FURTHER RESULTS ON 3-DIFFERENCE CORDIAL GRAPHS

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ABSTRACT. Let G be a (p,q) graph. Let $f : V(G) \to \{1,2,\ldots,k\}$ be a map where k is an integer $2 \leq k \leq p$. For each edge uv, assign the label $|f(u) - f(v)|$. f is called k-difference cordial labeling of G if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$ where $v_f(x)$ denotes the number of vertices labelled with x, $e_f(1)$ and $e_f(0)$ respectively denote the number of edges labelled with 1 and not labelled with 1. A graph with a k-difference cordial labeling is called a k-difference cordial graph. In this paper we investigate 3-difference cordial labeling behavior of slenting ladder, book with triangular pages, middle graph of a path, shadow graph of a path, triangular ladder, and the armed crown.

1. INTRODUCTION

Unless mentioned or otherwise, a graph in this paper shall mean a simple finite and undirected. For all terminology and notations in Graph Theory, we follow Harary [2]. The vertex and edge set of a graph G are denoted by $V(G)$ and $E(G)$ so that the order and size of G are respectively $|V(G)|$ and $|E(G)|$. Graph labeling is an assignment of integers to the vertices or edges, or both, subject to certain conditions. Graph labeling plays an important role of various fields of science and few of them are astronomy, coding theory, x-ray crystallography, radar, circuit design, communication network addressing, database management, secret sharing schemes, and models for constraint programming over finite domains [1]. The notion of difference cordial labeling was introduced by R. Ponraj, S. Sathish Narayanan and R. Kala in [3]. Seoud and Salman [11], studied the difference cordial labeling behavior of some families of graphs and they are ladder, triangular ladder, grid, step ladder and two sided step ladder graphs etc. Recently Ponraj et al. [4], introduced the concept of k-difference cordial labeling of graphs and studied the 3-difference cordial labeling behavior of star, m copies of star etc. In [5, 6, 7, 8, 9, 10] they discussed the 3-difference cordial labeling behavior of path, cycle, complete graph, complete bipartite graph,

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star, bistar, comb, double comb, quadrilateral snake, wheel, helms, flower graph, sunflower graph, lotus inside a circle, closed helm, double wheel, union of graphs with the star, union of graphs with splitting graph of star, union of graphs with subdivided star, union of graphs with bistar, $P_n \cup P_n$, $(C_n \odot K_1) \cup (C_n \odot K_1)$, $F_n \cup F_n$, $K_{1,n} \odot K_2$, $P_n \odot 3K_1$, mC_4, $spl(K_{1,n})$, $DS(B_{n,n})$, $C_n \odot K_2$, $C_4^{(t)}$, $S(K_{1,n})$, $S(B_{n,n})$, $DA(T_n) \odot K_1$, $DA(T_n) \odot 2K_1$, $DA(T_n) \odot K_2$, $DA(Q_n) \odot K_1$, $DA(Q_n) \odot 2K_1$, triangular snake, alternate triangular snake, alternate quadrilateral snake, irregular triangular snake, irregular quadrilateral snake, double triangular snake, double quadrilaterla snake, double alternate triangular snake, and double alternate quadrilateral snake, $T_n \odot K_1$, $T_n \odot 2K_1$, $T_n \odot K_2$, $A(T_n) \odot K_1$, $A(T_n) \odot 2K_1$, $A(T_n) \odot K_2$ and some more graphs. In this paper we investigate 3-difference cordial labeling behavior of slanting ladder, book with triangular pages, middle graph of a path, shadow graph of a path, triangular ladder, and the armed crown.

2. PRELIMINARIES

Definition 2.1. Let G be a (p, q) graph. Let $f : V(G) \rightarrow \{1, 2, \ldots, k\}$ be a map. For each edge uv, assign the label $|f(u) - f(v)|$. f is called a k-difference cordial labeling of G if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$ where $v_f(x)$ denotes the number of vertices labelled with x, $e_f(1)$ and $e_f(0)$ respectively denote the number of edges labelled with 1 and not labelled with 1. A graph with a k-difference cordial labeling is called a k-difference cordial graph.

Definition 2.2. Let G_1 and G_2 be two graphs with vertex sets V_1 and V_2 and edge sets E_1 and E_2 respectively. Then their join $G_1 + G_2$ is the graph whose vertex set is $V_1 \cup V_2$ and edge set is $E_1 \cup E_2 \cup \{uv : u \in V_1 \text{ and } v \in V_2\}$.

Definition 2.3. The cartesian product of two graphs G_1 and G_2 is the graph $G_1 \times G_2$ with the vertex set $V_1 \times V_2$ and two vertices $u = (u_1, u_2)$ and $v = (v_1, v_2)$ are adjacent whenever $[u_1 = v_1$ and u_2 adj $v_2]$ or $[u_2 = v_2$ and u_1 adj $v_1]$.

Definition 2.4. The graph $L_n = P_n \times P_2$ is called a ladder.

Definition 2.5. Slanting ladder is a graph obtained from two paths $u_1u_2 \ldots u_n$ and $v_1v_2 \ldots v_n$ by joining each u_i with v_{i+1}.

