# Eccentric Connectivity Index of Certain Graphs

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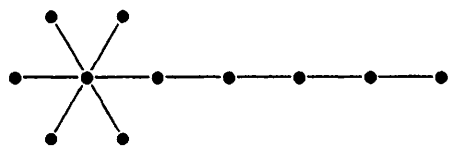
Abstract. The eccentric connectivity index of the molecular graph G was proposed by Sharma, Goswami, and Madan in 1997[17]. This index is defined as $\xi^c(G) = \sum_{v \in V(G)} \deg(v)\, ec(v)$, where $\deg(v)$ is the degree of vertex v in G and eccentricity $ec(v)$is the largest distance between $u$ and any other vertex $v$ of G. Thus, in this paper, we establishedthe general formulas for the eccentric connectivity index of joining special graph to their paths and of joining two different graphs by a path. Proofs were also provided.

Keywords: Eccentric Connectivity Index, Eccentricity, Special graphs.

## 1.Introduction

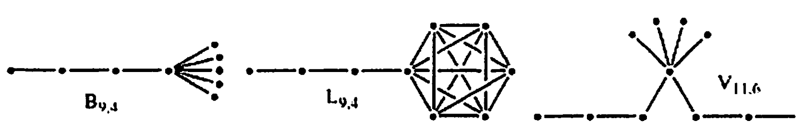
Topological indices, such as the eccentric connectivity index $\xi^c(G)$, are theoretical graph invariants designed to find relationships between the structure of chemical molecules and their physical properties. These indices have been used for isomer discrimination, chemical documentation, drug design, quantitative structure versus activity (or property) relationships (QSAR/QSPR's), combinatorial library design, and toxicology hazard assessments [4,5,7,13]. In pharmaceutical research, QSAR information is used to select the most promising compounds for a desired property.Thus, the number of compounds that need to be synthesized is decreased in designing new drugs[3,9,14]. Many topological indices have been defined and used. The Wiener index, Hosoya index, Randi´c's molecular connectivity index, Zagreb group parameters,Balaban's index and eccentric connectivity index have been previously investigated.This topological model has been shown to give a high degree of predictability in determining pharmaceutical properties and may provide insights about the development of safe and potent anti-HIV compounds. Ranjini and Lokesha [16] determined the eccentric connectivity index of the subdivision graph of complete graphs, tadpole graphs and wheel graphs. Yun Gao et al. [8] determined the eccentric connectivity index and augmented the eccentric connectivity index of fan graphs, wheel graphs, gear fan graphs, gear wheel graphs, and their r-corona graphs.

AleksandarIlic and Ivan Gutman [11]proved that the broom highest eccentric connectivity index among trees with a fixed maximum vertex degree. Such trees are characterized with minimum eccentric connectivity index(the broom graph is a tree consisting of a star $S_{\Delta+1}$ and a path of length $(n-\Delta-1)$ attached to a pendent vertex of the star) as shown in the figure below:



The broom $B_{11,6}$ .

Morgan et al. [15] obtained the exact lower bound of eccentric connectivity index in terms of order and showed that this bound is sharp. Moreover,they calculated the eccentric connectivity index for three classes of graphs, which are important in our theorems. The broom graph $B_{n,d}$consists of a path $P_d$, together with (n−d) end vertices that are adjacent to the same terminal vertex of $P_d$. The lollipop graph $L_{n,d}$ is obtained from a complete graph $K_{n-d}$ and a path $P_d$, by joining one of the terminal vertices of path to any vertex of the complete graph .The volcano graph $V_{n,d}$is the graph obtained from a path $P_{d+1}$ and a set S of( n − d − 1) vertices, by joining each vertex in S to a central vertex of $P_{d+1}$ , as shown in the figure below:




Graphs: $B_{9,4}$ , $L_{9,4}$ ,$V_{11,6}$

They found that the general formula for the eccentric connectivity index for this graphs as follows:

$$\xi^c(B_{n,d}) = \begin{cases} 2dn - n - \dfrac{d^2}{2} - d + 1\,; & for\ d\ even \\ \dfrac{1}{2}(3 - 2d - d^2 - 2n + 4dn); & for\ d\ odd; \end{cases}$$

$$\xi^c(L_{n,d}) = \begin{cases} \dfrac{1}{2}(2 - 2d + d^2 + 2d^3 - 2n + 2dn - 4d^2n + 2dn^2); & d\ even \\ \dfrac{1}{2}(3 - 2d + d^2 + 2d^3 - 2n + 2dn - 4d^2n + 2dn^2); & d\ odd \end{cases}$$

$$\xi^c(V_{n,d}) = \begin{cases} nd + n + \frac{d^2}{2} - 2d - 1; & for\ d\ even \\ nd + 2n + \frac{d^2}{2} - 3d - \frac{3}{2}; & for\ d\ odd. \end{cases}$$

Mohammad Ali Iranmanesh and RoghayehHafezieh [12] found the general formula for the eccentric connectivity index of families of graphs made by placing a complete graph($K_m$) instead of each vertex in path ($P_n$). Moreover, they introduced the general formula for the eccentric connectivity index of families of graphs made by placing cycle ($C_m$) instead of each vertex in path ($P_n$).HongboHuaa and Kinkar Ch. Dasb [8] compared the eccentric connectivity index and Zagreb indices for particular graph families.Libing Zhang and HongboHua [19] investigated the eccentric connectivity index of unicyclic graphs. The formulas for the eccentric connectivity index of the complete graph $K_n$, complete bipartite graph $K_{p,q}$, cycle graph $C_n$,star graph $S_n$, and path $P_n$ have been calculated independently by several authors [6,15,20]. Hence,for special classes of graphs, we obtained the following values for the eccentric connectivity index:

$$\xi^c(k_n) = n(n-1)\ ; \quad n \geq 2$$

$$\xi^c(k_{a,b}) = 4ab\ ; a,b \neq 1$$

$$\xi^c(S_n) = 3(n-1)\ ; \quad n \geq 3$$

$$\xi^c(C_n) = \begin{cases} n^2 & if\ n\ even \\ n(n-1) & if\ n\ odd. \end{cases}$$

$$\xi^c(P_n) = \begin{cases} \frac{1}{2}(3n^2 - 6n + 4); & if\ n\ even \\ \frac{3}{2}(n-1)^2\ ; & if\ n\ odd. \end{cases}$$

Studied on the eccentric connectivity index have been ongoing with current focus on nanotubes [1,2,18].In the current paper, we establishedthe general formulas for the eccentric connectivity index of graphs constructed by joining special graph to their path andby joining two different graphs by a path, and the proofs were provided. We denote the families of graphs made by joining $W_n$ to $P_r$ by A,for families of graphs made by joining fan graph ($F_n$) to path $P_r$ by B(We call this Umbrella Graph) and for families of graphs made by joining complete bipartite graph ($K_{n,m}$) to path $P_r$ by C. Moreover, we denote the families of graphs made by joining $K_n$ and $K_m$ to $P_r$ by D ;for families of graphs made by joiningW$_n$andW$_m$ to $P_r$ by E; forfamilies of graphs made by joining $F_n$ and $F_m$ to $P_r$ by G; and forfamilies of graphs made by joining $K_{n,m}$ and $K_{a,b}$ to $P_r$ by H.

## 2.CONSTRUCTION OF SPECIAL GRAPHS AND INTERNAL INDEICES

In this section we construct special graphs A, B, C, D, E, G and H as defined below, and determine their internal indices.

2.1.Families of graphs made by joining $W_n$ to $P_r$;

Graph A is constructed by joining the end-vertex of a path to any vertex on the circumference of a wheel graphs. In Fig.1, we have a graph A constructed in such a way for a path of order r and wheel graph of order n.

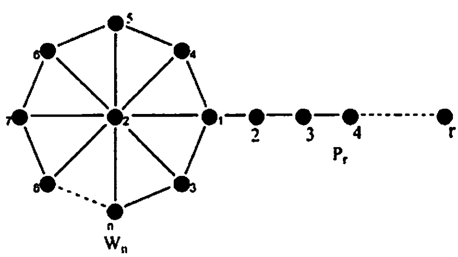



Fig.1. graph A

Definition 2.2.Let A be the graph constructed by joining the end-vertex of $P_r$ and an outside vertex of $W_n$as in Fig.1. We define its internal index$A_{n,r}$as follows:

$$A_{n,r} = d_{v_1}.ec(v_1) + d_{v_2}.ec(v_2) + \sum_{i=3}^{4} d_{v_i}.ec(v_i) + \sum_{i=5}^{n} d_{v_i}.ec(v_i)$$

$$= 4(r-1) + (n-1)r + 6r + (n-4)(3)(r+1)$$

$$= 4nr + 3n - 3r - 16.$$

2.3.Families of graphs made by joining $F_n$to $P_r$;

Graph B is constructed by joining the end-vertex of a path to center vertex in the Fan graph. In Fig.2, we have a graph B constructed in such a way for a path of order r and Fan graph of order n.

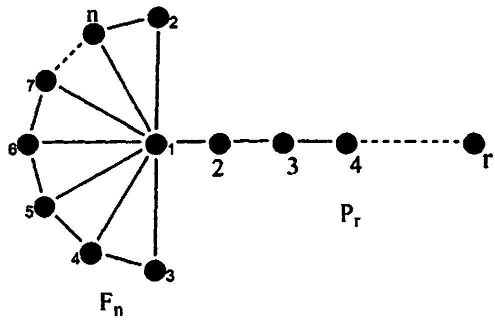



Fig.2. graph B

Definition 2.4. Let B be the graph constructed by joining the end-vertex of $P_r$ and center vertex of $F_n$ as in Fig.2. We define its internal index$B_{n,r}$as follows:

$$B_{n,r} = d_{v_1}.ec(v_1) + \sum_{i=2}^{3} d_{v_i}.ec(v_i) + \sum_{i=4}^{n} d_{v_i}.ec(v_i)$$

$$= (n)(r-1) + (2)(2)(r) + (n-3)(3)(r)$$

$$= 4nr - n - 5r.$$

2.5.Families of graphs made by joining $K_{n,m}$ to $P_r$;

Graph C is constructed by joining the end-vertex of a path to any terminal vertex of set vertices n in the complete bipartite graph. In Fig.3,we have a graph C constructed in such a way for a path of order r and complete bipartite graph of order$n + m$.

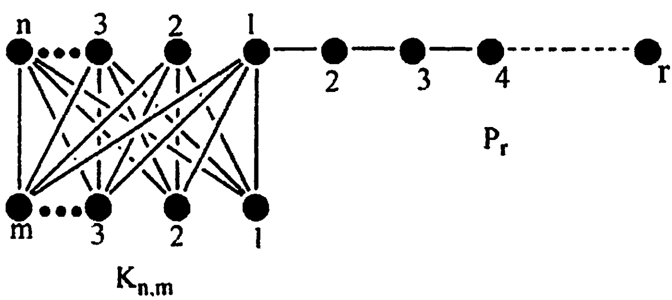



Fig.3. graph C

**Definition 2.6.** Let C be the graph constructed by joining the end-vertex of $P_r$ and any terminal vertex of set vertices n in$K_{n,m}$ as in Fig.3. We define its internal index$C_{n,m,r}$as follows:

$$C_{n,m,r} = d_{v_1}.ec(v_1) + \sum_{i=2}^{n} d_{v_i}.ec(v_i) + \sum_{j=1}^{m} d_{v_j}.ec(v_j)$$

$$= (m+1)(r-1) + (n-1)(m)(r+1) + mnr$$

$$= 2nmr + nm + r - 2m - 1.$$

2.7.Families of graphs made by joining $K_n$ and $K_m$ to $P_r$;

Graph D is constructed by joining each end-vertex of the path to any vertex of complete graph. In Fig.4, we have a graph D constructed in such a way for a path of order r and two complete graphs of order n, m respectively.

