A New operation on Icosikaitetragonal fuzzy number

V. Raju[1] and R. Jayagopal[2,*]
[1]Department of Mathematics,
Vels Institute of Science, Technology and Advanced Studies,
Chennai-600 117, India
[2]School of Advanced Sciences, Vellore Institute of Technology,
Chennai-600 127, India
[1]matraju@gmail.com and [2]jgopal89@gmail.com

Abstract

In this paper, we introduce a new form of fuzzy number named as Icosikaitetragonal fuzzy number with its membership function. It includes some basic arithmetic operations like addition, subtraction, multiplication and scalar multiplication by means of α-cut with numerical illustrations.

Keywords : Fuzzy arithmetic, Icosikaitetragonal fuzzy number and α-cut.

1 Introduction

Fuzzy numbers have been developed and utilized in various fields. Fuzzy set theory permits the moderate evaluation of the membership of elements in a set which is reported in the interval [0, 1]. This can be used in a broad range of domains where information is imprecise and incomplete. In our real life we are travelling with many uncertain situations. That decisions of human judgement are frequently vague due to our regular ways of using crisp values are inadequate. In the same manner using fuzzy numbers such as triangular, trapezoidal are not suitable for all cases where the uncertainties occur in twenty four different points. In this paper new operations

*Corresponding author: R. Jayagopal

of Icosikaitetragonal fuzzy numbers have been introduced with its basic membership functions. We have carried out basic arithmetic operations on Icosikaitetragonal fuzzy number using α-cut with numerical examples.

2 Preliminaries

In this section, we give the preliminaries that are required for this study.

Definition 2.1. [1, 2] *A fuzzy set A is defined by* $A = \{(x, \mu_A(x)) : x \in A, \mu_A(x) \in [0,1]\}$. *Here x is crisp set A and* $\mu_A(x)$ *is membership function in the interval* $[0, 1]$.

Definition 2.2. *A fuzzy number A is a convex normalized fuzzy set on the real line R such that*

- *there exists at least one* $x_0 \in R$ *with* $\mu_A(x_0) = 1$
- $\mu_A(x)$ *is piecewise continuous*

Definition 2.3. *The support of a fuzzy set A defined on X is a crisp set defined as* $support(A) = \{x \in X : \mu_A(x) > 0\}$

Definition 2.4. *An* α*-cut of fuzzy set A is crisp set defined as* $^{\alpha}[A] = \{x \in X \mid \mu_A(x) \geq \alpha\}$

Definition 2.5. *A fuzzy set A is a convex fuzzy set iff each of its* α*-cut* $^{\alpha}A$ *is a convex set.*

Definition 2.6. [3] *A fuzzy number* $A = (a, b, c)$, *where* $a \leq b \leq c$, *is triangular fuzzy number and its membership function is given by*

$$\mu_A(x) = \begin{cases} \frac{x-a}{b-a}, & for\ a \leq x \leq b \\ \frac{c-x}{c-b}, & for\ b \leq x \leq c \\ 0, & x > c \end{cases}$$

Definition 2.7. [4] *A fuzzy number* $A = (a, b, c, d)$, *where* $a \leq b \leq c \leq d$, *is trapezoidal fuzzy number and its membership function is given by*

$$\mu_A(x) = \begin{cases} 0, & for\ x < a \\ \frac{x-a}{b-a}, & for\ a \leq x \leq b \\ 1, & for\ b \leq x \leq c \\ \frac{d-x}{d-c}, & for\ c \leq x \leq d \\ 0, & x > d \end{cases}$$

3 Icosikaitetragonal fuzzy number

In this section, a new form of fuzzy number named Icosikaitetragonal fuzzy number is introduced which can be more useful in solving many decisions making problems. A fuzzy number $A_{ICKT} = (a_1, a_2, \ldots, a_{24})$ is said to be Icosikaitetragonal fuzzy number, where $a_1, a_2, \ldots, a_{24}$ are the real numbers which is given below $(0 \le k_1 \le k_2 \le k_3 \le k_4 \le k_5 \le 1)$

