Generalized Wiener Indices of Double Silicate Chain Graph

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Abstract

In this paper, the distance and degree based topological indices for double silicate chain graph are obtained.

Key Words: Double silicate chain graph, Gutman index, degree distance index, generalized Wiener polynomial.

1 Introduction

In the molecular representation of SiO_4 tetrahedra, the corner vertices represent oxygen ions and the center vertex represents the silicon ion. In graph theory, the corner vertices are known as oxygen nodes and center vertex as silicon node. When two oxygen node of two SiO_4 tetrahedra are fused, minerals are obtained [6]. While 'n' tetrahedral are arranged linearly single silicate chain of dimension 'n' is obtained and denoted by SSL(1,n).

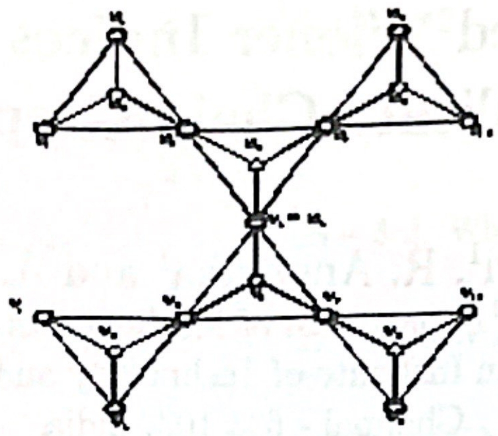

Figure 1.1: DSL(2,3)

When two copies of SSL(1, n) with the same order and size is combined in a particular pattern, the double silicate chain graph[1] is obtained. The graph DSL(2,3) is shown in Figure 1.1. In this paper Gutman index[5], degree distance [3] and generalized wiener polynomial[4,7] are determined for DSL(2,n).

2 Gutman index, Degree distance and generalized Wiener polynomial for DSL (2,n)

Theorem 2.1 Let G = DSL(2,n) be the double silicate chain graph. Then Gutman index is given by Gut(G) = $\frac{1}{4}$ ($384n^3$+$1017n^2$+$678n$-144) if n is even and $n \geq 4$ and Gut(G) = $\frac{1}{4}$ ($384n^3$+$1017n^2$+$804n$+387)if n is odd and $n \geq 5$

Proof. Let $V(G) = \{v_1, v_2, v_3, \ldots, v_{3n+1}\} \cup \{u_1, u_2, u_3, \ldots, u_{3n+1}\} \cup \{v_{3n} = u_{3n} : n = 2, 4, \ldots\}$ and $E(G) = \{v_iv_{i+1} : 1 \leq i \leq 3n\} \cup \{v_1v_3, v_5v_7, v_7v_9, \ldots, v_{3n-1}v_{3n+1}\} \cup \{v_2v_4, v_4v_6, v_6v_8, \ldots, v_{3n+2}v_{3n}\} \cup$ $\{v_1v_4, v_4v_7, v_7v_{10}, \ldots, v_{3n-2}v_{3n+1}\} \cup \{u_iu_{i+1} : 1 \ldots i \ldots 3n\} \cup$ $\{u_1u_3, u_5u_7, u_7u_9, \ldots, u_{3n-1}u_{3n+1}\} \cup \{u_2u_4, u_4u_6, u_6u_8, \ldots, u_{3n+2}u_{3n}\} \cup$

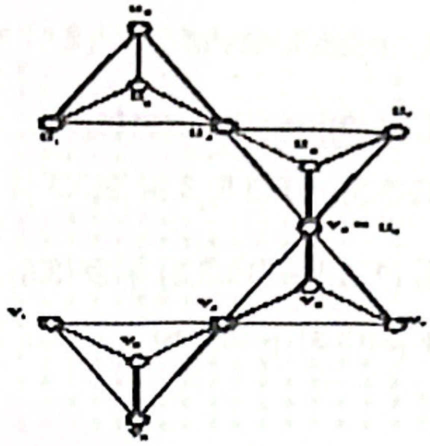

Figure 2.1: DSL(2,2)

