CONTEXT-FREE EQUI-TRIANGULAR ARRAY GRAMMARS

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Abstract:

In the field of membrane computing, P system is a versatile model of computing, introduced by Paun [6], based on a combination of (*i*) the biological features of functioning of living cells and the membrane structure and (*ii*) the theoretical concepts and results related to formal language theory. Among different application areas of the model of P system, Ceterchi et al. [2] proposed an array-rewriting P system generating picture arrays based on the well-established notions in the area of array rewriting grammars. Other theoretical models of picture generation such as hexagonal array grammars and iso-array grammars have also been introduced. In this paper we consider structures in the two-dimensional plane called equi-triangular arrays composed of equilateral triangles and define a new array grammar model, which we call as context-free equi-triangular array grammar and a corresponding P system, in order to generate such structures. We also examine the generative power of these new models of picture generation.

Keywords and Phrases: Formal languages, Context-free array grammars, P system

AMS classification: 68Q42, 68Q45

1. INTRODUCTION

A novel computing model, now known as P system, was proposed around the year 2000 by Paun [6] motivated by the structure of membranes and the functioning of living cells. The basic P system with its membranes arranged inside an outermost membrane, called skin membrane, has objects in its regions which can evolve by means of evolution rules in the regions. An example of a membrane structure, depicted in terms of Venn diagrams, is shown in Fig. 1. A membrane which does not have any other membrane inside it, is called an elementary membrane. The evolution rules are applied to the objects in the regions with maximal parallelism. The objects that are evolved in the regions continue to remain in the same region or enter an outer or inner region and the process can continue resulting in a computation which halts when no object can evolve further in the regions.

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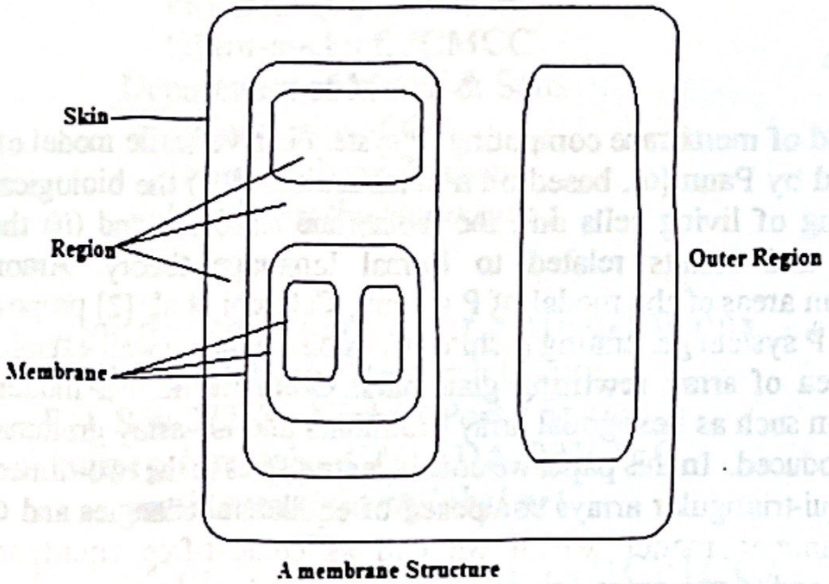


Fig. 1: A membrane structure in the basic P system model

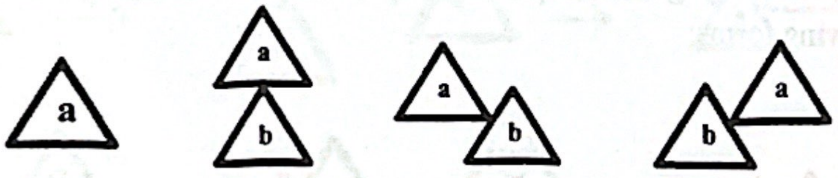
Among a variety of P systems, rewriting P systems [2, 6] both for strings and arrays have been considered. The objects in these P systems are strings or arrays and the evolution rules are string rewriting or array rewriting rules depending on the model. An important feature in such models is that an applicable rule present in a region is applied sequentially to a string or array and the evolved string or array is sent as a whole if sent to an adjacent region.

