Dominator Chromatic Number of Certain Graph

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Abstract: A proper vertex coloring of a graph where every node of the graph dominates all nodes of some color class is called the dominator coloring of the graph. The least number of colors used in the dominator coloring of a graph is called the dominator coloring number denoted by $\chi_d(G)$. The dominator coloring number and domination number of central, middle, total and line graph of quadrilateral snake graph are derived and the relation between them are expressed in this paper.

1. Introduction

A dominating set is a subset D_s of the vertex or node set of graph G which is such that each node in the graph either belongs to D_s or has a neighbour in D_s[9]. The domination number $\gamma(G)$ is the cardinality of a smallest dominating set of G[9]. A proper coloring of a graph G is a function $f: V \to Z_+$such that for $u, v \in V,\ f(u) \neq f(v)$ whenever u and v adjacent nodes in G.

A dominator coloring of a graph G is a proper coloring of graph such that every node or vertex of G dominates all nodes of at least one color class. The minimum cardinality of colors used in the graph for dominator coloring is called the dominator coloring number denoted by $\chi_d(G)$[4].

Gera in 2006 introduced the concept dominator coloring [4].The relationship between domination number, chromatic number and dominator chromatic number of various graphs were shown in [3], [5], [6]. The dominator coloring of prism graph, m-splitting graph and m-shadow graph of path graph, central graph, middle and total graphs, etc. were also studied in various papers[8], [7], [2], [1].

A quadrilateral snake $Q_n$consists of n blocks of quadrilateral with $3n+1$ vertices and 4n edges. It is obtained by replacing each edge of P_n by a cycle C_4.

The dominator coloring number of middle, central, total, and line graph of quadrilateral snake graph Q_n is obtained and a relationship between them is expressed in this paper.

2. Dominator Coloring number of Quadrilateral Snake graph

Theorem 2.1: If Q_n is a quadrilateral snake graph, then the dominator coloring number of Q_n is $\chi_d(Q_n) = \begin{cases} n+3 & when\ n \geq 3 \\ 2n & when\ n = 2 \end{cases}$

Proof:

The node set and the edge set of Q_n is given by

$V(Q_n) = \{v_1^1, v_i^j \,/\, 2 \leq i \leq 4, 1 \leq j \leq n\}$

$E(Q_n) = \{v_1^1 v_2^1, v_1^1 v_4^1\} \cup \{v_4^j v_2^{j+1}, v_4^j v_4^{j+1} / 1 \leq j \leq n-1\}$

$\cup \{v_i^j v_{i+1}^j \,/\, 2 \leq i \leq 3, 1 \leq j \leq n\}$

For dominator coloring of Q_n, the nodes are assigned colors as explained below

Case 1: When $n = 2$

The neighbouring nodes of v_2^1, v_3^1 are painted with color 1 and color 2 respectively. The uncoloured neighbouring nodes of v_2^2, v_3^2 are painted with color 3 and 4 respectively.

The node v_1^1 dominates color class 2. For $1 \leq j \leq n$ the nodes v_2^j, v_4^j are dominated by color class $2j-1$ and the nodes v_3^j are dominated by color class $2j$ respectively.

Every neighbouring node is given different color and also it is observed that every node of the graph dominates all the nodes of atleast one color class. Hence it is a dominator coloring of nodes and the dominator coloring number of Q_n is $\chi_d(Q_n) = 2n$.

Case 2: When $n > 2$

The node v_1^1 is painted with color 1 and for $1 \leq j \leq n$ the nodes v_2^j, v_3^j, v_4^j, are painted with colors n+2, n+3 and j+1 respectively.

The nodes v_1^1, v_2^1 are dominated by color class 1.Then for $1 \leq j \leq n$ the nodes v_4^j are dominated by color class $j+1$. And for $1 \leq j \leq n-1$ the nodes v_3^j, v_2^{j+1} are dominated by color class $j+1$ respectively. The node v_3^n is dominated by color class $n+1$.

Every neighbouring node is given different color and also it is observed that every node of the graph dominates all the nodes of atleast one color class. Hence it is a dominator coloring of nodes and the dominator coloring number of Q_n is $\chi_d(Q_n) = n+3$.

