

Even Arbitrary Supersubdivision of Corona Related MMD Graphs

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Abstract : A graph $G(V,E)$ with n vertices is said to have modular multiplicative divisor (MMD) labeling if there exist a bijection $f : V(G) \rightarrow \{1,2,\dots, n\}$ and the induced function $f^* : E(G) \rightarrow \{0,1,2,\dots, n-1\}$ where $f^*(uv) = f(u)f(v) \pmod n$ for all $uv \in E(G)$ such that n divides the sum of all edge labels of G . This paper studies MMD labeling of an even arbitrary supersubdivision (EASS) of corona related graphs.

1. Introduction

Throughout this paper we consider only finite, simple, connected and undirected graphs and follow Harary [2] for all terminology and notation in graph theory. Results concerning labeling of graphs are summarized in the survey paper [1]. Many authors [3] have constructed larger graphs using the operations like join, product, corona and k -multilevel corona etc. Sethuraman and Selvaraju [6] have introduced a new method of construction called an even arbitrary supersubdivision of graphs. A graph $G(V,E)$ with n vertices is said to have modular multiplicative divisor (MMD) labeling if there exist a bijection $f : V(G) \rightarrow \{1,2,\dots, n\}$ and the induced function $f^* : E(G) \rightarrow \{0,1,2,\dots, n-1\}$ where $f^*(uv) = f(u)f(v) \pmod n$ for all $uv \in E(G)$ such that n divides the sum of all edge labels of G . Existence and structural properties on MMD graphs can be found in [4] and [5]. In this paper we discuss MMD labeling of an even arbitrary supersubdivision (EASS) of corona of any two graphs.

For illustration MMD labeling of a graph with 5 vertices is shown in figure 1. Sum of all edge labels is equal to $0+0+3+2+3+2=10 \equiv 0 \pmod 5$.

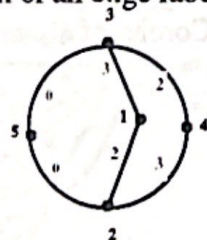


Fig. 1. MMD graph

2. Basic Definitions

Definition 2.1 A graph labeling is an assignment of integers to the vertices or edges, or both, subject to certain conditions.

Definition 2.2 The corona of two graphs G_1 and G_2 , is the graph $G = G_1 \circ G_2$ obtained from one copy of G_1 and $|V(G_1)|$ copies of G_2 , where the i^{th} vertex of G_1 is adjacent to every vertex in the i^{th} copy of G_2 .

Definition 2.3 Let $G = (V(G), E(G))$ be the graph with p vertices and q edges. If V_1 and V_2 are two partitions corresponding to a complete bipartite graph $K_{m,n}$ then V_1 is called m -vertices part and V_2 is called n -vertices part of $K_{m,n}$. A graph H is called a supersubdivision of G if H is obtained from G by replacing every edge e_i of G by a complete bipartite graph K_{2,m_i} for some $m_i, 1 \leq i \leq q$ in such a way that the ends of each e_i are merged with the two vertices of 2-vertices part of K_{2,m_i} after removing the edge e_i from graph G .

Definition 2.4 An arbitrary supersubdivision H of G is said to be an even arbitrary super subdivision of G if every edge of G is replaced by an arbitrary $K_{2,2m}$ for any arbitrary m . It is denoted by $EASS(G)$. For illustration $EASS$ of path P_4 with $m_1 = 2, m_2 = 4, m_3 = 6$ is shown in figure 2.

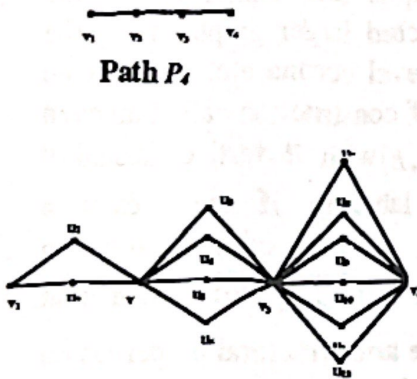


Fig. 2. $EASS(P_4)$

3. Main Results

Even arbitrary supersubdivision is an interesting method of construction of larger graphs. In this section we prove that an even arbitrary supersubdivision ($EASS$) of corona of any two graphs is a MMD graph. Corona of C_3 with C_3 is shown in figure 3.

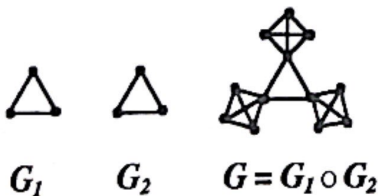


Fig. 3 Corona of G_1 and G_2 ($G_1 \circ G_2$)

Theorem: 3.1 An even arbitrary supersubdivision of corona of any two graphs admits MMD labeling.

Proof: Let G_1 and G_2 be two graphs with p_1, p_2 vertices respectively and q_1, q_2 edges respectively. Let G be the corona of G_1 and G_2 with $p_1(1+p_2)$ vertices and $q_1+p_1(q_2+p_2)$ edges. $EASS(G)$ is obtained by replacing every edge of G with K_{2,m_i} , $m_i \equiv 0 \pmod{2}$, $1 \leq i \leq |E(G)|$. Let u_j be the vertices of m_j -vertices part where $1 \leq j \leq m_1 + m_2 + \dots + m_{|E(G)|}$.