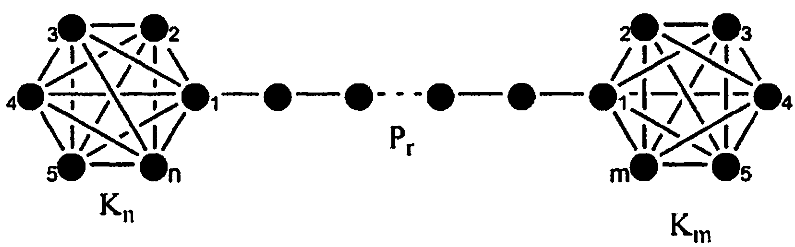



Fig.4. graph D

**Definition 2.8.** Let D be the graph constructed by joining each end-vertex of $P_r$ and any vertex of $K_n$ and $K_m$ as in Fig.4. For k=n, m we define $D_{k,r}$as follows:

$$D_{k,r} = d_{v_1}.ec(v_1) + \sum_{i=2}^{k} d_{v_i}.ec(v_i)$$

$$= kr + (k-1)(k-1)(r+1)$$

$$= kr + (k-1)^2(r+1)$$

2.9.Families of graphs made by joining $W_n$ and $W_m$ to $P_r$;

Graph E is constructed by joining each end-vertex of a path to any vertex on the circumference of a wheel graph. In Fig.5, we have a graph E constructed in such a way for a path of order r and two wheel graphs of order n, m respectively.

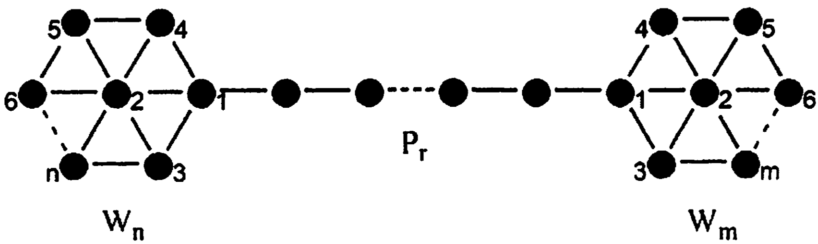



Fig.5. graph E

Definition 2.10. Let E be the graph constructed by joining each end-vertex of $P_r$ and any outside vertex of $W_n$ and$W_m$ as in Fig.5.For k=n, m we define $E_{k,r}$ as follows;

$$E_{k,r} = d_{v_1}.ec(v_1) + d_{v_2}.ec(v_2) + \sum_{i=3}^{4} d_{v_i}.ec(v_i) + \sum_{i=5}^{k} d_{v_i}.ec(v_i)$$

$$= 4(r+1) + (k-1)(r+2) + 2(3)(r+2) + (k-4)(3)(r+3)$$

$$= 4kr + 11n - 3r - 22.$$

2.11.Families of graphs made by joining $F_n$ and $F_m$ to$P_r$;

Graph G is constructed by joining each end-vertex of the path to center vertex of Fan graph. In Fig.6, we have a graph G constructed in such a way for a path of order r and two Fan graphs of order n, m respectively.

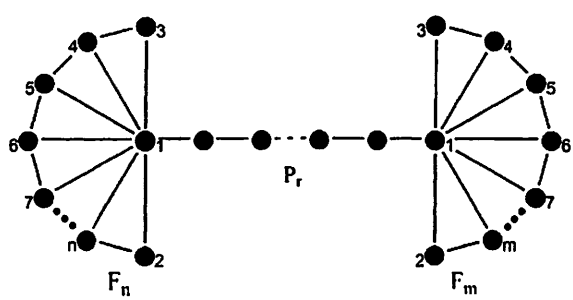



Fig.6. graph G

Definition 2.12.Let G be the graph constructed by joining each end-vertex of $P_r$ and center vertex of $F_n$ and$F_m$ as in Fig.6. For k=n, m we define $G_{k,r}$as follows;

$$G_{n,r} = k(r+1) + \sum_{i=2}^{3} d_{v_i}.ec(v_i) + \sum_{i=4}^{k} d_{v_i}.ec(v_i).$$

$$= k(r+1) + (2)(2)(r+1) + (k-3)(3)(r+1)$$

$$= 4kr + 3k - 5r - 5.$$

2.13.Families of graphs made by joining $K_{n,m}$ and $K_{a,b}$ to $P_r$;

Graph H is constructed by joining each end-vertex of a path to any terminal vertex of sets vertices n and a in the two complete bipartite graphs.In Fig.7, we have a graph H constructed in such a way for a path of order r and two complete bipartite graphs of order$n + m$, $a + b$.

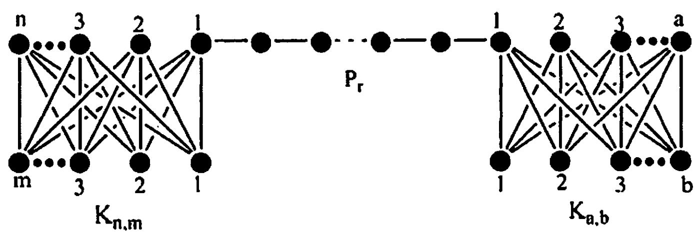



Fig.7. graph H

Definition 2.14. Let H be the graph constructed by joining each end-vertex of $P_r$ and any terminal vertex of sets vertices n and a respectivelyin $K_{n,m}$ and$K_{a,b}$respectively as Fig.7. For k=n, a and s=m, b we define $H_{k,s,r}$as follows:

$$H_{k,s,r} = d_{v_1}.ec(v_1) + \sum_{i=2}^{k} d_{v_i}.ec(v_i) + \sum_{j=1}^{s} d_{v_j}.ec(v_j)$$

$$= (s+1)(r+1) + (k-1)(s)(r+3) + ks(r+2)$$

$$= 2ksr + 5ks - 2s + r + 1.$$

Note. For a path of order r we have;$\xi^c(p_r) =$
$$\begin{cases} \frac{3}{2}r^2 - 3r + 2 \ ; & if\ r\ is\ even. \\ \frac{3}{2}r^2 - 3r + \frac{3}{2} \ ; & if\ r\ is\ odd. \end{cases}$$

## 3.THE ECCENTRIC CONNECTIVITY INDEX OF SOME SPECIALGRAPHS

In this section we establish the general formulas for eccentric connectivity index of the graphs A, B, C, D,E,G and H as follows:

Theorem 3.1.Let A be the graph in Fig.1. Then the eccentric connectivity index of this graph is as follows;

1.$\xi^c(A) = \frac{3}{2}r^2 + 4nr + 3n - 5r - 13$, $\quad if\ r\ is\ even\ ;\ r \geq 4, n \geq 5$.

2.$\xi^c(A) = \frac{3}{2}r^2 + 4nr + 3n - 5r - \frac{27}{2}$, $\quad if\ r\ is\ odd\ ;\ r \geq 3, n \geq 5$.

Proof.According to the definition of $A_{n,r}$, to compute for the eccentric connectivity index of A, this index must be computed for the vertices on the left of the path of lengthr $+ 2$ vertices. However, in this method, three vertices of $W_n$, are also computed to determine the indices of the vertices of the path. Thus, these three vertices must be subtracted from the first index. Therefore, we obtain the following:

1. If r is even, then

$$\xi^c(A) = \xi^c(p_{r+2}) + A_{n,r} - (5r - 1)$$

$$= \frac{3}{2}(r + 2)^2 - 3(r + 2) + 2 + (4nr + 3n - 3r - 16) - (5r - 1)$$

$$= \frac{3}{2}r^2 + 4nr + 3n - 5r - 13.$$

2. If r is odd, then

$$\xi^c(A) = \xi^c(p_{r+2}) + A_{n,r} - (5r - 1)$$

$$= \frac{3}{2}(r + 2)^2 - 3(r + 2) + \frac{3}{2} + (4nr + 3n - 3r - 16) - (5r - 1)$$

$$= \frac{3}{2}r^2 + 4nr + 3n - 5r - \frac{27}{2}. \qquad \blacksquare$$

Corollary 3.2.$If\ n \geq 5, r = 2\ then\ \xi^c(A) = 11n - 17.$

Corollary 3.3.$If\ n = 4, r \geq 2\ then\ \xi^c(A) = \begin{cases} \frac{3}{2}r^2 + 11r - 1; & if\ r\ is\ even. \\ \frac{3}{2}r^2 + 11r - \frac{3}{2}; & if\ r\ is\ odd. \end{cases}$