$$\mu_A(x) = \begin{cases} 0, \; for \; x < a_1 \\ k_1(\frac{x-a_1}{a_2-a_1}), \; for \; a_1 \le x \le a_2 \\ k_1 \;, \; for \; a_2 \le x \le a_3 \\ k_1 + (k_2 - k_1)(\frac{x-a_3}{a_4-a_3}), \; for \; a_3 \le x \le a_4 \\ k_2 \;, \; for \; a_4 \le x \le a_5 \\ k_2 + (k_3 - k_2)(\frac{x-a_5}{a_6-a_5}), \; for \; a_5 \le x \le a_6 \\ k_3 \;, \; for \; a_6 \le x \le a_7 \\ k_3 + (k_4 - k_3)(\frac{x-a_7}{a_8-a_7}), \; for \; a_7 \le x \le a_8 \\ k_4 \;, \; for \; a_8 \le x \le a_9 \\ k_4 + (k_5 - k_4)(\frac{x-a_9}{a_{10}-a_9}), \; for \; a_9 \le x \le a_{10} \\ k_5 \;, \; for \; a_{10} \le x \le a_{11} \\ k_5 + (1 - k_5)(\frac{x-a_{11}}{a_{12}-a_{11}}), \; for \; a_{11} \le x \le a_{12} \\ 1 \;, \; for \; a_{12} \le x \le a_{13} \\ k_5 + (1 - k_5)(\frac{a_{14}-x}{a_{14}-a_{13}}), \; for \; a_{13} \le x \le a_{14} \\ k_5 \;, \; for \; a14 \le x \le a_{15} \\ k_4 + (k_5 - k_4)(\frac{a_{16}-x}{a_{16}-a_{15}}), \; for \; a_{15} \le x \le a_{16} \\ k_4 \;, \; for \; a_{16} \le x \le a_{17} \\ k_3 + (k_4 - k_3)(\frac{a_{18}-x}{a_{18}-a_{17}}), \; for \; a_{17} \le x \le a_{18} \\ k_3 \;, \; for \; a_{18} \le x \le a_{19} \\ k_2 + (k_3 - k_2)(\frac{a_{20}-x}{a_{20}-a_{19}}), \; for \; a_{19} \le x \le a_{20} \\ k_2 \;, \; for \; a_{20} \le x \le a_{21} \\ k_1 + (k_2 - k_1)(\frac{a_{22}-x}{a_{22}-a_{21}}), \; for \; a_{21} \le x \le a_{22} \\ k_1, \; for \; a_{22} \le x \le a_{23} \\ k_1(\frac{a_{24}-x}{a_{24}-a_{23}}), \; for \; a_{23} \le x \le a_{24} \\ 0 \; for \; x > a_{24} \end{cases}$$

3.1 Arithmetic operations on Icosikaitetragonal fuzzy number

3.1.1 Addition

If $A = (a_1, a_2, \ldots, a_{24})$ and $B = (b_1, b_2, \ldots, b_{24})$ then $A+B = (a_1+b_1, a_2+b_2, \ldots, a_{24}+b_{24})$.

Example 3.1.1:
Let $A = (1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24)$ and $B = (1,3,5,7,9,11,13,15,17,19,21,23,25,27,29,31,33,35,37,39,41,43,45,47)$ then $A+B = (2,5,8,11,14,17,20,23,26,29,32,35,38,41,44,47,50,53,56,59,62,65,68,71)$.

3.1.2 Subtraction

If $A = (a_1, a_2, \ldots, a_{24})$ and $B = (b_1, b_2, \ldots, b_{24})$ then $A-B = (a_1-b_1, a_2-b_2, \ldots, a_{24}-b_{24})$.

Example 3.1.2:
Let $A = (1,3,5,7,9,11,13,15,17,19,21,23,25,27,29,31,33,35,37,39,41,43,45,47)$ and $B = (1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24)$ then $A-B = (0,1,2,\ldots,24)$.

3.1.3 Multiplication

If $A = (a_1, a_2, \ldots, a_{24})$ and $B = (b_1, b_2, \ldots, b_{24})$ then $A*B = (a_1*b_1, a_2*b_2, \ldots, a_{24}*b_{24})$.

Example 3.1.3:
Let $A = (1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24)$ and $B = (1,3,5,7,9,11,13,15,17,19,21,23,25,27,29,31,33,35,37,39,41,43,45,47)$ then $A*B = (1,6,15,\ldots,1128)$.

3.1.4 Scalar Multiplication

If $A = (a_1, a_2, \ldots, a_{24})$ and $k \geq 1$ then $k*A = (k*a_1, k*a_2, \ldots, k*a_{24})$.

Example 3.1.4:
Let $A = (1, 2, 3, \ldots, 24)$ and $k = 2$ then $2 * A = (2, 4, 6, \ldots, 48)$.

