Total Domination in Certain Nanotori

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Abstract

A set S of vertices in a graph G is said to be a dominating set if every vertex in $V(G) \setminus S$ is adjacent to some vertex in S. A dominating set S is called a total dominating set if each vertex of $V(G)$ is adjacent to some vertex in S. Molecules arranging themselves into predictable patterns on silicon chips could lead to microprocessors with much smaller circuit elements. Mathematically, assembling in predictable patterns is equivalent to packing in graphs. In this paper, we determine the total domination number for certain nanotori using packing as a tool.

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1 Introduction

A monitor in a network is a member of the network which is able to detect a faulty member among its neighbors. The problem of identify the faulty member can be modeled as a dominating sets in the network. A set S of vertices in a graph G is called a dominating set of G if every vertex in $V(G) \setminus S$ is adjacent to some vertex in S. Determining if an arbitrary

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graph has a dominating set of a given size is a well-known NP-complete problem [1]. A dominating set S of G is said to be a total dominating set of G if every vertex in $V(G)$ is adjacent to some vertex in S. The total domination in graphs was introduced by Cockayne et al. [2] in 1980 and has been extensively studied in the literature [3]. In 2009, Henning [4] gave a survey of selected recent results on total domination number.

Packing in graphs is used as tool to compute total domination in graphs. Mathematically, assembling in predictable patterns is equivalent to packing in graphs. Molecules arranging themselves into predictable patterns on silicon chips could lead to microprocessors with much smaller circuit elements. An H-packing of a graph G is a set of vertex disjoint subgraphs of G, each of which is isomorphic to a fixed graph H. From the optimization point of view, maximum H-packing problem is to find the maximum number of vertex disjoint copies of H in G called the packing number denoted by $\lambda(G,H)$. When there is no ambiguity $\lambda(G,H)$ is sometimes represented as λ. An H-packing in G is called perfect if it covers all vertices of G. If H is the complete graph K_2, the maximum H-packing problem becomes the familiar maximum matching problem. H-packing, is of practical interest in the areas of scheduling, wireless sensor tracking, code optimization and many others [8]. When H is a connected graph with at least three vertices, Kirkpatrick and Hell proved that the maximum H-packing problem is NP-complete [5].

All graphs considered in this paper are simple and connected. We give algorithms to find a perfect H-packing of certain nanotori, where H is the h-graph on six vertices and thus determine their packing numbers as well as the total domination numbers.

2 Main Results

We begin with certain known results.

Lemma 2.1. [6] *Let S be a total dominating set with this property that every vertex $u \in V(G)$ is dominated by exactly one vertex of S, then S is a minimum total dominating set.*

Lemma 2.2. [7] *If G is a k-regular graph with n vertices, then $\gamma_t(G) \geq \lceil \frac{n}{k} \rceil$.*

Theorem 2.3. [8] *Let G be a graph and H be a subgraph of G. Then $\lambda(G,H) \leq \left\lfloor \frac{V(G)}{V(H)} \right\rfloor$.*

Definition 2.4. *The h-graph is the tree on 6 vertices shown in Figure 1. It is a spanning subgraph of mesh graph $M_{3\times 2}$, with 3 rows and 2 columns.*

Theorem 2.5. [9] *In a 3-regular graph G, if there exists a perfect H-packing when $H \cong h$-graph, then $\gamma_{pr}(G) = \gamma_t(G) = 2\lambda$, where λ is the packing number of G.*

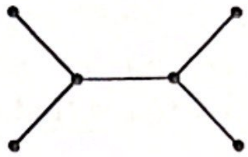

Figure 1: h-graph

2.1 H-Naphtalenic $[m, n]$ nanotori

A H-Naphtalenic $[m, n]$ nanotori is a trivalent decoration made by alternating squares C_4, pair of hexagons C_6 and octagons C_8 [10]. It is a 3-regular graph with m number of rows and n number of columns, each column comprising of the pair of hexagons C_6 viewed vetically and each row comprising of the pair of hexagons C_6 viewed horizontally. Each column of $G \cong H$-Naphtalenic $[m, n]$ nanotori comprises of 5 levels of disjoint set of vertices, viewed vertically and the $5n$ levels of vertices are labeled as level 1, level 2, ..., level $5n$ from left to right. See Figure 2(a). In this section, for convenience, we write H-Naphtalenic $[m, n]$ nanotori simply as Naphtalenic $[m, n]$ nanotori.

The Algorithm A given below computes the Packing in Naphtalenic $[m, n]$ nanotori with H, where $H \cong h$-graph.

Input: Let G be a Naphtalenic $[m, 3n]$ nanotori, $m, n \geq 1$.

Algorithm A: Select all the vertices from the level L_{3i-1}, $1 \leq i \leq 5n$ and call the set as S.

Output: S is a perfect H-packing of Naphtalenic $[m, 3n]$ nanotori.

Proof of Correctness: The vertices in the level L_{3i-1} are adjacent to the vertices in the levels L_{3i-2} and L_{3i}, $1 \leq i \leq 5n$. In each level L_{3i-1}, there are $2m$ vertices of G. Notice that, each L_{3i-1} contains exactly m vertex disjoint copies of edges (or P_2) in it. Let $e_1 = (u_1, v_1), e_2 = (u_2, v_2), \ldots, e_m = (u_m, v_m)$ be the m vertex disjoint copies of edges in L_{3i-1}. We see that $N[u_j, v_j] \cong H$, since each u_j and v_j is adjacent to a vertex in levels L_{3i-2} and L_{3i}, $1 \leq j \leq m$. See Figure 2(a). Thus the vertices in each level L_{3i-1}, $1 \leq i \leq 5n$, cover $4m$ vertices of G. Since $|V(G)| = 30mn$, $\lambda \geq \left\lfloor \frac{(2m+4m)(5n)}{6} \right\rfloor = \left\lfloor \frac{30mn}{6} \right\rfloor$. By Theorem 2.3, $\lambda \leq \left\lfloor \frac{30mn}{6} \right\rfloor$. Hence the proof.

Theorem 2.6. *Let G be a Naphtalenic $[m, 3n]$ nanotori. Then $\lambda(G, H) = \frac{10mn}{6}$.*

Theorem 2.7. *Let G be a Naphtalenic $[m, 3n]$ nanotori. Then $\gamma_t(G) = \frac{10mn}{3}$.*

Proof. By Theorem 2.5, it is easy to see that $\gamma_t(G) = 2\lambda = \frac{10mn}{3}$. □

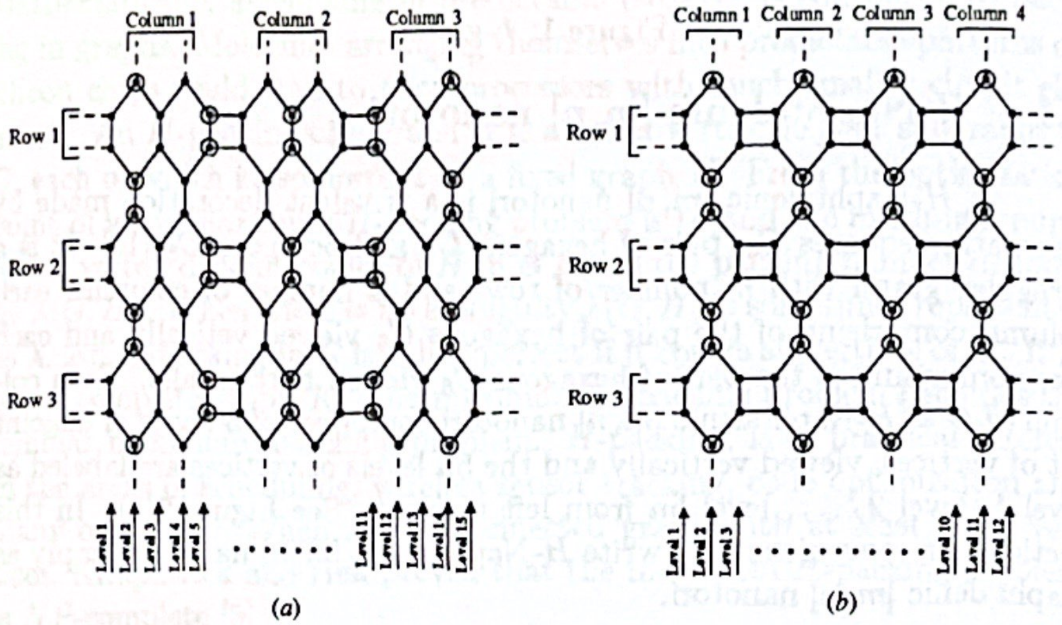

Figure 2: (a) Naphtalenic [3,3] nanotori and (b) $C_4C_6C_8[3, 4]$ nanotori

2.2 $C_4C_6C_8[m, n]$ nanotori

A $C_4C_6C_8[m, n]$ nanotori is a trivalent decoration made by alternating squares C_4, hexagons C_6 and octagons C_8 [11, 12]. It is a 3-regular graph with m number of rows and n number of columns, each column comprising of hexagons C_6 viewed vertically and each row comprising of hexagons C_6 viewed horizontally. Each column of $G \cong C_4C_6C_8[m, n]$ nanotori comprises of 3 levels of disjoint set of vertices, viewed vertically and the $3n$ levels of vertices are labeled as level 1, level 2, ..., level $3n$ from left to right. See Figure 2(b).

The Algorithm B given below computes the packing in $C_4C_6C_8[m, n]$ nanotori with H, where $H \cong h$-graph.

Input: Let G be a $C_4C_6C_8[m, n]$ nanotori, $m, n \geq 1$.

Algorithm B: Select all the vertices from the level L_{3i-1}, $1 \leq i \leq n$ and call the set as S.

Output: S is a perfect H-packing of $C_4C_6C_8[m,n]$ nanotori.

Proof of Correctness: The vertices in the level L_{3i-1}, dominate vertices in the levels L_{3i-2} and L_{3i}, $1 \le i \le n$. In each level L_{3i-1}, $1 \le i \le n$, there are $2m$ vertices of G (see Figure 2(b)). Now the vertices in each level L_{3i-1}, $1 \le i \le n$, cover $4m$ vertices of G. Since $|V(G)| = 6mn$, $\lambda \ge \left\lfloor \frac{(2m+4m)n}{6} \right\rfloor = \lfloor mn \rfloor$. By Theorem 2.3, $\lambda \le mn$. Hence the proof.

Theorem 2.8. *Let G be a $C_4C_6C_8[m,n]$ nanotori. Then $\lambda(G,H) = mn$.*

Theorem 2.9. *Let G be a $C_4C_6C_8[m,n]$ nanotori. Then $\gamma_t(G) = 2mn$.*

Proof. By Theorem 2.5, it is easy to see that $\gamma_t(G) = 2\lambda = 2mn$. □

2.3 $C_4C_8(S)[m,n]$ nanotori

A $C_4C_8(S)[m,n]$ nanotori is a trivalent decoration made by alternating squares C_4 and octagons C_8 [13]. It is a 3-regular graph with m number of rows and n number of columns, each column comprising of octagons C_8 viewed vertically and each row comprising of octagons C_8 viewed horizontally. Each column of $G \cong C_4C_8(S)[m,n]$ nanotori comprises of 4 levels of disjoint set of vertices, viewed vertically and the $4n$ levels of vertices are labeled as level 1, level 2, ..., level $4n$ from left to right. See Figure 3(a).

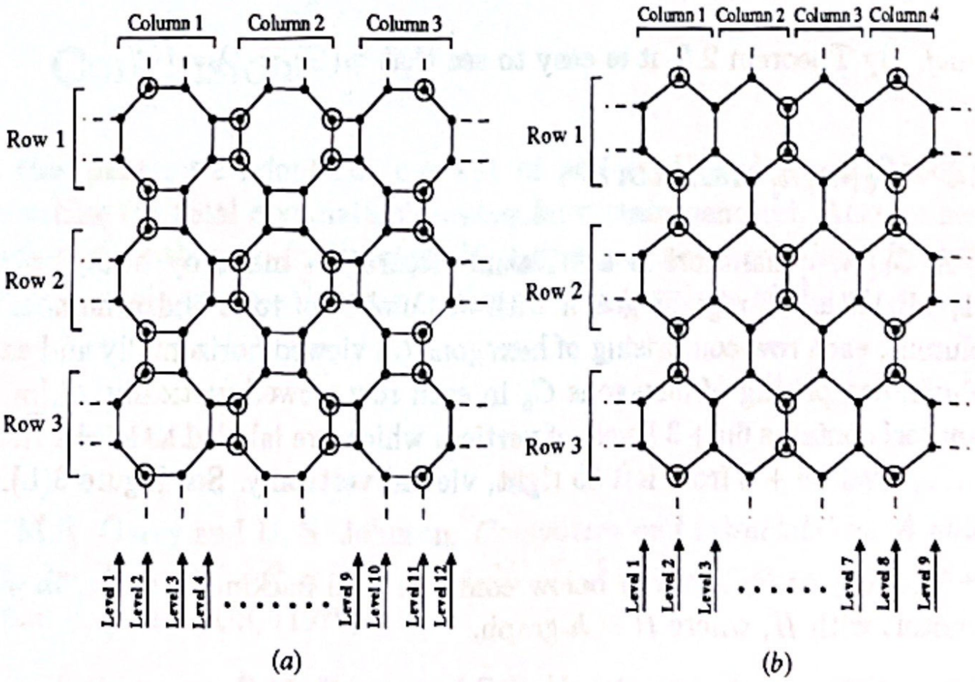

Figure 3: (a)$C_4C_8(S)[3,3]$ nanotori and (b) $C_6[3,4]$ nanotori

The Algorithm C given below computes the packing in $C_4C_8(S)[m, 3n]$ nanotori with H, where $H \cong h$-graph.

Input: Let G be a $C_4C_8(S)[m, 3n]$ nanotori, $m, n \geq 1$.

Algorithm C: Select all the vertices from the level L_{3i-1}, $1 \leq i \leq \frac{4n}{3}$ and call the set as S.

Output: S is a perfect H-packing of $C_4C_8(S)[m, 3n]$ nanotori.

Proof of Correctness: The vertices in the level L_{3i-1}, dominate vertices in the levels L_{3i-2} and L_{3i}, The vertices in the level L_{3i-1}, dominate vertices in the levels L_{3i-2} and L_{3i}, $1 \leq i \leq \frac{4n}{3}$. In each level L_{3i-1}, The vertices in the level L_{3i-1}, dominate vertices in the levels L_{3i-2} and L_{3i}, $1 \leq i \leq \frac{4n}{3}$, there are $2m$ vertices of G (see Figure 3(a)). Now the vertices in each level L_{3i-1}, The vertices in the level L_{3i-1}, dominate vertices in the levels L_{3i-2} and L_{3i}, $1 \leq i \leq \frac{4n}{3}$, cover $4m$ vertices of G. Since $|V(G)| = 8mn$, $\lambda \geq \left\lfloor \frac{(2m+4m)(4n/3)}{6} \right\rfloor = \lfloor \frac{8mn}{6} \rfloor$. By Theorem 2.3, $\lambda \leq \lfloor \frac{8mn}{6} \rfloor$. Hence the proof.

Theorem 2.10. *Let G be a $C_4C_8(S)[m, 3n]$ nanotori. Then $\lambda(G, H) = \frac{8mn}{6}$.*

Theorem 2.11. *Let G be a $C_4C_8(S)[m, 3n]$ nanotori. Then $\gamma_t(G) = \frac{8mn}{3}$.*

Proof. By Theorem 2.5, it is easy to see that $\gamma_t(G) = 2\lambda = \frac{8mn}{3}$. □

2.4 $C_6[m, n]$ nanotori

A $C_6[m, n]$ nanotori is a trivalent decoration made by hexagons C_6 [11, 14]. It is a 3-regular graph with m number of rows and n number of columns, each row comprising of hexagons C_6 viewed horizontally and each column comprising of hexagons C_6 in each row viewed vertically. $C_6[m, n]$ nanotori contains $6n+3$ levels of vertices which are labeled as level 1, level 2, ..., level $6n + 3$ from left to right, viewed vertically. See Figure 3(b).

The Algorithm D given below computes the packing in $C_6[m, 3n + 1]$ nanotori with H, where $H \cong h$-graph.

Input: Let G be a $C_6[m, 3n + 1]$ nanotori, $m, n \geq 1$.

Algorithm D: Select all the vertices from the level L_{3i-1}, $1 \leq i \leq \frac{4n+2}{6}$ and call the set as S.

Output: S is a perfect H-packing of $C_6[m, 3n+1]$ nanotori.

Proof of Correctness: The vertices in the level L_{3i-1}, dominate vertices in the levels L_{3i-2} and L_{3i}, $1 \leq i \leq \frac{4n+2}{6}$. In each level L_{3i-1}, $1 \leq i \leq \frac{4n+2}{6}$, there are $2m$ vertices of G (see Figure 3(b)). Now the vertices in each level L_{3i-1}, $1 \leq i \leq \frac{4n+2}{6}$, cover $4m$ vertices of G. Since $|V(G)| = m(4n+2)$, $\lambda \geq \left\lfloor \frac{(2m+4m)(4n+2)/6}{6} \right\rfloor = \left\lfloor \frac{m(4n+2)}{6} \right\rfloor$. By Theorem 2.3, $\lambda \leq \left\lfloor \frac{m(4n+2)}{6} \right\rfloor$. Hence the proof.

Theorem 2.12. *Let G be a $C_6[m, 3n+1]$ nanotori. Then* $\lambda(G, H) = \frac{m(4n+2)}{6}$.

Theorem 2.13. *Let G be a $C_6[m, 3n+1]$ nanotori. Then* $\gamma_t(G) = \frac{m(4n+2)}{3}$.

Proof. By Theorem 2.5, it is easy to see that $\gamma_t(G) = 2\lambda = \frac{m(4n+2)}{3}$. □

Remark 2.14. *The results obtained for H-Naphtalenic $[m, n]$ nanotori, $C_4C_6C_8[m, n]$ nanotori, $C_4C_8(S)[m, 3n]$ nanotori and $C_6[m, 3n+1]$ nanotori also hold good for H-Naphtalenic $[m, n]$ nanotubes, $C_4C_6C_8[m, n]$ nanotubes, $C_4C_8(S)[m, 3n]$ nanotubes and $C_6[m, 3n+1]$ nanotubes, respectively.*

3 Conclusion

In this paper, we adopt the concept of perfect H-packing as a tool to determine the total domination number for certain nanotori. Also we have devised algorithms to find perfect H-packings of nanotori where H is the h-graph on six vertices, leading to the packing numbers of the nanotori.

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