SSP-Cyclic Structure of Some Circulant Graphs

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Abstract

If every induced sub graph H of a graph G contains a minimal dominating set that intersects every maximal cliques of H, then G is SSP (super strongly perfect). This paper presents a cyclic structure of some circulant graphs and later investigates their SSP properties, while also giving attention to find the SSP parameters like colourability, cardinality of minimal dominating set and number of maximal cliques of circulant graphs.

1 Introduction

A graph is a set $G = (V, E)$ where V is a set of vertices and E is the set of edges expressed as unordered pairs such that an unordered pair containing two vertices denotes an edge between those two vertices. This paper deals exclusively with simple, finite, connected and undirected graphs.
A clique is a set of vertices every pair of which is adjacent. A clique is called a maximal clique if it does not exist exclusively within the vertex set of a bigger clique. A complete graph is a graph in which each pair of graph vertices is connected by an edge and it is denoted by K_n. A graph G is called a bigraph or bipartite graph, if V can be divided into two distinct subsets V_1 and V_2 such that every edge joins a vertex of V_1 to a vertex of V_2. (V_1, V_2) is called a bipartition of G. A closed path is called a cycle (A path is a walk in which all vertices are distinct. A walk on a graph is an alternating series of vertices and edges, beginning and ending with a vertex, in which each edge is incident with the vertex immediately preceding it and the vertex immediately following it). An odd/even cycle is a cycle with odd/even length (odd/even number of edges). A subset D of $V(G)$

is called a dominating set if every vertex in $V - D$ is adjacent to at least one vertex in D. A subset S of V is said to be a minimal dominating set if for any $u \in S$, $S - \{u\}$ is not a dominating set. Colouring of a graph G is assigning different colours to the vertices of the graph such that no two connected vertices get the same colour.

2 Overview of the Paper

There are many analyzations of graphs with their cyclic structure have been given [2, 3]. The structural analysis of graphs is essential in evaluating the structure of graphs. Circulant graphs have a vast number of uses and applications to telecommunication network, VLSI design, parallel and distributed computing [1]. The purpose of this paper is to investigate a structural analysis of cycles in some circulant graphs ($C_{in(1,2,\ldots\lfloor n/2\rfloor)}$, $C_{in(1)}n \geq 4$). Then it is extended the SSP parameters of some circulant graphs.

3 Super Strongly Perfect Graph (SSP)

If every induced sub graph H of a graph G contains a minimal dominating set that intersects every maximal cliques of H, then G is SSP. Figures 1, 2 demonstrate SSP and non-SSP graphs.

Illustration 1

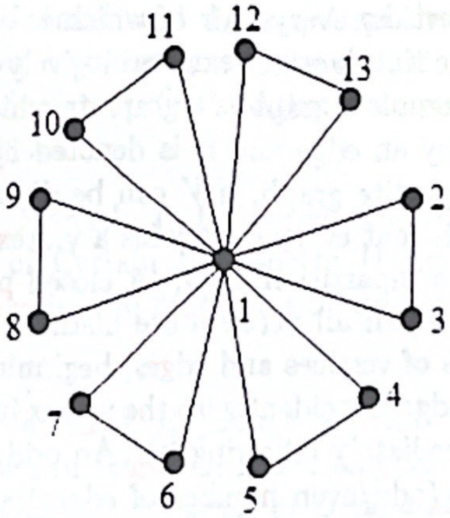

Figure 1: SSP Graph

Here, a minimal dominating set {1} meets every maximal cliques K_3.

Illustration 2

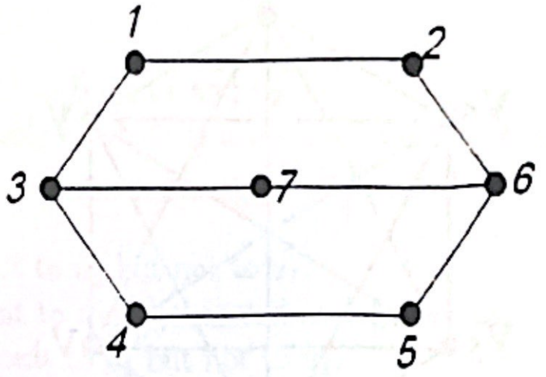

Figure 2: Non-SSP Graph

Here, a minimal dominating set {1, 4, 6} does not meet every maximal cliques K_2. ((i.e.,) clique 3-7 of G). Also, G does not have a minimal dominating set which meets every maximal cliques K_2.

3.1 Theorem [3]

A graph G with at least one maximal clique K_n, $n = 2, 3,...$ is n - colourable if and only if it is SSP.

4 Circulant Graph

A circulant graph is an undirected graph that has a cyclic group of symmetries which takes any vertex to any other vertex. It is a graph on n vertices such that the i^{th} vertex is connected to $(i+j)^{th}$ & $(i-j)^{th}$ vertices for each j in a list 1 and it is denoted by $C_{n(j_1,j_2,...j_m)}$, where n is the number of vertices and m is the number of jumps. The circulant graph $C_{in(1,2,...\lfloor n/2 \rfloor)}$ is the complete K_n and the graph $C_{in(1)}$ is the cyclic graph C_n [4]. Circilant graph $C_{i6(1,2,3)}$ is shown in the following figure 3.

Illustration 3

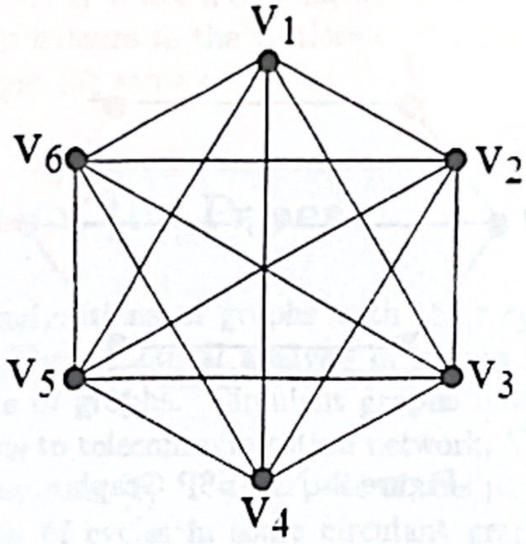

Figure 3: $C_{i6(1,2,3)}$

Here, a minimal dominating set $\{v_1\}$ is meeting the maximal clique K_6.

4.1 Theorem

Every circulant graph $C_{in(1,2,...\lfloor n/2 \rfloor)}$ is SSP.
Proof:

Let G be a circulant graph $C_{in(1,2,...\lfloor n/2 \rfloor)}$
$\Rightarrow$ From the definition of G (on n vertices), G is a complete graph K_n.
$\Rightarrow$ Every single vertex is a minimal dominating set that will meet all the maximal clique K_n (the graph itself).
$\Rightarrow$ G is SSP.

4.2 Theorem

Every circulant graph $C_{in(1)}, n \geq 4$, n is even, G is SSP.
Proof:

Let G be a circulant graph $C_{in(1)}, n \geq 4$, n is even.
$\Rightarrow$ G is an even cycle on n vertices with n maximal cliques K_2.
Let $S = \{u_i, u_{i+2}, u_{i+4}, ...u_{i+n-2}\}$ $i = 1, 2.$ be a minimal dominating set of G which is not minimum.
If G is not SSP, then there exists at least one edge $u_1u_2 \in V$ which is not met by the minimal dominating S of G.

This implies $u_1, u_2 \notin S$.
$\Rightarrow u_1, u_2 \in V - S$
$\Rightarrow$ There exists a vertex $u_3 \in S$ such that u_3 is adjacent to both u_1 & u_2 or u_1 or u_2.

Case 1:

If u_3 is adjacent to both u_1 and u_2.
Then we get a triangle, which is a contradiction since K_2 is the maximal clique of G.

Case 2:

If u_3 is adjacent to u_2 but not to u_1.
Then u_1 is adjacent to some other vertex, let it be u_4.
Clearly u_4 is adjacent to u_1 but not to u_2.
$\Rightarrow u_3, u_4 \in S$, which is a contradiction.
$\Rightarrow G$ is SSP.

4.3 Theorem

Every circulant graph $C_{in(1)}, n \geq 5$, n is odd, G is non-SSP.

Proof:

Let G be a circulant graph $C_{in(1)}, n \geq 5$, n is odd.
$\Rightarrow G$ is an even cycle on n vertices with n maximal cliques K_2.
Let $S = \{u_i, u_{i+2}, u_{i+4}, ...u_{i+n-3}\}$ $i = 1, 2.$ be a minimal dominating set of G.
To prove G is non - SSP.
If G is SSP, then every induced sub graphs H of G possesses a minimal dominating set that meets every maximal cliques of H.
Let the n maximal cliques K_2 of G be $u_1u_2, u_2u_3, u_3u_4...u_{n-1}u_n, u_nu_1$.
To meet the n maximal cliques K_2, (i.e.,) $\{u_iu_{i+1}\}$ i =1,2,...$n \in Z_n$, there should be a minimal dominating set $\{u_i, u_{i+2}, u_{i+4}, ..., u_{i+n-1}\}$ $i = 1$.
But $S = \{u_i, u_{i+2}, u_{i+4}...u_{i+n-3}\}$ $i = 1, 2.$ is a minimal dominating set.
$\Rightarrow \{u_i, u_{i+2}, u_{i+4}...u_{i+n-1}\}$ $i = 1.$ is not a minimal dominating set.
$\Rightarrow$ There does not exist a minimal dominating set that meets every maximal cliques of G.
$\Rightarrow G$ is non-SSP.

4.4 Theorem

Every circulant graph $C_{in(1)}, n \geq 4$, n is even, G has the following properties,

1) G has n maximal cliques K_2.
2) G has a minimal dominating set of cardinality$\lfloor \frac{n}{2} \rfloor$.

3) G is 2-colourable.

Proof:

Let G be circulant graph $C_{in(1)}, n \geq 4$, n is even.
As $C_{in(1)}$ is an even cycle, (1) and (3) are trivial.
Since G is SSP, there exists a minimal dominating set which meets all K_2.
Since G is bipartite with n vertices, there exists a bipartition (V_1, V_2) in G such that $|V_1| + |V_2| = n$.
Also, G is 2 colourable.
$\Rightarrow$The vertices from V_1 (or) V_2 will meet all the maximal cliques K_2.
$\Rightarrow |V_1| = n - |V_2|$.
$\Rightarrow |V_1| = n - \frac{n}{2}$.
$\Rightarrow |V_1| = \frac{n}{2}$ (or) $|V_2| = \frac{n}{2}$
$\Rightarrow$G has a minimal dominating set of cardinality $\lfloor \frac{n}{2} \rfloor$.

5 Conclusion

In this paper, it is analyzed the structural investigation of some circulant graphs($C_{in(1,2,...\lfloor n/2 \rfloor)}$, $C_{in(1)} n \geq 4$). Circulant graphs along with its cyclic structure. The basic approach to find the SSP parameters on circulant graphs is given. A nice consequence of this technique is that it can be easily modified to work for many other parameters of circulant graphs, e.g., to show that the number of Hamiltonian cycles, Eulerian tours and Eulerian orientations in these graphs. Even though the investigation was given only for undirected circulant graphs, it is very easy to extend this to directed circulant graphs and the remaining well known architectures as well.

References

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