{\pi}{2a}} \theta^2 * \frac{1 + cos(2a(\theta - \theta_0))}{2} d\theta \\ &= \frac{a}{\pi} \int_{\theta_0 - \frac{\pi}{2a}}^{\theta_0 + \frac{\pi}{2a}} \theta^2 d\theta + \frac{a}{\pi} \int_{\theta_0 - \frac{\pi}{2a}}^{\theta_0 + \frac{\pi}{2a}} \theta^2 cos(2a(\theta - \theta_0)) d\theta \end{split}$$

$$= \frac{a}{3\pi} ((\theta_0 + \frac{\pi}{2a})^3 - (\theta_0 - \frac{\pi}{2a})^3) + \frac{1}{2\pi} \int_{\theta_0 - \frac{\pi}{2a}}^{\theta_0 + \frac{\pi}{2a}} \theta^2 dsin(2a(\theta - \theta_0))$$

$$= \theta_0^2 + \frac{\pi^2}{12a^2} + 0 - \frac{1}{2\pi} \int_{\theta_0 - \frac{\pi}{2a}}^{\theta_0 + \frac{\pi}{2a}} 2\theta sin(2a(\theta - \theta_0))d\theta$$

$$= \theta_0^2 + \frac{\pi^2}{12a^2} - \frac{1}{2a^2}. \quad (11)$$

from the above equation we have:

$$\sigma_{\theta}^{2} = E(\theta - \theta_{0})^{2} = E(\theta^{2}) - E^{2}(\theta) = \frac{\pi^{2}}{12a^{2}} - \frac{1}{2a^{2}} = \frac{1}{2a^{2}} \left[\frac{\pi^{2}}{6} - 1\right]$$
 (12)

The first derivative of  $f(\theta)$  is:

$$f'(\theta) = -\frac{2a^2}{\pi} 2\cos(a(\theta - \theta_0))\sin(a(\theta - \theta_0))$$
 (13)

The second derivative is:

$$f''(\theta) = -\frac{4a^3}{\pi} cos(2a(\theta - \theta_0)) \tag{14}$$

the roots of the second derivative are:

$$\theta = \theta_0 + \frac{\pi}{4a}$$

$$\theta = \theta_0 - \frac{\pi}{4a}$$
(15)

The second derivative is negative for values

$$\theta_0 - \frac{\pi}{4a} < \theta < \theta_0 + \frac{\pi}{4a} \tag{16}$$

is positive for

$$\theta > \theta_0 + \frac{\pi}{4a}$$

$$\theta < \theta_0 - \frac{\pi}{4a}$$

$$(17)$$

and zero for

$$\theta = \theta_0 + \frac{\pi}{4a}$$

$$\theta = \theta_0 - \frac{\pi}{4a}$$
(18)

therefore the points

$$\theta = \theta_0 + \frac{\pi}{4a}$$

$$\theta = \theta_0 - \frac{\pi}{4a}$$

are points of inflection. Which means that  $f(\theta)$  is bell shaped with the mean and the mode at  $\theta_0$  and is symmetric about  $\theta_0$ .

The parameter a in the above circular probability function is inversely proportional to the noise. The higher the noise the smaller the parameter a. When the parameter  $a=\frac{1}{2}$ then the domain of the circular distribution is  $\theta_0-\pi<\theta<\theta_0+\pi$  and covers the full circle. On the other hand when the parameter a=2 then  $\theta_0-\frac{\pi}{4}<\theta<\theta_0+\frac{\pi}{4}$  and for larger values of the parameter a the probability distribution converges to a Dirac delta function.

Theorem 2. Let  $\phi$  be any angle in the interval  $[\theta_0 - \frac{\pi}{4a}, \theta_0 + \frac{\pi}{4a}]$  Let  $\theta$  be a random variable with probability density function  $f(\theta)$  then the:

$$p(\theta > \phi + \theta_0) = p(\theta < \theta_0 - \phi) = \frac{1}{2} - \frac{a\phi}{\pi} - \frac{\sin(2a\phi)}{2\pi}$$
 (19)

Proof

$$P(\theta > \phi + \theta_0) = \frac{2a}{\pi} \int_{\phi + \theta_0}^{\theta_0 + \frac{\pi}{2a}} \cos^2(a(\theta - \theta_0)) d\theta$$

$$= \frac{a}{\pi} \int_{\theta_0 + \phi}^{\theta_0 + \frac{\pi}{2a}} (1 + \cos(2a(\theta - \theta_0))) d\theta = \frac{a}{\pi} (\frac{\pi}{2a} - \phi) - \frac{1}{2\pi} \sin(2a\phi) =$$

$$\frac{1}{2} - \frac{a\phi}{\pi} - \frac{\sin(2a\phi)}{2\pi} \quad (20)$$

$$P(\theta < \theta_{0} - \phi) = \frac{2a}{\pi} \int_{\theta_{0} - \frac{\pi}{2a}}^{\theta_{0} - \phi} \cos^{2}(a(\theta - \theta_{0}))d\theta = \frac{a}{\pi} \int_{\theta_{0} - \frac{\pi}{2a}}^{\theta_{0} - \phi} (1 + \cos(2a(\theta - \theta_{0})))d\theta$$
$$= \frac{a}{\pi} (\frac{\pi}{2a} - \phi) + \frac{1}{2\pi} \sin(2a(-\phi)) - \frac{1}{2\pi} \sin((-\pi)) = \frac{1}{2} - \frac{a\phi}{\pi} - \frac{\sin(2a\phi)}{2\pi}$$