Definition 2.6. The Graph TL_n with vertex set $V(TL_n) = \{u_i, v_i : 1 \leq i \leq n\}$ and the edge set $E(TL_n) = \{u_iu_{i+1}, v_iv_{i+1}, u_iv_{i+1} : 1 \leq i \leq n-1\} \cup \{u_iv_i : 1 \leq i \leq n\}$ is called triangular ladders.

Definition 2.7. The shadow graph $D_2(G)$ of a connected graph G is constructed by taking 2 copies G_1 and G_2 of G and joining each vertex u in G_1 to the neighbors of the corresponding vertex v in G_2.

Definition 2.8. The middle graph $M(G)$ of a graph G is the graph whose vertex set is $V(G) \cup E(G)$ and in which two vertices are adjacent if and only if either they are adjacent edges of G or one is a vertex of G and the other is an edge incident with it.

Definition 2.9. Let G_1, G_2 respectively be (p_1, q_1), (p_2, q_2) graphs. The corona of G_1 with G_2, $G_1 \odot G_2$ is the graph obtained by taking one copy of G_1 and p_1 copies of G_2 and joining the i^{th} vertex of G_1 with an edge to every vertex in the i^{th} copy of G_2.

Definition 2.10. The graph $C_n \odot K_1$ is called a crown.

Definition 2.11. The graph $C_m \odot P_n$ is called a armed crown.

3. MAIN RESULTS

Theorem 3.1. *Slanting ladder SL_n is 3-difference cordial.*

Proof. Let $V(SL_n) = \{u_i, v_i : 1 \leq i \leq n\}$ and $E(SL_n) = \{u_iu_{i+1}, v_iv_{i+1}, u_iv_{i+1} : 1 \leq i \leq n-1\}$. Clearly, the order and size of SL_n are 2n and 3n-3 respectively.

Case 1. $n \equiv 0 \pmod 4$.

First we consider the vertices $u_1, u_2, u_3, ..., u_n$. Assign the label 2 to the vertices u_{12i+1}, u_{12i+5} and u_{12i+6} for all the values of i=0,1,2,.... and we assign the label 3 to the vertices u_{12i+2}, u_{12i+7} and u_{12i+10} for all the values of i=0,1,2,.... Then assign the label 3 to the vertices u_{12i} for i=0,1,2,..... For all the values of i=0,1,2,.... assign the label 1 to the vertices $u_{12i+3}, u_{12i+4}, u_{12i+8}, u_{12i+9}$ and u_{12i+11}. Now our attention is move to the vertices $v_1, v_2, v_3, ..., v_n$. Assign the label 1 to the vertices v_{12i+1}, v_{12i+5} for i=0,1,2,.... and we assign the label 1 to the vertices v_{12i} for i=1,2,3,... Then assign the label 3 to the vertices $v_{12i+2}, v_{12i+6}, v_{12i+8}$ and v_{12i+10} for i=0,1,2,..... For all the values of i=0,1,2,.... assign the label 2 to the vertices $v_{12i+3}, v_{12i+4}, v_{12i+7}, v_{12i+9}$ and v_{12i+11}.

Case 2. $n \equiv 1 \pmod 4$.

First we consider the vertices $u_1, u_2, u_3, ..., u_n$. Fix the label 3 to the vertex u_1. Assign the label 2 to the vertices u_{12i+2}, u_{12i+6} and u_{12i+7} for all the values of i=0,1,2,.... and assign the label 3 to the vertices u_{12i+3}, u_{12i+8} and u_{12i+11} for all the values of i=0,1,2,.... Then we assign the label 3 to the vertices u_{12i+1} for i=0,1,2,..... and we assign the label 1 to the vertices u_{12i} for all the values of i=1,2,.... For all the values of i= 0, 1,2,.... assign the label 1 to the vertices $u_{12i+4}, u_{12i+5}, u_{12i+9}$ and u_{12i+10}. Now we move to the vertices v_i. Fix the label 1 to the vertex v_1. Assign the label 1 to the vertices v_{12i+2} and v_{12i+6} for i=0,1,2,.... and assign the label 1 to the vertices v_{12i+1} for i=1,2,.... For all the values of i=0,1,2,.... and we assign the label 3 to the vertices $v_{12i+3}, v_{12i+7}, v_{12i+9}$ and v_{12i+11}. Then assign the label 2 to the vertices $v_{12i+4}, v_{12i+5}, v_{12i+8}$ and v_{12i+10} for i=0,1,2,..... and assign the label 2 to the vertices v_{12i} for all the values of i=1,2,3,.....

Case 3. $n \equiv 2 \pmod 4$.

First we consider the vertices $u_1, u_2, u_3, ..., u_n$. Fix the labels 3,1 to the vertices u_1, u_2 respectively. Assign the label 2 to the vertices u_{12i+3}, u_{12i+4} and u_{12i+11} for all the values of i=0,1,2,.... and assign the label 3 to the vertices u_{12i+5}, u_{12i+8} and u_{12i+10} for all the values of i=0,1,2,.... Then we assign the label 3 to the vertices $u_{12}, u_{24}, u_{36}, u_{48}, ...$ and we assign the label 1 to the vertices u_{12i+1} and u_{12i+2} for all the values of i= 1,2,.... For all the values of i=0,1,2,.... assign the label 1 to the vertices u_{12i+6}, u_{12i+7} and u_{12i+9}. Now our attention is move to the vertices $v_1, v_2, v_3, ..., v_n$. Fix the labels 1,2 to the vertices v_1, v_2 respectively. Assign the label 1 to the vertices v_{12i+3}, v_{12i+10} and v_{12i+11} for i=0,1,2,.... and we assign the label 3 to the vertices v_{12i+4}, v_{12i+6} and v_{12i+8} for i=0,1,2,.... Then we assign the label 3 to the vertices v_{12i} for all the values of i=1,2,3,..... and we assign the label 2 to the vertices v_{12i+5}, v_{12i+7} and v_{12i+9} for i=0,1,2,.... For all the values of i=1,2,.... assign the label 2 to the vertices v_{12i+1} and v_{12i+2} for i=0,1,2,.....

