Independence and Domination in 3-Connected Cubic Graphs

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Abstract. An infinite class of graphs is constructed to demonstrate that the difference between the independent domination number and the domination number of 3-connected cubic graphs may be arbitrarily large.

1. Introduction

A set D of vertices of a graph $G = (V, E)$ is a *dominating set* of G if each vertex in $V-D$ is adjacent to at least one vertex in D. A set I of vertices is *independent* in G if no two vertices in I are adjacent. If, in addition, I is also a dominating set then I is called an *independent dominating* set of G. Note that the independent dominating sets of G are exactly the maximal independent sets of G (see [3, p. 309]). The *domination number* $\gamma(G)$ (*independent domination number* $i(G)$) of G is the smallest number of vertices in a dominating (maximal independent) set of G. Since every maximal independent set is a minimal dominating set [3, p. 309], it follows that $\gamma(G) \leq i(G)$ for any graph G. A dominating set D of a graph G such that $|D| = \gamma(G)$ is also called a *γ-set* of G.

Various authors have found sufficient conditions under which equality of the domination and independent domination numbers occurs. In particular, Allan and Laskar [1] proved that if G is $K_{1,3}$-free, then $i(G) = \gamma(G)$. This extended an earlier result by Mitchell and Hedetniemi [6] that if G is the line graph of a tree, then $i(G) = \gamma(G)$. Harary and Livingston characterised the trees and the caterpillars for which $i = \gamma$ in [4] and [5] respectively.

The study of the difference between i and γ for cubic graphs was initiated by Barefoot, Harary and Jones [2] who constructed an infinite class of cubic graphs of connectivity 2 in which $i - \gamma$ becomes unbounded, and conjectured that a similar class of cubic graphs of connectivity 1 existed. This conjecture was settled when such a class of graphs was constructed in [7]. Barefoot et al. [2] further conjectured that the only cubic graphs of connectivity 3 for which $i \neq \gamma$, are the graphs

$K_{3,3}$ and $C_5 \times K_2$. That this conjecture is false can be seen by noting that (see [7]) if $k \equiv 5 \pmod{12}$, then $i(C_k \times K_2) = \lceil k/2 \rceil + 1$ while $\gamma(C_k \times K_2) = \lceil k/2 \rceil$.

In this paper we construct an infinite class of cubic graphs of connectivity 3 in which the difference $i - \gamma$ can be made arbitrarily large.

2. Construction

Let G_k have vertex set $U \cup V \cup W \cup X \cup Y \cup Z$, where

$$U = \bigcup_{j=1}^{k} U_j \quad \text{with} \quad U_j = \{u_{j1}, \ldots, u_{j4}\};$$

$$V = \bigcup_{j=1}^{k} V_j \quad \text{with} \quad V_j = \{v_{j1}, \ldots, v_{j3}\};$$

$$W = \bigcup_{j=1}^{k} W_j \quad \text{with} \quad W_j = \{w_{j1}, \ldots, w_{j3}\};$$

$$X = \bigcup_{j=1}^{k} X_j \quad \text{with} \quad X_j = \begin{cases} \{x_{j1}, x_{j2}\} & \text{if } j < k, \\ \{x_{j1}\} & \text{if } j = k; \end{cases}$$

$$Y = \bigcup_{j=1}^{k} Y_j \quad \text{with} \quad Y_j = \begin{cases} \{y_{j1}, y_{j2}\} & \text{if } j < k, \\ \{y_{j1}\} & \text{if } j = k; \end{cases}$$

$$Z = \bigcup_{j=1}^{k} Z_j \quad \text{with} \quad Z_j = \begin{cases} \{z_{j1}, z'_{j1}, z_{j2}, z'_{j2}\} & \text{if } j < k, \\ \{z_{j1}, z'_{j1}\} & \text{if } j = k. \end{cases}$$

Add edges such that for each $j \in \{1, \ldots, k\}$, the induced subgraph $\langle U_j \cup V_j \cup W_j \rangle$ ($\langle X_j \cup Y_j \cup Z_j \rangle$ respectively) is isomorphic to the graph H_j (F_j) depicted in Figure 1. For each $j \in \{1, \ldots, k-1\}$, add the edges $w_{j2}x_{j1}$, $x_{j1}w_{j3}$, $w_{j3}x_{j2}$, $x_{j2}w_{(j+1)1}$ and $z'_{j2}z_{(j+1)1}$. Finally, add the edges $w_{k3}z_{11}$, $w_{11}z'_{k1}$, $w_{k2}x_{k1}$ and $x_{k1}w_{k3}$. The graph G_2 is illustrated in Figure 2.