**Theorem 3.4.** Let B be the graph in Fig.2 . Then the eccentric connectivity index of this graph is as follows;

1. $\xi^c(B) = \left(\frac{3}{2}r^2 + 4nr + 2\right) - (n + 8r);$ *if r is even*

2. $\xi^c(B) = \left(\frac{3}{2}r^2 + 4nr + \frac{5}{2}\right) - (n + 8r);$ *if r is odd.*

**Proof.** According to the definition of $B_{n,r}$ , to compute for the eccentric connectivity index of B, this index must be computed for the vertices on the left of the path of length $r + 1$ vertices. However, in this method, two vertices of $F_n$, are also computed to determine the indices of the vertices of the path. Thus, these two vertices must be subtracted from the first index. Therefore, we obtain the following:

1. If r is even, then

$$\xi^c(B) = \xi^c(p_{r+1}) + B_{n,r} - (3r - 2)$$
$$= \frac{3}{2}(r + 1)^2 - 3(r + 1) + \frac{3}{2} + (4nr - n - 5r) - (3r - 2)$$
$$= \left(\frac{3}{2}r^2 + 4nr + 2\right) - (n + 8r).$$

2. If r is odd, then

$$\xi^c(B) = \xi^c(p_{r+1}) + B_{n,r} - (3r - 2)$$
$$= \frac{3}{2}(r + 1)^2 - 3(r + 1) + 2 + (4nr - n - 5r) - (3r - 2)$$
$$= \left(\frac{3}{2}r^2 + 4nr + \frac{5}{2}\right) - (n + 8r).$$ ∎

**Theorem 3.5.** Let C be the graph in Fig.3 . Then the eccentric connectivity index of this graph is as follows;

1. $\xi^c(C) = \frac{3}{2}r^2 + 2nmr + nm - 2m - r + 2;$ *if r is even* $; r \geq 4, n > 1.$

2. $\xi^c(C) = \frac{3}{2}r^2 + 2nmr + nm - 2m - r + \frac{3}{2};$ *if r is odd* $; n > 1.$

**Proof.** According to the definition of $C_{n,m,r}$ , to compute for the eccentric connectivity index of C, this index must be computed for the vertices on the left of the path of length $r + 2$ vertices. However, in this method, three vertices of

$K_{n,m}$, are also computed to determine the indices of the vertices of the path. Thus, these three vertices must be subtracted from the first index. Therefore, we obtain the following:

1. If r is even, then

$$\xi^c(C) = \xi^c(p_{r+2}) + C_{n,m,r} - (5r-1)$$

$$= \frac{3}{2}(r+2)^2 - 3(r+2) + 2 + (2nmr + nm + r - 2m - 1) - (5r-1)$$

$$= \frac{3}{2}r^2 + 2nmr + nm - 2m - r + 2\,.$$

2. If r is odd, then

$$\xi^c(C) = \xi^c(p_{r+2}) + C_{n,m,r} - (5r-1)$$

$$= \frac{3}{2}(r+2)^2 - 3(r+2) + \frac{3}{2} + (2nmr + nm + r - 2m - 1) - (5r-1)$$

$$= \frac{3}{2}r^2 + 2nmr + nm - 2m - r + \frac{3}{2}. \quad ■$$

Corollary 3.6. *If* $r = 2$, *then* $\xi^c(C) = 5nm - m + 5$.