Case 4. $n \equiv 3 \pmod 4$.

First we consider the vertices $u_1, u_2, u_3, ..., u_n$. Fix the labels 3,2,1 to the vertices u_1, u_2, u_3 respectively. Assign the label 2 to the vertices u_{12i+4}, u_{12i+5} for all the values of i=0,1,2,.... and we assign the label 2 to the vertices u_{12i} for all the values of i=1,2,.... Then we assign the label 3 to the vertices u_{12i+6}, u_{12i+9} and u_{12i+11} for i=0,1,2,..... and assign the label 3 to the vertices u_{12i+1} for all the values of i=1,2,.... For all the values of i=0,1,2,.... assign the label 1 to the vertices u_{12i+7}, u_{12i+8} and u_{12i+10}. Finally assign the label 1 to the vertices u_{12i+2} and u_{12i+3} for all the values of i=1,2,.... Next we move to the vertices v_i. Assign the labels 1,3,2 to the vertices v_1, v_2, v_3 respectively. Now we assign the label 1 to the vertices v_{12i+4} and v_{12i+11} for i=0,1,2,.... and we assign the label 1 to the vertices v_{12i} for all the values of i=1,2,3,..... Then we assign the label 3 to the vertices v_{12i+5}, v_{12i+7} and v_{12i+9} for i=1,2,.... and we assign the label 3 to the vertices v_{12i+1} for i=0,1,2,..... For all the values of i=0,1,2,.... assign the label 2 to the vertices v_{12i+6}, v_{12i+8} and v_{12i+10}. Finally we assign the label 2 to the vertices v_{12i+2} and v_{12i+3} for all the values of i=1,2,.....

Therefore,this vertex labeling is a 3- difference cordial labeling follows from table 1 and table 2.

Values of n	$e_f(0)$	$e_f(1)$
$n \equiv 0, 2 \pmod 4$	$\frac{3n-4}{2}$	$\frac{3n-2}{2}$
$n \equiv 1, 3 \pmod 4$	$\frac{3n-3}{2}$	$\frac{3n-3}{2}$

TABLE 1

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 0, 3, 6, 9 \pmod{12}$	$\frac{2n}{3}$	$\frac{2n}{3}$	$\frac{2n}{3}$
$n \equiv 2, 5 \pmod{12}$	$\frac{2n+2}{3}$	$\frac{2n-1}{3}$	$\frac{2n-1}{3}$
$n \equiv 4 \pmod{12}$	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$	$\frac{2n-2}{3}$
$n \equiv 7 \pmod{12}$	$\frac{2n-2}{3}$	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$
$n \equiv 8 \pmod{12}$	$\frac{2n-1}{3}$	$\frac{2n+2}{3}$	$\frac{2n-1}{3}$
$n \equiv 10, 1 \pmod{12}$	$\frac{2n+1}{3}$	$\frac{2n-2}{3}$	$\frac{2n+1}{3}$
$n \equiv 11 \pmod{12}$	$\frac{2n-1}{3}$	$\frac{2n-1}{3}$	$\frac{2n+2}{3}$

TABLE 2

□

Theorem 3.2. *Book with triangular pages $K_2 + mK_1$ is 3-difference cordial.*

Proof. Let $V(K_2+mK_1) = \{u, v\} \cup \{u_i : 1 \leq i \leq m\}$ and $E(K_2+mK_1) = \{uv\} \cup \{uu_i, vu_i : 1 \leq i \leq m\}$. Clearly, $|V(K_2+mK_1)| = m+2$ and $|E(K_2+mK_1)| = 2m+1$. Fix the labels 1,2 to the vertices u, v respectively. Now we consider the vertices $u_1, u_2, u_3, ..., u_n$. Fix the label 3 to the vertex u_1. Assign the labels 1,2,3 to the vertices u_2, u_3, u_4 respectively. Then we assign the labels 1,2,3 to the next three vertices u_5, u_6, u_7 respectively. Proceeding like this we assign the label to the next three vertices and so on. If all the vertices are labeled then we stop the process. Otherwise there exist nonlabelled vertices. If the number of non labeled vertices is less than or equal to two then we assign the labels 1,2 to the non labeled vertices. If only one non labeled vertex is exist then we assign the label 1 to that vertex. Therefore $e_f(0) = m$ and $e_f(1) = m+1$. The vertex condition is given in table 3.