On the other hand the problem of generation of pictures in the two-dimensional plane, considered as arrays in the rectangular grid, has been extensively investigated (see, for example [7]) with techniques that extend the ideas developed in formal string language theory [4]. Subsequent studies have been on hexagonal arrays or triangular arrays [1, 5] and others. In these studies the operation of concatenation or simply called catenation of strings is extended to arrays in different forms depending on the types of arrays.

In this paper we consider tiles that are equilateral triangles and pictures, called equitriangular arrays composed of such triangles catenated vertex to edge or equivalently edge to vertex and introduce a new model, called context-free equi-triangular array grammar generating such picture patterns based on equilateral triangles. We also define a rewriting P system, called equi-triangular array P system, with objects as equi-triangular arrays. We also study the generative capacity of the new models.

2. BASIC NOTIONS

We first introduce the notion of an equi-triangular array. We consider a labelled tile in the shape of an equilateral triangle of unit side length (Fig. 2A) and call it an et-tile. An equi-triangular array (et-array) is a region in the two-dimensional plane, composed of et-tiles catenated vertex to edge with a vertex joined to the mid-point of a side as shown in Figs. 2B, 2C and 2D. An et-array α is connected if for any two et-tiles A, B in α there is a path that starts in one of them and ends in another and passing through catenated adjacent et-tiles in the et-array α.



(A) (B) (C) (D)

Fig. 2 : (A) an et-tile (B), (C), (D) : Catenations of two et-tiles

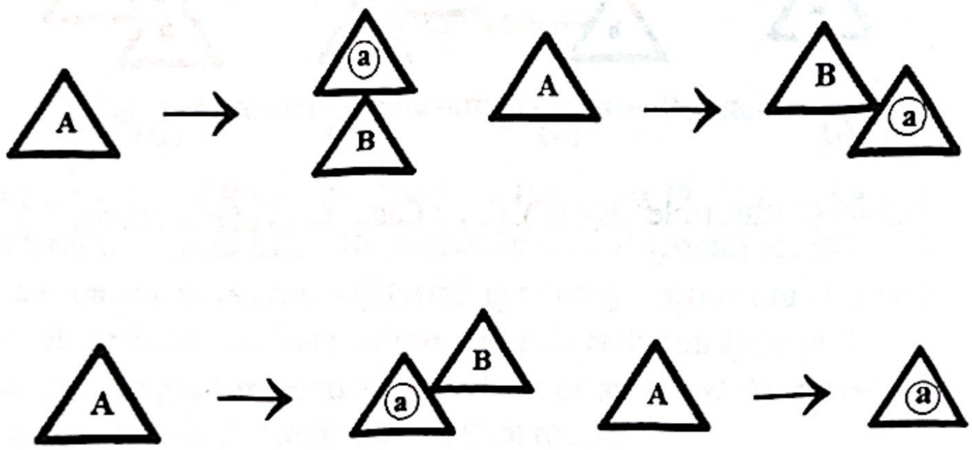
3. CONTEXT-FREE EQUI-TRIANGULAR ARRAY GRAMMAR

We now define a context-free equi-triangular array grammar for generating pictures of equi-triangular arrays.

Definition 3.1 A context-free equi-triangular array grammar (CFet-AG) is a 4-tuple $G = (N, T, P, S)$, where N is a finite nonempty set of et-tiles labelled by symbols, called non-terminals; T is a finite nonempty set of et-tiles labelled by symbols, called terminals and $N \cap T = \emptyset$. An et-tile in N labelled by S is the start tile. P consists of context-free rules of the form $X \rightarrow \alpha$, where X belongs to N and α is a finite connected et-array in the two-dimensional plane with each et-tile in α being labelled by a nonterminal or terminal symbol, with the symbol in one of the et-tiles of α being enclosed in a circle.

A derivation in G begins with the et-tile labelled by S. In a step of a direct derivation, a connected et-array β yields another et-array γ, if there is an et-tile in β with a nonterminal label X and a rule $X \rightarrow \alpha$ in G such that the region in the plane surrounding (and including the et-tile X) and geometrically identical to α, is empty so that X can be replaced by α to yield γ and in the replacement, the et-tile with the circled label occupies the et-tile with label X. and the rest of the et-tiles of α occupy their respective relative positions in relation to the et-tile of α enclosed in a circle . A derivation in G consists of a sequence of direct derivation steps yielding an et-array δ. The context-free et-array language of G, denoted by L (G), consists of such et-arrays δ such that all the et-tiles of δ are labelled by terminal symbols.