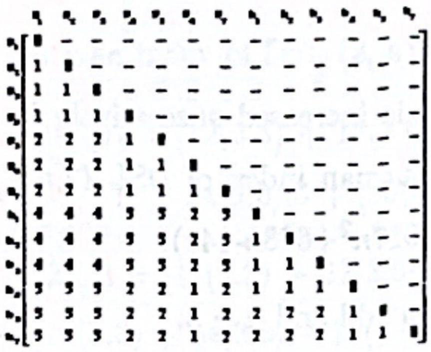

Figure 2.2: WM [DSL(2,2)]

$\{u_1u_4, u_4u_7, u_7u_{10}, \ldots, u_{3n-2}u_{3n+1}\}$ be the vertex set and edge set of G respectively. For the graph $G =$ DSL (2, n), $d(v_1) = d(v_{3n+1}) = d(v_{3n-1}) = 3$ for $n = 1, 2, 3, \ldots$, $d(v_j) = d(u_j) = 6$ for $j = 4, 7, 10, \ldots (3n-2)$ and $d(u_{3n}) = d(v_{3n}) = 6$ for $n = 2, 4, 6, \ldots$

The Gutman index is defined as Gut (G) = $\frac{1}{2}\sum_{u,v\in V(G)}(d(u)d(v)d(u,v)$, where d(u) and d(v) are the degrees of the vertices u and v, and d(u,v) is the distance between the vertices.

Case (i): When n is even.

The graph of DSL (2,2) and Wiener matrix of DSL(2,2) is shown in Figure 2.1 and 2.2 respectively.

The derivations of Gutman index of DSL (2,n) are given below:

For n = 2, Gut[DSL (2, 2)] = [8(3.3) + 14(3.6) + 2(6.6)]1 + [16(3.3)+ 10(3.6) + 1(6.6)]2 + [12(3.3) + 6(3.6)]3 + [9(3.3)]4 = 2088

For n = 4, Gut[DSL (2, 4)] = [10(3.3) + 28(3.6) + 10(6.6)]1 + [23(3.3) + 38(3.6) + 11(6.6)]2 + [34(3.3) + 46(3.6) + 7(6.6)]3 + [41(3.3) + 16(3.6)]4 + [12(3.3)]5 = 10854

For n = 6, Gut [DSL (2, 6)] = [12(3.3) + 42(3.6) + 18(6.6)]1 + [32(3.3) + 62(3.6) + 23(6.6)]2 + [52(3.3) + 82(3.6) + 22(6.6)]3 + [61(3.3) + 52(3.6) + 10(6.6)]4 + [34(3.3) + 32(3.6) + 5(6.6)]5 + [28(3.3) + 16(3.6)]6 + [12(3.3)]7 = 30870

If the dimension n is increased successively by 2, then the following expression gives the Gutman index of DSL (2,n), n is even and $n \geq 4$, Gut(G) = $\frac{1}{4}$ ($384n^3$+$1017n^2$+$678n$-144).

Case (ii): When n is odd and $n \geq 3$.

The graph of DSL (2, 3) and Wiener matrix of DSL (2,3) are as shown in Figure 1.1 and 2.3 respectively.

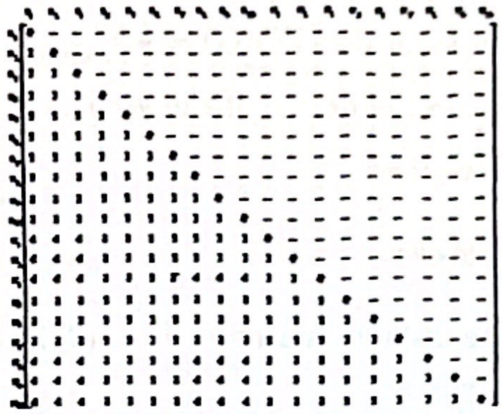

Figure 2.3: WM [DSL(2,3)]

The derivations of Gutman index of DSL (2, n) are given below:

For n = 3, Gut(DSL(2, 3)) = [12(3.3) + 18(3.6) + 6(6.6)]1 + [13(3.3) + 28(3.6) + 4(6.6)]2 + [30(3.3) + 24(3.6)]3 + [36(3.3)]4

For n = 5, Gut(DSL(2,5)) = [14(3.3) + 32(3.6) + 14(6.6)]1 + [22(3.3) + 52(3.6) + 16(6.6)]2 + [46(3.3) + 64(3.6) + 13(6.6)]3 + [60(3.3) + 40(3.6) + 2(6.6)]4 + [30(3.3) + 12(3.6)]5 + [18(3.3)]6 = 19458

For n = 7, Gut(DSL(2,7)) = [16(3.3) + 46(3.6) + 22(6.6)]1 + [31(3.3) + 76(3.6) + 28(6.6)]2 + [64(3.3) + 100(3.6) + 28(6.6)]3 + [82(3.3) + 72(3.6) + 14(6.6)]4 + [46(3.3) + 44(3.6) + 11(6.6)]5 + [38(3.3) + 40(3.6) + 2(6.6)]6 + [30(3.3) + 12(3.6)]7 + [18(3.3)]8 = 46890.

If the dimension n is increased successively by 2, then the following expression gives the Gutman index for DSL (2, n), n is odd and $n \geq 5$ Gut(G) = $\frac{1}{4}$ ($384n^3$+$1017n^2$+$804n$+387).

□

Corollary 2.2 Let G = DSL (2,n) be the Double silicate chain graph. Then the degree distance is given by DD(G) = $\frac{1}{2}$ ($88n^3+303n^2+212n+36$) if n is even, $n \geq 2$ and DD(G) = $\frac{1}{2}$ ($88n^3+315n^2+266n+219$) if n is odd and $n \geq 3$.

Proof. The vertex set V(G) and the edge set E(G) are as in Theorem 2.1. The degree distance is defined as $DD(G) = \frac{1}{2}\sum_{u,v\in V(G)}(d(u)+d(v)d(u,v)$ where d(u) and d(v) are the degree of the vertices u and v, d(u,v) is the distance between the vertices.

Case (i): When n is even.

The graph and the distance matrix of DSL (2, 2) are shown in Figure 2.1 and 2.2 respectively.

The derivations of degree distance of DSL (2, n) are given below:

DD[DSL(2,2)] = [8(3+3) + 14(3+6) + 2(6+6)]1 + [16(3+3) + 10(3+6) + 1(6+6)]2 + [12(3+3) + 6(3+6)]3 + [9(3+3)]4 = 1188

DD[DSL(2,4)] = [10(3+3) + 28(3+6) + 10(6+6)]1 + [23(3+3) + 38(3+6) + 11(6+6)]2 + [34(3+3) + 46(3+6) + 7(6+6)]3 + [41(3+3) + 16(3+6)]4 + [12(3+3)]5 = 5682

If the dimension n is increased successively by 2, then the following expression gives the degree distance index of DSL (2,n), n is even and $n \geq 2$. DD(G) = $\frac{1}{2}$ $(88n^3 + 303n^2 + 212n + 36)$ if n is even, $n \geq 2$.

Case (ii): When n is odd and $n \geq 3$.

The following degree distance index are calculated as explained in case(i)

DD(DSL(2,3)) = [12(3+3) + 18(3+6) + 6(6+6)]1 + [13(3+3) + 28(3+6) + 4(6+6)]2 + [30(3+3) + 24(3+6)]3 + [36(3+3)]4 = 3114.

DD(DSL(2,5)) = [14(3+3) + 32(3+6) + 14(6+6)]1 + [22(3+3) + 52(3+6) + 16(6+6)]2 + [46(3+3) + 64(3+6) + 13(6+6)]3 + [60(3+3) + 40(3+6) + 2(6+6)]4 + [30(3+3) + 12(3+6)]5 + [18(3+3) = 10212

If the dimension n is increased successively by 2, then the following expression gives the degree distance index for DSL (2,n), DD(G) = $\frac{1}{2}$ $(88n^3 + 315n^2 + 266n + 219)$, n is odd and $n \geq 3$. □

Theorem 2.3 Let G = DSL(2,n) be the double silicate chain graph.