Note that G_k is 3-regular. In the next three sections we prove that G_k is 3-connected and that if k is even, then $i(G_k) - \gamma(G_k) = k/2$.

3. The domination number of G_k

In order to determine the domination number of G_k, we first prove that the domination number of H_j is three and that any dominating set of G_k also contains at least three vertices of H_j for each j.

Lemma 1.

(a) *For each $j \in \{1, \ldots, k\}$, V_j is the only γ-set of H_j; hence $\gamma(H_j) = 3$.*

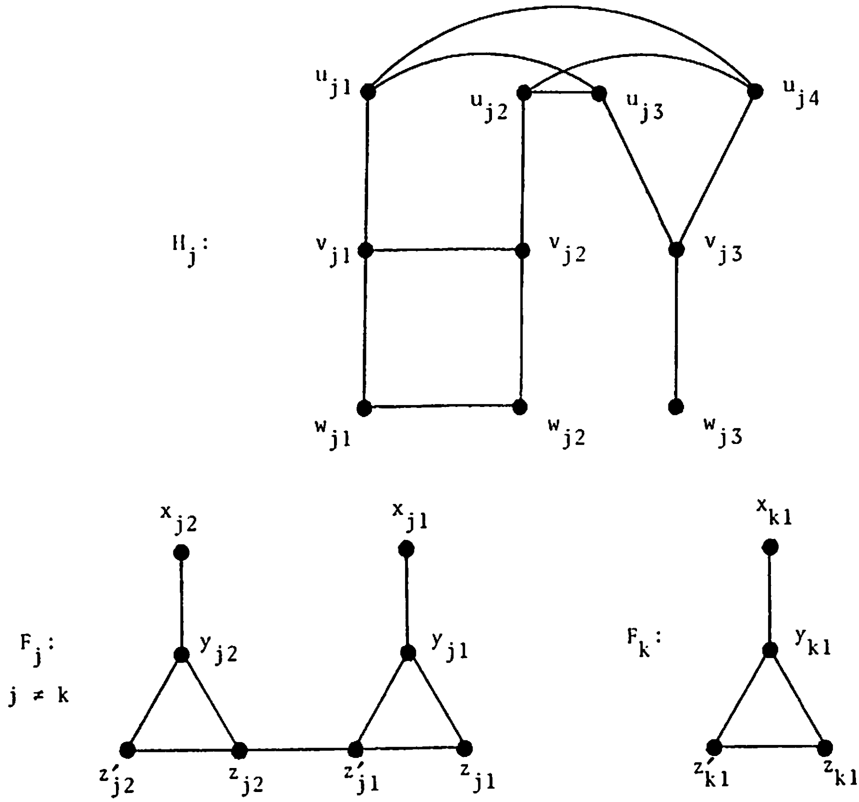

Figure 1. The graphs H_j and F_j used in the construction of G_k.

(b) *If D is any dominating set of G_k, then, for any $j \in \{1, \ldots, k\}$,*

$$|D \cap V(H_j)| \geq 3.$$

Proof: (a) Since V_j dominates H_j, it follows that $\gamma(H_j) \leq 3$. Suppose D with $|D| \leq 3$ is a dominating set of H_j. If $v_{j3} \notin D$, then $w_{j3} \in D$. Since $\{w_{j3}, u_{j1}, u_{j2}\}$ ($\{w_{j3}, u_{j3}, u_{j4}\}$ respectively) does not dominate H_j, D does not contain both u_{j1} and u_{j2} (u_{j3} and u_{j4}). Hence in order to dominate u_{j3} and u_{j4}, D contains exactly one of u_{j1} and u_{j2}. But if $u_{j1} \in D$ ($u_{j2} \in D$ respectively), then no single vertex dominates $\{u_{j2}, v_{j2}, w_{j2}, w_{j1}\}$ ($\{u_{j1}, v_{j1}, w_{j1}, w_{j2}\}$). Hence $v_{j3} \in D$.

If $v_{j1} \notin D$, then since w_{j1} is dominated, at least one of w_{j1} and w_{j2} is in D. If $w_{j1} \in D$ ($w_{j2} \in D$ respectively), then no single vertex dominates $\{u_{j1}, u_{j2}, v_{j2}\}$ ($\{u_{j1}, u_{j2}, v_{j1}\}$). Hence D contains v_{j1} and, similarly, v_{j2}. Hence $D = V_j$ and $\gamma(H_j) = 3$.