Corollary 3.7. *If* $n = 1$, *then* $\xi^c(C) = \begin{cases} \frac{3}{2}r^2 + 2mr - m - 3r + 1 \;; & \text{if } r \text{ is even.} \\ \frac{3}{2}r^2 + 2mr - m - 3r + \frac{3}{2} \;; & \text{if } r \text{ is odd.} \end{cases}$

Theorem 3.8. Let D be the graph in Fig.4 . Then the eccentric connectivity index of this graph is as follows;

1. $\xi^c(D) = \frac{3}{2}r^2 + r(n+m-3) + (r+1)(n^2+m^2-2n-2m+2);$

*if r is even.*

2. $\xi^c(D) = \frac{3}{2}r^2 - \frac{1}{2} + r(n+m-3) + (r+1)(n^2+m^2-2n-2m+2);$

*if r is odd.*

Proof. According to the definition of $D_{k,r}$, to compute for the eccentric connectivity index of D, this index must be computed for the vertices on the left and right of the path of length $r + 2$ vertices. However, in this method, two vertices of $K_n$and two vertices of $K_m$, are also computed to determine the indices of the vertices of the path. Thus, these vertices must be subtracted from the first index. Therefore, we obtain the following:

1. If r is even, then

$$\xi^c(D) = \xi^c(p_{r+2}) + D_{n,r} + D_{m,r} - (6r + 2)$$

$$= \frac{3}{2}(r+2)^2 - 3(r+2) + 2 + nr + (n-1)^2(r+1)$$

$$+mr + (m-1)^2(r+1) \; - (6r+2)$$

$$= \frac{3}{2}r^2 + 3r + 2 + (r+1)(n^2 - 2n + 1 + m^2 - 2m + 1)$$

$$+nr + mr - 6r - 2$$

$$= \frac{3}{2}r^2 + r(n + m - 3) + (r+1)(n^2 + m^2 - 2n - 2m + 2)$$

2. If r is odd, then

$$\xi^c(D) = \xi^c(p_{r+2}) + D_{n,r} + D_{m,r} - (6r + 2)$$

$$= \frac{3}{2}(r+2)^2 - 3(r+2) + \frac{3}{2} + nr + (n-1)^2(r+1)$$

$$+mr + (m-1)^2(r+1) \; - (6r+2)$$

$$= \frac{3}{2}r^2 + 3r + \frac{3}{2} + (r+1)(n^2 - 2n + 1 + m^2 - 2m + 1)$$

$$+nr + mr - 6r - 2$$

$$= \frac{3}{2}r^2 - \frac{1}{2} + r(n + m - 3) + (r+1)(n^2 + m^2 - 2n - 2m + 2) \blacksquare$$

**Theorem 3.9.** Let E be the graph in Fig.5 . Then the eccentric connectivity index of this graph is as follows;

1. $\xi^c(E) = \frac{3}{2}r^2 - 7r + (4r + 11)(n + m) - 48$;   *if r is even.*

2. $\xi^c(E) = \frac{3}{2}r^2 - 7r + (4r + 11)(n + m) - \frac{97}{2}$;   *if r is odd.*

Proof.According to the definition of $E_{k,r}$ , to compute for the eccentric connectivity index of E, this index must be computed for the vertices on the left and right of the path of length $r + 4$ vertices. However, in this method, three vertices of $W_n$ and two vertices of $W_m$, are also computed to determine the indices of the vertices of the path. Thus, these vertices must be subtracted from the first index. Therefore, we obtain the following:

1. If r is even, then

$$\xi^c(E) = \xi^c(p_{r+4}) + E_{n,r} + E_{m,r} - (10r + 18)$$

$$= \frac{3}{2}(r + 4)^2 - 3(r + 4) + 2 + (4nr + 11n - 3r - 22)$$

$$+(4mr + 11m - 3r - 22) - (10r + 18)$$

$$= \frac{3}{2}r^2 - 7r + (4r + 11)(n + m) - 48.$$

2. If r is odd, then

$$\xi^c(E) = \xi^c(p_{r+4}) + E_{n,r} + E_{m,r} - (10r + 18)$$

$$= \frac{3}{2}(r + 4)^2 - 3(r + 4) + \frac{3}{2} + (4nr + 11n - 3r - 22)$$

$$+(4mr + 11m - 3r - 22) - (10r + 18)$$

$$= \frac{3}{2}r^2 - 7r + (4r + 11)(n + m) - \frac{97}{2} \blacksquare$$

Theorem 3.10. Let G be the graph in Fig.6 . Then the eccentric connectivity index of this graph is as follows;

1. $\xi^c(G) = \frac{3}{2}r^2 + (4r + 3)(n + m) - 13r - 10;$ $\quad$ *if r is even.*