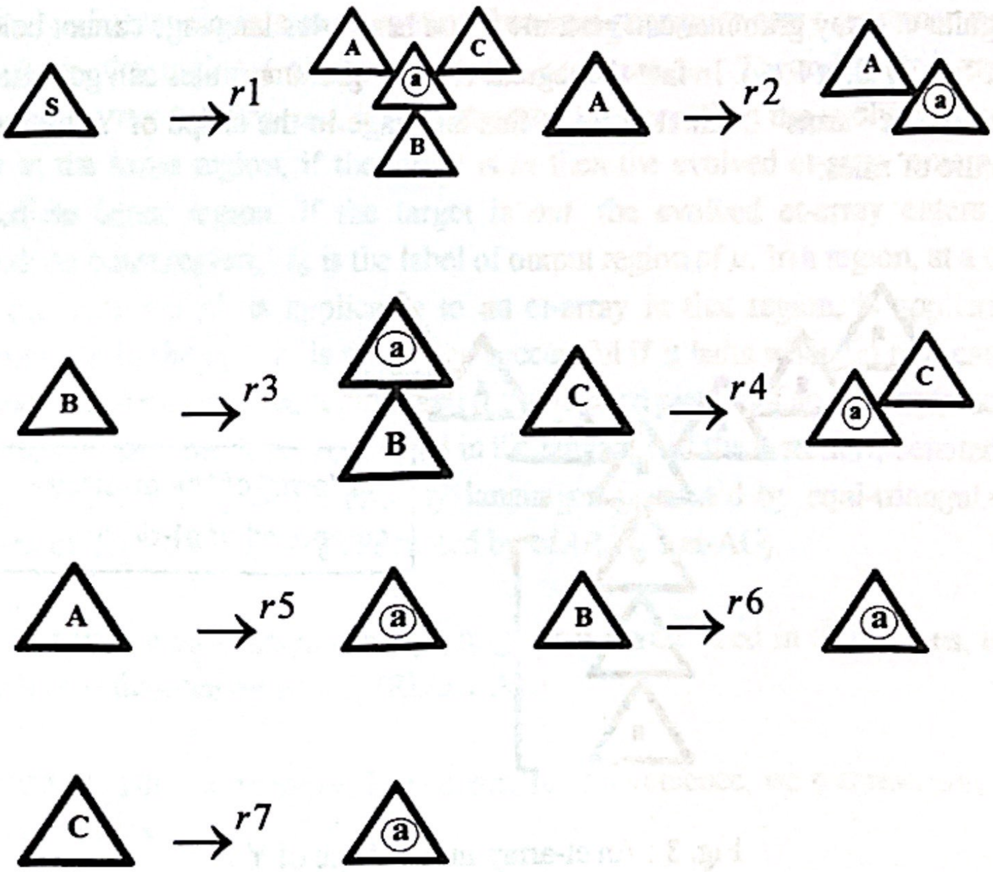
A context-free equi-triangular array grammar (CFet-AG) is called a regular equi-triangular array grammar (REGetAG) if the rules of the grammar are of the following forms:



The language generated by a regular equi-triangular array grammar is called a regular et-array language. The family of context-free et-array languages and the family of regular et-array languages are respectively denoted by $\mathcal{L}$(et-CF) and $\mathcal{L}$(et-Reg).

For convenience, throughout the rest of the paper, we simply mention only the labels in listing the et-tiles labelled by non-terminals and terminals. We now illustrate CFet-AG with an example.

Example 3.1 We consider a context-free equi-triangular array grammar G_1 = (N, T, P, S) where N = $\{S, A, B, C\}$, T = $\{a\}$, and the CF rules r1,r2,...r7of P are given below:



In a derivation starting with the start symbol  , we obtain an et-array in the shape of the letter Y as shown in Fig. 3A, on applying the rule $r1$ for S, followed by the application of the rules $r2, r3, r4$, each of them certain number of times (not necessarily equal number) and terminating the derivation by the application of the rules $r5, r6, r7$, each exactly once. The language generated by G is the set of all et-arrays of this form with the "arms" of all possible lengths where by length we mean the number of et-tiles.

Similar to the string case, the CF rules have more generative power than the regular type of rules in et-AG as shown in the following theorem.