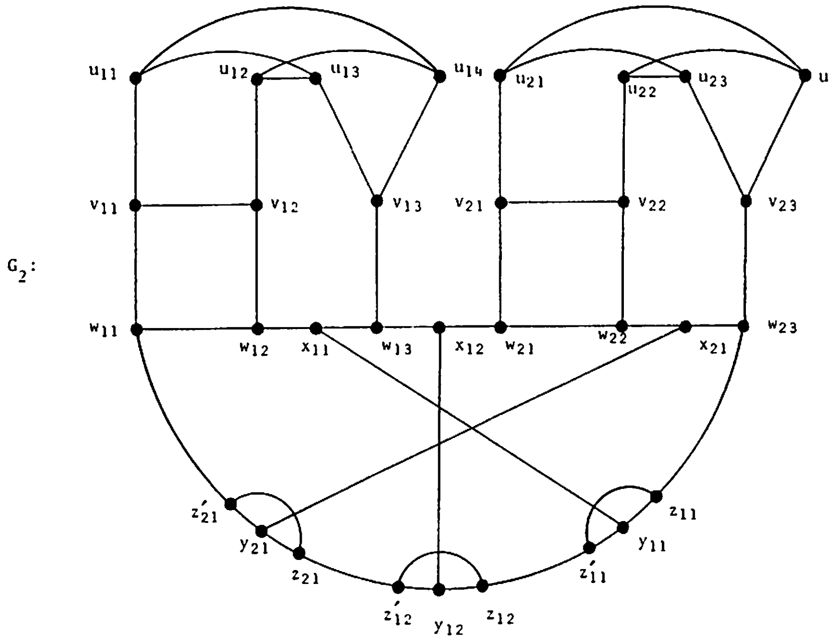

Figure 2. The graph G_k for $k = 2$.

(b) Note that $\gamma(\langle U_j \cup V_j \rangle) = 3$ and that no vertex of $G_k - V(H_j)$ is adjacent to a vertex in $U_j \cup V_j$. Also, no vertex in W_j is adjacent to more than one vertex in $U_j \cup V_j$. Hence in order to dominate $U_j \cup V_j$, any dominating set of G_k contains at least three vertices of H_j. ∎

Theorem 1. *For every integer* $k \geq 1, \gamma(G_k) = 5k - 1$.

Proof: The set $V \cup Y$ dominates G_k and therefore $\gamma(G_k) \leq 5k - 1$. Let D be any dominating set of G_k. By Lemma 1(b), $|D \cap (U \cup V \cup W)| \geq 3k$. Also, if $D \cap \{y_{j\ell}, z_{j\ell}, z'_{j\ell}\} = \phi$ for some j and ℓ, then $x_{j\ell}$ belongs to D in order to dominate $y_{i\ell}$. Hence $|D \cap (X \cup Y \cup Z)| \geq 2k - 1$ and it follows that $|D| \geq 5k - 1$. ∎

4. The Independent Domination Number of G_k (k Even)

In this section we prove that if k is even, then $i(G_k) = (11k/2) - 1$. It can be shown in a similar way that $i(G_k) = (11(k-1)/2) - 1$ if k is odd. However, for the sake of brevity and since the latter result is not required for the purpose of this paper, the proof is omitted here. We first need three lemmas.

Lemma 2. *If I is an an independent dominating set of G_k, then for any $j \in \{1, \ldots, k\}$, $|I \cap V(H_j)| \geq 4$ unless at least one of w_{j1} and w_{j2} is dominated by $I - V(H_j)$ (in which case $|I \cap V(H_j)| \geq 3$).*

Proof: Let I be an independent dominating set of G_k such that neither w_{j1} nor

w_{j2} is adjacent to a vertex in $I - V(H_j)$. As in the proof of Lemma 1(a), if either w_{j3} or v_{j3} belongs to I, then $|I \cap V(H_j)| \geq 4$. Hence suppose $I \cap \{w_{j3}, v_{j3}\} = \phi$. In order to dominate v_{j3} at least one of u_{j3} and u_{j4} belongs to I. Since I is independent, $I \cap \{u_{j1}, u_{j2}\} = \phi$ and hence $\{u_{j3}, u_{j4}\} \subseteq I$ since $\{u_{j3}, u_{j4}\}$ is dominated. Then $\{v_{j1}, w_{j2}\} \subseteq I$ or $\{v_{j2}, w_{j1}\} \subseteq I$ since $\{v_{j1}, v_{j2}, w_{j1}, w_{j2}\}$ is dominated; consequently $|I| \geq 4$. The last part of the statement follows directly from Lemma 1(b). ∎