2. $\xi^c(G) = \frac{3}{2}r^2 + (4r + 3)(n + m) - 13r - \frac{21}{2};$ $\quad$ *if r is odd.*

Proof.According to the definition of $G_{k,r}$ , to compute for the eccentric connectivity index of G, this index must be computed for the vertices on the left and right of the path of length $r + 2$ vertices. However, in this method, two vertices of $F_n$ and two vertices of $F_m$, are also computed to determine the indices of the vertices of the path. Thus, these vertices must be subtracted from the first index. Therefore, we obtain the following:

1. If r is even, then

$$\xi^c(G) = \xi^c(p_{r+2}) + G_{n,r} + G_{m,r} - (6r + 2)$$

$$= \frac{3}{2}(r+2)^2 - 3(r+2) + 2 + (4nr + 3n - 5r - 5)$$

$$+(4mr + 3m - 5r - 5) - (6r + 2)$$

$$= \frac{3}{2}r^2 + (4r + 3)(n + m) - 13r - 10.$$

2. If r is odd, then

$$\xi^c(G) = \xi^c(p_{r+2}) + G_{n,r} + G_{m,r} - (6r + 2)$$

$$= \frac{3}{2}(r+2)^2 - 3(r+2) + \frac{3}{2} + (4nr + 3n - 5r - 5)$$

$$+(4mr + 3m - 5r - 5) - (6r + 2)$$

$$= \frac{3}{2}r^2 + (4r + 3)(n + m) - 13r - \frac{21}{2}.$$

Theorem 3.11. Let H be the graph in Fig.7 . Then the eccentric connectivity index of this graph is as follows:

$$\xi^c(H) = \frac{3}{2}r^2 + r + (2r + 5)(nm + am) - 2(m + b) - 2; \;\; r\ even; n, a > 1.$$

$$\xi^c(H) = \frac{3}{2}r^2 + r + (2r + 5)(nm + am) - 2(m + b) - \frac{5}{2}; \;\; odd; \; n, a > 1.$$

Proof.According to the definition of $H_{k,s,r}$ , to compute for the eccentric connectivity index of H, this index must be computed for the vertices on the left and right of the path of length $r + 4$ vertices. However, in this method, two vertices of $K_{n,m}$and two vertices of $K_{a,b}$, are also computed to determine the indices of the vertices of the path. Thus, these vertices must be subtracted from the first index. Therefore, we obtain the following:

1. If r is even, then

$$\xi^c(H) = \xi^c(p_{r+4}) + H_{n,m,r} + H_{a,b,r} - (10r + 18)$$

$$= \frac{3}{2}(r+4)^2 - 3(r+4) + 2 + (2nmr + 5nm - 2m + r + 1)$$

$$+(2abr + 5ab - 2b + r + 1) - (10r + 18)$$

$$= \frac{3}{2}r^2 + r + (2r + 5)(nm + am) - 2(m + b) - 2.$$

2. If r is odd, then

$$\xi^c(H) = \xi^c(p_{r+4}) + H_{n,m,r} + H_{a,b,r} - (10r + 18)$$

$$= \frac{3}{2}(r + 4)^2 - 3(r + 4) + \frac{3}{2} + (2nmr + 5nm - 2m + r + 1)$$

$$+(2abr + 5ab - 2b + r + 1) - (10r + 18)$$

$$= \frac{3}{2}r^2 + r + (2r + 5)(nm + am) - 2(m + b) - \frac{5}{2}. \blacksquare$$

Corollary 3.12.*If* $n, a = 1,$ *then*

$$\xi^c(H) = \begin{cases} \frac{3}{2}r^2 + 2mr + 2br + m + b - r\,; & \text{if } r \text{ is even.} \\ \frac{3}{2}r^2 + 2mr + 2br + m + b - r - \frac{1}{2}; & \text{if } r \text{ is odd.} \end{cases}$$

## 4.CONCLUSIONS

In this paper, we focus on finding the general formulas for Eccentric Connectivity Index of some familiesof graphs made by joining special graphs topath. We also found eccentric connectivity index of two different special graphs joining by path.

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