Thus $\chi_d(Q_n) = \begin{cases} n+3 & when\ n \geq 3 \\ 2n & when\ n = 2 \end{cases}$

3. Dominator Coloring number of Middle, Central, Total and Line graph of Quadrilateral Snake graph

Theorem 3.1: If MQ_n is the middle graph of quadrilateral snake graph Q_n then its dominator coloring number is $\chi_d(MQ_n) = \left\lceil \frac{n+1}{2} \right\rceil + n + 3$

Proof:

The middle graph MQ_n of a quadrilateral snake graph Q_n is obtained by the subdivision of each edge of Q_n exactly once and connecting by an edge of all the newly added nodes of adjacent edges of Q_n. Let the new nodes obtained by the subdivision of edges be $\{e_i^j / 1 \le i \le 4,\ 1 \le j \le n\}$.

The node set and edge set of the middle graph of the quadrilateral snake graph MQ_n are

$V(MQ_n) = \{v_1^1\} \cup \{v_i^j / 2 \le i \le 4, 1 \le j \le n\} \cup \{e_i^j / 1 \le i \le 4,\ 1 \le j \le n\}$

$E(MQ_n) = \{v_1^1 e_1^1, e_1^1 v_2^1, v_1^1 e_4^1, e_4^n v_4^n\} \cup \{ v_i^j e_i^j, e_i^j v_{i+1}^j / 2 \le i \le 4, 1 \le j \le n\} \cup \{e_4^j v_4^j,\ v_4^j e_4^{j+1}, e_3^j e_1^{j+1} / 1 \le j \le n-1\} \cup \{e_i^j e_{i+1}^j, e_1^j e_4^j / 1 \le i \le 3, 1 \le j \le n\}$

For dominator coloring of MQ_n, the nodes are assigned colors as explained below

The nodes v_1^1, v_2^j, v_3^j for $1 \le j \le n$ and v_4^j for $1 \le j \le n-1$ are painted with color 1. The node v_4^n is painted with color 1 $when\ j\ (mod\ 2) \equiv 1$ and color $n + \left\lceil \frac{j+1}{2} \right\rceil + 3$ when $j (mod\ 2) \equiv 0$.

For $1 \le j \le n$, the nodes e_1^j, e_3^j are painted with color 2 when $j\ (mod\ 2) \equiv 1$ and color 3 when $j (mod\ 2) \equiv 0$ and the nodes e_2^j are painted with color $j + 3$ respectively. For $1 \le j \le n$ the nodes e_4^j are painted with color $n + \left\lceil \frac{j+1}{2} \right\rceil + 3$ when $j (mod\ 2) \equiv 1$ and color 2 when $j\ (mod\ 2) \equiv 0$ respectively.

The node v_1^1 is dominated by color class $n+4$. Then for $1 \le j \le n$ the nodes v_4^j, e_4^j are dominated by color class $n + \left\lceil \frac{j+1}{2} \right\rceil + 3$ and the nodes v_2^j, v_3^j, e_1^j, e_2^j, e_3^j are dominated by color class $j+3$ respectively.

Every neighbouring node is given different color and also it is observed that every node of the graph dominates all the nodes of atleast one color class. Thus it is a dominator coloring of nodes and the dominator coloring number of middle graph of Q_n is $\chi_d (MQ_n) = \left\lceil \frac{n+1}{2} \right\rceil + n + 3$.

Theorem 3.2: If TQ_n is the total graph of quadrilateral snake graph Q_n then its dominator coloring number is

$$\chi_d (TQ_n) = \begin{cases} n + \left\lceil \dfrac{n+1}{2} \right\rceil + 3 & when\ n(mod\ 2) \equiv 1 \\ n + \left\lceil \dfrac{n+1}{2} \right\rceil + 2 & when\ n(mod\ 2) \equiv 0 \end{cases}$$

Proof:

The total graph TQ_n of quadrilateral snake graph Q_n is obtained by the subdivision of each edge of Q_n exactly once and connecting by an edge of all the newly added nodes of adjacent edges of Q_n and all the neighbouring edges of Q_n. Let the new nodes obtained by the subdivision of edges of $E(Q_n)$ be $\{e_i^j / 1 \le i \le 4,\ 1 \le j \le n\}$.

The total graph of the quadrilateral snake graph TQ_n has its node set and edge set given by

$V(TQ_n) = \{v_1^1\} \cup \{v_i^j / 2 \le i \le 4,\ 1 \le j \le n\} \cup \{e_i^j / 1 \le i \le 4,\ 1 \le j \le n\}$

$E(TQ_n) = E(Q_n) \cup E(MQ_n)$

i.e., $E(TQ_n) = \{v_1^1 v_2^1, v_1^1 v_4^1\} \cup \{v_1^1 e_1^1, e_1^1 v_2^1, v_1^1 e_4^1, e_4^n v_4^n\} \cup$
$\{v_4^j v_2^{j+1}, v_4^j v_4^{j+1} / 1 \le j \le n-1\} \cup \{v_i^j v_{i+1}^j / 2 \le i \le 3, 1 \le j \le n\} \cup$
$\{v_i^j e_i^j, e_i^j v_{i+1}^j / 2 \le i \le 4, 1 \le j \le n\} \cup \{e_4^j v_4^j, v_4^j e_4^{j+1}, e_3^j e_1^{j+1} / 1 \le j \le n-1\} \cup$
$\{e_i^j e_{i+1}^j, e_1^j e_4^j / 1 \le i \le 3, 1 \le j \le n\}$

For dominator coloring of TQ_n, the nodes are assigned colors as explained below

The node v_1^1 is painted with color 1. For $1 \le j \le n$, the nodes v_2^j are painted with color 3 when $j (mod\ 4) \equiv 1\ or\ 2$ and color 1 when $j (mod\ 4) \equiv 0\ or\ 3$, the nodes v_3^j are painted with color 1 when $j (mod\ 4) \equiv 1$, color 3 when $j (mod 4) \equiv 3$ and color 2 when $j (mod\ 4) \equiv 0\ or\ 2$ and the nodes v_4^j are painted with color $n + \left\lceil \frac{j+1}{2} \right\rceil + 3$ when $j (mod\ 4) \equiv 1\ or\ 3$, color 3 when $j (mod\ 4) \equiv 2$ and color 1 when $j (mod\ 4) \equiv 0$ respectively.

For $1 \le j \le n$, the nodes e_1^j, e_3^j are painted with color 2 when $j (mod\ 4) \equiv 1\ or\ 3$, color 1 when $j (mod\ 4) \equiv 2$ and color 3 when $j (mod\ 4) \equiv 0$, the nodes e_4^j are painted with color 2 when $j (mod\ 4) \equiv 0\ or\ 2$, color 1 when $j (mod\ 4) \equiv 3$ and color 3 when $j (mod\ 4) \equiv 1$ and the nodes e_2^j are painted with color $j + 3$ respectively.

The node v_1^1 is dominated by color class $n + 4$. For $1 \le j \le n$ the nodes v_2^j, v_3^j, e_1^j, e_2^j, e_3^j are dominated by color class $j + 3$ and the nodes v_4^j , e_4^j are dominated by color class $n + \left\lceil \frac{j+1}{2} \right\rceil + 3$ when j is odd and by color class $n + \left\lceil \frac{j+1}{2} \right\rceil + 2$ when j is even respectively.

Every neighbouring node is given different color and also it is observed that every node of the graph dominates all the nodes of atleast one color class. Hence it is a dominator coloring of nodes and the dominator coloring number of total graph of Q_n is $\chi_d(TQ_n) = \begin{cases} n + \left\lceil \frac{n+1}{2} \right\rceil + 3 & when\ n(mod\ 2) \equiv 1 \\ n + \left\lceil \frac{n+1}{2} \right\rceil + 2 & when\ n(mod\ 2) \equiv 0 \end{cases}$

Theorem 3.3: If CQ_n is the central graph of quadrilateral snake graph Q_n then its dominator coloring number is $\chi_d(CQ_n) = 3n + 1$

Proof:

The central graph CQ_n of quadrilateral snake graph Q_n is obtained by the subdivision of each edge of Q_n exactly once and connecting by an edge all the non-adjacent nodes of Q_n in CQ_n. Let the new nodes obtained by the subdivision of edges $E(Q_n)$ be $\{e_i^j \,/\, 1 \le i \le 4\,,\ 1 \le j \le n\}$. The node set and edge set of the central graph of the quadrilateral snake graph CQ_n are

$V(CQ_n) = \{v_1^1\} \cup \{v_i^j \,/\, 2 \le i \le 4\,, 1 \le j \le n\} \cup \{e_i^j \,/\, 1 \le i \le 4\,, 1 \le j \le n\}$

$E(CQ_n) = \{v_1^1 e_1^1, e_1^1 v_2^1, v_1^1 e_4^1, e_4^n v_4^n\} \cup \{\, v_i^j e_i^j\,, e_i^j v_{i+1}^j / 2 \le i \le 4\,, 1 \le j \le n\} \cup$
$\{e_4^j v_4^j,\ v_4^j e_4^{j+1}, e_3^j e_1^{j+1} /\ 1 \le j \le n-1\} \cup E(G^c)$

For dominator coloring of CQ_n, the nodes are assigned colors as explained below

The node v_1^1 is painted with color 1. For $1 \le j \le n$, the nodes v_2^j, v_3^j, v_4^j are painted with color $3j-1$, $3j$, $3j+1$ respectively. The nodes e_i^1 for $1 \le i \le 4$ are painted with color 5. For $1 \le i \le 4$, $2 \le j \le n$, the nodes e_i^j are painted color 1.

The node v_1^1 is dominated by color class 2. For $1 \le j \le n$ the nodes v_2^j, e_1^j, e_3^j are dominated by color class $3j-1$ and the nodes v_3^j, v_4^j, e_3^j, e_4^j are dominated by color class $3j+1$ respectively.

Every neighbouring node is given different color and also it is observed that every node of the graph dominates all the nodes of atleast one color class. Hence it is a dominator coloring of nodes and the dominator coloring number of central graph of Q_n is $\chi_d(CQ_n) = 3n+1$.

Theorem 3.4: If LQ_n is the line graph of Q_n then its dominator coloring number is

$$\chi_d(LQ_n) = \begin{cases} 2n & when\ 2 \le n \le 3 \\ n+3 & when\ n > 3 \end{cases}$$

Proof:

The line graph LQ_n of a quadrilateral snake graph Q_n is a graph obtained by taking the edges of Q_n as the nodes of LQ_n and connecting two nodes of LQ_n by an edge whenever the corresponding edges of Q_n are adjacent.

Let the new nodes obtained by the replacing the edges $E(Q_n)$ be $\{e_i^j \,/\, 1 \le i \le 4\,,\ 1 \le j \le n\}$. The node set and edge set of the line graph of the quadrilateral snake graph LQ_n are given by

$V(LQ_n) = \{e_i^j \,/\, 1 \le i \le 4\,,\ 1 \le j \le n\}$

$E(LQ_n) = \{e_1^j e_4^j / 1 \le j \le n\} \cup \{e_i^j e_{i+1}^j / 1 \le i \le 3\,, 1 \le j \le n\} \cup$
$\{e_3^j e_1^{j+1} / 1 \le j \le n-1\}$

For dominator coloring of LQ_n, the nodes are assigned colors as explained below

Case 1: When $2 \le n \le 3$

For $1 \le j \le n$, the e_1^j, e_3^j are painted with color $2j$ and the nodes e_2^j, e_4^j are painted with color $2j-1$ respectively. For $1 \le j \le n$ the nodes e_1^j, e_3^j are dominated by color class $2j-1$ and the nodes e_2^j, e_4^j are dominated by color class $2j$ respectively.

Every neighbouring node is given different color. Also it is observed that every node of the graph dominates all the nodes of atleast one color class. Hence it is a dominator coloring of nodes and the dominator coloring number of line graph of Q_n is $\chi_d(LQ_n) = 2n$ when $2 \leq n \leq 3$.

Case 2: When $n > 3$

For $1 \leq j \leq n$, let the nodes e_1^j be painted with color $j+2$, the nodes e_2^j and the nodes e_4^j are painted with color 1 when $j(mod\ 2) \equiv 1$ and color 2 when $j(mod\ 2) \equiv 0$ respectively. For $1 \leq j \leq n-1$ the nodes e_3^j are painted with color 2 when $j\ (mod\ 2) \equiv 1$ and color 1 when $j\ (mod\ 2) \equiv 0$. The node e_3^n is painted with color $n+3$.

For $1 \leq j \leq n$ the nodes e_1^j, e_2^j, e_4^j are dominated by color class $j+2$ and the nodes e_3^j are dominated by color class $j+3$ respectively.

Every neighbouring node is given different color and also it is observed that every node of the graph dominates all the nodes of atleast one color class. Thus it is a dominator coloring of nodes and the dominator coloring number of line graph of Q_n is $\chi_d(LQ_n) = n+3$ when $n > 3$.

Hence the dominator coloring number of line graph of quadrilateral snake graph G is $\chi_d(LQ_n) = \begin{cases} 2n & when\ 2 \leq n \leq 3 \\ n+3 & when\ n > 3 \end{cases}$.

4. Relationship between Dominator Coloring number and Domination Number

Lemma 4.1: If MQ_n is the middle graph of quadrilateral snake graph Q_n then its domination number is $\gamma(MQ_n) = \begin{cases} n + \left\lceil \frac{n}{2} \right\rceil & when\ n(mod2) \equiv 1 \\ n + \left\lceil \frac{n+1}{2} \right\rceil & when\ n(mod2) \equiv 0 \end{cases}$

Proof:

Case 1: $when\ n(mod\ 2) \equiv 1$

Let $D_s = \left\{e_2^{2i-1}, e_4^{2i-1}, e_2^{2i} / 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor\right\} \cup \{e_2^n, e_4^n\}$. Clearly each node in V- D_s has atleast a neighbour in D_s and the dominating set D_s has the minimum cardinality.

Hence the domination number of $M(Q_n)$ is given by $\gamma(M(Q_n)) = n + \left\lceil \frac{n}{2} \right\rceil$ $when\ n(mod2) \equiv 1$.

Case 2: $when\ n(mod\ 2) \equiv 0$

Let $D_s = \left\{e_2^{2i-1}, e_4^{2i-1}, e_2^{2i} / 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor\right\} \cup \{e_4^n\}$. Clearly each node in V- D_s has atleast a neighbour in D_s and the dominating set D_s has the minimum cardinality. Hence the domination number of $M(Q_n)$ is given by $\gamma(M(Q_n)) = n + \left\lceil \frac{n+1}{2} \right\rceil$ $when\ n(mod2) \equiv 0$.

Hence the domination number of MQ_n is

$$\gamma(MQ_n) = \begin{cases} n + \left\lceil \frac{n}{2} \right\rceil & when\, n(mod2) \equiv 1 \\ n + \left\lceil \frac{n+1}{2} \right\rceil & when\, n(mod2) \equiv 0 \end{cases}$$

Corollary 4.2: If MQ_n is the middle graph of quadrilateral snake graph Q_n then it satisfies the relation $\chi_d(MQ_n) = \gamma(MQ_n) + 3$

Proof is obtained using theorem 3.1 and lemma 4.1

Lemma 4.3: If CQ_n is the central graph of quadrilateral snake graph Q_n then its domination number is $\gamma(CQ_n) = 2n$

Proof:

Let $D_s = \{v_3^i, v_4^i, v_1^n, v_3^n / 1 \le i \le n-1\}$.

Clearly each node in *V-D_s* has atleast a neighbour in D_s and the dominating set *DS* has the minimum cardinality.

Hence the domination number of CQ_n is given by $\gamma(CQ_n) = 2n$.

Corollary 4.4: If CQ_n is the central graph of quadrilateral snake graph then it satisfies the relation $\chi_d(CQ_n) = \gamma(CQ_n) + 3$

Proof is obtained using theorem 3.3 and lemma 4.3

Lemma 4.5: If LQ_n is the line graph of quadrilateral snake graph Q_n then its domination number is $\gamma(LQ_n) = n + 1$

Proof:

Let $D_s = \{e_1^i / 1 \le i \le n\} \cup \{e_3^n\}$. Clearly each node in *V-D_s* has atleast a neighbour in D_s and the dominating set D_s has the minimum cardinality.

Hence the domination number of line graph of quadrilateral snake graph Q_n is $\gamma(LQ_n) = n + 1$.

Corollary 4.6: If LQ_n is the line graph of quadrilateral snake graph Q_n then it satisfies the relation $\chi_d(LQ_n) = \gamma(LQ_n) + \left\lceil \frac{n}{2} \right\rceil$

Proof is obtained using theorem 3.4 and lemma 4.5.

5. Relation between Dominator coloring number of quadrilateral snake graph with Middle graph, Total graph, Central graph and Line graph

Theorem 5.1: If MQ_n is the middle graph of a quadrilateral snake graph Q_n then the dominator coloring number satisfies the relation

$$\chi_d(MQ_n) = \begin{cases} \chi_d(Q_n) + 3 & when\, n = 2 \\ \chi_d(Q_n) + \left\lceil \frac{n+1}{2} \right\rceil & when\ n > 2 \end{cases}$$

Proof: This theorem is obtained using theorem 2.1 and 3.1.

Theorem 5.2: If TQ_n is the total graph of quadrilateral snake graph Q_n then the dominator coloring number satisfies the relation

$$\chi_d(TQ_n) = \begin{cases} \chi_d(Q_n) + 2 & when\, n = 2 \\ \chi_d(Q_n) + \left\lceil \frac{n}{2} \right\rceil & when\ n > 2 \end{cases}$$

Proof: This theorem is obtained using theorem 2.1 and 3.2.

Theorem 5.3: If CQ_n is the central graph of quadrilateral snake graph Q_n then the dominator coloring number satisfies the relation

$$\chi_d(CQ_n) = \begin{cases} \chi_d(Q_n) + 3 & when\, n = 2 \\ \chi_d(Q_n) + 2n - 2 & when\, n > 2 \end{cases}$$

Proof: This theorem is obtained using theorem 2.1 and 3.3.

Theorem 5.4: If LQ_n is the line graph of quadrilateral snake graph Q_n then the dominator coloring number satisfies the relation $\chi_d(LQ_n) = \chi_d(Q_n)$

Proof: This theorem is obtained using theorem 2.1 and 3.4.

Theorem 5.5: If MQ_n and TQ_n are the middle graph and the total graph of quadrilateral snake graph Q_n then the dominator coloring number satisfies the relation

$$\chi_d(MQ_n) = \begin{cases} \chi_d(TQ_n) & when\, n(mod\ 2) \equiv 1 \\ \chi_d(TQ_n) + 1 & when\, n(mod\ 2) \equiv 0 \end{cases}$$

Proof: This theorem is obtained using theorem 3.1 and 3.2.

Theorem 5.6: If MQ_n and CQ_n are the middle graph and the central graph of quadrilateral snake graph Q_n then the dominator coloring number satisfies the relation

$$\chi_d(C(Q_n)) = \begin{cases} \chi_d(MQ_n) + 3\left(\left\lfloor\frac{n}{2}\right\rfloor - 1\right) + 2 & when\, n(mod\ 2) \equiv 1 \\ \chi_d(MQ_n) + 3\left(\left\lfloor\frac{n}{2}\right\rfloor - 1\right) & when\, n(mod\ 2) \equiv 0 \end{cases}$$

Proof: This theorem is obtained using theorem 3.1 and 3.3.

Theorem 5.7: If MQ_n and LQ_n are the middle graph and the line graph of quadrilateral snake graph Q_n then the dominator coloring number satisfies the relation

$$\chi_d(MQ_n) = \begin{cases} \chi_d(LQ_n) + \left\lceil\frac{n+1}{2}\right\rceil & when\, n > 3 \\ \chi_d(LQ_n) + 3 & when\, n = 2 \\ \chi_d(LQ_n) + 2 & when\, n = 3 \end{cases}$$

Proof: This theorem is obtained using theorem 3.1 and 3.4.

Theorem 5.8: If TQ_n and CQ_n are the total graph and the central graph of quadrilateral snake graph Q_n then the dominator coloring number satisfies the relation

$$\chi_d(CQ_n) = \begin{cases} \chi_d(TQ_n) + 3\left\lceil\frac{n-1}{2}\right\rceil - 1 & when\, n(mod\ 2) \equiv 1 \\ \chi_d(TQ_n) + 3\left\lceil\frac{n-1}{2}\right\rceil - 2 & when\, n(mod\ 2) \equiv 0 \end{cases}$$

Proof: This theorem is obtained using theorem 3.2 and 3.3.

Theorem 5.9: If $TQ_n$and and $LQ_n$are the total graph and the line graph of quadrilateral snake graph Q_n then the dominator coloring number satisfies the relation

$$\chi_d(TQ_n) = \begin{cases} \chi_d(LQ_n) + 2 & when\ 2 \le n \le 3 \\ \chi_d(LQ_n) + \left\lceil \frac{n}{2} \right\rceil & when\ n > 3 \end{cases}$$

Proof: This theorem is obtained using theorem 3.2 and 3.4.

Theorem 5.10: If CQ_n and LQ_n are the central graph and the line graph of quadrilateral snake graph Q_n then the dominator coloring number satisfies the relation

$$\chi_d(CQ_n) = \begin{cases} \chi_d(LQ_n) + n + 1 & when\ 2 \le n \le 3 \\ \chi_d(LQ_n) + 2n - 2 & when\ n > 3 \end{cases}$$

Proof: This theorem is obtained using theorem 3.3 and 3.4.

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