4 Alpha Cut

Definition 4.1. *For $\alpha \in [0,1]$, the α-cut of an Icosikaitetragonal fuzzy number $A = (a_1, a_2, \ldots, a_{24})$ is defined as*

$$[A]_\alpha = \begin{cases} [a_1 + (\frac{\alpha}{k_1})(a_2 - a_1), a_{24} - (\frac{\alpha}{k_1})(a_{24} - a_{23})], \ for\ \alpha \in [0, k_1] \\ [a_3 + (\frac{\alpha-k_1}{k_2-k_1})(a_4 - a_3), a_{22} - (\frac{\alpha-k_1}{k_2-k_1})(a_{22} - a_{21})], \ for\ \alpha \in [k_1, k_2] \\ [a_5 + (\frac{\alpha-k_2}{k_3-k_2})(a_6 - a_5), a_{20} - (\frac{\alpha-k_2}{k_3-k_2})(a_{20} - a_{19})], \ for\ \alpha \in [k_2, k_3] \\ [a_7 + (\frac{\alpha-k_3}{k_4-k_3})(a_8 - a_7), a_{18} - (\frac{\alpha-k_3}{k_4-k_3})(a_{18} - a_{17})], \ for\ \alpha \in [k_3, k_4] \\ [a_9 + (\frac{\alpha-k_4}{k_5-k_4})(a_{10} - a_9), a_{16} - (\frac{\alpha-k_4}{k_5-k_4})(a_{16} - a_{15})], \ for\ \alpha \in [k_4, k_5] \\ [a_{11} + (\frac{\alpha-k_5}{1-k_5})(a_{12} - a_{11}), a_{14} - (\frac{\alpha-k_5}{1-k_5})(a_{14} - a_{13})], \ for\ \alpha \in [k_5, k_1] \end{cases}$$

4.1 Operations of Icosikaitetragonal fuzzy number using α-cut

The alpha cut of Icosikaitetragonal fuzzy number $[A] = (a_1, a_2, \ldots, a_{24})$ for all $\alpha \in [0,1]$ where $k_1 = \frac{1}{6}, k_2 = \frac{2}{6}, k_3 = \frac{3}{6}, k_4 = \frac{4}{6}, k_5 = \frac{5}{6}$ is given below

$$[A]_\alpha = \begin{cases} [a_1 + (6\alpha)(a_2 - a_1), a_{24} - (6\alpha)(a_{24} - a_{23})], \ for\ \alpha \in [0, 1/6] \\ [a_3 + (6\alpha - 1)(a_4 - a_3), a_{22} - (6\alpha - 1)(a_{22} - a_{21})], \ for\ \alpha \in [1/6, 2/6] \\ [a_5 + (6\alpha - 2)(a_6 - a_5), a_{20} - (6\alpha - 2)(a_{20} - a_{19})], \ for\ \alpha \in [2/6, 3/6] \\ [a_7 + (6\alpha - 3)(a_8 - a_7), a_{18} - (6\alpha - 3)(a_{18} - a_{17})], \ for\ \alpha \in [3/6, 4/6] \\ [a_9 + (6\alpha - 4)(a_{10} - a_9), a_{16} - (6\alpha - 4)(a_{16} - a_{15})], \ for\ \alpha \in [4/6, 5/6] \\ [a_{11} + (6\alpha - 5)(a_{12} - a_{11}), a_{14} - (6\alpha - 5)(a_{14} - a_{13})], \ for\ \alpha \in [5/6, 1] \end{cases}$$

and

$$[B]_\alpha = \begin{cases} [b_1 + (6\alpha)(b_2 - b_1), b_{24} - (6\alpha)(b_{24} - b_{23})], \ for\ \alpha \in [0, 1/6] \\ [b_3 + (6\alpha - 1)(b_4 - b_3), b_{22} - (6\alpha - 1)(b_{22} - b_{21})], \ for\ \alpha \in [1/6, 2/6] \\ [b_5 + (6\alpha - 2)(b_6 - b_5), b_{20} - (6\alpha - 2)(b_{20} - b_{19})], \ for\ \alpha \in [2/6, 3/6] \\ [b_7 + (6\alpha - 3)(b_8 - b_7), b_{18} - (6\alpha - 3)(b_{18} - b_{17})], \ for\ \alpha \in [3/6, 4/6] \\ [b_9 + (6\alpha - 4)(b_{10} - b_9), b_{16} - (6\alpha - 4)(b_{16} - b_{15})], \ for\ \alpha \in [4/6, 5/6] \\ [b_{11} + (6\alpha - 5)(b_{12} - b_{11}), b_{14} - (6\alpha - 5)(b_{14} - b_{13})], \ for\ \alpha \in [5/6, 1] \end{cases}$$