Then if n is even, $n \geq 4$ the generalized wiener polynomial of G is given by

$W_\lambda P[DSL(2,n):x] = [48 + (n\text{-}4)12]x^{1^\lambda} + [72 + (\frac{n-4}{2})18 + (\frac{n-4}{2})27]$ $x^{2^\lambda} + [87 + (\frac{n-4}{2})36 + (\frac{n-4}{2})33]x^{3^\lambda} + [57 + (\frac{n-4}{2})45 + (\frac{n-4}{2})21]x^{4^\lambda} + [12$ $+ (\frac{n-4}{2})30 + (\frac{n-4}{2})29]x^{5^\lambda} + [18 + (\frac{n-4}{2})26 + (\frac{n-6}{2})36]x^{6^\lambda} + [12 + (\frac{n-6}{2})30$ $+ (\frac{n-6}{2})29]x^{7^\lambda} + [18 + (\frac{n-6}{2})26 + (\frac{n-8}{2})36]x^{8^\lambda} + \ldots + [18 + (\frac{n-(n-2)}{2})26$ $+ (\frac{n-n}{2})29]x^{n^\lambda} + [12 + (\frac{n-n}{2})30 + (\frac{n-n}{2})29]x^{n+1^\lambda}$

and if n is odd, $n \geq 5$

$W_\lambda P[DSL(2,n):x] = [60 + (n\text{-}5)12]x^{1^\lambda} + [80 + (\frac{n-5}{2})27 + (\frac{n-5}{2})18]$ $x^{2^\lambda} + [123 + (\frac{n-5}{2})33 + (\frac{n-5}{2})36]x^{3^\lambda} + [102 + (\frac{n-5}{2})21 + (\frac{n-5}{2})45]x^{4^\lambda}$ $+ [42 + (\frac{n-5}{2})29 + (\frac{n-5}{2})30]x^{5^\lambda} + [18 + (\frac{n-5}{2})26 + (\frac{n-5}{2})36]x^{6^\lambda} + [12 +$ $(\frac{n-5}{2})30 + (\frac{n-7}{2})29]x^{7^\lambda} + \ldots + [12 + (\frac{n-(n-2)}{2})30 + (\frac{n-n}{2})29]x^{n^\lambda} + [18$ $+ (\frac{n-n}{2})26 + (\frac{n-n}{2})36]x^{n+1^\lambda}$

Proof. The vertex set V(G) and the edge set E(G) are as in Theorem 2.1. The generalized wiener polynomial of G is defined by $W_\lambda P(G:x)$ $= \sum_x d(u,v)^\lambda$, where λ is any real number. The derivation of generalized wiener polynomial of DSL (2,n) is considered in two different cases by forming the distance matrix.

Case (i): When n is even.

The graph DSL (2,2) and the corresponding Wiener matrix are as shown in Figure 2.1 and Figure 2.2 respectively. The following polynomial can be written

$W_\lambda P(G:x) = 24x^{1^\lambda} + 27x^{2^\lambda} + 18x^{3^\lambda} + 9x^{4^\lambda}$; $W(G) = 168$.

From the wiener matrix of DSL (2, n), the following polynomial can be written:

If G = DSL(2,4), then $W_\lambda P(G:x) = 48x^{1^\lambda} + 72x^{2^\lambda} + 87x^{3^\lambda} + 57x^{4^\lambda}$ $+ 12x^{5^\lambda}$; $W(G) = 741$.

If G = DSL(2,6), then $W_\lambda P(G:x) = 72x^{1^\lambda} + 117x^{2^\lambda} + 156x^{3^\lambda} +$

$123x^{4^\lambda} + 71x^{5^\lambda} + 44x^{6^\lambda} + 12x^{7^\lambda}$; $W(G) = 1969$.