In what follows, we shall also denote z'_{k1} by x_{02} for the sake of convenience. For each $j \in \{1, \ldots, k\}$, let L_j denote the subgraph of G_k induced by $V(H_j) \cup \{x_{(j-1)2}, x_{j1}\}$.

Lemma 3. *Let I be an an independent dominating set of G_k and let*

$$\begin{aligned} A_{j1} &= \{x_{(j-1)2}, x_{j1}\}; \\ A_{j2} &= \{x_{(j-1)2}, w_{j2}\} \quad \text{and} \\ A_{j3} &= \{w_{j1}, x_{j1}\}. \end{aligned}$$

If, for any $j \in \{1, \ldots, k\}$ and any $m \in \{1, 2, 3\}$, the set A_{jm} is contained in I, then

$$|V(L_j) \cap I| \geq 5.$$

Proof: If $A_{j1} \subseteq I$, the result follows immediately from Lemma 1(b). Suppose $A_{j2} \subseteq I$. If v_{j3} belongs to I, then either $\{u_{j1}, u_{j2}\}$ or $\{v_{j1}, u_{j2}\}$ is contained in I to dominate $\{u_{j1}, u_{j2}\}$. If $\{v_{j3}, w_{j3}\} \cap I = \phi$, then, as in the proof of Lemma 2, $\{u_{j3}, u_{j4}, v_{j1}\} \subseteq I$. Finally, if w_{j3} belongs to I, then one of $\{u_{j3}, u_{j4}, v_{j1}\}$, $\{u_{j1}, u_{j2}\}$ and $\{v_{j1}, u_{j2}\}$ is contained in I. In each of the above cases, $|V(L_j) \cap I| \geq 5$. Similar arguments yield the desired result if $A_{j3} \subseteq I$.

Lemma 4. *Let I be an independent dominating set of G_k such that for some $j \in \{2, \ldots, k-1\}$,*

$$\begin{aligned} I \cap \{y_{(j-1)1}, z_{(j-1)1}, z'_{(j-1)1}\} &= \phi \quad \text{and} \\ I \cap \{y_{j2}, z_{j2}, z'_{j2}\} &= \phi. \end{aligned}$$

Then $|I \cap V(L_j)| \geq 5$.

Proof: If I does not contain any of the vertices $\{y_{j2}, y_{(j-1)1}, z_{j2}, z_{(j-1)1}, z'_{j2}, z'_{(j-1)1}\}$, then $x_{(j-1)1}$ and x_{j2} belong to I since $y_{(j-1)1}$ and y_{j2} are dominated. Also, since $z'_{(j-1)1}$ and z_{j2} are dominated, $z_{(j-1)2}$ and z'_{j1} belong to I. Therefore $I \cap \{y_{(j-1)2}, y_{j1}\} = \phi$ because I is independent. Hence in order to dominate $x_{(j-1)2}$ (x_{j1} respectively), I contains exactly one of the vertices $x_{(j-1)2}$ and w_{j1} (w_{j2} and x_{j1} respectively), but not both w_{j1} and w_{j2}. The desired result is obtained by applying Lemma 3. ∎

Our next result concerns the path $P_{4\ell-1}$ and will also be used in our determination of the lower bound for the independent domination number of G_k.

Proposition 1. *Suppose $\ell + s + 1$ vertices are chosen on the path $P_{4\ell-1}$ which has vertex sequence $1, \dots, 4\ell - 1$, in such a way that each of the $\ell + s$ subpaths between pairs of consecutive chosen vertices has length at least three. Let a be the number of these $\ell + s$ subpaths with length exactly three and vertex sequence $2i - 1, \dots, 2i + 2$ for some $i \in \{1, \dots, 2\ell - 2\}$. Then*