4.1.1 Addition

Let $A = (a_1, a_2, \ldots, a_{24})$ and $B = (b_1, b_2, \ldots, b_{24})$ be two Icosikaitetragonal fuzzy numbers. Let us add the α-cuts of $[A]_\alpha$ and $[B]_\alpha$ of A and B using interval arithmetic as defined below

$$[A]_\alpha + [B]_\alpha = \begin{cases} [a_1 + (6\alpha)(a_2 - a_1), a_{24} - (6\alpha)(a_{24} - a_{23})] \\ \quad + [b_1 + (6\alpha)(b_2 - b_1), b_{24} - (6\alpha)(b_{24} - b_{23})], \\ \quad for\ \alpha \in [0, 1/6] \\ [a_3 + (6\alpha - 1)(a_4 - a_3), a_{22} - (6\alpha - 1)(a_{22} - a_{21})] \\ \quad + [b_3 + (6\alpha - 1)(b_4 - b_3), b_{22} - (6\alpha - 1)(b_{22} - b_{21})], \\ \quad for\ \alpha \in [1/6, 2/6] \\ [a_5 + (6\alpha - 2)(a_6 - a_5), a_{20} - (6\alpha - 2)(a_{20} - a_{19})] \\ \quad + [b_5 + (6\alpha - 2)(b_6 - b_5), b_{20} - (6\alpha - 2)(b_{20} - b_{19})], \\ \quad for\ \alpha \in [2/6, 3/6] \\ [a_7 + (6\alpha - 3)(a_8 - a_7), a_{18} - (6\alpha - 3)(a_{18} - a_{17})] \\ \quad + [b_7 + (6\alpha - 3)(b_8 - b_7), b_{18} - (6\alpha - 3)(b_{18} - b_{17})], \\ \quad for\ \alpha \in [3/6, 4/6] \\ [a_9 + (6\alpha - 4)(a_{10} - a_9), a_{16} - (6\alpha - 4)(a_{16} - a_{15})] \\ \quad + [b_9 + (6\alpha - 4)(b_{10} - b_9), b_{16} - (6\alpha - 4)(b_{16} - b_{15})], \\ \quad for\ \alpha \in [4/6, 5/6] \\ [a_{11} + (6\alpha - 5)(a_{12} - a_{11}), a_{14} - (6\alpha - 5)(a_{14} - a_{13})] \\ \quad + [b_{11} + (6\alpha - 5)(b_{12} - b_{11}), b_{14} - (6\alpha - 5)(b_{14} - b_{13})], \\ \quad for\ \alpha \in [5/6, 1] \end{cases}$$

Example 4.1.1: Let $A = (1, 2, 3, \ldots, 24)$ and $B = (1, 3, 5, \ldots, 47)$

For $\alpha \in [0, 1/6]$, $[A]_\alpha = [1 + 6\alpha, 24 - 6\alpha]$ and $[B]_\alpha = [1 + 12\alpha, 47 - 12\alpha]$ then $[A]_\alpha + [B]_\alpha = [2 + 18\alpha, 71 - 18\alpha]$. When $\alpha = 0$, $[A]_\alpha + [B]_\alpha = [2, 71]$ and When $\alpha = 1/6$, $[A]_\alpha + [B]_\alpha = [5, 68]$.

For $\alpha \in [1/6, 2/6]$, $[A]_\alpha = [2 + 6\alpha, 23 - 6\alpha]$ and $[B]_\alpha = [3 + 12\alpha, 45 - 12\alpha]$ then $[A]_\alpha + [B]_\alpha = [5 + 18\alpha, 68 - 18\alpha]$. When $\alpha = 1/6$, $[A]_\alpha + [B]_\alpha = [8, 65]$ and When $\alpha = 2/6$, $[A]_\alpha + [B]_\alpha = [11, 62]$.

For $\alpha \in [2/6, 3/6]$, $[A]_\alpha = [3 + 6\alpha, 22 - 6\alpha]$ and $[B]_\alpha = [5 + 12\alpha, 43 - 12\alpha]$ then $[A]_\alpha + [B]_\alpha = [8 + 18\alpha, 65 - 18\alpha]$. When $\alpha = 2/6$, $[A]_\alpha + [B]_\alpha = [14, 59]$ and When $\alpha = 3/6$, $[A]_\alpha + [B]_\alpha = [17, 56]$.