If G = DSL(2,8), then $W_\lambda P(G : x) = 96x^{1^\lambda} + 162x^{2^\lambda} + 225x^{3^\lambda} + 189x^{4^\lambda} + 130x^{5^\lambda} + 106x^{6^\lambda} + 71x^{7^\lambda} + 44x^{8^\lambda} + 12x^{9^\lambda}$; $W(G) = 4094$.

If n is increased successively by one, the generalized Wiener polynomial of G = DSL(2,n) is

$W_\lambda P[DSL(2,n) : x] = [48 + (n\text{-}4)12]x^{1^\lambda} + [72 + (\frac{n-4}{2})18 + (\frac{n-4}{2})27]x^{2^\lambda} + [87 + (\frac{n-4}{2})36 + (\frac{n-4}{2})33]x^{3^\lambda} + [57 + (\frac{n-4}{2})45 + (\frac{n-4}{2})21]x^{4^\lambda} + [12 + (\frac{n-4}{2})30 + (\frac{n-4}{2})29]x^{5^\lambda} + [18 + (\frac{n-4}{2})26 + (\frac{n-6}{2})36]x^{6^\lambda} + [12 + (\frac{n-6}{2})30 + (\frac{n-6}{2})29]x^{7^\lambda} + [18 + (\frac{n-6}{2})26 + (\frac{n-8}{2})36]x^{8^\lambda} + \ldots + [18 + (\frac{n-(n-2)}{2})26 + (\frac{n-n}{2})29]x^{n^\lambda} + [12 + (\frac{n-n}{2})30 + (\frac{n-n}{2})29]x^{n+1^\lambda}$

Case (ii): When n is odd.

The wiener matrix WM of DSL(2,3) is shown in Figure 1.1. The following polynomial can be written from the Wiener matrix WM of DSL(2,3) in Figure 2.3.

If G = DSL(2,3), then $W_\lambda P(G : x) = 36x^{1^\lambda} + 45x^{2^\lambda} + 54x^{3^\lambda} + 36x^{3^\lambda}$; $W(G) = 432$.

From the wiener matrix WM of DSL(2,n), the following polynomial can be written:

If G = DSL(2,5), then $W_\lambda P(G : x) = 60x^{1^\lambda} + 90x^{2^\lambda} + 123x^{3^\lambda} + 102x^{4^\lambda} + 42x^{5^\lambda} + 18x^{6^\lambda}$; $W(G) = 1334$.

Therefore, in general

$W_\lambda P[DSL(2,n) : x] = [60 + (n\text{-}5)12]x^{1^\lambda} + [80 + (\frac{n-5}{2})27 + (\frac{n-5}{2})18]x^{2^\lambda} + [123 + (\frac{n-5}{2})33 + (\frac{n-5}{2})36]x^{3^\lambda} + [102 + (\frac{n-5}{2})21 + (\frac{n-5}{2})45]x^{4^\lambda} + [42 + (\frac{n-5}{2})29 + (\frac{n-5}{2})30]x^{5^\lambda} + [18 + (\frac{n-5}{2})26 + (\frac{n-5}{2})36]x^{6^\lambda} + [12 + (\frac{n-5}{2})30 + (\frac{n-7}{2})29]x^{7^\lambda} + \ldots + [12 + (\frac{n-(n-2)}{2})30 + (\frac{n-n}{2})29]x^{n^\lambda} + [18 + (\frac{n-n}{2})26 + (\frac{n-n}{2})36]x^{n+1^\lambda}$

□

3 Conclusion

In this paper degree distance, Gutman index and generalized wiener polynomial are derived for double silicate network graphs. By putting $\lambda = 1, -1, -2$ the wiener index, reciprocal wiener index, and Harary index respectively can be obtained from the generalized Wiener polynomial. The hyper Wiener index $WW(G) = \frac{1}{2}W_1 + \frac{1}{2}W_2$ and Tratch-Stankevich-Zefirov index TSZ (G) = $\frac{1}{6}W_1 + \frac{1}{2}W_2 + \frac{1}{3}W_3$ can also be derived from the generalized Wiener polynomial.

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