Nature of m	$v_f(1)$	$v_f(2)$	$v_f(3)$
$m \equiv 0 \pmod 3$	$\frac{n+3}{3}$	$\frac{n+3}{3}$	$\frac{n}{3}$
$m \equiv 1 \pmod 3$	$\frac{n+2}{3}$	$\frac{n+2}{3}$	$\frac{n+2}{3}$
$m \equiv 2 \pmod 3$	$\frac{n+4}{3}$	$\frac{n+1}{3}$	$\frac{n+1}{3}$

TABLE 3

□

Theorem 3.3. *The middle graph of the path P_n, MP_n is 3-difference cordial.*

Proof. Let $V(MP_n) = \{u_i : 1 \leq i \leq n\} \cup \{v_i : 1 \leq i \leq n-1\}$ and $E(MP_n) = \{u_iu_{i+1}, u_iv_i, u_{i+1}v_i : 1 \leq i \leq n-1\} \cup \{v_iv_{i+1} : 1 \leq i \leq n-2\}$. Therefore $|V(MP_n)| = 2n-1$ and $|E(MP_n)| = 4n-5$.

Case 1. $n \equiv 0 \pmod 4$.
First we consider the vertices $u_1, u_2, u_3, ..., u_n$. Fix the labels 1,3,2,1 to the vertices u_1, u_2, u_3, u_4 respectively. Assign the label 2 to the vertices u_{12i+5}, u_{12i+8} and u_{12i+11} for all the values of i=0,1,2,.... and we assign the label 2 to the vertices u_{12i+2} for all the values of i=1,2,.... For all the values of i=0,1,2,.... assign the label 3 to the vertices u_{12i+6}, u_{12i+7} and u_{12i+10}. Then we assign the label 3 to the vertices u_{12i} and u_{12i+3} for all the values of i=1,2,.... and assign the label 1 to the vertices u_{12i+9} for all the values of i=0,1,2,..... Finally assign the label 1 to the vertices u_{12i+1} and u_{12i+4} for all the values of i=1,2,.... Now our attention is move to the vertices v_i. Fix the labels 1,3,2 to the vertices v_1, v_2, v_3 respectively. Assign the label 1 to the vertices $v_{12i+4}, v_{12i+6}, v_{12i+8}$ and v_{12i+9} for all the values of i=0,1,2,.... and we assign the label 1 to the vertices v_{12i+1} for all the values of i=1,2,.... Then we assign the label 3 to the vertices v_{12i+5} for all the values of i=0,1,2,.... and assign the label 3 to the vertices v_{12i} and v_{12i+2} for i=1,2,.... For all the values of i=0,1,2,..... assign the label 2 to the vertices v_{12i+7}, v_{12i+10} and v_{12i+11}. Finally we assign the label 2 to the vertices v_{12i+3} for all the values of i=1,2,....
Case 2. $n \equiv 1 \pmod 4$.
Consider the vertices $u_1, u_2, u_3, ..., u_n$. Fix the label 1 to the vertex u_1. For all the values of i=0,1,2,.... assign the label 2
to the vertices $u_{12i+2}, u_{12i+5}, u_{12i+8}$ and u_{12i+11}. Assign the label 3 to the vertices $u_{12i+3}, u_{12i+4}, u_{12i+7}$ and u_{12i+9} for all the values of i=0,1,2,.... and we assign the label 3 to the vertices u_{12i} for all the values of i=1,2,.... Then we assign the label 1 to the vertices u_{12i+6} and u_{12i+10} for all the values of i=0,1,2,.... and assign the label 1 to the vertices $u_{13}, u_{25}, u_{37}, ...$. Next we move to the vertices v_i. For all the values of i=0,1,2,.... assign the label 1 to the vertices $v_{12i+1}, v_{12i+3}, v_{12i+5}, v_{12i+6}$ and v_{12i+10}. Now we assign the label 3 to the vertices v_{12i+2}, v_{12i+9} and v_{12i+11} for all the values of i=0,1,2,.... and assign the label 2 to the vertices v_{12i+4}, v_{12i+7} and v_{12i+8} for all the values of i=0,1,2,.... Finally we assign the label 2 to the vertices v_{12i} for all the values of i=1,2,....
Case 3. $n \equiv 2 \pmod 4$.
First we consider the vertices $u_1, u_2, u_3, ..., u_n$.
Fix the label 1,3 to the vertices u_1, u_2 respectively. For all the values of i=0,1,2,.... assign the label 1 to the vertices u_{12i+3}, u_{12i+6} and u_{12i+11}. Assign the label 2 to the vertices u_{12i+4}, u_{12i+7} and u_{12i+10} for all the values of i=0,1,2,.... and we assign the label 2 to the vertices u_{12i+1} for all the values of i=1,2,.... Then we assign the label 3 to the vertices u_{12i+5}, u_{12i+8} and u_{12i+9} for all the values of i=0,1,2,.... and assign the label 3 to the vertices u_{12i} and u_{12i+2} for all the values of i=1,2,..... Now our attention move to the vertices v_i. Fix the label 2 to the vertex v_1. Assign the label 3 to the vertices v_{12i+2}, v_{12i+4} and v_{12i+7} for all the values of i=0,1,2,....

and we assign the label 1 to the vertices $v_{12i+3}, v_{12i+6}, v_{12i+8}, v_{12i+10}$ and v_{12i+11} for all the values of i=0,1,2,.... For all the values of i=0,1,2,.... assign the label 2 to the vertices v_{12i+5} and v_{12i+9}. Finally we assign the label 2 to the vertices v_{12i} and v_{12i+1} for all the values of i=1,2,....

