On some transitive combinatorial structures and codes constructed from the symplectic group $S(6,2)$

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Abstract

We describe the construction of transitive 2-designs and strongly regular graphs defined on the conjugacy classes of the maximal and second maximal subgroups of the symplectic group $S(6,2)$. Furthermore, we present linear codes invariant under the action of the group $S(6,2)$ obtained as the codes of the constructed designs and graphs.

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1 Introduction

An incidence structure is an ordered triple $\mathcal{D} = (\mathcal{P}, \mathcal{B}, \mathcal{I})$ where $\mathcal{P}$ and $\mathcal{B}$ are non-empty disjoint sets and $\mathcal{I} \subseteq \mathcal{P} \times \mathcal{B}$. The elements of the set $\mathcal{P}$ are called points, the elements of the set $\mathcal{B}$ are called blocks and $\mathcal{I}$ is called an incidence relation. If $|\mathcal{P}| = |\mathcal{B}|$, then the incidence structure is called a symmetric one. The incidence matrix of an incidence structure is a $b \times v$ matrix $[m_{ij}]$ where b and v are the numbers of blocks and points respectively, such that $m_{ij} = 1$ if the point P_j and the block x_i are incident, and $m_{ij} = 0$ otherwise. An isomorphism from one incidence structure to other is a bijective mapping of points to points and blocks to blocks which preserves incidence. An isomorphism from an incidence structure $\mathcal{D}$ onto itself is called an automorphism of $\mathcal{D}$. The set of all automorphisms forms a group called the full automorphism group of $\mathcal{D}$ and is denoted by $Aut(\mathcal{D})$.

A $t-(v,k,\lambda)$ design is a finite incidence structure $\mathcal{D} = (\mathcal{P}, \mathcal{B}, \mathcal{I})$ satisfying the following requirements:

1. $|\mathcal{P}| = v$,
2. every element of $\mathcal{B}$ is incident with exactly k elements of $\mathcal{P}$,
3. every t elements of $\mathcal{P}$ are incident with exactly λ elements of $\mathcal{B}$.

A $2-(v,k,\lambda)$ design is called a block design. Note that this definition allows $\mathcal{B}$ to be a multiset. If $\mathcal{B}$ is a set, then $\mathcal{D}$ is called a simple design. If a design $\mathcal{D}$ consists of k copies of some simple design $\mathcal{D}'$, then it is denoted by $\mathcal{D} = k\mathcal{D}'$.

Let $\Gamma = (\mathcal{V}, \mathcal{E}, \mathcal{I})$ be a finite incidence structure. Γ is a graph if each element of $\mathcal{E}$ is incident with exactly two elements of $\mathcal{V}$. The elements of $\mathcal{V}$ are called vertices and the elements of $\mathcal{E}$ are called edges. Two vertices u and v are called adjacent or neighbours if they are incident with the same edge. The number of neighbours of a vertex v is called the degree of v. If all the vertices of the graph Γ have the same degree k, then Γ is called k-regular. We define a square $\{0,1\}$-matrix $A = (a_{uv})$ labelled by the vertices of Γ in such a way that $a_{uv} = 1$ if and only if the vertices u and v are adjacent. The matrix A is called the adjacency matrix of the graph Γ.

A graph Γ is called a strongly regular graph with parameters (n,k,λ,μ), and it is denoted by $SRG(n,k,\lambda,\mu)$, if Γ is k-regular graph with n vertices and if any two adjacent vertices have λ common neighbours and any two non-adjacent vertices have μ common neighbours.

The code $C_{\mathbb{F}}$ of the design $\mathcal{D}$ over the finite field $\mathbb{F}$ is the space spanned by the incidence vectors of the blocks over $\mathbb{F}$. If $\mathcal{Q}$ is any subset of $\mathcal{P}$, then we will denote the incidence vector of $\mathcal{Q}$ by $v^{\mathcal{Q}}$. Thus $C_{\mathbb{F}} = \langle v^B \mid B \in \mathcal{B} \rangle$ is a subspace of $\mathbb{F}^{\mathcal{P}}$, the full vector space of functions from $\mathcal{P}$ to $\mathbb{F}$. Similarly, we can span a code by the incidence vectors of the points over some finite field $\mathbb{F}$. All our codes will be linear codes, i.e. subspaces of the ambient vector space. If a code C, over a field of order q, is of length n, dimension k, and minimum weight d, then we write $[n, k, d]_q$ to show this information. An automorphism of a code is any permutation of the coordinate positions that maps codewords to codewords. Two codes over a field of prime order are equivalent if one of the codes can be obtained from the other by permuting the coordinates and multiplication of components by non-zero elements.

The code of a graph Γ over the finite field $\mathbb{F}$ is the row span of an adjacency matrix A over the field $\mathbb{F}$.

Let G be a group and S be a subset of G. The conjugacy class of S is denoted by $ccl_G(S)$ and $|ccl_G(S)| = [G{:}N_G(S)]$.

In this paper we consider block designs and strongly regular graphs constructed from the symplectic group $S(6,2)$. $S(6,2)$ is the simple group of order 1451520, and it has eight distinct classes of maximal subgroups: $M_8 \cong U(4,2){:}Z_2$, $M_7 \cong S_8$, $M_6 \cong E_{32}{:}S_6$, $M_5 \cong U(3,3){:}Z_2$, $M_4 \cong E_{64}{:}L(3,2)$, $M_3 \cong ((E_{16}{:}Z_2) \times E_4){:}(S_3 \times S_3)$, $M_2 \cong S_3 \times S_6$, and $M_1 \cong L(2,8){:}Z_3$. Up to conjugation, $S(6,2)$ has 30 second maximal subgroups, given in Table 4.

We define incidence structures on the elements of the conjugacy classes of the maximal and second maximal subgroups of $S(6,2)$. This construction will result with 2-designs and strongly regular graphs on which the group $S(6,2)$ acts transitively. Note that 2-designs and strongly regular graphs on which the group $S(6,2)$ acts primitively (*i.e.* the points and the blocks labelled by the elements of the conjugacy classes of the maximal subgroups in $S(6,2)$) are described in [6].

Generators of the group $S(6,2)$, and its maximal subgroups are available on the Internet: http://brauer.maths.qmul.ac.uk/Atlas/.

The construction employed in this paper is introduced in [8] and it is a generalization of the construction described in [6] and [7]. While in [6] and [7] we constructed the block designs and strongly regular graphs from primitive groups, in [8] we described a construction of combinatorial structures from transitive groups. Using this method, in this paper, we construct block designs with 28, 36, 63, 120 and 378 points, and strongly regular graphs on 378, 630 and 1120 vertices from the simple group $S(6,2)$. Moreover, we study the linear codes generated by the incidence matrices of the block designs. The linear codes are spanned by incidence vectors of the points and the blocks. Additionally, we consider linear codes obtained from the adjacency matrices of the strongly regular graphs. Note that the linear codes that admit a primitive action of $S(6,2)$ are described in [3] and [4].

For basic definitions and group theoretical notation we refer the reader to [5] and [15].

All the structures are obtained by using the programs Magma ([1]), GAP ([14]) and GAP package Design ([16]).

The paper is organized as follows: in Section 2 we briefly describe the method of construction of transitive designs and graphs used in this paper, and in Section 3 we describe structures constructed on the conjugacy classes of the maximal and second maximal subgroups under the action of the symplectic group $S(6,2)$ and codes of the constructed designs and graphs.

2 The construction

The construction of primitive symmetric 1-designs and regular graphs for which a stabilizer of a point and a stabilizer of a block are conjugate is presented in [10], [11] and [12]. The generalization, *i.e.* the method for constructing not necessarily symmetric but still primitive 1-designs, is presented in [6] and [7]. In [8] we presented a construction of not necessarily primitive, but still transitive block designs:

Theorem 1 *Let G be a finite permutation group acting transitively on the sets Ω_1 and Ω_2 of size m and n, respectively. Let $\alpha \in \Omega_1$ and $\Delta_2 = \bigcup_{i=1}^{s} \delta_i G_\alpha$, where $\delta_1, ..., \delta_s \in \Omega_2$ are representatives of distinct G_α-orbits. If $\Delta_2 \neq \Omega_2$ and*

$$\mathcal{B} = \{\Delta_2 g : g \in G\},$$

then $\mathcal{D}(G, \alpha, \delta_1, ..., \delta_s) = (\Omega_2, \mathcal{B})$ is a $1-(n, |\Delta_2|, \frac{|G_\alpha|}{|G_{\Delta_2}|} \sum_{i=1}^{s} |\alpha G_{\delta_i}|)$ design with $\frac{m \cdot |G_\alpha|}{|G_{\Delta_2}|}$ blocks. The group $H \cong G / \bigcap_{x \in \Omega_2} G_x$ acts as an automorphism group on $(\Omega_2, \mathcal{B})$, transitively on points and blocks of the design.

If $\Delta_2 = \Omega_2$ then the set $\mathcal{B}$ consists of one block, and $\mathcal{D}(G, \alpha, \delta_1, ..., \delta_s)$ is a design with parameters $1-(n, n, 1)$.

The construction described in Theorem 1 gives us all simple designs on which the group G acts transitively on the points and blocks, *i.e.* if a group G acts transitively on the points and blocks of a simple 1-design $\mathcal{D}$, then $\mathcal{D}$ can be obtained as described in Theorem 1.