Let $N = |V(EASS(G))| = |V(G)| + m_1 + m_2 + \dots + m_{|E(G)|}$. The vertices of m_j -vertices part are labeled as follows.

$$f(u_i) = i, \quad 1 \leq i \leq \frac{m_1}{2}, \quad f(u_{\frac{m_1}{2}+i}) = N - i, \quad 1 \leq i \leq \frac{m_1}{2}$$

$$f(u_{m_1+m_2+\dots+m_{j-1}+\frac{m_j}{2}+i}) = N - \frac{m_1+m_2+\dots+m_{j-1}}{2} - i,$$

$$1 \leq i \leq \frac{m_j}{2}, \quad 2 \leq j \leq |E(G)|$$

$$f(u_{m_1+m_2+\dots+m_j+i}) = \frac{m_1+m_2+\dots+m_j}{2} + i, \quad 1 \leq i \leq \frac{m_j}{2}, \quad 1 \leq j \leq |E(G)|-1$$

Remaining vertices of $EASS(G)$ can be labeled with the remaining integers in any order.

Sum of all edge labels of $EASS(G)$ is equal to $N(m_1 + m_2 + \dots + m_{|E(G)|})$ which is congruent to 0 (mod N). Hence the graph $EASS(G)$ admits MMD labeling.

Example: 3.2 An even arbitrary supersubdivision of corona of G_1 and G_2 is shown in figure 4.

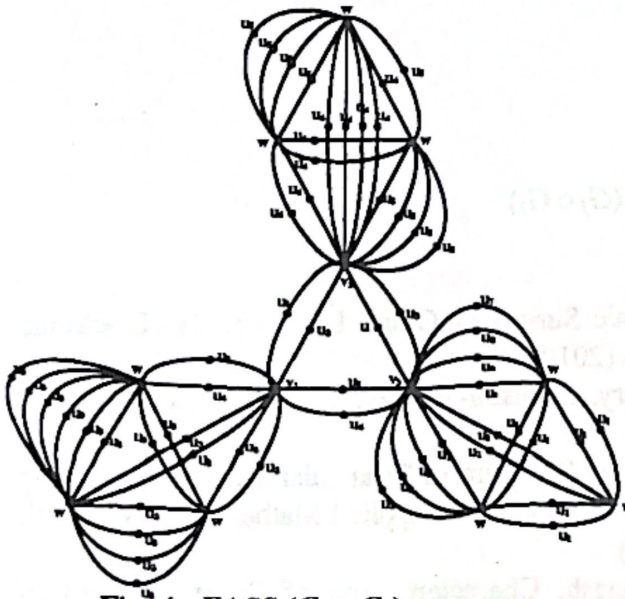


Fig. 4. $EASS(G_1 \circ G_2)$

Example: 3.3 MMD labeling of an even arbitrary supersubdivision of corona of G_1 and G_2 is shown in figure 5.

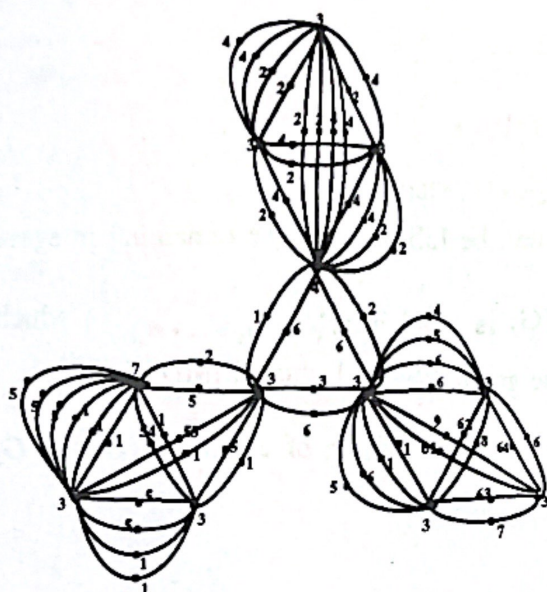


Fig. 5. MMD labeling of $EASS (G_1 \circ G_2)$

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Abstract

The k -th domination number of a graph is the cardinality of a minimum dominating set having size which is equal to k times the graph. In this paper, we define a special family of Hahn graphs and describe the k -th domination number of these graphs. We show that the construction considered as a graph in families of Hahn graphs but with different domination number.

1 Introduction

The graphs which are dealt with in this paper, are connected finite graphs. Let $G = (V, E)$. Domination theory in graphs is an important tool used in estimation of the number of vertices and edges of a graph. The parameter is here to found to be applied in domination theory and graph colouring theory. For a detailed survey on domination theory, we referred to [1, 3, 5]. Many different domination parameters have been defined and there are found to have many applications in the real world. The k -th domination problem, Maximum k -th domination problem, k -th domination number and k -th domination edge number are some of the interesting biological networks. In 1957, Ore defined the parameter k -th domination number [4]. Let G be a graph with V as vertex set and