Transversal Total Domination in Graphs

Nayaka S.R., Anwar Alwardi and Puttaswamy

Abstract

Let $G = (V, E)$ be a graph. A total dominating set of G which intersects every minimum total dominating set in G is called a transversal total dominating set. The minimum cardinality of a transversal total dominating set is called the transversal total domination number of G, denoted by $\gamma_{tt}(G)$. In this paper, we begin to study this parameter. We calculate $\gamma_{tt}(G)$ for some families of graphs. Further some bounds and relations with other domination parameters are obtained for $\gamma_{tt}(G)$.

2010 Mathematics Subject Classification : Primary 05C50, 05C69.

Keywords and Phrases : Total Dominating set, Transversal total dominating set, Transversal total domination number.

1 Introduction

Let $G = (V, E)$ be any graph with n vertices and m edges. For any graph theoretic terminologies not defined here, refer to the book by Bondy and Murthy [1]. One of the fastest growing areas in graph theory is the study of domination and related subset problems such as independence, covering and matching. In fact, there are scores of graph theoretic concepts involving domination, covering and independence. The bibliography in domination maintained by Haynes et al. [2] currently has over 1200 entries; Hedetniemi and Laskar [4] edited a recent issue of Discrete Mathematics devoted entirely to domination, and a survey of advanced topics in domination is given in the book by Haynes et al. [3]. In spite of all possible variations, so we may find only a limited number of basic domination parameters means that the domination parameter defined for all non-trivial connected graphs. For instance, we have the domination parameters like total domination, independent domination, connected domination, isolated domination and paired domination are some basic domination parameters. Recently, one more basic domination parameter was introduced by Ismail Sahul Hamid [5] namely independent transversal domination. This parameter is a combination of two graph invariants namely independence and domination.

In this paper by a graph G, we mean a non-trivial, finite, undirected graph with neither loops nor multiple edges.

2 Basic definitions and Notations

Let G be any graph and v be any vertex in G. Then, the open neighborhood of a vertex v is denoted by $N(v)$ and is defined by $N(v) = \{u \in V | uv \in E\}$, the set of all vertices adjacent to v. The closed neighborhood of v is denoted $N[v]$ and defined by $N[v] = N(v) \cup \{v\}$. For any subset S of G, the open and closed neighborhoods of S in G is defined by $N(S) = \cup_{v \in S} N(v)$ and $N(S) = \cup_{v \in S} N[v]$. A graph H is called the subgraph of G if G contains all the vertices and edges of H. For any subset S of $V(G)$, the sub-graph induced by the set S is denoted $\langle S \rangle$. For a graph G, $\delta = \delta(G)$ denotes the minimum degree of a vertex in G and $\Delta = \Delta(G)$ denotes maximum degree of a vertex in G.

A subset S of vertices is called a dominating set if every vertex in $V(G)$ is either belongs to S or adjacent to some vertex in S. The least cardinality of a minimal dominating set is called the domination number, denoted by $\gamma(G)$. A dominating set S in G is called an independent transversal dominating set if it intersects every maximum independent set in G. The minimum cardinality of an independent transversal dominating set of G is called the independent transversal domination number in G, denoted by $\gamma_{it}(G)$. The Line Graph H of a graph G is a graph with $V(H) = E(G)$ and any two vertices in H are adjacent if the corresponding edges are incident in G.

Definition 2.1. *The multi-star graph $K_m(a_1, a_2, \ldots, a_m)$ is a graph of order $a_1 + a_2 + \cdots + a_m + m$ formed by joining $a_1, a_2, \ldots, a_m$ end-edges to m vertices of K_m. For example, $K_2(a_1, a_2)$ is a double star.*

Definition 2.2. *The join $G = G_1 \vee G_2$ of graphs G_1 and G_2 with disjoint vertex sets V_1 and V_2 and edge sets E_1 and E_2 is the graph union $G_1 \cup G_2$ together with all the edges joining V_1 and V_2.*

Definition 2.1. *The corona $G_1 \circ G_2$ is defined as the graph G obtained by taking one copy of G_1 of order n_1 and n_1 copies of G_2, and then joining the i'th node of G_1 to every node in the i'th copy of G_2.*

As usual given a real number x, $\lfloor x \rfloor$ denotes the greatest integer less than x and $\lceil x \rceil$ denotes the smallest integer greater than x.