Remark 1 *Let $\Omega_1 = \alpha G$, and let us define the function $f : \Omega_1 \to \mathcal{B}$ such that $\alpha g \mapsto \Delta_2 g$. That function is not neccessarily an injection. Further, let us define the design such that for each element $\alpha g \in \Omega_1$ the block $\Delta_2 g$ is constructed. This design consists of m blocks and has parameters $1-$*

$(n, |\Delta_2|, \sum_{i=1}^{s} |\alpha G_{\delta_i}|)$. *The constructed design may or may not be simple; depending on whether the function f is an injection or not.*

Remark 2 *If* $\Omega_1 = \Omega_2$ *and* Δ_2 *is a union of self-paired and mutually paired orbits of* G_α*, then the design* $\mathcal{D}(G, \alpha, \delta_1, ..., \delta_s)$ *is a symmetric self-dual design and the incidence matrix of that design is the adjacency matrix of a* $|\Delta_2|-$*regular graph.*

We can use Theorem 1 to construct 1-design as follows. Let M be a finite group and H_1, H_2, and G be subgroups of M. G acts transitively on the class $ccl_G(H_i)$, $i = 1, 2$, by conjugation and

$$|ccl_G(H_1)| = [G:N_G(H_1)] = m,$$

$$|ccl_G(H_2)| = [G:N_G(H_2)] = n.$$

Let us denote the elements of $ccl_G(H_1)$ by $H_1^{g_1}, H_1^{g_2}, \ldots, H_1^{g_m}$, and the elements of $ccl_G(H_2)$ by $H_2^{h_1}, H_2^{h_2}, \ldots, H_2^{h_n}$.

By taking into consideration the Remark 1, the 1-designs can be constructed in the following way (but the resulting 1-designs may have repeated blocks):

- the point set of the design is $ccl_G(H_2)$,
- the block set is $ccl_G(H_1)$,
- the block $H_1^{g_i}$ is incident with the point $H_2^{h_j}$ if and only if $H_2^{h_j} \cap H_1^{g_i} \cong G_s$, $s = 1, \ldots, k$, where $\{G_1, ..., G_k\} \subset \{H_2^x \cap H_1^y \mid x, y \in G\}$.

We denote a 1-design constructed in this way by $\mathcal{D}(G, H_2, H_1; G_1, ..., G_k)$.

Let M be a finite group and H and G be subgroups of M. One can construct regular graph in the following way:

- the vertex set of the graph is $ccl_G(H)$,
- the vertex H^{g_i} is adjacent to the vertex H^{g_j} if and only if $H^{g_i} \cap H^{g_j} \cong G_s$, $s = 1, \ldots, k$, where $\{G_1, ..., G_k\} \subset \{H^x \cap H^y \mid x, y \in G\}$.

A regular graph constructed in this way is denoted by $\Gamma(G, H; G_1, ..., G_k)$.

Remark 3 *Note that we only consider* 1*-designs that are* 2*-designs and regular graphs that are strongly regular.*

Remark 4 *If* H_1 *and* H_2 *are subgroups of* G *such that* $M_1 = N_G(H_1)$ *and* $M_2 = N_G(H_2)$ *are maximal subgroups of* G*, then the group* G *acts primitively on* $ccl_G(H_1)$ *and* $ccl_G(H_2)$*. If* $\mathcal{D}(G, H_2, H_1; G_1, ..., G_l)$ *is a design constructed from the group* G *by the method described in Theorem 1, then* *there exist groups $G'_1, ..., G'_k$ ($k \geq l$) such that* $\{G'_i \cap H_1 \cap H_2 \mid i = 1, ..., k\} = \{G_1, ..., G_l\}$ *and* $\{G'_1, ..., G'_k\} \subseteq \{ M_1^{g_i} \cap M_2^{g_j} \mid g_i, g_j \in G\}$. *Using the describe method, one can construct primitive design* $\mathcal{D}(G, M_2, M_1; G'_1, ..., G'_k)$. *Note that the designs* $\mathcal{D}(G, H_2, H_1; G_1, ..., G_l)$ *and* $\mathcal{D}(G, M_2, M_1; G'_1, ..., G'_k)$ *are isomorphic because there exists bijective mapping that maps the class* $ccl_G(M_1)$ *onto the class* $ccl_G(H_1)$ *and the class* $ccl_G(M_2)$ *onto the class* $ccl_G(H_2)$, *such that the intersection of the images of the groups* M_1 *and* M_2 *is equal to the intersection of the groups* H_1 *and* H_2.

By Remark 4, we did not need to construct structures whose points and blocks are labelled by the elements of the conjugacy classes of the subgroups which normalizer is a maximal subgroup in $S(6, 2)$ because they are already described in [6]. Below we give a list of structures constructed in [6] from the group $S(6, 2)$: primitive block designs (Table 1) and primitive strongly regular graphs (Table 2).

Table 1: Primitive block designs constructed from the group $S(6, 2)$

Design	Parameters	The full automorphism group
$\mathcal{D}(S(6,2), M_8, M_6; E_{16}{:}S_5)$	$(28, 12, 11)$	$S(6,2)$
$\mathcal{D}(S(6,2), M_8, M_3; ((E_8.Z_{12}){:}Z_6){:}Z_2)$	$(28, 4, 5)$	$S(6,2)$
$\mathcal{D}(S(6,2), M_8, M_2; E_{27}{:}(D_8 \times Z_2))$	$(28, 10, 40)$	$S(6,2)$
$\mathcal{D}(S(6,2), M_7, M_6; A_6{:}E_4)$	$(36, 16, 12)$	$S(6,2)$
$\mathcal{D}(S(6,2), M_7, M_4; E_8{:}L(2,7))$	$(36, 8, 6)$	$S(6,2)$
$\mathcal{D}(S(6,2), M_7, M_3; E_{16}{:}S_4)$	$(36, 12, 33)$	$S(6,2)$
$\mathcal{D}(S(6,2), M_7, M_2; S_5 \times S_3)$	$(36, 6, 8)$	$S(6,2)$
$\mathcal{D}(S(6,2), M_6, M_6; Ex_{32}^{+}{:}(D_{12} \times Z_2), E_{32}{:}S_6)$	$(63, 31, 15)$	$PGL(6,2)$

Table 2: Primitive strongly regular graphs constructed from the group $S(6, 2)$

Graph	Parameters	The full automorphism group
$\Gamma(S(6,2), M_6; Ex_{32}^{+}{:}(D_{12} \times Z_2))$	$(63, 30, 13, 15)$	$S(6,2)$
$\Gamma(S(6,2), M_5; (Ex_{27}^{+}{:}Z_2).Z_4)$	$(120, 56, 28, 24)$	$O_8^{+}(2) : Z_2$
$\Gamma(S(6,2), M_4; L(2,7))$	$(135, 64, 28, 32)$	$O_8^{+}(2) : Z_2$

More details about the listed designs and graphs can be found in [6].

3 Transitive combinatorial structures constructed from the group $S(6, 2)$

In this section we consider transitive structures constructed from a simple group G isomorphic to the symplectic group $S(6, 2)$. We describe structures constructed on the conjugacy classes of the maximal and second maximal subgroups of the group G. The maximal and second maximal subgroups of the group $S(6,2)$ are listed in Table 3 and Table 4, respectively.

Table 3: Maximal subgroups of the group $S(6,2)$ (up to conjugation)

Subgroup	Structure of the subgroup	Size	Size of G-conjugacy class
M_8	$U(4,2){:}Z_2$	51840	28
M_7	S_8	40320	36
M_6	$E_{32}{:}S_6$	23040	63
M_5	$U(3,3){:}Z_2$	12096	120
M_4	$E_{64}{:}L(3,2)$	10752	135
M_3	$((E_{16}{:}Z_2) \times E_4){:}(S_3 \times S_3)$	4608	315
M_2	$S_3 \times S_6$	4320	336
M_1	$L(2,8){:}Z_3$	1512	960

Table 4: Second maximal subgroups of the group $S(6,2)$ (up to conjugation)

Subgroup	Structure of the group	Size	Size of G-conjugacy class
H_1	$E_{16}{:}S_5$	1920	378
H_2	$((Z_2 \times D_8){:}Z_2){:}(S_3 \times S_3)$	1152	1260
H_3	$Z_2 \times S_6$	1440	1008
H_4	$(E_9{:}Z_3){:}GL(2,3)$	1296	1120
H_5	$E_{27}{:}(Z_2 \times S_4)$	1296	1120
H_6	$(S_4 \times S_4){:}Z_2$	1152	1260
H_7	S_7	5040	288
H_8	$E_8{:}(Z_2 \times S_4)$	384	3780
H_9	$S_5 \times S_3$	720	2016
H_{10}	$PSL(32){:}Z_2$	336	4320
H_{11}	$(E_{32}{:}A_5){:}Z_2$	3840	378
H_{12}	$Z_2 \times ((E_{16}{:}A_5){:}Z_2)$	3840	378
H_{13}	$Z_2 \times ((S_4 \times S_4){:}Z_2)$	2304	630
H_{14}	$E_{32}{:}(Z_2{:}S_4)$	1536	945
H_{15}	$E_8{:}(D_8 \times S_4)$	1536	945
H_{16}	$Z_2 \times S_6$	1440	1008
H_{17}	$(E_9{:}Z_3){:}QD_{16}$	432	3360
H_{18}	$(SL(23){:}Z_4){:}Z_2$	192	7560
H_{19}	$E_4{:}(Z_2 \times S_4)$	192	7560
H_{20}	$E_{32}{:}(Z_2 \times S_4)$	1536	945
H_{21}	$(E_{64}{:}Z_7){:}Z_3$	1344	1080
H_{22}	$E_8.PSL(32)$	1344	1080
H_{23}	$E_8{:}PSL(32)$	1344	1080
H_{24}	$Z_2 \times S_3 \times S_4$	288	5040
H_{25}	$S_5 \times S_3$	720	2016
H_{26}	$((S_3 \times S_3){:}Z_2) \times S_3$	432	3360
H_{27}	$Z_2 \times S_4 \times S_3$	288	5040
H_{28}	$(E_8{:}Z_7){:}Z_3$	168	8640
H_{29}	$(Z_9{:}Z_3){:}Z_2$	54	26880
H_{30}	$E_{21}{:}Z_2$	42	34560

Below we give the intersections of the elements of the conjugacy classes that we use for construction of strongly regular graphs and block designs.