$$a \geq 2s + 1.$$

Proof: Let a subpath of length three of the type mentioned in the hypothesis be called a subpath of type α. Further, among the $\ell + s$ subpaths, suppose there are also
b subpaths of length three with vertex sequence $2i, \dots, 2i + 3$, called subpaths of type β;
c subpaths of length at least five-type γ subpaths.
Hence there are
$\ell + s - (a + b + c)$ subpaths of length exactly four, of type δ.
Then the $\ell + s$ subpaths induced by the pairs of chosen vertices form a sequence of length $\ell + s$ with elements from the set $\{\alpha, \beta, \gamma, \delta\}$.
By counting chosen vertices as well as the vertices between them, we obtain

$$\begin{aligned} 4\ell - 1 &\geq \ell + s + 1 + 2(a + b) + 3(\ell + s - a - b - c) + 4c \\ &= 4\ell + 4s - (a + b) + c + 1 \end{aligned}$$

so that

$$(a + b) \geq 4s + c + 2. \tag{1}$$

Now notice that between any two β's there must be an α or a γ. Hence

$$a + c \geq b - 1. \tag{2}$$

By adding (1) and (2) we obtain that

$$a \geq 2s + 1.$$

∎

We are now ready to determine the independent domination number of G_k for any even positive integer k.

Theorem 2. *For any positive integer ℓ, $i(G_{2\ell}) = 11\ell - 1$.*

Proof: Since the set

$$\Big(\bigcup_{j=1}^{\ell} \{u_{(2j-1)1}, u_{(2j)1}, u_{(2j-1)2}, v_{(2j)2}, v_{(2j-1)3}, w_{(2j-1)2}, w_{(2j)3}, x_{(2j-1)2}, z'_{(2j-1)1}, z_{(2j)1}\}\Big) \bigcup \left(\bigcup_{j=1}^{\ell-1} \{y_{(2j)2}\}\right)$$

is an independent dominating set of $G_{2\ell}$ of cardinality $11\ell - 1$, it follows that $i(G_{2\ell}) \leq 11\ell - 1$.

Suppose there exists an independent dominating set I of $G_{2\ell}$ with $|I| \leq 11\ell - 2$. Lemma 2 implies that $|I \cap V(L_j)| \geq 4$ for each $j \in \{1, \ldots, 2\ell\}$ and therefore

$$|I \bigcap \left(\bigcup_{j=1}^{2\ell} V(L_j) \right)| \geq 8\ell.$$

Let s be a non-negative integer such that

$$|I \cap \left(\bigcup_{j=1}^{2\ell} V(L_j) \right)| = 8\ell + s. \tag{3}$$

Then

$$|I \cap (Y \cup Z)| \leq \begin{cases} 3\ell - s - 2 & \text{if } z'_{(2\ell)1} \notin I \\ 3\ell - s - 1 & \text{if } z'_{(2\ell)1} \in I \end{cases}$$

and hence there are at least

$$\begin{aligned} (4\ell - 1) - \begin{cases} 3\ell - s - 2 & \text{if } z'_{(2\ell)1} \notin I \\ 3\ell - s - 1 & \text{if } z'_{(2\ell)1} \in I \end{cases} \\ = \begin{cases} \ell + s + 1 & \text{if } z'_{(2\ell)1} \notin I \\ \ell + s & \text{if } z'_{(2\ell)1} \in I \end{cases} \end{aligned}$$

pairs of integers (j, t) with $j \in \{1, \ldots, 2\ell\}$, $t \in \{1, 2\}$ (but $(j, t) \neq (2\ell, 2)$) such that

$$I \cap \{Z'_{jt}, y_{jt}, z_{jt}\} = \phi$$

Let $B_{jt} = \{z'_{jt}, y_{jt}, z_{jt}\}$. If $I \cap (B_{j1} \cup B_{j2}) = \phi$ for some j ($I \cap (B_{j2} \cup B_{(j+1)1}) = \phi$ for some j, respectively), then z'_{j1} and z'_{j2} (z_{j2} and $z_{(j+1)1}$) are not dominated. If $I \cap (B_{jt} \cup B_{(j+1)t}) = \phi$ for some j and some t, then since z'_{jt} and $z_{(j+1)t}$ are dominated, both z_{j2} and z'_{j2} if $t = 1$ (both $z_{(j+1)1}$ and $z'_{(j+1)1}$ if $t = 2$) belong to I. This is impossible since I is independent.

Let P be the path with $4\ell - 1$ vertices obtained from the subgraph $\langle Y \cup Z \rangle$ of $G_{2\ell}$ by identifying the vertices of B_{jt} for each j and t. Say P has vertex sequence $(b_{11}, b_{12}, b_{21}, \ldots, b_{(2\ell)1})$, where b_{jt} is the vertex obtained by identifying z_{jt}, y_{jt} and z'_{jt}. By the above, if $I \cap (B_{jt} \cup B_{j't'}) = \phi$ for some j, j' and some t, t', then the distance between b_{jt} and $b_{j't'}$ on P is at least three. We consider two cases.

Case 1. $z'_{(2\ell)1} \notin I$

Then $I \cap B_{jt} = \phi$ for at least $\ell + s + 1$ pairs of integers (j, t). Consider the set Q consisting of the corresponding $\ell + s + 1$ vertices b_{jt} on P. By Proposition 1, there exist at least $2s+1$ pairs of consecutive vertices of Q of the form $b_{j1}, b_{(j+1)2}$ for some $j \in \{1, \ldots, 2\ell-2\}$. By Lemma 4 there exist at least $2s+1$ subgraphs L_j such that

$$|I \cap V(L_j)| \geq 5.$$

So, if we let

$$S = \{j \in \{1, \ldots, 2\ell\} : |I \cap V(L_j)| \geq 5\}.$$

then $|S| \geq 2s+1$. However, it follows from the definition of s (see (3)) that $|S| \leq s$. This contradiction establishes Case 1.

Case 2. Let S be defined as in Case 1. If $1 \in S$, let $S' = S - \{1\}$ and note that $s' = |S'| \leq s-1$. Then $I \cap B_{jt} = \phi$ for at least $\ell + s' + 1$ pairs of integers (j, t) and we may proceed exactly as in Case 1 to prove that this is impossible.

Hence suppose $1 \notin S$. Since $z'_{(2\ell)1} = x_{02} \in I$ and $|V(L_1) \cap I| = 4$, it follows from Lemma 3 that $I \cap \{x_{11}, w_{12}\} = \phi$. Now $I \cap B_{11} \neq \phi$ for otherwise, since y_{11} is dominated, x_{11} must belong to I which is impossible. Furthermore, if $I \cap B_{12} = \phi$, then x_{12} belongs to I since y_{12} is dominated; hence $w_{13} \notin I$ because I is independent. Similarly, $y_{11} \notin I$ since z'_{11} belongs to I in order to dominate z_{12} But now I contains neither x_{11} nor any of its neighbours, which is impossible; hence $I \cap B_{12} \neq \phi$. It is also clear that $I \cap B_{(2\ell-1)2} \neq \phi$ for otherwise $z'_{(2\ell-1)2}$ is not dominated.

Consider the subpath P' of P with $4(\ell-1)-1$ vertices and vertex sequence $(b_{21}, \ldots, b_{(2\ell-1)1})$. Recall that $I \cap B_{jt} = \phi$ for at least $\ell + s$ pairs of integers (j, t) and, by the preceding paragraph, the set Q' of corresponding vertices of P is contained in P'. Notice that $\ell + s = (\ell - 1) + s + 1$. We may therefore apply Proposition 1 to P' and assert that there are at least $2s+1$ pairs of consecutive vertices of Q' of the form $b_{j1}, b_{(j+1)2}$ for $j \in \{2, \ldots, 2\ell-3\}$. This gives a contradiction as in Case 1.

Therefore $i(G_{2\ell}) \geq 11\ell - 1$ and hence $i(G_{2\ell}) = 11\ell - 1$. ∎

5. The Connectivity of G_k

It remains to be shown that G_k is 3-connected. In order to do this, we use Tutte's characterisation of 3-connected graphs (see [8]) which we state here in slightly different form. By $N(u)$ we denote the *open neighborhood* of a vertex u of a graph $G = (V, E)$, where $N(u) = \{w \in V : uw \in E\}$.

Tutte's Theorem. *A graph G is 3-connected if and only if G is a wheel or a wheel can be obtained from G by a seqence of operations of the following two types:*

1. *the deletion of an edge;*
2. *the identification of two adjacent vertices u and v of degrees at least three such that $N(u) \cap N(v) = \phi$ (where the resulting loop is deleted).*

By inspecting the graph G_2, or G_k for other small values of k, it seems reasonable to expect that G_k is 3-connected. We now prove that this is indeed the case.

Theorem 3. *For every integer $k \geq 1$, the graph G_k is 3-connected.*

Proof: We prove that G_k is 3-connected by giving the sequence of operations of the above types by which the wheel $W_{2k}(= K_1 + C_{2k-1})$ is obtained from G. Any vertex obtained by identifying two adjacent vertices may be referred to by using any of its former labels.