3 Transversal Total Domination Number

Here, we define the transversal total domination number of a graph and also we determine the transversal total domination number for some standard families of graphs

Definition 3.1. *A total dominating set S intersecting every minimum total dominating set in G is called a transversal total dominating set. The minimum cardinality of a transversal total dominating set in G is called the transversal total domination number of G and is denoted by $\gamma_{tt}(G)$. Any transversal total dominating set S of G with $|S| = \gamma_{tt}(G)$ is called a γ_{tt}-set.*

Observation 3.1. *Let G be a complete graph with n vertices. Then $\gamma_{tt}(G) = n - 1$.*

Observation 3.2. *Let $G \cong K_{1,n-1}$ be a star with $n \geq 2$ vertices. Then $\gamma_{tt}(G) = 2$.*

Observation 3.3. *Let G be any graph having unique vertex of degree $n-1$. Then $\gamma_{tt}(G) = \gamma_t(G) = \gamma(G) + 1$.*

Proposition 3.1. *Let $G \cong K_{m,n}$ be a complete bipartite graph. Then*

$$\gamma_{tt}(K_{m,n}) = min\{m, n\} + 1$$

Proof. Let $G \cong K_{m,n}$ be a complete bipartite graph and let $\{V_1, V_2\}$ be the partition of vertex set of G such that $|V_1| = m$ and $|V_2| = n$. Assume $m \leq n$. For any vertex $v \in V_2$, take $S = V_1 \cup \{v\}$. Then S is a total dominating set intersecting every minimum total dominating set in G. Hence $\gamma_{tt}(G) \leq m + 1$. On the other hand, for any two vertices u and v taken from V_1 and V_2 respectively, the set $\{u, v\}$ will be the minimum total dominating set in G and so G contains exactly mn minimum total dominating sets. Thus, every vertex in V_1 corresponds to a distinct minimum total dominating set in G. Further, V_1 is not a total dominating set as it contains an isolated vertex, so we must have $\gamma_{tt}(G) \geq m + 1$. Therefore $\gamma_{tt}(G) = m + 1$. i.e., $\gamma_{tt}(G) = min\{m, n\} + 1$. □

Proposition 3.2. *Let $G \cong K_{m_1,m_2,\ldots,m_r}$ be a multipartite graph with $m_1 \leq m_2, \ldots, \leq m_r$. Then $\gamma_{tt}(G) = m_1 + m_2 + \ldots + m_{r-1}$.*

Proof. Let $G \cong K_{m_1,m_2,\ldots,m_r}$ be a multi-partite graph and let $\{V_1, V_2, \ldots, V_r\}$ be the partition of vertex set of G with $|V_i| = m_i, 1 \leq i \leq r$. Then, for each i, every vertex of V_i corresponds to a minimum total dominating set in G. Let S be any transversal total dominating set of G. Suppose $i \neq j$ and let u, v be the vertices of V_i and V_j respectively, then either u or v must be in S. For otherwise $\{u, v\}$ will be the minimum total dominating set in G not intersected by S, a contradiction. This contradiction shows that no two sets V_i, V_j can lie outside S. Therefore, by the minimality we have $\gamma_{tt}(G) \geq m_1 + m_2 + \cdots + m_{r-1}$. On the other hand $\cup_{i=1}^{r-1} V_i$ is a transversal total dominating set in G of cardinality $m_1 + m_2 + \cdots + m_{r-1}$. This proves that $\gamma_{tt}(G) = m_1 + m_2 + \cdots + m_{r-1}$. □

Theorem 3.1. *Let G be path with $n \geq 2$ vertices. Then*

$$\gamma_{tt}(P_n) = \begin{cases} 2, & \text{if } n = 2, 3; \\ \frac{n}{2}, & \text{if } n \equiv 0 \ (\text{mod } 4); \\ 3 + \lfloor \frac{n-4}{2} \rfloor, & \text{otherwise.} \end{cases}$$

Proof. Let $G \cong P_n$ be a path with $n \geq 2$ vertices and let $V(P_n) = \{v_1, v_2, \ldots, v_n\}$. If $n \in \{2, 3\}$ then clearly $\gamma_{tt}(G) = 2$. Assume $n \notin \{2, 3\}$. We first note that any minimum total dominating set in G contains the vertices $\{v_2, v_3\}$. First, suppose $n \equiv 0 \,(mod\, 4)$. Then $S = \{v_{4i-2}, v_{4i-1} | 1 \leq i \leq \frac{n}{4}\}$ is a unique total dominating set in G. Hence $\gamma_{tt}(G) = \gamma_t(G) = \frac{n}{2}$. For otherwise, since any minimum total dominating set in G contains the vertices from the set $\{v_2, v_3\}$, it follows that $\gamma_{tt}(G) = 2 + \gamma_t(G')$ where G' is the graph obtained by removing the vertices v_1, v_2 and its neighbors from G. Then $G' \cong P_{n-4}$ and so $\gamma_{tt}(G) = 2 + \gamma_t(P_{n-4}) = 3 + \lfloor \frac{n-4}{2} \rfloor$. □

The Line graph of a Path P_n with n vertices is the path graph P_{n-1} with $n - 1$ vertices. So we have the following corollary.