Case 4. $n \equiv 3 \pmod 4$.

First we consider the vertices $u_1, u_2, u_3, ..., u_n$. Fix the label 1,3,3 to the vertices u_1, u_2, u_3 respectively. Assign the label 2 to the vertices u_{12i+4}, u_{12i+7} and u_{12i+10} for all the values of i=0,1,2,..... and we assign the label 2 to the vertices $u_{13}, u_{25}, u_{37}, ...$ For all the values of i=0,1,2,.... assign the label 3 to the vertices $u_{12i+5}, u_{12i+6}, u_{12i+9}$ and u_{12i+11}. Then we assign the label 3 to the vertices u_{12i+2} for all the values of i=1,2,..... and assign the label 1 to the vertices u_{12i+8} for all the values of i=0,1,2,..... Now we assign the label 1 to the vertices u_{12i} and u_{12i+3} for all the values of i=1,2,..... Next we move to the vertices v_i. Fix the labels 1,2 to the vertex v_1, v_2 respectively. For all the values of i=0,1,2,.... assign the label 1 to the vertices $v_{12i+3}, v_{12i+5}, v_{12i+7}$ and v_{12i+8}. Assign the label 1 to the vertices v_{12i} for i=1,2,3,... and we assign the label 3 to the vertices v_{12i+4} and v_{12i+11} for all the values of i=0,1,2,.... and assign the label 2 to the vertices v_{12i+6}, v_{12i+9} and v_{12i+10} for all the values of i=0,1,2,..... Finally we assign the label 2 to the vertices v_{12i+2} for all the values of i=1,2,.... Then assign the label 3 to the vertices $v_{14}, v_{26}, v_{38}, v_{50}...$ .The edge and vertex conditions are given in table 4 and table 5.

Values of n	$e_f(0)$	$e_f(1)$
$n \equiv 0, 1, 3 \pmod 4$	$2n-2$	$2n-3$
$n \equiv 2 \pmod 4$	$2n-3$	$2n-2$

TABLE 4

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 0, 9 \pmod{12}$	$\frac{2n}{3}$	$\frac{2n}{3}$	$\frac{2n-3}{3}$
$n \equiv 1, 4 \pmod{12}$	$\frac{2n+1}{3}$	$\frac{2n-2}{3}$	$\frac{2n-2}{3}$
$n \equiv 2, 5, 8, 11 \pmod{12}$	$\frac{2n-1}{3}$	$\frac{2n-1}{3}$	$\frac{2n-1}{3}$
$n \equiv 3, 6 \pmod{12}$	$\frac{2n}{3}$	$\frac{2n-3}{3}$	$\frac{2n}{3}$
$n \equiv 7, 10 \pmod{12}$	$\frac{2n-2}{3}$	$\frac{2n-2}{3}$	$\frac{2n+1}{3}$

TABLE 5

□

Theorem 3.4. *$D_2(P_n)$ is 3-difference cordial.*

Proof. Let $V(D_2(P_n)) = \{u_i, v_i : 1 \leq i \leq n\}$ and $E(D_2(P_n)) = \{u_iu_{i+1}, v_iv_{i+1}, u_iv_{i+1}, u_{i+1}v_i : 1 \leq i \leq n-1\}$. Therefore $D_2(P_n)$ has 2n vertices and 4n-4 edges.

Case 1. $n \equiv 0 \pmod 4$.

First we consider the vertices $u_1, u_2, u_3, ..., u_n$. Fix the label 2 to the vertices $u_{12i+1}, u_{12i+4}, u_{12i+7}$ and u_{12i+9} for all the values of i=0,1,2,.... and assign the label 2 to the vertices $u_{12}, u_{24}, u_{36}, u_{48}, ..$. For all the values of i=0,1,2,.... assign the label 3 to the vertices $u_{12i+2}, u_{12i+8}, u_{12i+10}$ and u_{12i+11}. Then we assign the label 1 to the vertices u_{12i+3}, u_{12i+5} and u_{12i+6} for all the values of i=0,1,2,.... Now we move to the vertices v_i. Assign the label 1 to the vertices v_{12i+1}, v_{12i+4}, v_{12i+7} and v_{12i+9} for i=1,2,.... and assign the label 1 to the vertices v_{12i} i=1,2,.... For all the values of i=0,1,2,..... assign the label 3 to the vertices $v_{12i+2}, v_{12i+5}, v_{12i+8}$ and v_{12i+11}. Then we assign the label 2 to the vertices v_{12i+3}, v_{12i+6} and v_{12i+10} for all the values of i=0,1,2,....

Case 2. $n \equiv 1 \pmod 4$.

Consider the vertices $u_1, u_2, u_3, ..., u_n$. Fix the label 2 to the vertex u_1. Assign the label 1 to the vertices u_{12i+2} and u_{12i+3} for all the values of i=0,1,2,.... and assign the label 1 $u_{12}, u_{24}, u_{36}, u_{48}, ..$ For all the values of i=0,1,2,.... assign the label 2 to the vertices $u_{12i+4}, u_{12i+6}, u_{12i+9}$ and u_{12i+10}. Then we assign the label 2 to the vertices u_{12i+1} for all the values of i=1,2,.... and we assign the label 3 to the vertices $u_{12i+5}, u_{12i+7}, u_{12i+8}$ and u_{12i+11} for all the values of i=0,1,2,.... Next our attention move to the vertices v_i. Fix the label 1 to the vertex v_1. For all the values of i=0,1,2,..... assign the label 3 to the vertices v_{12i+2}, v_{12i+5}, v_{12i+8} and v_{12i+11}. Now we assign the label 2 to the vertices v_{12i+3} and v_{12i+7} for i=1,2,.... Finally assign the label 1 to the vertices $v_{12i+4}, v_{12i+6}, v_{12i+9}$ and v_{12i+10} for all the values of i=0,1,2,..... and assign the label 1 to the vertices v_{12i+1} for all the values of i=1,2,3,....