For $\alpha \in [3/6, 4/6]$, $[A]_\alpha = [4 + 6\alpha, 21 - 6\alpha]$ and $[B]_\alpha = [7 + 12\alpha, 41 - 12\alpha]$ then $[A]_\alpha + [B]_\alpha = [11 + 18\alpha, 62 - 18\alpha]$. When $\alpha = 3/6$, $[A]_\alpha + [B]_\alpha = [20, 53]$ and When $\alpha = 4/6$, $[A]_\alpha + [B]_\alpha = [23, 50]$.

For $\alpha \in [4/6, 5/6]$, $[A]_\alpha = [5 + 6\alpha, 20 - 6\alpha]$ and $[B]_\alpha = [9 + 12\alpha, 39 - 12\alpha]$

then $[A]_\alpha+[B]_\alpha = [14+18\alpha, 59-18\alpha]$. When $\alpha = 4/6$, $[A]_\alpha+[B]_\alpha = [26, 47]$ and When $\alpha = 5/6$, $[A]_\alpha + [B]_\alpha = [29, 44]$.

For $\alpha \in [5/6, 1]$, $[A]_\alpha = [6+6\alpha, 19-6\alpha]$ and $[B]_\alpha = [11+12\alpha, 37-12\alpha]$ then $[A]_\alpha+[B]_\alpha = [17+18\alpha, 56-18\alpha]$. When $\alpha = 5/6$, $[A]_\alpha+[B]_\alpha = [32, 41]$ and When $\alpha = 1$, $[A]_\alpha + [B]_\alpha = [35, 38]$.

Hence, $[A]_\alpha + [B]_\alpha = [2, 5, 8, 11, \ldots, 71]$.

4.1.2 Subtraction

Let $A = (a_1, a_2, \ldots, a_{24})$ and $B = (b_1, b_2, \ldots, b_{24})$ be two Icosikaitetragonal fuzzy numbers. Let us subtract the α-cuts of $[A]_\alpha$ and $[B]_\alpha$ of A and B using interval arithmetic as defined below

$$[A]_\alpha - [B]_\alpha = \begin{cases} [a_1 + (6\alpha)(a_2 - a_1), a_{24} - (6\alpha)(a_{24} - a_{23})] \\ \quad - [b_1 + (6\alpha)(b_2 - b_1), b_{24} - (6\alpha)(b_{24} - b_{23})], \\ \quad for\ \alpha \in [0, 1/6] \\ [a_3 + (6\alpha - 1)(a_4 - a_3), a_{22} - (6\alpha - 1)(a_{22} - a_{21})] \\ \quad - [b_3 + (6\alpha - 1)(b_4 - b_3), b_{22} - (6\alpha - 1)(b_{22} - b_{21})], \\ \quad for\ \alpha \in [1/6, 2/6] \\ [a_5 + (6\alpha - 2)(a_6 - a_5), a_{20} - (6\alpha - 2)(a_{20} - a_{19})] \\ \quad - [b_5 + (6\alpha - 2)(b_6 - b_5), b_{20} - (6\alpha - 2)(b_{20} - b_{19})], \\ \quad for\ \alpha \in [2/6, 3/6] \\ [a_7 + (6\alpha - 3)(a_8 - a_7), a_{18} - (6\alpha - 3)(a_{18} - a_{17})] \\ \quad - [b_7 + (6\alpha - 3)(b_8 - b_7), b_{18} - (6\alpha - 3)(b_{18} - b_{17})], \\ \quad for\ \alpha \in [3/6, 4/6] \\ [a_9 + (6\alpha - 4)(a_{10} - a_9), a_{16} - (6\alpha - 4)(a_{16} - a_{15})] \\ \quad - [b_9 + (6\alpha - 4)(b_{10} - b_9), b_{16} - (6\alpha - 4)(b_{16} - b_{15})], \\ \quad for\ \alpha \in [4/6, 5/6] \\ [a_{11} + (6\alpha - 5)(a_{12} - a_{11}), a_{14} - (6\alpha - 5)(a_{14} - a_{13})] \\ \quad - [b_{11} + (6\alpha - 5)(b_{12} - b_{11}), b_{14} - (6\alpha - 5)(b_{14} - b_{13})], \\ \quad for\ \alpha \in [5/6, 1] \end{cases}$$

Example 4.1.2: Let $A = (1, 3, 5, \ldots, 47)$ and $B = (1, 2, 3, \ldots, 24)$

For $\alpha \in [0, 1/6]$, $[A]_\alpha = [1 + 12\alpha, 47 - 12\alpha]$ and $[B]_\alpha = [1 + 6\alpha, 24 - 6\alpha]$ then $[A]_\alpha - [B]_\alpha = [6\alpha, 23 - 6\alpha]$. When $\alpha = 0$, $[A]_\alpha - [B]_\alpha = [0, 23]$ and When $\alpha = 1/6$, $[A]_\alpha - [B]_\alpha = [1, 22]$.