Theorem 3.1 $\mathcal{L}$(et-Reg) $\subset$ $\mathcal{L}$(et-CF)

Proof. The inclusion in the statement directly follows from the definitions as a regular type of rule in an et-array grammar is a special kind of CF type of rule.

The proper inclusion can be seen as follows: The language of et-arrays generated by the CF et-array grammar in Example 3.1 is in the family $\mathcal{L}$(et-CF) but no regular et-array grammar can generate it and hence this language cannot belong to the family $\mathcal{L}$(et-Reg). In fact the regular et-array grammar rules can generate only two of the "arms" in an et-array in this language in the shape of Y, by the very nature of rules.

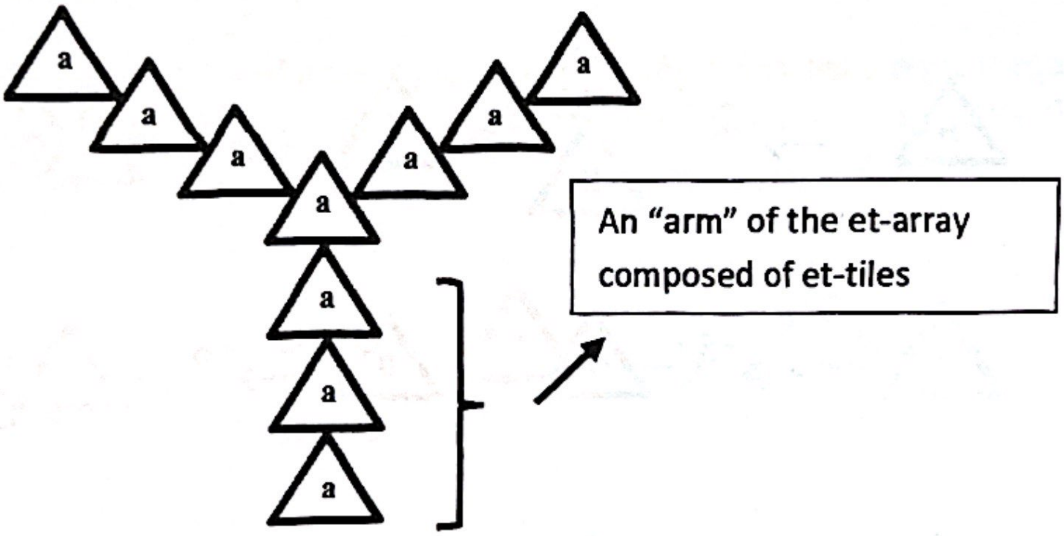


Fig. 3 : An et-array in the shape of Y

4. EQUI-TRIANGULAR ARRAY P SYSTEM

We now introduce a rewriting P system with objects in the regions as et-arrays and evolution rules as the context-free equi-triangular array grammar rules. The system we introduce is a rewriting P system with the property that only any one of the applicable rules in a region is applied to an et-array in that region while the "maximal parallelism" feature which is characteristic of a P system amounts to the fact that all et-arrays in the regions of a rewriting P system evolve at the same time if applicable rules are available in the regions.

Definition 4.1

A context-free equi-triangular array P system of degree $m \geq 1$, is defined as $\Pi = (V, T, \mu, F_1, F_2,\ldots, F_m, R_1, R_2,\ldots, R_m, i_0)$, where V is the finite set of et-tiles, $T \subset V$ is the finite set of et-tiles, called terminal et-tiles, μ is a membrane structure with m labelled regions, each having a distinct label taken from the set $\{1, \ldots, m\}$; $F_1, F_2,\ldots, F_m$ are finite sets of et- arrays over V , with F_i in the region i of μ; $R_1, R_2,\ldots, R_m$ are finite sets of context-free equi-triangular array grammar rules, with R_i in the region i of the membrane structure μ. The rules have target indications $tar \in \{here, out, in\}$. If the target is *here*, then the evolved et-array stays in the same region; if the target is *in* then the evolved et-array enters the immediate inner region; if the target is *out*, the evolved et-array enters the immediate outer region; i_0 is the label of output region of μ. In a region, at a time only one rule, which is applicable to an et-array in that region, is applied. A computation in the system is said to be successful if it halts when no rule can be applied to the objects in the regions and the generated picture of an et-array reaches the output membrane and is collected in the language of the system Π, denoted by et-$AL(\Pi)$. The family of all et-array languages generated by equi-triangular P systems of degree at most m, is denoted by $etAP_m$ (CFet-AG).