1. Identify z_{11} and w_{k3}, w_{11} and z'_{k1}, and then for $j \in \{1, \ldots, k-1\}$, the vertices z'_{j1} with z_{j2} and z'_{j2} with $z_{(j+1)1}$.
2. Delete the edges $z_{j1}z_{j2}$ for $j \in \{1, \ldots, k-1\}$ and the edge $z_{k1}w_{11}$.
3. Identify z_{j1} and y_{j1}, $j \in \{1, \ldots, k\}$.
4. For $j \in \{1, \ldots, k-1\}$, identify y_{j1} and z_{j2} Also identify y_{k1} and w_{11}.
5. Delete the edges $y_{j1}y_{(j+1)1}1, j \in \{1, \ldots, k-1\}$.
6. For $j \in \{1, \ldots, k\}$ and $\ell \in \{1, 2, 3\}$, identify $u_{j\ell}$ and $v_{j\ell}$.
7. For $j \in \{1, \ldots, k\}$, delete $v_{j1}v_{j2}$.
8. For $j \in \{1, \ldots, k\}$ and $\ell \in \{1, 2, 3\}$, identify $v_{k\ell}$ and $w_{j\ell}$.
9. Delete the edges $w_{j1}w_{j3}$ and $w_{j2}w_{j3}$, $j \in \{1, \ldots, k\}$.
10. Identify u_{j4} and w_{j3} for $j \in \{1, \ldots, k\}$.

The graph obtained from G_2 by the above sequence of operations is illustrated in Figure 3. We continue as follows.

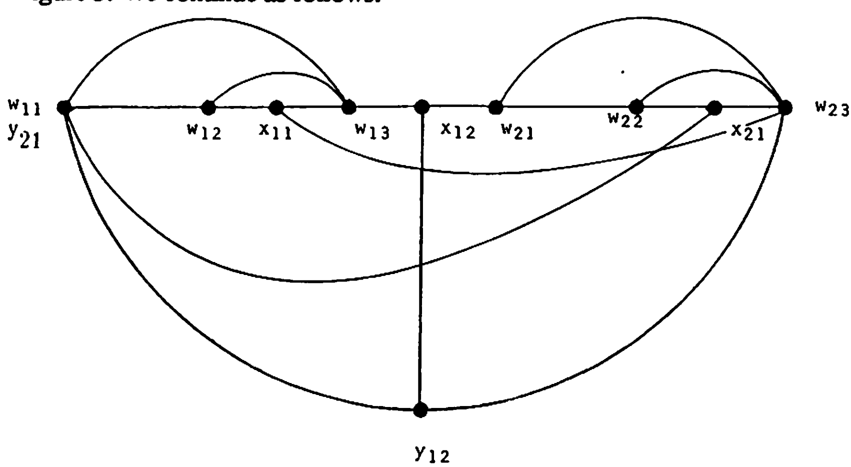

Figure 3. The graph obtained from G_2 by operations 1–10.

11. Identify x_{j2} and $w_{(j+1)1}$, $j \in \{1, \ldots, k-1\}$.
12. Identify w_{j3} and $w_{(j+1)1}$, $j \in \{1, \ldots, k-1\}$.

13. Delete $w_{k1}w_{k3}$ and $w_{j1}w_{(j+1)1}$, $j \in \{1, \ldots, k-1\}$.
14. Identify w_{j1} and w_{j2}, $j \in \{1, \ldots, k\}$.
15. Delete $w_{j2}w_{(j+1)2}$, $j \in \{1, \ldots, k-1\}$.
16. Identify x_{k1} and w_{k3}, and x_{11} and w_{22}.
17. Delete $w_{22}w_{k3}$ (which is the same as $x_{11}x_{k1}$).
18. Finally, identify in sequence y_{12} and y_{21}; y_{21} and y_{22}; ...; $y_{(k-1)1}$ and $y_{(k-1)2}$.

The resulting graph is the wheel W_{2k}. ∎

We summarise the preceding results as follows:

Theorem 4. *For every positive integer k, the graph G_{2k} constructed above is a 3-connected cubic graph with $\gamma(G_{2k}) = 10k - 1$ and $i(G_{2k}) = 11k - 1$; hence*

$$i(G_{2k}) - \gamma(G_{2k}) = k.$$

∎

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