Case 3. $n \equiv 2 \pmod 4$.

Now we consider the vertices $u_1, u_2, u_3, ..., u_n$. Fix the labels 3,2 to the vertex u_1, u_2 respectively. For all the values of i=0,1,2,.... assign the label 2 to the vertices $u_{12i+3}, u_{12i+6}, u_{12i+7}$ and u_{12i+10}. Assign the label 2 to the vertices u_{12i+2} for all the values of i=1,2,.... and we assign the label 3 to the vertices u_{12i+4} and u_{12i+8} for all the values of i=0,1,2,...... Then we assign the label 3 to the vertices u_{12i+1} for all the values of i=1,2,.... and we assign the label 1 to the vertices u_{12i+5}, u_{12i+9} and u_{12i+11} for all the values of i=0,1,2,.... Finally assign the label 1 to the vertices u_{12i+2} for all the values of i=1,2,.... Next we move to the vertices v_i. Fix the label 1 to the vertices v_1, v_2 respectively. Assign the label 3 to the vertices v_{12i+4}, v_{12i+8} and v_{12i+11} for all the values of i=0,1,2,..... and assign the label 3 to the vertices v_{12i+1} for i=1,2,.... Then we assign the label 2 to the vertices v_{12i+5} and v_{12i+9} for all the values of i=0,1,2,..... and assign the label 2 to the vertices v_{12i} for i=1,2,3,... For all the values of i=0,1,2,...... assign the label 1 to the vertices v_{12i+6}, v_{12i+7} and v_{12i+10}. Finally assign the label 1 to the vertices $u_{14}, u_{26}, u_{38}, u_{50}, ..$

Case 4. $n \equiv 3 \pmod 4$. Consider the vertices $u_1, u_2, u_3, ..., u_n$. Fix the labels 3,1,2 to the vertex u_1, u_2, u_3 respectively. Assign the label 2 to the vertices u_{12i+4}, u_{12i+7} and u_{12i+11} for all the values of i=0,1,2,...... and assign the label 2 to the vertices u_{12i} and u_{12i+3} for all the values of i=1,2,.... Then we assign the label 3 to the vertices u_{12i+5} and u_{12i+10} for all the values of i=0,1,2,...... and assign the label 3 to the vertices u_{12i+1} for all the values of i=1,2,.... For all the values of i=1,2,.... assign the label 1 to the vertices u_{12i+6}, u_{12i+8} and u_{12i+9}. Then assign the label 1 to the vertices u_{12i+2} for all the values of i=1,2,.... Now our move is to the vertices v_i. Fix the labels 3,2,1 to the vertices v_1, v_2, v_3 respectively. Assign the label 1 to the vertices v_{12i+4}, v_{12i+7} and v_{12i+11} for all the values of i=0,1,2,..... and assign the label 1 to the vertices v_{12i+3} for i=1,2,.... Then we assign the label 3 to the vertices v_{12i+5}, v_{12i+8} and v_{12i+10} for all the values of i=0,1,2,..... For all the values of i=0,1,2,...... assign the label 2 to the vertices v_{12i+6} and v_{12i+9} assign the label 3 to the vertices v_{12i}, v_{12i+1} i=1,2,3,..... . Finally assign the label 2 to the vertices $v_{14}, v_{26}, v_{38}, v_{50}, ...$ Clearly $e_f(0) = e_f(1) = 2n-2$ and the vertex condition is given in table 6.

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 0, 3, 6, 9 \pmod{12}$	$\frac{2n}{3}$	$\frac{2n}{3}$	$\frac{2n}{3}$
$n \equiv 1, 4, 7, 10 \pmod{12}$	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$	$\frac{2n-2}{3}$
$n \equiv 2, 5, 8, 11 \pmod{12}$	$\frac{2n+2}{3}$	$\frac{2n-1}{3}$	$\frac{2n-1}{3}$

TABLE 6

□

Example 3.1. *A 3-difference cordial labeling of $D_2(P_4)$ as shown in figure 1.*

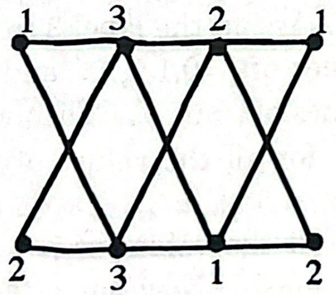

FIGURE 1

Theorem 3.5. *The triangular ladder TL_n is 3-difference cordial.*

Proof. Let $V(TL_n) = \{u_i, v_i : 1 \leq i \leq n\}$ and $E(TL_n) = \{u_iu_{i+1}, v_iv_{i+1}: 1 \leq i \leq n-1\} \cup \{u_iv_i : 1 \leq i \leq n\}$. Therefore TL_n has 2n vertices and 4n-3 edges.

Case 1. $n \equiv 0 \pmod 4$.

Consider the vertices $u_1, u_2, u_3, ..., u_n$. Assign the label 1 to the vertices u_{12i+1} and u_{12i+11} for all the values of i=0,1,2,.... and assign the label 3 to the vertices $u_{12i+2}, u_{12i+6}, u_{12i+9}$ and u_{12i+10} for all the values of i=0,1,2,.... For all the values of i=0,1,2,.... assign the label 2 to the vertices $u_{12i+3}, u_{12i+4}, u_{12i+5}, u_{12i+7}$ and u_{12i+8}. Then we assign the label 2 to the vertices u_{12i} for all the values of i=1,2,.... Now our attention move to the vertices v_i. For all the values of i=0,1,2,.... assign the label 1 to the vertices $v_{12i+1}, v_{12i+4}, v_{12i+5}, v_{12i+7}, v_{12i+9}$ and v_{12i+11}. Then we assign the label 2 to the vertices v_{12i+2} and v_{12i+10} for i=0,1,2,.... and assign the label 3 to the vertices v_{12i+3}, v_{12i+6} and v_{12i+8} for all the values of i=0,1,2,.... Finally assign the label 3 to the vertices v_{12i} for all the values of i=1,2,....

Case 2. $n \equiv 1 \pmod 4$.

Consider the vertices $u_1, u_2, u_3, ..., u_n$. Fix the label 2 to the vertex u_1. Assign the label 2 to the vertices $u_{12i+2}, u_{12i+4}, u_{12i+5}$ and u_{12i+9} for all the values of i=0,1,2,.... and assign the label 2 to the vertices u_{12i} and u_{12i+1} for all the values of i=1,2,.... Then we assign the label 3 to the vertices $u_{12i+3}, u_{12i+6}, u_{12i+7}$ and u_{12i+11} for all the values of i=0, 1,2,.... For all the values of i=0,1,2,.... assign the label 1 to the vertices u_{12i+8} and u_{12i+10}. Next we move to the vertices v_i. Fix the label 1 to the vertex v_1. Assign the label 1 to the vertices $v_{12i+2}, v_{12i+4}, v_{12i+6}, v_{12i+8}$ and v_{12i+10} for all the values of i=1,2,.... and assign the label 1 to the vertices v_{12i+1} for all i=1,2,.... Then we assign the label 3 to the vertices v_{12i+3}, v_{12i+5} and v_{12i+9} for all the values of i=0,1,2,.... and assign the label 3 to the vertices $v_{12}, v_{24}, v_{36}, v_{48}, ..$. For all the values of i=0,1,2,.... assign the label 2 to the vertices v_{12i+7} and v_{12i+11}.

Case 3. $n \equiv 2 \pmod 4$.

First we consider the vertices $u_1, u_2, u_3, ..., u_n$. Fix the label 2 to the vertices u_1, u_2 respectively. Assign the label 3 to the vertices u_{12i+3}, u_{12i+4} and u_{12i+8} for all the values of i=0,1,2,.... and assign the label 3 to the vertices u_{12i} for all the values of i=1,2,.... Then we assign the label 1 to the vertices u_{12i+5} and u_{12i+7} for all the values of i=0,1,2,.... and we assign the label 2 to the vertices $u_{12i+6}, u_{12i+9}, u_{12i+10}$ and u_{12i+11} for all the values of i=0,1,2,..... For all the values of i=0,1,2,.... assign the label 2 to the vertices u_{12i+1} and u_{12i+2}. Now our attention move to the vertices v_i. Fix the labels 3,1 to the vertices v_1, v_2 respectively. Assign the label 1 to the vertices $v_{12i+3}, v_{12i+5}, v_{12i+7}, v_{12i+10}$ and v_{12i+11} for all the values of i=0,1,2,.... and assign the label 1 to the vertices v_{12i+1} for all i=1,2,....

Then we assign the label 2 to the vertices v_{12i+4} and v_{12i+8} for all the values of i=0,1,2,.... and assign the label 3 to the vertices v_{12i+6} and v_{12i+9} for all the values of i=0,1,2,.... For all the values of i=1,2,.... assign the label 3 to the vertices v_{12i} and v_{12i+2}.

Case 4. $n \equiv 3 \pmod 4$.

Consider the vertices $u_1, u_2, u_3, ..., u_n$. Fix the labels 1,3,2 to the vertices u_1, u_2, u_3 respectively. Assign the label 1 to the vertices u_{12i+4} for all the values of i=0,1,2,.... and assign the label 1 to the vertices u_{12i+2} for all the values of i=1,2,.... Then we assign the label 3 to the vertices u_{12i+5} and u_{12i+9} for all the values of i=0,1,2,.... and assign the label 3 to the vertices u_{12i} and u_{12i+1} for all the values of i=1,2,..... For all the values of i=0,1,2,.... assign the label 2 to the vertices $u_{12i+6}, u_{12i+7}, u_{12i+8}, u_{12i+10}$ and u_{12i+11}. Then we assign the label 2 to the vertices u_{12i+3} for i=1,2,3,... Next we move to the vertices v_i. Fix the labels 1,2,3 to the vertices v_1, v_2, v_3 respectively. Assign the label 1 to the vertices $v_{12i+4}, v_{12i+7}, v_{12i+8}$ and v_{12i+10} for all the values of i=0,1,2,.... and we assign the label 1 to the vertices v_{12i} and v_{12i+2} for all i=1,2,.... Then we assign the label 2 to the vertices v_{12i+5} for all the values of i=0,1,2,.... and assign the label 2 to the vertices v_{12i+1} for all the values of i=1,2,..... For all the values of i=0,1,2,.... assign the label 3 to the vertices v_{12i+6}, v_{12i+9} and v_{12i+11}. Finally we assign the label 3 to the vertices v_{12i+3} for i=1,2,3,.... Therefore the edge condition is $e_f(0) = 2n - 2$ and $e_f(1) = 2n - 1$ and the vertex condition is given in table 7.

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 0 \pmod 3$	$\frac{2n}{3}$	$\frac{2n}{3}$	$\frac{2n}{3}$
$n \equiv 1 \pmod 3$	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$	$\frac{2n-2}{3}$
$n \equiv 2 \pmod 3$	$\frac{2n-1}{3}$	$\frac{2n+2}{3}$	$\frac{2n-1}{3}$

TABLE 7

□

Theorem 3.6. *The armed crown AC_n is 3-difference cordial.*

Proof. Let $u_1u_2u_3....u_nu_1$ be the cycle of length n. Let $V(AC_n) = V(C_n) \cup \{v_i, w_i : 1 \le i \le n\}$ and $E(AC_n) = E(C_n) \cup \{u_iv_i, v_iw_i : 1 \le i \le n\}$. Therefore AC_n has 3n vertices and 3n edges. First we consider the cycle vertices $u_1, u_2, u_3, ..., u_n$. Assign the label 1 to all the vertices $u_1, u_2, u_3, ..., u_n$. Next we move to the vertices v_i. Assign the label 3 to the vertices $v_1, v_3, v_5, ...$ and we assign the label 2 to the vertices $v_2, v_4, v_6, ...$ Now our attention move to the vertices w_i. Assign the label 2 to the vertices $w_1, w_3, w_5, ...$ then we assign the label 3 to the vertices $w_2, w_4, w_6, ...$ Therefore the vertex condition is $v_f(1) = v_f(2) = v_f(3) = n$. Also the edge condition is given in table 8. □

Values of n	$e_f(0)$	$e_f(1)$
$n \equiv 0 \pmod 2$	$\frac{3n}{2}$	$\frac{3n}{2}$
$n \equiv 1 \pmod 2$	$\frac{3n+1}{2}$	$\frac{3n-1}{2}$

TABLE 8

Example 3.2. *The 3-difference cordial labeling of AC_6 as shown in figure 2.*

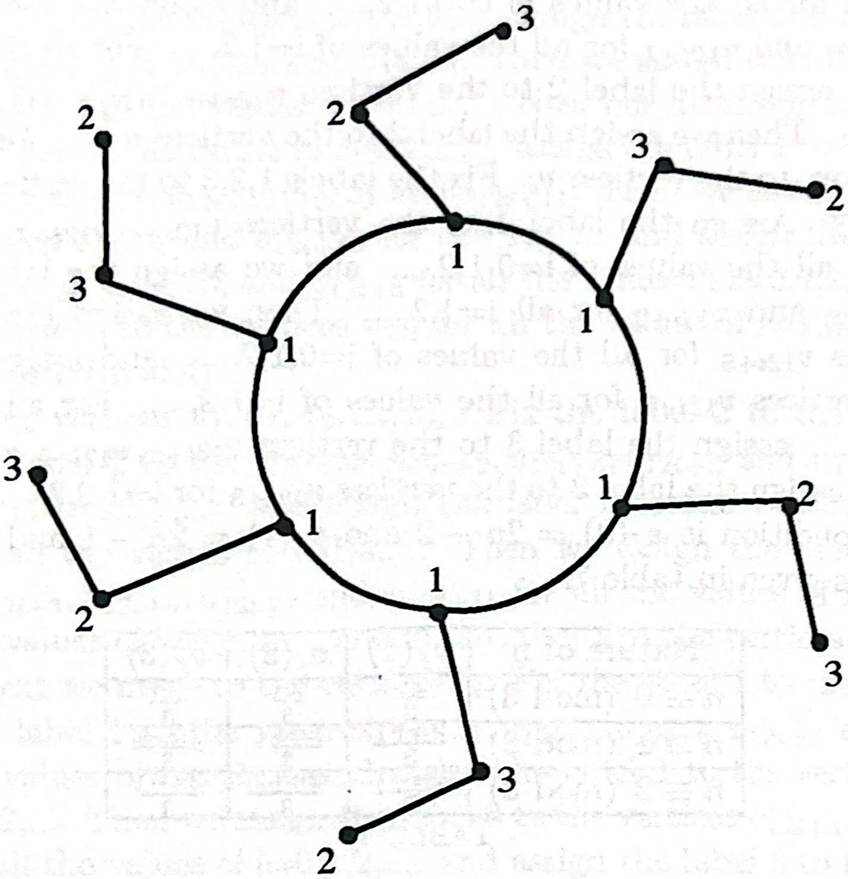

FIGURE 2 .

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4. INTRODUCTION

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