For $\alpha \in [1/6, 2/6]$, $[A]_\alpha = [3+12\alpha, 45-12\alpha]$ and $[B]_\alpha = [2+6\alpha, 23-6\alpha]$ then $[A]_\alpha - [B]_\alpha = [1 + 6\alpha, 22 - 6\alpha]$. When $\alpha = 1/6$, $[A]_\alpha - [B]_\alpha = [2, 21]$ and When $\alpha = 2/6$, $[A]_\alpha - [B]_\alpha = [3, 20]$.

For $\alpha \in [2/6, 3/6]$, $[A]_\alpha = [5+12\alpha, 43-12\alpha]$ and $[B]_\alpha = [3+6\alpha, 22-6\alpha]$ then $[A]_\alpha - [B]_\alpha = [2+6\alpha, 21-6\alpha]$. When $\alpha = 2/6$, $[A]_\alpha - [B]_\alpha = [4, 19]$ and When $\alpha = 3/6$, $[A]_\alpha - [B]_\alpha = [5, 18]$.

For $\alpha \in [3/6, 4/6]$, $[A]_\alpha = [7+12\alpha, 41-12\alpha]$ and $[B]_\alpha = [4+6\alpha, 21-6\alpha]$ then $[A]_\alpha - [B]_\alpha = [3+6\alpha, 20-6\alpha]$. When $\alpha = 3/6$, $[A]_\alpha - [B]_\alpha = [6, 17]$ and When $\alpha = 4/6$, $[A]_\alpha - [B]_\alpha = [7, 16]$.

For $\alpha \in [4/6, 5/6]$, $[A]_\alpha = [9+12\alpha, 39-12\alpha]$ and $[B]_\alpha = [5+6\alpha, 20-6\alpha]$ then $[A]_\alpha - [B]_\alpha = [4+6\alpha, 19-6\alpha]$. When $\alpha = 4/6$, $[A]_\alpha - [B]_\alpha = [8, 15]$ and When $\alpha = 5/6$, $[A]_\alpha - [B]_\alpha = [9, 14]$.

For $\alpha \in [5/6, 1]$, $[A]_\alpha = [11+12\alpha, 37-12\alpha]$ and $[B]_\alpha = [6+6\alpha, 19-6\alpha]$ then $[A]_\alpha - [B]_\alpha = [5+6\alpha, 18-6\alpha]$. When $\alpha = 5/6$, $[A]_\alpha - [B]_\alpha = [10, 13]$ and When $\alpha = 1$, $[A]_\alpha - [B]_\alpha = [11, 12]$. Hence, $[A]_\alpha - [B]_\alpha = [0, 1, 2, 3, \ldots, 23]$.

4.1.3 Multiplication

Let $A = (a_1, a_2, \ldots, a_{24})$ and $B = (b_1, b_2, \ldots, b_{24})$ be two Icosikaitetragonal fuzzy numbers. Let us multiply the α-cuts of $[A]_\alpha$ and $[B]_\alpha$ of A and B using interval arithmetic as defined below

$$[A]_\alpha * [B]_\alpha = \begin{cases} [a_1 + (6\alpha)(a_2 - a_1), a_{24} - (6\alpha)(a_{24} - a_{23})] \\ \quad * [b_1 + (6\alpha)(b_2 - b_1), b_{24} - (6\alpha)(b_{24} - b_{23})], \\ \quad for\ \alpha \in [0, 1/6] \\ [a_3 + (6\alpha - 1)(a_4 - a_3), a_{22} - (6\alpha - 1)(a_{22} - a_{21})] \\ \quad * [b_3 + (6\alpha - 1)(b_4 - b_3), b_{22} - (6\alpha - 1)(b_{22} - b_{21})], \\ \quad for\ \alpha \in [1/6, 2/6] \\ [a_5 + (6\alpha - 2)(a_6 - a_5), a_{20} - (6\alpha - 2)(a_{20} - a_{19})] \\ \quad * [b_5 + (6\alpha - 2)(b_6 - b_5), b_{20} - (6\alpha - 2)(b_{20} - b_{19})], \\ \quad for\ \alpha \in [2/6, 3/6] \\ [a_7 + (6\alpha - 3)(a_8 - a_7), a_{18} - (6\alpha - 3)(a_{18} - a_{17})] \\ \quad * [b_7 + (6\alpha - 3)(b_8 - b_7), b_{18} - (6\alpha - 3)(b_{18} - b_{17})], \\ \quad for\ \alpha \in [3/6, 4/6] \\ [a_9 + (6\alpha - 4)(a_{10} - a_9), a_{16} - (6\alpha - 4)(a_{16} - a_{15})] \\ \quad * [b_9 + (6\alpha - 4)(b_{10} - b_9), b_{16} - (6\alpha - 4)(b_{16} - b_{15})], \\ \quad for\ \alpha \in [4/6, 5/6] \\ [a_{11} + (6\alpha - 5)(a_{12} - a_{11}), a_{14} - (6\alpha - 5)(a_{14} - a_{13})] \\ \quad * [b_{11} + (6\alpha - 5)(b_{12} - b_{11}), b_{14} - (6\alpha - 5)(b_{14} - b_{13})], \\ \quad for\ \alpha \in [5/6, 1] \end{cases}$$

