On $(r, 2, k)$-Regular Fuzzy Graphs

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Abstract

In this paper, $(r, 2, k)$- regular fuzzy graphs and totally $(r, 2, k)$-regular fuzzy graphs are defined and $(r, 2, k)$- regular fuzzy graphs and totally $(r, 2, k)$- regular fuzzy graphs are compared through various examples. A necessary and sufficient condition under which they are equivalent is provided. Also $(r, 2, k)$-regularity on some fuzzy graphs whose underlying crisp graphs are a path on four vertices, a Barbell graph $B_{n,n}(n > 1)$ and a cycle is studied with some specific membership functions.

Key Words :degree of a vertex, regular fuzzy graphs, total degree, totally regular fuzzy graph, d_2 degree of a vertex in graphs, semiregular graphs, $(2, k)$-regular fuzzy graphs, totally (2,k)-regular fuzzy graphs.

AMS Subject Code Classification 2010: 05C12, 05C72.

1 Introduction

In 1965, Lofti A.Zadeh[12] introduced the concept of a fuzzy subset of a set as a method for representing the phenomena of uncertainty in real life situation. Azriel Rosenfeld introduced fuzzy graphs in 1975[12]. It has been growing fast and has numerous applications in various fields. Nagoor Gani and Radha [11] introduced regular fuzzy graphs, total degree and totally regular fuzzy graphs. Alison Northup introduced Semiregular graphs that we call it as $(2,k)$-regular graphs and studied some properties on $(2,k)$-regular graphs[2]. N.R. Santhi Maheswari and C. Sekar introduced d_2 of a vertex in graphs[13] and also discussed some properties on d_2 of a vertex in graphs[15] and introduced $(r,2,k)$-regular graphs and also discussed some properties on $(r,2,k)$-regular graphs[14]. Also we introduced d_2 degree of a vertex in fuzzy graphs, total d_2-degree of a vertex in fuzzy graphs, $(2,k)$-regular fuzzy graphs and totally $(2,k)$-regular fuzzy graphs.

In this paper, we introduce $(r,2,k)$-regular fuzzy graphs and totally $(r,2,k)$-regular fuzzy graphs and also discuss some properties on $(r,2,k)$-regular fuzzy graphs. We make comparative study between $(r,2,k)$-regular fuzzy graphs and totally $(r,2,k)$-regular fuzzy graphs. Then we provide a necessary and sufficient condition under which they are equivalent. Also $(r,2,k)$-regularity on fuzzy graphs whose underlying crisp graphs are a path on four vertices, a Barbell graph $B_{n,n}(n>1)$ and a cycle is studied with some specific membership functions.

2 Preliminaries

We present some known definitions and results for a ready reference to go through the work presented in this paper.

Definition 2.1. *For a given graph G, the d_2-degree of a vertex v in G, denoted by $d_2(v)$ means number of vertices at a distance two away from v.*

Definition 2.2. *A graph G is said to be $(2,k)$-regular (d_2-regular) if $d_2(v)=k$, for all v in G. We observe that $(2,k)$-regular graphs and semiregular graphs and d_2-regular graphs are same.*

Definition 2.3. *A graph G is said to be $(r,2,k)$-regular if $d(v)=r$ and $d_2(v)=k$, for all v in G.*

Definition 2.4. *A Fuzzy graph denoted by $G:(\sigma,\mu)$ on graph $G^*:(V,E)$. is a pair of functions (σ,μ) where $\sigma:V\to[0,1]$ is a fuzzy subset of a non empty set V and $\mu:V\times V\to[0,1]$ is a symmetric fuzzy relation on σ such that for all u, v in V the relation $\mu(u,v)=\mu(uv)\leq\sigma(u)\wedge\sigma(v)$ is satisfied. A fuzzy graph G is complete if $\mu(u,v)=\mu(uv)=\sigma(u)\wedge\sigma(v)$ for all $u,v\in V$ where uv denotes the edge between u and v. $G^*:(V,E)$ is called the underlying crisp graph of the fuzzy graph $G:(\sigma,\mu)$, where σ and μ are called membership functions.*

Definition 2.5. *Let $G : (\sigma, \mu)$ be a fuzzy graph. The degree of a vertex u is $d_G(u) = \sum\limits_{u \neq v} \mu(uv)$ for $uv \in E$ and $\mu(uv) = 0$ for uv not in E; this is equivalent to $d_G(u) = \sum\limits_{uv \in E} \mu(uv)$.*

Definition 2.6. *The strength of connectedness between two vertices u and v is $\mu^{\infty}(u, v) = sup\{\mu^k(u, v)/k = 1, 2, \dots\}$ where $\mu^k(u, v) = sup\{\mu(uu_1) \wedge \mu(u_1u_2) \wedge \dots \cdots \wedge \mu(u_{k-1}v)/u, u_1, u_2, \dots, u_{k-1}, v$ is a path connecting u and v of length $k\}$.*

Definition 2.7. *Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$. If $d(v) = k$ for all $v \in V$, then G is said to be regular fuzzy graph of degree k.*

Definition 2.8. *Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$. The total degree of a vertex u is defined as $td(u) = \sum \mu(u, v) + \sigma(u) = d(u) + \sigma(u)$, $uv \in E$. If each vertex of G has the same total degree k, then G is said to be totally regular fuzzy graph of degree k or k-totally regular fuzzy graph.*

Definition 2.9. *Let $G : (\sigma, \mu)$ be a fuzzy graph. The d_2-degree of a vertex u in G is $d_2(u) = \sum \mu^2(u, v)$, where $\mu^2(u, v) = sup\{\mu(u, u_1) \wedge \mu(u_1, v)\}$. Also $\mu(uv) = 0$, for uv not in E.*

Definition 2.10. *Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$. If $d_2(u) = k$ for all $u \in V$, then G is said to be $(2, k)$-regular fuzzy graph [15].*

Definition 2.11. *Let $G : (\sigma, \mu)$ be fuzzy graph on $G^* : (V, E)$. The total d_2-degree of a vertex $u \in V$ is defined as $td_2(u) = \sum \mu^2(u, v) + \sigma(u) = d_2(u) + \sigma(u)$.*

Definition 2.12. *If each vertex of G has the same total d_2-degree k, then G is said to be totally $(2, k)$-regular fuzzy graph.*

Remark 2.13. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ denote two fuzzy graphs. Let $G_1^* : (V_1, E_1)$ and $G_2^* : (V_2, E_2)$ be respectively the underlying crisp graphs such that $|V_i| = p_i, i = 1, 2$. Also $d^*_{Gi}(u_i)$ denote degree of u_i in G_i^*.*

3 $(r, 2, k)$-regular fuzzy graphs.

In this section , we introduce $(r, 2, k)$-regular fuzzy graph and study some properties of $(r, 2, k)$-regular fuzzy graph through various examples.

Definition 3.1. *Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$. If $d(v) = r$ and $d_2(v) = k$, for all $v \in V$, then G is said to be $(r, 2, k)$-regular fuzzy graph. That is, G is r-regular and $(2, k)$-regular fuzzy graph.*

Example 3.2. *Let $G^* : (V, E)$ where $V = \{u, v, w, x\}$ and $E = \{uv, vw, wx, xu\}$.*

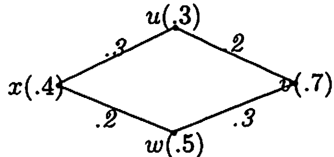

Figure 1

Here, $d(u) = .5, d(v) = .5, d(w) = .5, d(x) = .5$. Each vertex has the same degree .5. So G is a regular fuzzy graphs of degree .5. Also $d_2(u) = .2, d_2(v) = .2, d_2(w) = .2, d_2(x) = .2$. Each vertex has the same d_2- degree .2. So G is a .5 regular and $(2, .2)$-regular fuzzy graph. Hence G is a $(.5, 2, .2)$-regular fuzzy graph.

Example 3.3. *Non regular which are $(2, k)$-regular fuzzy graphs.*

1.Let $G : (\sigma, \mu)$ be a fuzzy graph such that $G^* : (V, E)$ is a path on four vertices.

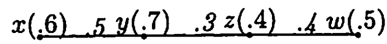

Figure 2

Here, $d_2(x) = .3, d_2(y) = .3, d_2(z) = .3, d_2(w) = .3$. So G is a $(2, .3)$-regular. But $d(x) = .5, d(y) = .8, d(z) = .7, d(w) = .4$. All the vertices in G has distinct degree. So G is a non regular and $(2, .3)$-regular fuzzy graph.

2. Let $G : (\sigma, \mu)$ be a fuzzy graph such that $G^* : (V, E)$ is a Barbell graph $B_{2,2}$ of order 6.

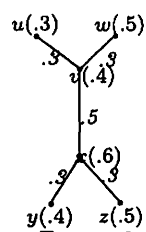

Figure 3

Here, $d_2(u) = .6, d_2(v) = .6, d_2(w) = .6, d_2(x) = .6, d_2(y) = .6.d_2(z) = .6$. This graph is a $(2, .6)$-regular fuzzy graph. But $d(u) = .3, d(v) = 1.1, d(w) = .3, d(x) = .1.1, d(y) = .3d(z) = .3$. So G is a non regular and $(2, .6)$-regular fuzzy graph.

3. Let $G : (\sigma, \mu)$ be a fuzzy graph such that $G^* : (V, E)$ is an odd cycle of length five.

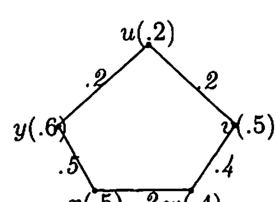

Figure 4

Here, $d_2(u) = .4, d_2(v) = .4, d_2(w) = .4, d_2(x) = .4, d_2(y) = .4$. *So G is* $(2, .4)$*-regular fuzzy graph. But* $d(u) = .4, d(v) = .6, d(w) = .6, d(x) = .7, d(y) = .7$. *So G is a non regular* $(2, .4)$*-regular fuzzy graph.*

Remark 3.4. *If* $G : (\sigma, \mu)$ *is a fuzzy graph such that the underlying crisp graph* $G^* : (V, E)$ *is a graph with more than two vertices having a pedant edge, then G is always non regular.*

Example 3.5. *Regular which is not a* $(2, k)$*-regular fuzzy graph.*
Let the fuzzy graph $G : (\sigma, \mu)$ *whose underlying crisp is given in figure 5 .*

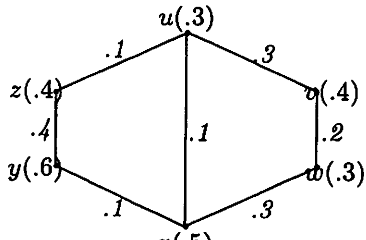

Figure 5

Here, $d_2(u) = .3, d_2(v) = .3, d_2(w) = .3, d_2(x) = .3, d_2(y) = .2, d_2(z) = .2$ *and* $d(u) = .5, d(v) = .5, d(w) = .5, d(x) = .5, d(y) = .5. \Rightarrow$ *G is a regular fuzzy graph but not a* $(2, k)$*-regular fuzzy graph.*

4 Totally $(r, 2, k)$-regular fuzzy graphs.

In this section, we introduce totally $(r, 2, k)$- regular fuzzy graphs and we make a comparative study between $(r, 2, k)$-regular fuzzy graphs and totally $(r, 2, k)$-regular fuzzy graphs. A necessary and sufficient condition under which they are equivalent is provided