The intersection of two different elements, one from the conjugacy class of the maximal subgroup M_i and the other from the conjugacy class of the second maximal subgroup H_j, denoted by $P^k_{i,j}$, is isomorphic to:

- $P^1_{8,6} \cong S_4 \times E_4$, $P^2_{8,6} \cong E_9{:}D_8$
- $P^1_{8,7} \cong S_6$, $P^2_{8,7} \cong S_5 \times Z_2$
- $P^1_{8,8} \cong S_4 \times E_4$, $P^2_{8,8} \cong Z_2 \times D_8$
- $P^1_{8,9} \cong S_5 \times Z_2$, $P^2_{8,9} \cong Z_2 \times S_3 \times S_3$, $P^3_{8,9} \cong S_4 \times Z_2$
- $P^1_{8,11} \cong E_{16}{:}(Z_5{:}Z_4)$, $P^2_{8,11} \cong S_5 \times Z_2$
- $P^1_{8,14} \cong (D_8 \times D_8){:}Z_2$, $P^2_{8,14} \cong S_4 \times E_4$
- $P^1_{8,15} \cong E_{16}{:}(A_4{:}Z_2)$, $P^2_{8,15} \cong S_4 \times D_8$, $P^3_{8,15} \cong S_4 \times E_4$
- $P^1_{8,16} \cong (S_3 \times S_3){:}E_4$, $P^2_{8,16} \cong S_5$, $P^3_{8,16} \cong S_5 \times Z_2$
- $P^1_{8,18} \cong (Z_3 \times Q_8){:}Z_2$, $P^2_{8,18} \cong D_8$
- $P^1_{8,19} \cong QD_{16}$, $P^2_{8,19} \cong D_{12}$
- $P^1_{8,24} \cong D_8 \times S_3$, $P^2_{8,24} \cong Z_2 \times S_3 \times S_3$, $P^3_{8,24} \cong Z_2 \times D_8$
- $P^1_{8,26} \cong D_8 \times S_3$, $P^2_{8,26} \cong (S_3 \times S_3){:}D_{12}$, $P^3_{8,26} \cong E_4 \times S_3$
- $P^1_{8,27} \cong Z_2 \times S_3 \times S_3$, $P^2_{8,27} \cong D_8 \times S_3$, $P^3_{8,27} \cong S_4 \times Z_2$, $P^4_{8,27} \cong E_4 \times S_3$
- $P^1_{8,1} \cong Ex^+_{32}{:}S_3$, $P^2_{8,1} \cong S_5$, $P^3_{8,1} \cong E_{16}{:}S_5$
- $P^1_{8,2} \cong E_{16}{:}(A_4{:}Z_2)$, $P^2_{8,2} \cong ((Z_2 \times D_8){:}Z_2){:}(S_3 \times S_3)$, $P^3_{8,2} \cong S_4 \times Z_2$
- $P^1_{8,3} \cong S_5$, $P^2_{8,3} \cong Z_2 \times S_6$, $P^3_{8,3} \cong S_4 \times E_4$
- $P^1_{8,30} \cong Z_6$, $P^2_{8,30} \cong Z_2$
- $P^1_{7,1} \cong S_5$, $P^2_{7,1} \cong S_4 \times E_4$
- $P^1_{7,2} \cong S_4 \times E_4$, $P^2_{7,2} \cong S_4 \times Z_2$
- $P^1_{7,5} \cong (S_3 \times S_3){:}E_4$, $P^2_{7,5} \cong S_4 \times Z_2$
- $P^1_{7,8} \cong (D_8 \times D_8){:}Z_2$, $P^2_{7,8} \cong E_{16}{:}(A_4{:}Z_2)$, $P^3_{7,8} \cong S_4 \times Z_2$, $P^4_{7,8} \cong Z_2 \times D_8$
- $P^1_{7,9} \cong S_3 \times S_4$, $P^2_{7,9} \cong S_5 \times S_3$, $P^3_{7,9} \cong E_4 \times S_3$
- $P^1_{7,10} \cong S_4$, $P^2_{7,10} \cong PSL(3,2){:}Z_2$, $P^3_{7,10} \cong D_{16}$
- $P^1_{7,14} \cong E_4{:}(Z_2 \times S_4)$, $P^2_{7,14} \cong (D_8 \times D_8){:}Z_2$, $P^3_{7,14} \cong S_4 \times E_4$
- $P^1_{7,15} \cong (D_8 \times D_8){:}Z_2$, $P^2_{7,15} \cong S_4 \times D_8$, $P^3_{7,15} \cong S_4 \times E_4$
- $P^1_{7,16} \cong S_5 \times Z_2$, $P^2_{7,16} \cong (S_3 \times S_3){:}E_4$, $P^3_{7,16} \cong E_9 \times D_8$
- $P^1_{7,18} \cong D_{16}$, $P^2_{7,18} \cong D_8$
- $P^1_{7,19} \cong D_{12}$, $P^2_{7,19} \cong D_{16}$, $P^3_{7,19} \cong S_4$
- $P^1_{7,20} \cong E_{16}{:}(A_4{:}Z_2)$, $P^2_{7,20} \cong D_8 \times D_8$, $P^3_{7,20} \cong Ex^+_{32}{:}S_3$
- $P^1_{7,21} \cong (E_8{:}Z_7){:}Z_3$, $P^2_{7,21} \cong E_4 \times A_4$
- $P^1_{7,24} \cong D_8 \times S_3$, $P^2_{7,24} \cong E_4 \times S_3$, $P^3_{7,24} \cong Z_2 \times D_8$
- $P^1_{7,26} \cong Z_2 \times S_3 \times S_3$, $P^2_{7,26} \cong (S_3 \times S_3){:}E_4$, $P^3_{7,26} \cong Z_2 \times D_8$

- $P^1_{7,27} \cong Z_2 \times S_3 \times S_3$, $P^2_{7,27} \cong E_4 \times S_3$, $P^3_{7,27} \cong S_3 \times S_4$, $P^4_{7,27} \cong Z_2 \times D_8$
- $P^1_{7,28} \cong Z_7{:}Z_3$, $P^2_{7,28} \cong Z_6$
- $P^1_{7,3} \cong S_4 \times E_4$, $P^2_{7,3} \cong Z_2 \times S_6$, $P^3_{7,3} \cong E_9{:}D_8$
- $P^1_{7,6} \cong E_9{:}D_8$, $P^2_{7,6} \cong (S_4 \times S_4){:}Z_2$, $P^3_{7,6} \cong E_8{:}D_8$
- $P^1_{7,23} \cong E_4{:}(Z_2 \times S_4)$, $P^2_{7,23} \cong E_8{:}L(2,7)$, $P^3_{7,23} \cong S_4 \times Z_2$
- $P^1_{7,29} \cong Z_6$, $P^2_{7,29} \cong Z_2$
- $P^1_{7,30} \cong Z_3$, $P^2_{7,30} \cong D_{14}{:}Z_3$, $P^3_{7,30} \cong Z_2$
- $P^1_{6,1} \cong S_4 \times E_4$, $P^2_{6,1} \cong Ex^+_{32}{:}S_3$, $P^3_{6,1} \cong E_{16}{:}S_5$, $P^4_{6,1} \cong S_5$
- $P^1_{6,3} \cong S_4 \times E_4$, $P^2_{6,3} \cong Z_2 \times S_6$, $P^3_{6,3} \cong S_5$, $P^4_{6,3} \cong E_9 \times D_8$
- $P^1_{6,6} \cong S_4 \times E_4$, $P^2_{6,6} \cong E_8{:}D_8$, $P^3_{6,6} \cong (S_4 \times S_4){:}Z_2$, $P^4_{6,6} \cong E_9{:}D_8$
- $P^1_{6,14} \cong Z_2 \times (E_{16}{:}Z_2)$, $P^2_{6,14} \cong E_4 \times (E_8{:}D_8)$, $P^3_{6,14} \cong E_{32}{:}(Z_2 \times S_4$, $P^4_{6,14} \cong S_4 \times Z_2$
- $P^1_{6,19} \cong E_8$, $P^2_{6,19} \cong Ex^+_{32}$, $P^3_{6,19} \cong (Z_4 \times Z_4){:}D_{12}$, $P^4_{6,19} \cong S_3$
- $P^1_{6,27} \cong S_4 \times E_4$, $P^2_{6,27} \cong E_{16}$, $P^3_{6,27} \cong Z_2 \times S_3 \times S_4$, $P^4_{6,27} \cong E_8 \times S_3$, $P^5_{6,27} \cong S_3 \times S_3$, $P^6_{6,27} \cong D_{12}$
- $P^1_{5,10} \cong D_{16}$, $P^2_{5,10} \cong S_4$, $P^3_{5,10} \cong D_{12}$, $P^4_{5,10} \cong S_3$, $P^5_{5,10} \cong PSL(3,2){:}Z_2$