Example 4.1.3: Let $A = (1, 2, 3, \ldots, 24)$ and $B = (1, 3, 5, \ldots, 47)$

For $\alpha \in [0,1/6]$, $[A]_\alpha = [1+6\alpha, 24-6\alpha]$ and $[B]_\alpha = [1+12\alpha, 47-12\alpha]$ then $[A]_\alpha * [B]_\alpha = [72\alpha^2+18\alpha+1, 72\alpha^2-570\alpha+1128]$. When $\alpha = 0$, $[A]_\alpha * [B]_\alpha = [1,1128]$ and When $\alpha = 1/6$, $[A]_\alpha * [B]_\alpha = [6,1035]$.

For $\alpha \in [1/6,2/6]$, $[A]_\alpha = [2+6\alpha, 23-6\alpha]$ and $[B]_\alpha = [3+12\alpha, 45-12\alpha]$ then $[A]_\alpha * [B]_\alpha = [72\alpha^2+42\alpha+6, 72\alpha^2-546\alpha+1035]$. When $\alpha = 1/6$, $[A]_\alpha * [B]_\alpha = [15,946]$ and When $\alpha = 2/6$, $[A]_\alpha * [B]_\alpha = [28,861]$.

For $\alpha \in [2/6,3/6]$, $[A]_\alpha = [3+6\alpha, 22-6\alpha]$ and $[B]_\alpha = [5+12\alpha, 43-12\alpha]$ then $[A]_\alpha * [B]_\alpha = [72\alpha^2+66\alpha+15, 72\alpha^2-522\alpha+946]$. When $\alpha = 2/6$, $[A]_\alpha * [B]_\alpha = [45,780]$ and When $\alpha = 3/6$, $[A]_\alpha * [B]_\alpha = [66,703]$.

For $\alpha \in [3/6,4/6]$, $[A]_\alpha = [4+6\alpha, 21-6\alpha]$ and $[B]_\alpha = [7+12\alpha, 41-12\alpha]$ then $[A]_\alpha * [B]_\alpha = [72\alpha^2+90\alpha+28, 72\alpha^2-498\alpha+861]$. When $\alpha = 3/6$, $[A]_\alpha * [B]_\alpha = [91,630]$ and When $\alpha = 4/6$, $[A]_\alpha * [B]_\alpha = [120,561]$.

For $\alpha \in [4/6,5/6]$, $[A]_\alpha = [5+6\alpha, 20-6\alpha]$ and $[B]_\alpha = [9+12\alpha, 39-12\alpha]$ then $[A]_\alpha * [B]_\alpha = [72\alpha^2+114\alpha+45, 72\alpha^2-474\alpha+780]$. When $\alpha = 4/6$, $[A]_\alpha * [B]_\alpha = [153,496]$ and When $\alpha = 5/6$, $[A]_\alpha * [B]_\alpha = [190,435]$.

For $\alpha \in [5/6,1]$, $[A]_\alpha = [6+6\alpha, 19-6\alpha]$ and $[B]_\alpha = [11+6\alpha, 37-12\alpha]$ then $[A]_\alpha * [B]_\alpha = [72\alpha^2+138\alpha+66, 72\alpha^2-450\alpha+703]$. When $\alpha = 5/6$, $[A]_\alpha * [B]_\alpha = [231,378]$ and When $\alpha = 1$, $[A]_\alpha * [B]_\alpha = [276,325]$.

Hence, $[A]_\alpha * [B]_\alpha = [1,6,15,28,45,66,91,120,153,190,231,276,325,378,435,496,561,630,703,780,861,946,1035,1128]$.