Corollary 3.1. *For a path P_n with $n \geq 3$ vertices, we have*

$$\gamma_{tt}(L(P_n)) = \begin{cases} 2, & \text{if } n = 3, 4; \\ \frac{n-1}{2}, & \text{if } n \equiv 1 \ (\text{mod } 4); \\ 3 + \lfloor \frac{n-5}{2} \rfloor, & \text{otherwise.} \end{cases}$$

Theorem 3.2. *For any cycle C_n with $n \geq 3$, we have*

$$\gamma_{tt}(C_n) = \begin{cases} n - 1, & \text{if } n = 3, 4; \\ \frac{n}{2} & \text{if } n > 5 \text{ and } n \equiv 0 (mod\, 4); \\ 3 + \lfloor \frac{n-4}{2} \rfloor & \text{otherwise.} \end{cases}$$

Proof. Let $G \cong C_n$ be a cycle with n vertices and let $V(C_n) = \{v_1, v_2, \ldots, v_n\}$. First, if $n = 3$, then G is a complete graph and so $\gamma_{tt}(G) = 2$. Next, suppose $n = 4$. Then every pair of vertices will be a total dominating set and so for any vertices u, v of G at least one vertex must in a transversal total dominating set. Therefore $\gamma_{tt}(G) = n - 1$. Finally, assume that $n > 5$. Consider a sequence $\{v_1, v_2\}$ of two consecutive vertices in C_n. Then, any minimum total dominating set in C_n should contain at least one of the vertex $v_i (1 \leq i \leq 2)$. Therefore $\gamma_{tt}(G) = 2 + \gamma_t(H)$ where H is the graph obtained by removing the vertices of the sequence and its neighbors from G. Clearly $H \cong P_{n-4}$. Therefore, $\gamma_{tt}(C_n) = 2 + \gamma_t(P_{n-4}) = 3 + \lceil \frac{n-4}{2} \rceil$. □

Proposition 3.3. *Let G be a firefly graph having t pendant paths. Then* $\gamma_{tt}(G) = t + 1$.

Proof. Let G be a firefly graph with the vertex set $V(G) = \{v_1, v_2, \ldots, v_n\}$. Suppose v_n is the vertex of G that is common to the triangles, pendent edges and pendent paths in G. Then, clearly, the vertex v_1 dominates the graph except the pendant paths. Let S be the set of support vertices of the pendant paths. Then $|S| = t$ and $S \cup \{v_1\}$ is a unique total dominating set in G. Therefore, $\gamma_{tt}(G) = \gamma(G) = t + 1$. □

Proposition 3.4. *Let $G = K_m(a_1, a_2, \ldots, a_m)$ be a multi-star graph. Then $\gamma_{tt}(G) = m$.*

Proof. Let $G = K_m(a_1, a_2, \ldots, a_m)$ be a multi-star graph as shown in the Figure 1. Then G contains a unique minimum total dominating set consisting of all support vertices of G. Hence $\gamma_{tt}(G) = \gamma(G) = m$.

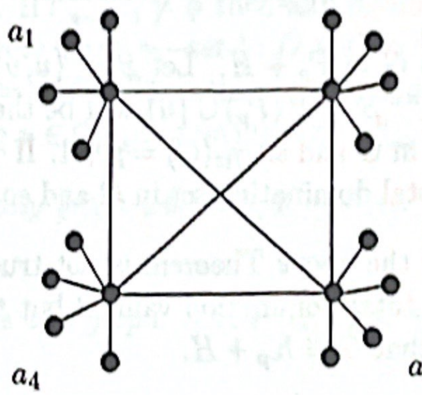

Figure 1:

□

Theorem 3.3. *Let G_1 and G_2 be two connected graphs with n_1 and n_2 vertices respectively. Then $\gamma_{tt}(G_1 \vee G_2) = min\{n_1, n_2\}$ if and only if $\gamma_t(G_1), \gamma_t(G_2) \geq 3$. Further, for $i, j \in \{1, 2\}$, if $n_i < n_j$ and G_i contains an isolated vertex, then $\gamma_{tt}(G_1 \vee G_2) = n_i + 1$.*

Proof. Let G_1 and G_2 be two connected graphs with n_1 and n_2 vertices respectively. Suppose $\gamma_t(G_1), \gamma_t(G_2) \geq 3$. Then by the definition of join of two graphs each vertex of G_1 is adjacent to every vertex in G_2 and vice versa. Hence $\gamma_t(G_1 \vee G_2) = 2$ and the γ_t-set is obtained by choosing one vertex from $V(G_1)$ and another vertex from $V(G_2)$. Clearly G contains at least $n_1 n_2$ minimum total dominating sets. By our assumption $G_1 \vee G_2$ cannot have any other choices for γ_t-set. Moreover, each vertex in G_i $(i = 1, 2)$ corresponds to a minimum total dominating set. As G_1 and G_2 are connected graphs, it follows that $\gamma_{tt}(G_1 \vee G_2) = min\{n_1, n_2\}$.

Conversely, on contradiction, we assume that $\gamma_t(G_1) = \gamma_t(G_2) = 2$. Any γ_t-set of G_i $(i = 1, 2)$ is also a γ_t-set of $G_1 \vee G_2$. Therefore $\gamma_{tt}-$set of $G_1 \vee G_2$ contains at least one vertex from each graph and so $\gamma_t(G_1 \vee G_2) \geq min\{n_1, n_2\} + 1$, a contradiction. This contradiction shows that $\gamma_t(G_1), \gamma_t(G_2) \geq 3$.

Next, Suppose $n_1 < n_2$ and G_1 contains an isolated vertex, then $V(G_1)$ will be a minimum dominating set but not a total dominating set as G_1 contains isolated vertex. