Even Arbitrary Supersubdivision of Corona Related MMD Graphs

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Abstract : A graph $G(V,E)$ with n vertices is said to have modular multiplicative divisor (MMD) labeling if there exist a bijection $f:V(G)\rightarrow\{1,2,\ldots,n\}$ and the induced function $f^{*}:E(G)\rightarrow\{0,1,2,\ldots,n-1\}$ where $f^{*}(uv)=f(u)f(v)(\text{mod } n)$ for all $uv\in E(G)$ such that n divides the sum of all edge labels of G. This paper studies MMD labeling of an even arbitrary supersubdivision (*EASS*) of corona related graphs.

1. Introduction

Throughout this paper we consider only finite, simple, connected and undirected graphs and follow Harary [2] for all terminology and notation in graph theory. Results concerning labeling of graphs are summarized in the survey paper [1]. Many authors [3] have constructed larger graphs using the operations like join, product, corona and k-multilevel corona etc. Sethuraman and Selvaraju [6] have introduced a new method of construction called an even arbitrary supersubdivision of graphs. A graph $G(V,E)$ with n vertices is said to have modular multiplicative divisor (MMD) labeling if there exist a bijection $f:V(G)\rightarrow\{1,2,\ldots,n\}$ and the induced function $f^{*}:E(G)\rightarrow\{0,1,2,\ldots,n-1\}$ where $f^{*}(uv)=f(u)f(v)(\text{mod} n)$ for all $uv\in E(G)$ such that n divides the sum of all edge labels of G. Existence and structural properties on MMD graphs can be found in [4] and [5]. In this paper we discuss MMD labeling of an even arbitrary supersubdivision (*EASS*) of corona of any two graphs.

For illustration MMD labeling of a graph with 5 vertices is shown in figure 1. Sum of all edge labels is equal to 0+0+3+2+3+2=10 ≡ 0 (*mod 5*).

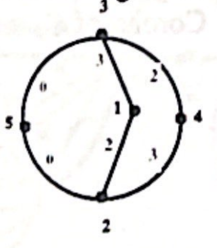


Fig. 1. MMD graph

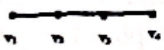
2. Basic Definitions

Definition 2.1 A graph labeling is an assignment of integers to the vertices or edges, or both, subject to certain conditions.

Definition 2.2 The *corona of two graphs* G_1 and G_2, is the graph $G = G_1 \circ G_2$ obtained from one copy of G_1 and $|V(G_1)|$ copies of G_2, where the i^{th} vertex of G_1 is adjacent to every vertex in the i^{th} copy of G_2.

Definition 2.3 Let $G = (V(G), E(G))$ be the graph with p vertices and q edges. If V_1 and V_2 are two partitions corresponding to a complete bipartite graph $K_{m,n}$ then V_1 is called m–vertices part and V_2 is called n–vertices part of $K_{m,n}$. A graph H is called a supersubdivision of G if H is obtained from G by replacing every edge e_i of G by a complete bipartite graph $K_{2,mi}$ for some m_i, $1 \leq i \leq q$ in such a way that the ends of each e_i are merged with the two vertices of 2-vertices part of $K_{2,mi}$ after removing the edge e_i from graph G.

Definition 2.4 An arbitrary supersubdivision H of G is said to be an even arbitrary super subdivision of G if every edge of G is replaced by an arbitrary $K_{2,2m}$ for any arbitrary m. It is denoted by *EASS(G)*. For illustration *EASS* of path P_4 with $m_1 = 2$, $m_2 = 4$, $m_3 = 6$ is shown in figure 2.



Path P_4

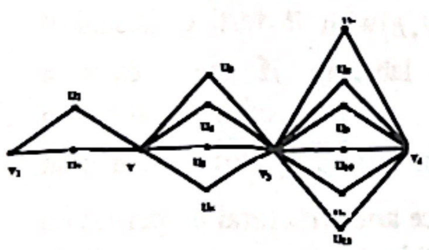


Fig. 2. EASS (P_4)

3. Main Results

Even arbitrary supersubdivision is an interesting method of construction of larger graphs. In this section we prove that an even arbitrary supersubdivision (*EASS*) of corona of any two graphs is a MMD graph. Corona of C_3 with C_3 is shown in figure 3.

G_1 $\quad G_2 \quad G = G_1 \circ G_2$

Fig. 3 Corona of G_1 and G_2 ($G_1 \circ G_2$)

Theorem: 3.1 An even arbitrary supersubdivision of corona of any two graphs admits MMD labeling.

Proof: Let $G_1$and G_2 be two graphs with p_1, p_2 vertices respectively and q_1, q_2 edges respectively. Let G be the corona of $G_1$and G_2 with $p_1(1+ p_2)$ vertices and $q_1+ p_1$ (q_2+p_2) edges. *EASS (G)* is obtained by replacing every edge of G with $K_{2,mi}$, $m_i \equiv 0 (\text{mod } 2)$, $1 \leq i \leq |E(G)|$. Let u_j be the vertices of m_i-vertices part where $1 \leq j \leq m_1 + m_2 + ... + m_{|E(G)|}$.

Let $N = |V(EASS(G))| = |V(G)| + m_1 + m_2 + ... + m_{|E(G)|}$. The vertices of m_i-vertices part are labeled as follows.

$f(u_i) = i, \quad 1 \leq i \leq \frac{m_1}{2}, \qquad f(u_{\frac{m_1}{2}+i}) = N - i, \ 1 \leq i \leq \frac{m_1}{2}$

$f(u_{m_1+m_2+...+m_{j-1}+\frac{m_j}{2}+i}) = N - \frac{m_1 + m_2 + ... + m_{j-1}}{2} - i,$

$1 \leq i \leq \frac{m_j}{2}, \quad 2 \leq j \leq |E(G)|$

$f(u_{m_1+m_2+...+m_j+i}) = \frac{m_1 + m_2 + ... + m_j}{2} + i, \ 1 \leq i \leq \frac{m_{j+1}}{2}, \ 1 \leq j \leq |E(G)| - 1$

Remaining vertices of *EASS(G)* can be labeled with the remaining integers in any order.

Sum of all edge labels of *EASS(G)* is equal to $N(m_1 + m_2 + ... + m_{|E(G)|})$ which is congruent to 0 (mod N). Hence the graph *EASS(G)* admits MMD labeling.

Example: 3.2 An even arbitrary supersubdivision of corona of G_1 and G_2 is shown in figure 4.

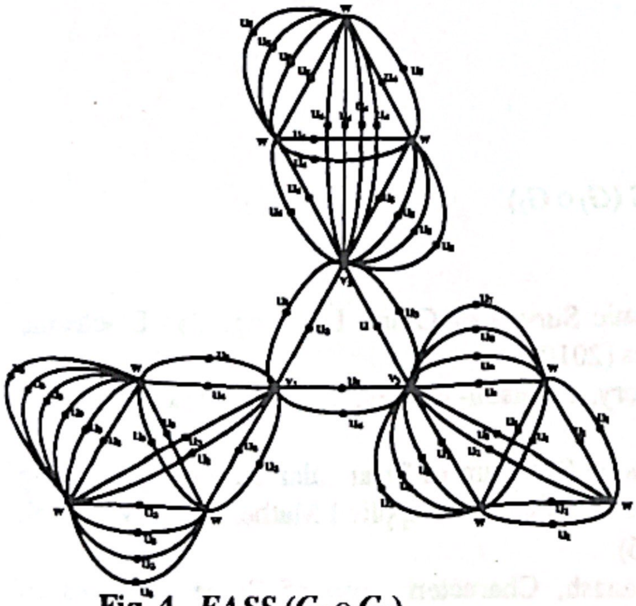


Fig. 4. *EASS* $(G_1 \circ G_2)$

Example: 3.3 MMD labeling of an even arbitrary supersubdivision of corona of $G_1$and G_2 is shown in figure 5.

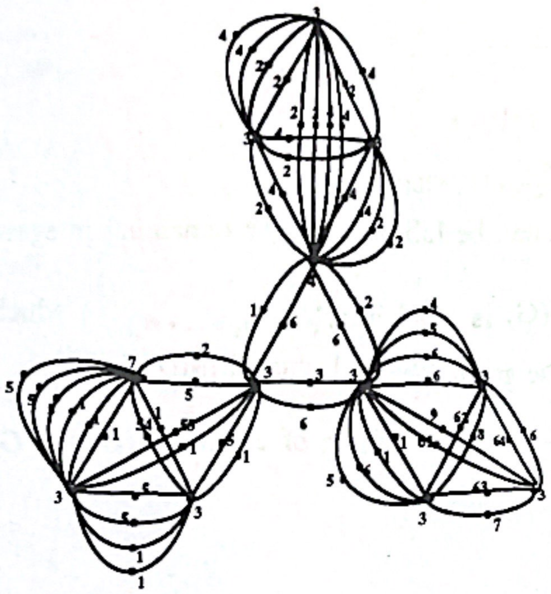


Fig. 5. MMD labeling of *EASS* ($G_1 \circ G_2$)

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