If regular type equi-triangular array grammar rules are used in the regions, then the family is denoted by $etAP_m$ (REGet-AG).

We illustrate with an example. Here again, for convenience, we mention only the labels of et-tiles.

EXAMPLE 4.1 Consider an equi-triangular array P system Π_1 , having two membranes and rules that are regular et-array grammar kind of rules, given by $\Pi_1 = (\{A,C, a\}, \{a\}, \mu, F_1, F_2, R_1, R_2, 2)$ where the membrane structure $\mu = [_1\ [_2\ ]_2\ ]_1$ consists of two membranes with membrane labelled 2 inside the membrane labelled 1; F_1 consists of an et-array initially present in region 1 as in Fig. 4A and F_2 is empty indicating that the region with label 2 has no initial et-array in it; the region 2 is the output region. The rule sets are as given below: the set R_1 consists of rules r_2 , r_5 with target *in* in region 1 while the set R_2 consists of rules r_4 , r_7 with target *out, here* respectively, in region 2.

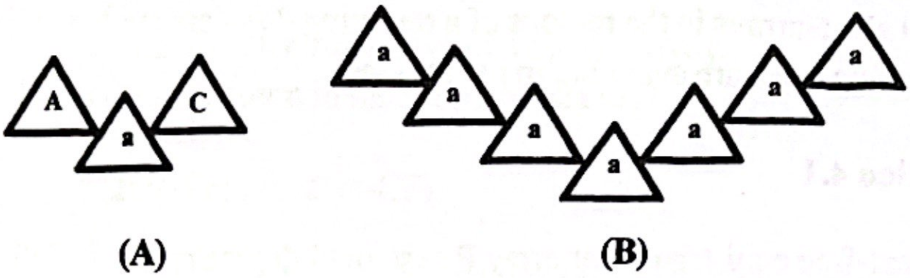


Fig. 4: (A) Initial array in region 1 in Π_1 in Example 4.1

(B) An et-array generated by the P system in Example 4.1

In a computation in the P system Π_1 , since there is an initial et-array (as in Fig. 4A) only in region 1, the left "arm" of the et-array "grows" by the application of the rule r_2 , adding one et-tile. Then the et-array is sent to region 2 as the target in the rule is *in*. In region 2, if the rule r_4 is applied, then the right "arm" grows by one et-tile and the et-array evolved is sent to region 1 due to target *out*. This process can repeat growing the left and right arms equally. If in region 1, the rule r_5 is applied, the growth in the left arm is terminated and the et-array enters region 2 where the growth in the right arm is terminated by an application of the rule r_7, yielding an et-array in the output region 2, which is in the shape of the letter V as shown in Fig. 4(B). The generated language consists of such et-arrays. Thus this language is in the family $etAP_2$ (REG et-AG).

THEOREM 4.1 $etAP_1$ (REGet-AG) $\subset$ $etAP_2$ (REGet-AG)

Proof. The inclusion follows from definitions. For proving proper inclusion, we consider the language of et-arrays in the shape of V, as in Fig. 4(B) with the left and right arms of any length but equal length. This language is in the family $etAP_2$ (REGet-AG) as shown in Example 4.1 but this language cannot belong to $etAP_1$ (REGet-AG) since the feature of equal growth in the arms cannot be handled if there is only one region and the application of the regular type of rules cannot be controlled in order to grow the arms equally.

THEOREM 4.2 $etAP_3$ (REGet-AG) - $\mathcal{L}$(et-CF) $\neq$ ϕ where ϕ is the empty set.

Proof. We consider the language of et-arrays in the shape of Y, as in Fig. 3 but the lengths of all the three "arms" of the et-array are equal. This language is not in the family $\mathcal{L}$(et-CF). In fact it cannot be obtained by any context-free equi-triangular array grammar since the equal length feature of the arms cannot be handled by the rules in such a grammar as the rules of the context-free equi-triangular array grammar can be independently applied in growing the arms. But we can construct a Regular equi-triangular array P system with three membranes with membrane structure $[_1 [_2 [_3]_3]_2]_1$.

5. CONCLUDING REMARKS

Here we have defined a new model of generation of picture languages of et-arrays called context-free equi-triangular array grammar and a corresponding membrane computing model, called et-array P system and obtained results on the generative powers of these models. It remains to compare these models with other models such as the picture models in [3].

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