Definition 4.1. *If each vertex of G has the same total degree r and total* d_2*-degree k, then G is said to be totally* $(r, 2, k)$*-regular fuzzy graph.*

Example 4.2. *Let* $G : (\sigma, \mu)$ *be a fuzzy graph such that* $G^* : (V, E)$ *is an even cycle of length six.*

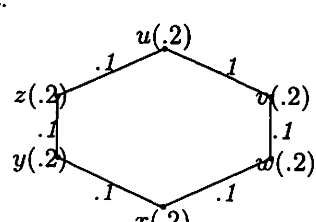

Figure 6

Here, $d_2(u) = .2, d_2(v) = .2, d_2(w) = .2, d_2(x) = .2, d_2(y) = .2$ *and* $d(u) = .2, d(v) = .2, d(w) = .2, d(x) = .2, d(y) = .2.$ $\Rightarrow$ *G is a* $(.2, 2, .2)$*-regular fuzzy graph.*
Here, $td_2(u) = .4, td_2(v) = .4, td_2(w) = .4, td_2(x) = .4, td_2(y) = .4$ *and* $td(u) = .4, td(v) = .4, td(w) = .4, td(x) = .4, td(y) = .4.$ $\Rightarrow$ *G is a totally* $(.4, 2, .4)$*-regular fuzzy graph.*

Example 4.3. *Consider the fuzzy graph* $G : (\sigma, \mu)$ *such that the underlying crisp graph* $G^* : (V, E)$ *is an even cycle of length six.*

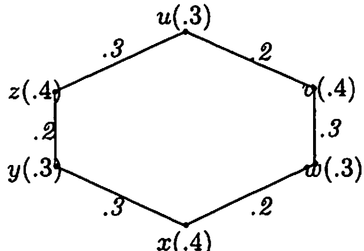

Figure 7

Here, $d_2(u) = .4, d_2(v) = .4, d_2(w) = .4, d_2(x) = .4, d_2(y) = .4, d_2(z) = .4,$ *and* $d(u) = .5, d(v) = .5, d(w) = .5, d(x) = .5, d(y) = .5, d(z) = .5 \Rightarrow G$ *is a* $(.5, 2, .4)$*-regular fuzzy graph. Here,* $td_2(u) = .7, td_2(v) = .8, td_2(w) = .7, td_2(x) = .8, td_2(y) = .7, td_2(z) = .8$ *and* $td(u) = .8, td(v) = .9, td(w) = .8, td(x) = .9, td(y) = .8, td(x) = .9 \Rightarrow G$ *is not a totally* $(r, 2, k)$*-regular fuzzy graph.*

Theorem 4.4. *Let* $G : (\sigma, \mu)$ *be a fuzzy graph on* $G^* : (V, E)$*. Then* σ *is a constant function iff the following conditions are equivalent.*

1. $G : (\sigma, \mu)$ *is a* $(r, 2, k)$*-regular fuzzy graph.*
2. $G : (\sigma, \mu)$ *is a totally* $(r, 2, k)$*-regular fuzzy graph.*

Proof. Suppose that σ is a constant function. Let $\sigma(u) = c$, constant for all $u \in V$. Assume that $G : (\sigma, \mu)$ is a $(r, 2, k)$-regular fuzzy graph. Then $d(u) = r$ and $d_2(u) = k$, for all $u \in V$.

$$\text{So } td(u) = d(u) + \sigma(u) \text{ and } td_2(u) = d_2(u) + \sigma(u), \text{ for all } u \in V$$
$$\Rightarrow td(u) = r + c \text{ and } td_2(u) = k + c, \text{ for all } u \in V.$$

Hence $G : (\sigma, \mu)$ is a totally $(r + c, 2, k + c)$- regular fuzzy graph. Thus (1) $\Rightarrow$ (2) is proved. Now suppose G is a totally $(r, 2, k)$-regular fuzzy graph.

$$\Rightarrow td_2(u) = k \text{ and } td(u) = r, \text{ for all } u \in V.$$
$$\Rightarrow d_2(u) + \sigma(u) = k \text{ and } d(u) + \sigma(u) = r, \text{ for all } u \in V.$$
$$\Rightarrow d_2(u) + c = k \text{ and } d(u) + \sigma(u) = r, \text{ for all } u \in V.$$
$$\Rightarrow d_2(u) = k - c \text{ and } d(u) = r - c \text{ for all } u \in V.$$

Hence $G : (\sigma, \mu)$ is a $(r - c, 2, k - c)$-regular fuzzy graph. Hence (1) and (2) are equivalent. Conversely assume that (1) and (2) are equivalent. Suppose σ is not a constant function. Then $\sigma(u) \neq \sigma(w)$, for at least one pair $u, w \in V$.

Let $G : (\sigma, \mu)$ be a $(r, 2, k)$-regular fuzzy graph. Then $d_2(u) = d_2(w) = k$ and $d(u) = d(w) = r$. So $td_2(u) = d_2(u) + \sigma(u) = k + \sigma(u)$ and $td_2(w) = d_2(w) + \sigma(w) = k + \sigma(w)$ and $td(u) = d(u) + \sigma(u) = r + \sigma(u)$ and $td(w) = d(w) + \sigma(w) = r + \sigma(w)$.

Since $\sigma(u) \neq \sigma(w) \Rightarrow k + \sigma(u) \neq k + \sigma(w)$ and $r + \sigma(u) \neq r + \sigma(w) \Rightarrow td_2(u) \neq td_2(w)$ and $td(u) \neq td(w)$. So $G : (\sigma, \mu)$ is not a totally $(r, 2, k)$-regular fuzzy graph which is a contradiction to our assumption.

Let $G : (\sigma, \mu)$ be a totally $(r, 2, k)$-regular fuzzy graph. Then $td_2(u) = td_2(w)$ and $td(u) = td(w)$.

$$\Rightarrow d_2(u) + \sigma(u) = d_2(w) + \sigma(w) \text{ and } d(u) + \sigma(u) = d(w) + \sigma(w)$$
$$\Rightarrow d_2(u) - d_2(w) = \sigma(w) - \sigma(u) \neq 0 \text{ and } d(u) - d(w) = \sigma(w) - \sigma(u) \neq 0$$
$$\Rightarrow d_2(u) \neq d_2(w) \text{ and } d(u) \neq d(w).$$

So $G : (\sigma, \mu)$ is not a $(r, 2, k)$-regular fuzzy graph which is a contradiction to our assumption. Hence σ is a constant function. □

Theorem 4.5. *If a fuzzy graph $G : (\sigma, \mu)$ is both $(r, 2, k)$-regular and totally $(r, 2, k)$-regular then σ is a constant function.*

Proof. Let G be $(r_1, 2, k_1)$-regular and totally $(r_2, 2, k_2)$-regular fuzzy graph. Then $d_2(u) = k_1$ and $td_2(u) = k_2, d(u) = r_1$ and $td(u) = r_2$, for all $u \in V$. Now $td_2(u) = k_2$ and $td(u) = r_2$, for all $u \in V$.

$$\Rightarrow d_2(u) + \sigma(u) = k_2 \text{ and } d(u) + \sigma(u) = r_2, \text{ for all } u \in V.$$
$$\Rightarrow k_1 + \sigma(u) = k_2 \text{ and } r_1 + \sigma(u) = r_2, \text{ for all } u \in V.$$
$$\Rightarrow \sigma(u) = k_2 - k_1 \text{ and } \sigma(u) = r_2 - r_1, \text{ for all } u \in V.$$

Hence σ is a constant function. □

Remark 4.6. *The converse of the above theorem is not true.*
Let the following graph $G^* : (V, E)$ where $V = \{u, v, w, x, y\}$ and $E =$

$\{uv, vw, wx, xy, yu\}$. *Here,* $d_2(u) = .4, d_2(v) = .4, d_2(w) = .3, d_2(x) = .3, d_2(y) = .4$ *and* $td_2(u) = .8, td_2(v) = .8, td_2(w) = .7, td_2(x) = .7, td_2(y) = .8$.

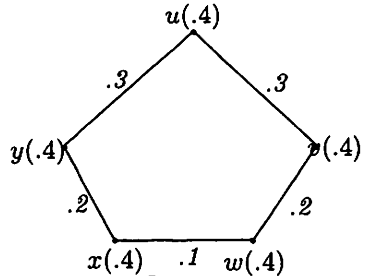

Figure 8

Here, σ *is a constant function but* G *is neither a* $(r, 2, k)$*-regular fuzzy graph nor a totally* $(r, 2, k)$*-regular fuzzy graph.*

5 $(r, 2, k)$- regularity on a Fuzzy graph obtained from a cycle with some specific membership function.

Theorem 5.1, 5.4, 5.6 and Theorem 5.8 provide $(r, 2, k)$-regularity on a fuzzy graph $G : (\sigma, \mu)$ such that $G^* : (V, E)$ is a cycle.

Theorem 5.1. *Let* $G : (\sigma, \mu)$ *be a fuzzy graph such that* $G^* : (V, E)$ *is a cycle of length* ≥ 4*. If* μ *is a constant function, then* $G : (\sigma, \mu)$ *is a* $(r, 2, k)$*-regular fuzzy graph.*

Proof. If μ is a constant function say $\mu(uv) = c$, for $uv \in E$, then $d(u) = 2c$ and $d_2(v) = 2c$, for all $v \in V$. Hence $G : (\sigma, \mu)$ is $(2c, 2, 2c)$-regular fuzzy graph. □

Remark 5.2. *Converse of the Theorem 5.1 need not be true.*
Let $G : (\sigma, \mu)$ *be fuzzy graph such that* $G^* : (V, E)$ *is cycle of length six.*

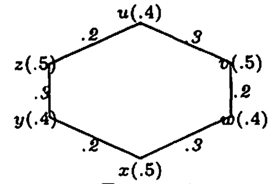

Figure 9

Here, $d_2(u) = .4, d_2(v) = .4, d_2(w) = .4, d_2(x) = .4, d_2(y) = .4 d_2(z) = .4$ *and* $d(u) = .5, d(v) = .5, d(w) = .5, d(x) = .5, d(y) = .5, d(z) = .5$ *and* *so $G : (\sigma, \mu)$ is a $(.5, 2, .4)$-regular fuzzy graph. But μ is not a constant function.*

Remark 5.3. *Even if μ is a constant function, then $G : (\sigma, \mu)$ need not be totally $(r, 2, k)$-regular fuzzy graph.*
For example, let $G : (\sigma, \mu)$ be a fuzzy graph such that $G^* : (V, E)$ is an even cycle.

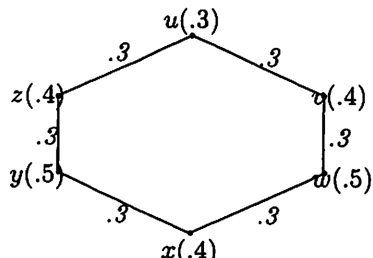

Figure 10

Here, $td_2(u) = .9, td_2(v) = 1, td_2(w) = 1.1, td_2(x) = 1, td_2(y) = 1.1, td_2(z) = 1$ and $td(u) = .9, td(v) = 1, td(w) = 1.1, td(x) = 1, td(y) = 1.1, td(z) = 1$ and so $G : (\sigma, \mu)$ is not totally $(r, 2, k)$-regular fuzzy graph. But μ is a constant function.

Theorem 5.4. *Let $G : (\sigma, \mu)$ be a fuzzy graph such that $G^* : (V, E)$ is an even cycle. If the alternate edges have the same membership values, then $G : (\sigma, \mu)$ is a $(r, 2, k)$-regular fuzzy graph.*

Proof. If the alternate edges have the same membership values, then

$$\mu(e_i) = \begin{cases} c_1, & \text{if } i \text{ is odd} \\ c_2, & \text{if } i \text{ is even.} \end{cases}$$

Here, $d(u) = c_1 + c_2$, for all $u \in V$.
If $c_1 = c_2$, then μ is a constant function. So G is a $(2c_1, 2, 2c_1)$-regular fuzzy graph.
If $c_1 < c_2$, then $d(v) = c_1 + c_2$ and $d_2(v) = 2c_1$, for all $v \in V$. So G is a $(c_1 + c_2, 2, 2c_1)$-regular fuzzy graph.
If $c_1 > c_2$, then $d(v) = c_1 + c_2$ and $d_2(v) = 2c_2$, for all $v \in V$. So G is a $(c_1 + c_2, 2, 2c_2)$-regular fuzzy graph. □

Remark 5.5. *Even if the alternate edges have the same membership values, then $G : (\sigma, \mu)$ need not be totally $(r, 2, k)$-regular fuzzy graph.*
For example, let $G : (\sigma, \mu)$ be a fuzzy graph such that $G^* : (V, E)$ is an even cycle.

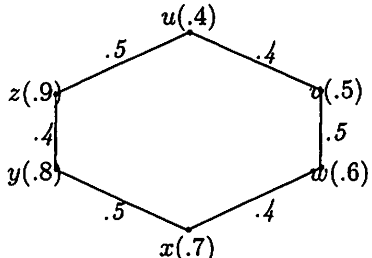

Figure 11

In the Figure 11, $td_2(u) = 1.2, td_2(v) = 1.3, td_2(w) = 1.4, td_2(x) = 1.5, td_2(y) = 1.6, td_2(z) = 1.7$ *and* $td(u) = 1.3, td(v) = 1.4, td(w) = 1.5, td(x) = 1.6, td(y) = 1.7, td(z) = 1.8$ *and so G is not a totally* $(r, 2, k)$*-regular fuzzy graph.*

Remark 5.6. *The above theorem 5.4 is not true for the fuzzy graph* $G : (\sigma, \mu)$ *where* $G^* : (V, E)$ *is any odd cycle of length* ≥ 5. *This result is true only when* μ *is a constant function.*
For, let $G : (\sigma, \mu)$ be a fuzzy graph such that $G^* : (V, E)$ is an odd cycle of length ≥ 5.

Let the alternate edges have the same membership values. That is
$\mu(e_i) = \begin{cases} c_1, & \text{if } i \text{ is odd} \\ c_2, & \text{if } i \text{ is even.} \end{cases}$ where $c_1 \neq c_2$,

Clearly $d(v_1) = 2c_1$ and $d(v_i) = c_1 + c_2$, for all $i = 2, 3, 4, \ldots, 2n+1$. $d(v_1) \neq d(v_i)$, for all $i = 2, 3, 4, \ldots, 2n+1$. So G is non regular graph and $d_2(v) = 2c_1$, for all $v \in V$. So G is not a $(r, 2, k)$-regular fuzzy graph.
Illustration Let $G : (\sigma, \mu)$ be a fuzzy graph such that $G^* : (V, E)$ is an odd cycle of length $= 5$.

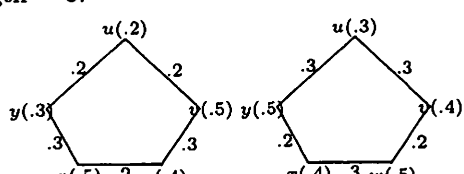

Figure 12

Graphs given in Figure 12 are not $(r, 2, k)$-regular graphs.

Acknowledgement: The authors are thankful to the anonymous refree for his /her valuable suggestions.

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