4.1.4 Scalar Multiplication

Let $A = (a_1, a_2, \ldots, a_{24})$ be an Icosikaitetragonal fuzzy number and k be an positive integer. Let us find the scalar multiplication of α-cuts of A_α of A using interval arithmetic as defined below

$$k * [A]_\alpha = \begin{cases} ka_1 + k(6\alpha)(a_2 - a_1), ka_{24} - k(6\alpha)(a_{24} - a_{23}) \\ \quad for\ \alpha \in [0,1/6] \\ ka_3 + k(6\alpha - 1)(a_4 - a_3), ka_{22} - k(6\alpha - 1)(a_{22} - a_{21}) \\ \quad for\ \alpha \in [1/6,2/6] \\ ka_5 + k(6\alpha - 2)(a_6 - a_5), ka_{20} - k(6\alpha - 2)(a_{20} - a_{19}) \\ \quad for\ \alpha \in [2/6,3/6] \\ ka_7 + k(6\alpha - 3)(a_8 - a_7), ka_{18} - k(6\alpha - 3)(a_{18} - a_{17}) \\ \quad for\ \alpha \in [3/6,4/6] \\ ka_9 + k(6\alpha - 3)(a_{10} - a_9), ka_{16} - k(6\alpha - 4)(a_{16} - a_{15}) \\ \quad for\ \alpha \in [4/6,5/6] \\ ka_{11} + k(6\alpha - 5)(a_{12} - a_{11}), ka_{14} - k(6\alpha - 5)(a_{14} - a_{13}) \\ \quad for\ \alpha \in [5/6,1] \end{cases}$$

Example 4.1.4: Let $A = (1, 2, 3, \ldots, 24)$ and k=2.

For $\alpha \in [0, 1/6]$, $[A]_\alpha = k[1 + 6\alpha, 24 - 6\alpha]$. When $\alpha = 0$, $[A]_\alpha = [2, 48]$ and When $\alpha = 1/6$, $[A]_\alpha = [4, 46]$.

For $\alpha \in [1/6, 2/6]$, $[A]_\alpha = k[2 + 6\alpha, 23 - 6\alpha]$. When $\alpha = 1/6$, $[A]_\alpha = [6, 44]$ and When $\alpha = 2/6$, $[A]_\alpha = [8, 42]$.

For $\alpha \in [2/6, 3/6]$, $[A]_\alpha = k[3 + 6\alpha, 22 - 6\alpha]$. When $\alpha = 2/6$, $[A]_\alpha = [10, 40]$ and When $\alpha = 3/6$, $[A]_\alpha = [12, 38]$.

For $\alpha \in [3/6, 4/6]$, $[A]_\alpha = k[4 + 6\alpha, 21 - 6\alpha]$. When $\alpha = 3/6$, $[A]_\alpha = [14, 36]$ and When $\alpha = 4/6$, $[A]_\alpha = [16, 34]$.

For $\alpha \in [4/6, 5/6]$, $[A]_\alpha = k[5 + 6\alpha, 20 - 6\alpha]$. When $\alpha = 4/6$, $[A]_\alpha = [18, 32]$ and When $\alpha = 5/6$, $[A]_\alpha = [20, 30]$.

For $\alpha \in [5/6, 1]$, $[A]_\alpha = k[6+6\alpha, 19-6\alpha]$. When $\alpha = 5/6$, $[A]_\alpha = [22, 28]$ and When $\alpha = 1$, $[A]_\alpha = [24, 26]$.

Hence, $[A]_\alpha * [B]_\alpha = [2, 4, 6, \ldots, 48]$.

5 Conclusion

Icosikaitetragonal fuzzy number was introduced in this paper and also some basic arithmetic operations were worked out such as addition, subtraction, multiplication and scalar multiplication. Icosikaitetragonal fuzzy number can be used to the problem which has twenty four points in representation. In future it may be applied in many decision making problems as well as in optimization techniques problems.

References

[1] L. A. Zadeh, *Fuzzy sets*, Information and Control, Vol. 8, no. 3, 338-353, 1965.

[2] L. A. Zadeh, *Fuzzy sets as a basis for a theory of possibility*, Fuzzy Sets and Systems, Vol. 1, no. 1, 3-28, 1978.

[3] D. D. Stephen and K. Latha, *Some types of types-2 triangular fuzzy matrices*, International njournal of pure and applied mathematics, Vol. 82, no. 1, 21-32, 2013.

[4] A. Bansal, *Trapezoidal fuzzy numbers* (a, b, c, d), Arithmetic behaviour international journal of physical and mathematical sciences, ISSN: 2010-1791.