The intersection of two different elements $H_i^x \in ccl_G(H_i)$ and $H_j^y \in ccl_G(H_j)$, denoted by $P^k_{i,j}$, is isomorphic to:

- $P^1_{1,12} \cong S_4 \times Z_2$, $P^2_{1,12} \cong S_5$, $P^3_{1,12} \cong E_{16}{:}(A_4{:}Z_2)$, $P^4_{1,12} \cong S_4$, $P^5_{1,12} \cong E_{16}{:}S_5$, $P^6_{1,12} \cong Ex^+_{32}{:}S_3$, $P^7_{1,12} \cong E_4 \times S_3$

The intersection of two different elements from the class $ccl_G(H_i)$, denoted by P^k_i, is isomorphic to:

- $P^1_{12} \cong S_5$, $P^2_{12} \cong Ex^+_{32}{:}S_3$, $P^3_{12} \cong S_4 \times E_4$, $P^4_{12} \cong E_8{:}(E_4 \times S_4)$, $P^5_{12} \cong S_4$, $P^6_{12} \cong E_8 \times S_3$, $P^7_{12} \cong Z_2 \times ((E_{16}{:}A_5){:}Z_2)$
- $P^1_{13} \cong E_8{:}D_8$, $P^2_{13} \cong E_9{:}D_8$, $P^3_{13} \cong E_8{:}S_3$, $P^4_{13} \cong Z_2 \times ((D_8 \times D_8){:}Z_2)$, $P^5_{13} \cong D_8$, $P^6_{13} \cong Ex^+_{32}$, $P^7_{13} \cong Z_2 \times ((S_4 \times S_4){:}Z_2)$
- $P^1_4 \cong S_3$, $P^2_4 \cong D_{12}$,$P^3_4 \cong GL(2,3)$, $P^4_4 \cong (E_9{:}Z_2) \times S_3$, $P^5_4 \cong (E_9{:}Z_3){:}GL(2,3)$, $P^6_4 \cong QD_{16}$, $P^7_4 \cong Z_2$

Further on, we describe 2-designs and strongly regular graphs obtained from $G-$conjugacy classes of the maximal and second maximal subgroups. In the following tables (Table 5, 6, 7 and 8) we give the list of the constructed designs and strongly regular graphs and some of their properties. The group G acts on all constructed designs, primitively on points, and transitively but imprimitively on blocks.

Table 5: Transitive block designs constructed from the group $S(6,2)$, $v = 28$

$\mathcal{D}$	Construction	Parameters	Simple design	Corresp. $\mathcal{D}'$	Aut($\mathcal{D}$)
$\mathcal{D}_1$	$\mathcal{D}(G, M_8, H_6; P^1_{8,6})$	(28, 12, 110)	no	(28, 12, 11)	$S(6,2)$
$\mathcal{D}_2$	$\mathcal{D}(G, M_8, H_7; P^1_{8,7})$	(28, 7, 16)	yes		$S(6,2)$
$\mathcal{D}_3$	$\mathcal{D}(G, M_8, H_8; P^1_{8,8})$	(28, 4, 60)	no	(28, 4, 5)	$S(6,2)$
$\mathcal{D}_4$	$\mathcal{D}(G, M_8, H_9; P^1_{8,9})$	(28, 3, 16)	yes		$S(6,2)$
$\mathcal{D}_5$	$\mathcal{D}(G, M_8, H_9; P^2_{8,9})$	(28, 10, 240)	no	(28, 10, 40)	$S(6,2)$
$\mathcal{D}_6$	$\mathcal{D}(G, M_8, H_9; P^1_{8,9}, P^2_{8,9})$	(28, 13, 416)	yes		$S(6,2)$
$\mathcal{D}_7$	$\mathcal{D}(G, M_8, H_{11}; P^1_{8,11})$	(28, 12, 66)	no	(28, 12, 11)	$S(6,2)$
$\mathcal{D}_8$	$\mathcal{D}(G, M_8, H_{14}; P^1_{8,14})$	(28, 12, 165)	no	(28, 12, 11)	$S(6,2)$
$\mathcal{D}_9$	$\mathcal{D}(G, M_8, H_{15}; P^1_{8,15})$	(28, 4, 15)	no	(28, 4, 5)	$S(6,2)$
$\mathcal{D}_{10}$	$\mathcal{D}(G, M_8, H_{15}; P^2_{8,15})$	(28, 8, 70)	yes		$S(6,2)$
$\mathcal{D}_{11}$	$\mathcal{D}(G, M_8, H_{16}; P^1_{8,16})$	(28, 10, 120)	no	(28, 10, 40)	$S(6,2)$
$\mathcal{D}_{12}$	$\mathcal{D}(G, M_8, H_{16}; P^2_{8,16})$	(28, 12, 176)	no	(28, 12, 11)	$S(6,2)$
$\mathcal{D}_{13}$	$\mathcal{D}(G, M_8, H_{16}; P^3_{8,16})$	(28, 6, 40)	yes		$S(6,2)$
$\mathcal{D}_{14}$	$\mathcal{D}(G, M_8, H_{18}; P^1_{8,18})$	(28, 4, 120)	no	(28, 4, 5)	$S(6,2)$
$\mathcal{D}_{15}$	$\mathcal{D}(G, M_8, H_{19}; P^1_{8,19})$	(28, 12, 660)	no	(28, 12, 11)	$S(6,2)$
$\mathcal{D}_{16}$	$\mathcal{D}(G, M_8, H_{24}; P^1_{8,24})$	(28, 6, 200)	yes		$S(6,2)$
$\mathcal{D}_{17}$	$\mathcal{D}(G, M_8, H_{24}; P^2_{8,24})$	(28, 4, 80)	no	(28, 4, 5)	$S(6,2)$
$\mathcal{D}_{18}$	$\mathcal{D}(G, M_8, H_{24}; P^1_{8,24}, P^2_{8.24})$	(28, 10, 600)	no	(28, 10, 40)	$S(6,2)$
$\mathcal{D}_{19}$	$\mathcal{D}(G, M_8, H_{26}; P^1_{8,26})$	(28, 9, 320)	yes		$S(6,2)$
$\mathcal{D}_{20}$	$\mathcal{D}(G, M_8, H_{26}; P^1_{8,26}, P^2_{8.26})$	(28, 10, 400)	no	(28, 10, 40)	$S(6,2)$
$\mathcal{D}_{21}$	$\mathcal{D}(G, M_8, H_{27}; P^1_{8,27})$	(28, 4, 80)	yes		$S(6,2)$
$\mathcal{D}_{22}$	$\mathcal{D}(G, M_8, H_{27}; P^2_{8,27}, P^3_{8,27})$	(28, 12, 880)	no	(28, 12, 11)	$S(6,2)$
$\mathcal{D}_{23}$	$\mathcal{D}(G, M_8, H_{27}; P^4_{8,27})$	(28, 12, 880)	yes		$S(6,2)$
$\mathcal{D}_{24}$	$\mathcal{D}(G, M_8, H_1; P^1_{8,1})$	(28, 10, 45)	yes		$S(6,2)$
$\mathcal{D}_{25}$	$\mathcal{D}(G, M_8, H_2; P^1_{8,2})$	(28, 3, 10)	yes		$S(6,2)$
$\mathcal{D}_{26}$	$\mathcal{D}(G, M_8, H_2; P^1_{8,2}, P^2_{8,2})$	(28, 4, 20)	no	(28, 4, 5)	$S(6,2)$
$\mathcal{D}_{27}$	$\mathcal{D}(G, M_8, H_3; P^1_{8,3}, P^2_{8,3})$	(28, 13, 208)	yes		$S(6,2)$
$\mathcal{D}_{28}$	$\mathcal{D}(G, M_8, H_{30}; P^1_{8,30})$	(28, 7, 1920)	no	(28, 7, 16)	$S(6,2)$

Table 6: Transitive block designs constructed from the group $S(6,2)$, $v = 36$

$\mathcal{D}$	Construction	Parameters	Simple design	Corresp. $\mathcal{D}'$	Aut($\mathcal{D}$)
$\mathcal{D}_{29}$	$\mathcal{D}(G, M_7, H_1; P^1_{7,1})$	(36, 16, 72)	no	(36, 16, 12)	$S(6,2)$
$\mathcal{D}_{30}$	$\mathcal{D}(G, M_7, H_2; P^1_{7,2})$	(36, 12, 132)	no	(36, 12, 33)	$S(6,2)$
$\mathcal{D}_{31}$	$\mathcal{D}(G, M_7, H_5; P^1_{7,5})$	(36, 9, 64)	yes		$S(6,2)$
$\mathcal{D}_{32}$	$\mathcal{D}(G, M_7, H_8; P^1_{7,8})$	(36, 3, 18)	yes		$S(6,2)$
$\mathcal{D}_{33}$	$\mathcal{D}(G, M_7, H_8; P^1_{7,8}, P^2_{7,8})$	(36, 4, 36)	no	(36, 4, 9)	$S(6,2)$
$\mathcal{D}_{34}$	$\mathcal{D}(G, M_7, H_8; P^3_{7,8})$	(36, 8, 168)	no	(36, 8, 6)	$S(6,2)$
$\mathcal{D}_{35}$	$\mathcal{D}(G, M_7, H_8; P^2_{7,8}, P^3_{7,8})$	(36, 9, 216)	yes		$S(6,2)$
$\mathcal{D}_{36}$	$\mathcal{D}(G, M_7, H_8; P^1_{7,8}, P^3_{7,8})$	(36, 11, 330)	yes		$S(6,2)$
$\mathcal{D}_{37}$	$\mathcal{D}(G, M_7, H_8; P^1_{7,8}, P^2_{7,8}, P^3_{7,8})$	(36, 12, 396)	no	(36, 12, 33)	$S(6,2)$
$\mathcal{D}_{38}$	$\mathcal{D}(G, M_7, H_9; P^1_{7,9})$	(36, 5, 32)	yes		$S(6,2)$
$\mathcal{D}_{39}$	$\mathcal{D}(G, M_7, H_9; P^1_{7,9}, P^2_{7,9})$	(36, 6, 48)	no	(36, 6, 8)	$S(6,2)$
$\mathcal{D}_{40}$	$\mathcal{D}(G, M_7, H_{10}; P^1_{7,10})$	(36, 14, 624)	yes		$S(6,2)$
$\mathcal{D}_{41}$	$\mathcal{D}(G, M_7, H_{10}; P^1_{7,10}, P^2_{7,10})$	(36, 15, 720)	yes		$S(6,2)$
$\mathcal{D}_{42}$	$\mathcal{D}(G, M_7, H_{14}; P^1_{7,14})$	(36, 8, 42)	no	(36, 8, 6)	$S(6,2)$
$\mathcal{D}_{43}$	$\mathcal{D}(G, M_7, H_{14}; P^2_{7,14})$	(36, 12, 99)	yes		$S(6,2)$
$\mathcal{D}_{44}$	$\mathcal{D}(G, M_7, H_{14}; P^3_{7,14})$	(36, 16, 180)	no	(36, 16, 12)	$S(6,2)$
$\mathcal{D}_{45}$	$\mathcal{D}(G, M_7, H_{15}; P^1_{7,15})$	(36, 12, 99)	no	(36, 12, 33)	$S(6,2)$
$\mathcal{D}_{46}$	$\mathcal{D}(G, M_7, H_{15}; P^2_{7,15})$	(36, 8, 42)	yes		$S(6,2)$
$\mathcal{D}_{47}$	$\mathcal{D}(G, M_7, H_{16}; P^1_{7,16})$	(36, 6, 24)	no	(36, 6, 8)	$S(6,2)$
$\mathcal{D}_{48}$	$\mathcal{D}(G, M_7, H_{16}; P^2_{7,16})$	(36, 10, 72)	yes		$S(6,2)$
$\mathcal{D}_{49}$	$\mathcal{D}(G, M_7, H_{16}; P^1_{7,16}, P^2_{7,16})$	(36, 16, 192)	no	(36, 16, 12)	$S(6,2)$
$\mathcal{D}_{50}$	$\mathcal{D}(G, M_7, H_{18}; P^1_{7,18})$	(36, 12, 792)	no	(36, 12, 33)	$S(6,2)$
$\mathcal{D}_{51}$	$\mathcal{D}(G, M_7, H_{19}; P^1_{7,19})$	(36, 16, 720)	no	(36, 16, 12)	$S(6,2)$
$\mathcal{D}_{52}$	$\mathcal{D}(G, M_7, H_{19}; P^2_{7,19})$	(36, 12, 396)	no	(36, 12, 99)	$S(6,2)$
$\mathcal{D}_{53}$	$\mathcal{D}(G, M_7, H_{20}; P^1_{7,20})$	(36, 4, 9)	yes		$S(6,2)$
$\mathcal{D}_{54}$	$\mathcal{D}(G, M_7, H_{21}; P^1_{7,21})$	(36, 8, 48)	no	(36, 8, 6)	$S(6,2)$
$\mathcal{D}_{55}$	$\mathcal{D}(G, M_7, H_{24}; P^1_{7,24})$	(36, 6, 120)	no	(36, 6, 8)	$S(6,2)$
$\mathcal{D}_{56}$	$\mathcal{D}(G, M_7, H_{24}; P^2_{7,24})$	(36, 12, 528)	no	(36, 12, 33)	$S(6,2)$
$\mathcal{D}_{57}$	$\mathcal{D}(G, M_7, H_{24}; P^1_{7,24}, P^2_{7,24})$	(36, 18, 1224)	yes		$S(6,2)$
$\mathcal{D}_{58}$	$\mathcal{D}(G, M_7, H_{24}; P^3_{7,24})$	(36, 18, 1224)	yes		$S(6,2)$
$\mathcal{D}_{59}$	$\mathcal{D}(G, M_7, H_{26}; P^1_{7,26})$	(36, 6, 80)	no	(36, 6, 8)	$S(6,2)$
$\mathcal{D}_{60}$	$\mathcal{D}(G, M_7, H_{26}; P^2_{7,26})$	(36, 3, 16)	yes		$S(6,2)$
$\mathcal{D}_{61}$	$\mathcal{D}(G, M_7, H_{26}; P^1_{7,26}, P^2_{7,26})$	(36, 9, 192)	no	(36, 9, 64)	$S(6,2)$
$\mathcal{D}_{62}$	$\mathcal{D}(G, M_7, H_{27}; P^1_{7,27})$	(36, 4, 48)	yes		$S(6,2)$
$\mathcal{D}_{63}$	$\mathcal{D}(G, M_7, H_{27}; P^2_{7,27})$	(36, 12, 528)	yes		$S(6,2)$
$\mathcal{D}_{64}$	$\mathcal{D}(G, M_7, H_{27}; P^2_{7,27}, P^3_{7,27})$	(36, 14, 728)	yes		$S(6,2)$
$\mathcal{D}_{65}$	$\mathcal{D}(G, M_7, H_{27}; P^2_{7,27}, P^1_{7,27})$	(36, 16, 960)	no	(36, 16, 12)	$S(6,2)$
$\mathcal{D}_{66}$	$\mathcal{D}(G, M_7, H_{27}; P^2_{7,27}, P^1_{7,27}, P^3_{7,27})$	(36, 18, 1224)	no	(36, 18, 153)	$S(6,2)$

Table 7: Transitive block designs constructed from the group $S(6,2)$, $v = 36$ (continued from the previous page)

$\mathcal{D}$	Construction	Parameters	Simple design	Corresp. $\mathcal{D}'$	Aut($\mathcal{D}$)
$\mathcal{D}_{67}$	$\mathcal{D}(G, M_7, H_{27}; P^4_{7,27})$	(36, 18, 1224)	no	(36, 18, 153)	$S(6,2)$
$\mathcal{D}_{68}$	$\mathcal{D}(G, M_7, H_{28}; P^1_{7,28})$	(36, 8, 384)	no	(36, 8, 6)	$S(6,2)$
$\mathcal{D}_{69}$	$\mathcal{D}(G, M_7, H_3; P^1_{7,3})$	(36, 15, 168)	yes		$S(6,2)$
$\mathcal{D}_{70}$	$\mathcal{D}(G, M_7, H_6; P^1_{7,6})$	(36, 16, 120)	no	(36, 16, 12)	$S(6,2)$
$\mathcal{D}_{71}$	$\mathcal{D}(G, M_7, H_6; P^1_{7,6}, P^2_{7,6})$	(36, 18, 153)	yes		$S(6,2)$
$\mathcal{D}_{72}$	$\mathcal{D}(G, M_7, H_6; P^3_{7,6})$	(36, 18, 153)	yes		$S(6,2)$
$\mathcal{D}_{73}$	$\mathcal{D}(G, M_7, H_{23}; P^1_{7,23})$	(36, 7, 36)	yes		$S(6,2)$
$\mathcal{D}_{74}$	$\mathcal{D}(G, M_7, H_{29}; P^1_{7,29})$	(36, 9, 1536)	no	(36, 9, 64)	$S(6,2)$
$\mathcal{D}_{75}$	$\mathcal{D}(G, M_7, H_{30}; P^1_{7,30})$	(36, 14, 4992)	no	36, 14, 624	$S(6,2)$
$\mathcal{D}_{76}$	$\mathcal{D}(G, M_7, H_{30}; P^1_{7,30}, P^2_{7,30})$	(36, 15, 5760)	no	(36, 15, 720)	$S(6,2)$

Table 8: Transitive block designs constructed from the group $S(6,2)$, $v = 63, 120, 378$

$\mathcal{D}$	Construction	Parameters	Simple design	Corresp. $\mathcal{D}'$	Aut($\mathcal{D}$)
$\mathcal{D}_{77}$	$\mathcal{D}(G, M_6, H_1; P^1_{6,1}, P^2_{6,1}, P^3_{6,1})$	(63, 31, 90)	no	(63, 31, 15)	$PGL(6,2)$
$\mathcal{D}_{78}$	$\mathcal{D}(G, M_6, H_3; P^1_{6,3}, P^2_{6,3})$	(63, 31, 240)	no	(63, 31, 15)	$PGL(6,2)$
$\mathcal{D}_{79}$	$\mathcal{D}(G, M_6, H_6; P^1_{6,6}, P^2_{6,6}, P^3_{6,6})$	(63, 31, 150)	no	(63, 31, 15)	$PGL(6,2)$
$\mathcal{D}_{80}$	$\mathcal{D}(G, M_6, H_{14}; P^1_{6,14},, P^2_{6,14} P^3_{6,14})$	(63, 31, 225)	no	(63, 31, 15)	$PGL(6,2)$
$\mathcal{D}_{81}$	$\mathcal{D}(G, M_6, H_{19}; P^1_{6,19}, P^2_{6,19}, P^3_{6,19})$	(63, 31, 900)	no	(63, 31, 15)	$PGL(6,2)$
$\mathcal{D}_{82}$	$\mathcal{D}(G, M_6, H_{27}; P^1_{6,27}, P^2_{6,27}, P^3_{6,27}, P^4_{6,27})$	(63, 31, 1200)	no	(63, 31, 15)	$PGL(6,2)$
$\mathcal{D}_{83}$	$\mathcal{D}(G, M_5, H_{10}; P^1_{5,10}, P^2_{5,10})$	(120, 35, 360)	yes		$O^+(8,2){:}Z_2$
$\mathcal{D}_{84}$	$\mathcal{D}(G, H_1, H_{12}; P^1_{1,12}, P^2_{1,12}, P^3_{1,12})$	(378, 117, 36)	yes		$O(7,3){:}Z_2$

Table 9: Strongly regular graphs constructed from the group $S(6,2)$ from the conjugacy classes of the second maximal subgroups

Graph Γ	Construction	Parameters of Γ	Aut(Γ)
Γ_1	$\Gamma(G_2, H_{12}; P^1_{12}, P^2_{12})$	(378, 52, 26, 4)	S_{28}
Γ_2	$\Gamma(G_2, H_{12}; P^1_{12}, P^3_{12}, P^4_{12})$	(378, 117, 36, 36)	$O_7(3){:}Z_2$
Γ_3	$\Gamma(G_2, H_{13}; P^1_{13}, P^2_{13})$	(630, 68, 34, 4)	S_{36}
Γ_4	$\Gamma(G_2, H_4; P^1_4, P^2_4, P^3_4, P^4_4)$	(1120, 390, 146, 130)	$O^+_8(3).D_8$

Remarks on the constructed structures:

- Block designs $\mathcal{D}_1, \mathcal{D}_7, \mathcal{D}_8, \mathcal{D}_{12}, \mathcal{D}_{15}, \mathcal{D}_{22}$ are made of the copies of the design with parameters 2-$(28, 12, 11)$ which is isomorphic to the derived design of the symplectic SDP design with the parameters 2-$(64, 28, 12)$ (see [13]).

- Block designs $\mathcal{D}_3, \mathcal{D}_9, \mathcal{D}_{14}, \mathcal{D}_{17}, \mathcal{D}_{26}$ are made of the copies of the design with parameters 2-$(28, 4, 5)$ which is simple design isomorphic to the design constructed in [6].

- Block designs $\mathcal{D}_5, \mathcal{D}_{11}, \mathcal{D}_{18}, \mathcal{D}_{20}$ are composed of the copies of the design with parameters 2-$(28, 10, 40)$ which is simple design isomorphic to the design constructed in [6].

- $\mathcal{D}_{28}$ is composed of the copies of the design with parameters 2-$(28, 7, 6)$ isomorphic to the design $\mathcal{D}_2$. $\mathcal{D}_2$ is isomorphic to the design described in [8].

- $\mathcal{D}_{29}, \mathcal{D}_{44}, \mathcal{D}_{49}, \mathcal{D}_{51}, \mathcal{D}_{65}$ and $\mathcal{D}_{70}$ are made of the copies of the design with parameters 2-$(36, 16, 12)$ which is isomorphic to the residual design of the symplectic SDP design with the parameters 2-$(64, 28, 12)$ (see [13]).

- $\mathcal{D}_{30}, \mathcal{D}_{37}, \mathcal{D}_{45}, \mathcal{D}_{50}, \mathcal{D}_{56}, \mathcal{D}_{34}, \mathcal{D}_{42}, \mathcal{D}_{54}, \mathcal{D}_{68}, \mathcal{D}_{39}, \mathcal{D}_{47}, \mathcal{D}_{55}$ and $\mathcal{D}_{59}$ are made of the copies of the designs isomorphic to the primitive designs constructed from the group $S(6, 2)$ (see [6]).

- Designs $\mathcal{D}_{77}, \mathcal{D}_{78}, \mathcal{D}_{79}, \mathcal{D}_{80}, \mathcal{D}_{81}, \mathcal{D}_{82}$ are made of the copies of the design with parameters 2-$(63, 31, 15)$, which is isomorphic to the design described in [9].

- According to [2], the strongly regular graph Γ_1 is the unique $T(28)$ graph and Γ_3 is the unique $T(36)$ graph.

- Strongly regular graphs Γ_2 and Γ_4 are isomorphic to the graphs mentioned in [2].

- We did not find any other information about other designs constructed in this paper.

We describe codes of the constructed simple designs and their complements. If A is an incidence matrix of a 2-(v, k, λ) design $\mathcal{D}$ and p is a prime that does not divide $r - \lambda$, then $\text{rank}_p(A) \geq v - 1$ (see [17]). If $\text{rank}_p(A) < v - 1$ then p divides $r - \lambda$, hence the code of a design $\mathcal{D}$ is interesting only when p divides $r - \lambda$.

In Tables 10, 11 and 12 we present the non-trivial codes of the constructed simple block designs and their complements. Further, in Table 13, we present information about the non-trivial codes obtained from the strongly regular graphs constructed in this paper. Note that for some codes we could not determine the automorphism group in a reasonable amount of computational time.

Table 10: Non-trivial codes, spanned by the blocks of the incidence matrices of the designs

Design	Parameters	$\|Aut(C)\|$	$Aut(C)$
$\mathcal{D}_{10}$	$[28, 21, 4]_2$	1451520	$S(6, 2)$
$\mathcal{D}_{40}$	$[36, 15, 8]_2$	1451520	$S(6, 2)$
$\mathcal{D}_{46}$	$[36, 21, 6]_2$	1451520	$S(6, 2)$
$\mathcal{D}_{53}$	$[36, 29, 4]_2$	1451520	$S(6, 2)$
$\mathcal{D}_{84}$	$[378, 27, 117]_3$	9170703360	$O(7, 3){:}Z_2$
$\mathcal{D}_{83}$	$[120, 84, 8]_5$	$\geq$ 1451520	–

Table 11: Non-trivial codes, spanned by the points of the incidence matrices of the designs

Design	Parameters	$\|Aut(C)\|$	$Aut(C)$
$\mathcal{D}_2$	$[288, 28, 48]_2$	1451520	$S(6, 2)$
$\mathcal{D}_4$	$[2016, 28, 216]_2$	1451520	$S(6, 2)$
$\mathcal{D}_6$	$[2016, 28, 336]_2$	1451520	$S(6, 2)$
$\mathcal{D}_{10}$	$[945, 21, 210]_2$	1451520	$S(6, 2)$
$\mathcal{D}_{13}$	$[1008, 21, 216]_2$	1451520	$S(6, 2)$
$\mathcal{D}_{16}$	$[5040, 21, 1080]_2$	1451520	$S(6, 2)$
$\mathcal{D}_{19}$	$[3360, 28, 1080]_2$	1451520	$S(6, 2)$
$\mathcal{D}_{21}$	$[5040, 27, 720]_2$	1451520	$S(6, 2)$
$\mathcal{D}_{23}$	$[5040, 27, 1280]_2$	1451520	$S(6, 2)$
$\mathcal{D}_{24}$	$[378, 27, 52]_2$	1451520	$S(6, 2)$
$\mathcal{D}_{27}$	$[1008, 28, 144]_2$	1451520	$S(6, 2)$
$\mathcal{D}_{24}$	$[378, 28, 117]_3$	9170703360	$O(7, 3){:}Z_2$
$\mathcal{D}_{31}$	$[1120, 36, 160]_2$	1451520	$S(6, 2)$
$\mathcal{D}_{38}$	$[2016, 36, 280]_2$	1451520	$S(6, 2)$
$\mathcal{D}_{40}$	$[4320, 15, 1680]_2$	1451520	$S(6, 2)$
$\mathcal{D}_{41}$	$[4320, 36, 720]_2$	$\geq$ 1451520	–
$\mathcal{D}_{43}$	$[945, 15, 315]_2$	1451520	$S(6, 2)$
$\mathcal{D}_{48}$	$[1008, 21, 280]_2$	1451520	$S(6, 2)$
$\mathcal{D}_{53}$	$[945, 29, 105]_2$	$\geq$ 1451520	–
$\mathcal{D}_{57}$	$[5040, 21, 1760]_2$	$\geq$ 1451520	–
$\mathcal{D}_{60}$	$[3360, 36, 280]_2$	$\geq$ 1451520	–
$\mathcal{D}_{62}$	$[5040, 35, 560]_2$	$\geq$ 1451520	–
$\mathcal{D}_{63}$	$[5040, 35, 1024]_2$	$\geq$ 1451520	–
$\mathcal{D}_{64}$	$[5040, 21, 1800]_2$	$\geq$ 1451520	–
$\mathcal{D}_{69}$	$[1008, 36, 112]_2$	$\geq$ 1451520	–
$\mathcal{D}_{71}$	$[630, 35, 68]_2$	$\geq$ 1451520	–
$\mathcal{D}_{72}$	$[630, 35, 68]_2$	$\geq$ 1451520	–
$\mathcal{D}_{73}$	$[1080, 36, 180]_2$	$\geq$ 1451520	–
$\mathcal{D}_{83}$	$[4320, 120]_2$	$\geq$ 1451520	–

Table 12: Non-trivial codes, spanned by the points of the incidence matrices of the complement designs

Design	Parameters	$\|\mathrm{Aut}(C)\|$	$\mathrm{Aut}(C)$
$\mathcal{D}_{10}^c$	$[945, 21, 336]_2$	1451520	$S(6,2)$
$\mathcal{D}_{13}^c$	$[1008, 21, 280]_2$	1451520	$S(6,2)$
$\mathcal{D}_{16}^c$	$[5040, 21, 1760]_2$	1451520	$S(6,2)$
$\mathcal{D}_{21}^c$	$[5040, 27, 1280]_2$	1451520	$S(6,2)$
$\mathcal{D}_{23}^c$	$[5040, 27, 1280]_2$	1451520	$S(6,2)$
$\mathcal{D}_{40}^c$	$[4320, 15, 2016]_2$	1451520	$S(6,2)$
$\mathcal{D}_{43}^c$	$[945, 15, 400]_2$	1451520	$S(6,2)$
$\mathcal{D}_{48}^c$	$[1008, 21, 216]_2$	1451520	$S(6,2)$
$\mathcal{D}_{53}^c$	$[945, 29, 192]_2$	≥ 1451520	–
$\mathcal{D}_{57}^c$	$[5040, 21, 1080]_2$	≥ 1451520	–
$\mathcal{D}_{62}^c$	$[5040, 35, 1024]_2$	≥ 1451520	–
$\mathcal{D}_{63}^c$	$[5040, 35, 1024]_2$	≥ 1451520	–
$\mathcal{D}_{64}^c$	$[5040, 21, 1920]_2$	1451520	$S(6,2)$

Table 13: Non-trivial codes, spanned by the rows of the adjacency matrices of the graphs

Graph	Parameters	$\|\mathrm{Aut}(C)\|$	$\mathrm{Aut}(C)$
Γ_1	$[378, 26, 52]_2$	28!	S_{28}
Γ_1	$[378, 351, 4]_3$	≥ 1451520	–
Γ_2	$[378, 27, 117]_3$	9170703360	$O(7,3){:}Z_2$
Γ_3	$[630, 34, 68]_2$	36!	S_{36}
Γ_4	$[1120, 300]_3$	≥ 1451520	–

Appendix

In this section we give the generators of all maximal and second maximal subgroups used in this paper.

$S(6,2)$:

$g_1 = (2,3)(6,7)(9,10)(12,14)(17,19)(20,22)$

$g_2 = (1,2,3,4,5,6,8)(7,9,11,13,16,18,14)(10,12,15,17,20,19,21)(22,23,24,25,26,27,28)$

M_1 :

$g_1 = (1,9,4,21,13,11,6,25,12)(2,15,26,19,10,7,17,16,28)(5,18,8,20,14,22,24,23,27)$

$g_2 = (1,2,10)(3,12,15)(4,11,7)(5,24,16)(6,28,13)(8,19,21)(9,23,20)(14,26,22)(17,25,27)$

M_2 :

$g_1 = (1,24,3)(2,8,23,13)(4,25,7,17)(5,28,26,6,12,27,9,21,20,19,18,11)(10,16)(14,15,22)$

$g_2 = (1,26,12,24,18,20)(2,17,10,4,23,8)(3,9,5)(6,15)(7,13,25)(11,22)(14,21)(27,28)$

M_3 :

$g_1 = (1,27,6,16,9,19,17,20,26,7,11,5)(2,28,12,8)(3,15,18,14,24,23)(4,25,10,22)$

$g_2 = (1,11)(2,12,28,8)(3,27,18,7)(4,24,5,21)(6,25,26,22)(9,17)(10,15,19,13)(14,20,23,16)$
$g_3 = (1,25)(2,28)(3,14)(4,10)(5,20)(7,16)(8,12)(13,23)(15,24)(17,22)(18,21)(19,27)$
M_4 :
$g_1 = (1,16,26,3,24,6,18,8)(2,23,19,10,25,7,15,13)(4,21,5,12,9,11,17,20)(14,22,27,28)$
$g_2 = (1,2,26,22,13,9,8)(3,12,15,21,14,10,6)(4,17,20,19,18,28,23)(5,24,25,11,27,7,16)$
M_5 :
$g_1 = (1,25,2,21,28,9,17,14)(4,11,24,5,23,22,27,7)(6,20,10,26,8,18,15,12)(13,19)$
$g_2 = (1,21,7,20,26,23)(2,15,25)(3,14,6,5,22,9)(4,8,28,16,17,27)(10,24,18,13,12,11)$
M_6 :
$g_1 = (1,18)(2,26,10,24)(3,4,25)(5,14,22,16)(6,15,21,27,23,7,28,12,19,17,20,11)(8,13,9)$
$g_2 = (1,26,10,8,9,13)(2,4,3,25,18,24)(5,22,21,20,28,16)(6,7,17,11,23,12)(14,19,27)$
M_7 :
$g_1 = (1,26,13,27,22,21,20)(2,7,25,18,28,10,3)(4,15,19,24,5,14,8)(6,23,12,16,17,11,9)$
$g_2 = (1,6,24,15,25,16)(2,19,9,20,4,12)(3,18,8,26,10,13)(5,27,22,14,11,23)(17,28,21)$
M_8 :
$g_1 = (1,6,23,22,10,15,7,5,18,4,25,24)(3,9,16,8,19,11,14,27,17,28,26,20)(12,21,13)$
$g_2 = (1,4,25,12,9,26)(3,16,20,17,22,13)(5,6,7,8,27,24)(10,14,15,23,28,21)(11,19,18)$
H_1 :
$g_1 = (1,2,25)(5,14,27)(7,23,21)(10,13,18)(15,19,20)(17,28,22)$
$g_2 = (1,3,21,13,12,27,23,7,25,5,10,18)(2,9,6,14)(4,28,26,15)(8,19,17,24,22,20)$
H_2 :
$g_1 = (1,10,24,18,21,15,11,20)(2,19,23,26,7,13,25,12)(3,8)(4,27,28,17,9,22,14,5)$
$g_2 = (1,13)(2,10,23,26,24,20)(3,8)(4,5,27,9,17,22)(7,15,25,12,11,18)(19,21)$
$g_3 = (1,23)(2,7)(4,12)(5,19)(8,16)(9,26)(10,28)(13,17)(14,15)(18,22)(20,27)(21,25)$
H_3 :
$g_1 = (1,7,23,8,24,3)(2,25,16,11,6,21)(4,14,18)(5,12,13,10,17,20)(9,22,27,28,26,15)$
$g_2 = (1,25)(2,11,6,8)(3,16,7,24)(4,14,15,10)(5,18,27,13)(9,26,12,28)(17,22)(21,23)$
H_4 :
$g_1 = (1,3,21,26,8,12)(2,23,9,13,15,4)(5,6,17)(7,22,20,19,27,11)(10,18,14,25,24,28)$
$g_2 = (1,9)(2,10,19,26,15,22,25,5)(3,11,27,23,18,17,7,14)(4,21,12,28,6,8,24,13)$
H_5 :
$g_1 = (1,17,3,28,2,19)(5,20,12,15,14,22)(6,18,7,9,21,10)(8,25)(11,26)(24,27)$
$g_2 = (1,20,3,10)(4,28,23,6,25,5,27,19,24,21,11,12)(7,13,14,8,17,26)(9,18,15,22)$
H_6 :
$g_1 = (1,8,18,23,3,26,9,27)(2,25,17,20,13,10,16,11)(4,28,14,22,24,19,7,15)(5,12,6,21)$
$g_2 = (1,24,27,3,7,26)(2,10,6,11,17,21)(4,8,15,14,23,22)(5,20,16,12,13,25)(18,28,19)$
H_7 :
$g_1 = (1,27,8,2,10,13,19,18,14,6,11,5)(3,7,26,28)(4,25,15,20)(9,23,12)(16,24,22)(17,21)$
$g_2 = (1,28)(2,17)(3,19)(4,23)(13,16)(24,27)$
H_8 :
$g_1 = (1,7)(3,14)(4,22)(5,6)(8,28)(9,27)(10,25)(11,20)(15,24)(16,17)(18,23)(19,26)$
$g_2 = (1,7,9,24,3,14,18,4)(2,25,16,11,13,10,17,20)(5,12,6,21)(8,22,27,28,26,15,23,19)$
$g_3 = (1,14,16)(2,28,8)(3,7,17)(4,25,15)(5,6,21)(9,23,20)(10,22,24)(11,18,27)(13,19,26)$
H_9 :

$g_1 = (1,10,9,18,2,3)(4,24)(5,23,14)(7,21,27)(8,26)(11,20,28,16,17,15)(13,25)(19,22)$
$g_2 = (1,27,18,9,7,3)(2,19,8)(4,21,24)(6,28,15)(10,26,22)(11,23,16,17,14,20)(12,25,13)$
H_{10} :
$g_1 = (1,2,26,22,12,27)(3,8,17,24,18,16)(4,13,9,7,21,19)(5,20,6)(10,14,23,25,15,28)$
$g_2 = (1,21)(3,6)(4,17)(5,9)(8,16)(10,13)(11,24)(12,18)(14,27)(15,19)(20,26)(22,28)$
H_{11} :
$g_1 = (1,4,26,23,19,27,7,24)(2,12,5,17,13,21,6,16)(3,9,14,18,28,22,8,15)(11,20)$
$g_2 = (1,19,22)(2,21,6,13,12,5)(3,8,4)(7,28,24)(10,20,16,25,11,17)(14,26,15)(18,23,27)$
H_{12} :
$g_1 = (1,7)(3,14)(4,9)(5,6)(8,28)(10,11)(15,23)(16,17)(18,24)(19,26)(20,25)(22,27)$
$g_2 = (1,9,27,4,23,19,15,28,8,14,26,18)(2,20,13,11)(3,7,22,24)(10,16,12,25,17,21)$
H_{13} :
$g_1 = (2,12)(3,15,23,28)(4,19,22,26)(5,20,10,17)(6,11,25,16)(8,14,24,18)(9,27)(13,21)$
$g_2 = (1,7)(2,21)(3,24)(4,22)(5,20)(6,11)(8,23)(10,17)(12,13)(14,15)(16,25)(18,28)$
$g_3 = (1,27,14)(2,5,25)(4,26)(6,10,13)(7,23,9)(8,28,19,24,15,22)(11,16)(17,20)$
H_{14} :
$g_1 = (1,2,24,16,4,19)(3,28,17,27,13,23)(5,11,18,26,14,7)(6,25,20,15,12,9)(8,21,22)$
$g_2 = (1,20)(2,24,6,19,15,12,4,25)(3,21,23,26,18,8,14,5)(7,10,17,13,27,22,11,28)$
H_{15} :
$g_1 = (2,23,11,7,25,24)(3,6)(4,19,27,18,14,15)(5,26)(9,13,22,20,28,10)(12,17)$
$g_2 = (1,24,23,2)(3,8)(4,5,28,22)(6,16)(7,21,11,25)(9,17,14,27)(10,12,15,26)(18,20)$
$g_3 = (1,24)(2,7)(4,10)(5,19)(8,16)(9,15)(11,21)(12,27)(13,17)(14,18)(20,28)(22,26)$
H_{16} :
$g_1 = (1,2,9,4,13)(3,22,25,7,10,27,8,12,18,21)(5,16,14,15,28)(6,20,11,17,23)(19,24)$
$g_2 = (3,27)(4,26)(7,18)(8,19)(10,21)(11,16)(12,25)(15,28)(17,20)(22,24)$
H_{17} :
$g_1 = (2,5,4,6,10,12,7,26,18,3,22,17)(8,21,16)(9,15,25,28,14,11,20,27,23,13,19,24)$
$g_2 = (2,27,13,26,21,9,12,19)(3,20,10,28,15,6,16,23)(4,22)(5,25,18,24,11,17,8,14)$
H_{18} :
$g_1 = (1,20)(2,15)(3,22)(5,10)(7,21)(8,13)(9,12)(11,27)(14,18)(16,24)(17,28)(23,26)$
$g_2 = (1,26,27,2,15,28,14,12,11,5,10,21)(3,7,23,18,25,24,16,17,9,20,4,22)(6,19,8)$
H_{19} :
$g_1 = (1,3)(2,27)(4,16)(5,12)(7,10)(8,20)(11,26)(13,23)(14,25)(15,22)(17,24)(19,28)$
$g_2 = (1,6,22,5,18,19)(2,8,4,25,17,16)(3,21,15,12,9,28)(7,10,13)(11,14,20,26,27,24)$
$g_3 = (1,28,5,15,3,19,12,22)(2,11,8,7,27,26,20,10)(4,24,13,14,16,17,23,25)(9,18)$
H_{20} :
$g_1 = (1,21)(2,24)(3,8,6,16)(4,18,9,20)(5,10,17,15)(7,11)(12,22,26,27)(13,14,19,28)$
$g_2 = (1,26,24,18)(2,19,25,15)(3,16,8,6)(4,17,9,5)(7,13,23,10)(11,20,21,12)(14,22)(27,28)$
H_{21} :
$g_1 = (1,7,23,20,3,17)(4,15,6)(5,8,12,13,10,24)(9,27,14,16,18,11)(19,28,21,25,26,22)$
$g_2 = (1,25,13,4,14,7,21)(2,23,18,8,20,24,5)(3,22,9,12,28,6,17)(10,16,19,26,15,11,27)$
H_{22} :
$g_1 = (1,9,20)(2,7,10,15,27,22)(3,21,6,18,8,4)(5,24,11,26,12,17)(13,19,14,28,25,23)$
$g_2 = (1,28,16,13)(2,11,4,14)(3,7,27,18)(5,9,26,20)(6,17,15,23)(12,24,25,19)$

H_{23} :
$g_1 = (1,17,9,11)(2,10,28,19)(4,22,5,25)(6,21,26,24)(7,27)(8,13,12,15)(14,16)(20,23)$
$g_2 = (1,20,9)(2,13,27,15,28,7)(3,6,5,18,4,26)(8,17,24,21,11,12)(10,14,19,22,23,25)$
H_{24} :
$g_1 = (1,9,26,13,2,4)(3,24,25,10,8,18)(5,17,6,23,20,14)(7,27,19,12,21,22)(15,28,16)$
$g_2 = (1,9,4,13,2,26)(3,22,25,21,24,7)(5,20,6,23,17,14)(8,12,10,19,18,27)(11,28,15)$
$g_3 = (1,10)(2,18)(3,13)(4,8)(5,14)(6,23)(7,21)(9,25)(12,27)(15,28)(17,20)(24,26)$
H_{25} :
$g_1 = (1,21,4,7,2,22)(3,25,24)(5,15,23)(6,28,14)(8,18,10)(9,12,26,27,13,19)(11,20,17)$
$g_2 = (1,26,4,13)(3,27)(5,28,23,11)(6,17,20,14)(7,24,22,25)(8,12,18,19)(10,21)(15,16)$
H_{26} :
$g_1 = (1,7)(2,12,9,21,13,27)(3,10,25)(4,19)(5,6,23)(8,24)(11,17,28,16,20,15)(22,26)$
$g_2 = (1,8,26,18,4,24)(2,25,9,10,13,3)(5,20,15)(6,28,16)(7,22,19)(11,17,23)(12,27,21)$
$g_3 = (1,9,26,13)(2,4)(3,19,25,7)(5,28,16,20)(6,23,15,17)(8,21)(10,22)(12,18,27,24)$
H_{27} :
$g_1 = (1,3,12)(2,10,21)(4,24,22,26,8,19)(6,14,23)(7,9,25)(11,20,15,16,17,28)(13,18,27)$
$g_2 = (1,3)(2,25)(4,8)(5,6)(7,27)(9,18)(10,13)(11,16)(12,21)(14,23)(17,20)(24,26)$
$g_3 = (1,21)(2,7)(4,22)(5,14)(9,27)(10,18)(12,13)(15,20)(17,28)(19,26)$
H_{28} :
$g_1 = (1,7,15,18,13,8,24)(2,3,10,5,6,16,14)(4,20,27,19,11,22,21)(9,23,25,17,28,26,12)$
$g_2 = (1,24,13,9,4,10)(2,11,26,15,3,5)(6,28,16,12,22,20)(7,23,14)(8,19,18,21,25,17)$
H_{29} :
$g_1 = (1,4,21,26,12,23,24,17,13)(2,15,6,22,25,8,16,11,20)(3,19,10,27,9,14,7,28,5)$
$g_2 = (1,11,16,27,14,12)(2,3,5,4,26,25)(6,23,9,8,21,28)(7,10,17,24,15,22)(13,19,20)$
H_{30} :
$g_1 = (1,16)(2,25)(3,11)(4,6)(5,8)(7,27)(9,20)(12,24)(14,23)(15,19)(17,18)(21,26)$
$g_2 = (1,21,11)(2,27,17)(3,18,10)(4,25,22)(5,23,14)(6,26,19)(7,28,9)(8,12,13)(15,16,20)$

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