The above theorem has applications in calculating the probability of phase shift due to noise in the various digital modulation schemes. Thus the binary phase shift keying if the transmitted bit is 1 the phase is 0. If due to noise there is a shift in the phase in the positive or negative direction by more than  $\frac{\pi}{2}$  then the demodulator would register the transmitted bit as

0. The probability for that to happen is  $P((\theta - \theta_0 < -0.5\pi) + P(\theta - \theta_0 >$  $(0.5\pi) = 2 * (0.5(1-a) - \frac{\sin(a\pi)}{2\pi})$ . For a=0.5 this probability is 2 \* (0.25 - $1/(2\pi)$ ) and the probability for a shift in either direction is  $0.5-1/\pi$ , for a= 1 the probability is 0. In the QPSK (Quad phase shift keying), a symbol consists of two bits 00 at  $\frac{\pi}{4}$ , 10 at  $\frac{\pi}{4} + \frac{\pi}{2}$ , 11 at  $\frac{\pi}{4} + \pi$  01 at  $\frac{3\pi}{4} + \pi$ . If there is a shift greater than  $\frac{\pi}{4}$  either in the positive or negative direction due to noise then the decoder will register the incorrect symbol. Thus the probability for the decoder to register the incorrect symbol is:  $P((\theta - \theta_0 <$  $-0.25\pi$ ) +  $P(\theta - \theta_0 > 0.25\pi) = 2.0 * (0.5 - \frac{a}{4} - \frac{\sin(0.5a\pi)}{2\pi})$  For a=0.5 this probability is  $P((\theta - \theta_0 < -0.25\pi) + P(\theta - \theta_0 > 0.25\pi) = 2(\frac{3}{8} - \frac{\sqrt{2}}{4\pi})$ , for a=1  $P((\theta - \theta_0 < -0.25\pi) + P(\theta - \theta_0 > 0.25\pi) = 2(0.5 - 0.25 - \frac{1}{2\pi}) = 0.5 - \frac{1}{\pi}$ . and for a= 2 we have  $P((\theta - \theta_0 < -0.25\pi) + P(\theta - \theta_0 > 0.25\pi) = 0$ . For every increase in the number of bits per symbol by one, the angle between two symbols decreases by  $\frac{1}{2}$ . Thus if we transmit symbols with 3 bits each, the first symbol consists of the 3-bits 000, the second symbol which is 45 degrees to the left of the first symbol on the circle is 001, the third symbol which 45 degrees to the left of the second symbol is 011, and so on until all 8-symbols are placed on the circle. The probability of shifting during transmission increases as the number of bits per symbol increase. Thus our approach is to detect the number of errors during transmission and as these number of errors increase to reduce the number of bits per symbol transmitted until the number of transmission errors decreases and the throughput increases. The analytics we provide here could be programed in a DSP, or an FPGA, or be part of any ASICS, and can be used to find the optimal number of bits per symbol to be transmitted and keep adjusting real time.

### 4 Conclusion and Future Work

Digital wireless communication is enormous area that is rapidly growing. Digital communication is a field in which theoretical ideas have had an unusually powerful impact on system design and practice. Digital communication takes better advantage of the bandwidth than analog communication, it requires less power than analog to transmit. It is however far more sophisticated than analog transmission, and it is a software hardware problem in the category of client, server. The advantage of that is, that it could include forward error correction and automatic repeat request, compression, and cryptography. The signal can be compressed and treated digitally in a way that during transmission has the statistical signature of noise, so when intercepted the adversary thinks that it is noise. More work is needed in this area to get a better understanding of the noise sources, their effect on the transmitted signal, and ways to overcome them. The most difficult

thing is to reproduce in the laboratory all the noise factors and their effect on the transmitted signal. Our theory leads to a real time algorithm that can be embedded on a chip for real time adjustment of the number of bits per symbol to be transmitted, and thus provide more efficient modulation.

#### References

- [1] R. Gallager, "Course Materials for 6.450 Principals of Digital Communications I", Fall 2006. MIT OpenCourseWare (http://ocw.mit.edu/), Massachusetts Institute of Technology.
- [2] S. G. Wilson, "Digital Modulation and Coding," C. Bauer, M. Zitter-bart," Prentice Hall, 1996 pp. 1-677
- [3] J. Kurzweil "An Introduction to Digital Communications,", J. Wiley, 2000, pp. 1-534.
- [4] J. Proakis, "Digital Communications," Second Edition, McGraw-Hill Book Co., 1989, pp. 1-905.
- [5] ITU-T Focus Group on Future Networks (FG FN) OD-66 Draft Deliverable on "Overview of Energy Savings of Networks," October 2010.
- [6] S. Kent, C. Lynn, and K. Seo, "Secure Border Gateway Protocol (S-BGP)," IEEE J. Sel. Areas Commun., vol. 18, no. 4, pp. 582-592,
- [7] A. Muller, C. Carle, and A. Klenk, "Behavior and classification of NAT devices and implications for NAT traversal," IEEE Network, Vol. 22, no 5, pp 14-19,
- [8] S. Novaczki, L. Bokor, G. Jeney and S. Imre, "Design and Evaluation of a Novel HIP Based Network Mobility Protocol," Journal of Networks (JNW), vol. 3, no 1, pp. 10-24, 2008.
- [9] R. Wakikawa, V. Devarapalli, G. Tsirtsis, T. Ernst, and K. Nagami, "Multiple Care-of Addresses Registration," RFC 3775, June, 2004.
- [10] E. A. Yfantis, and A. Fayed, "Authentication and Secure Robot Communication," International Journal of Robotic Systems, Vol. 11:10 2014, pp. 1-6.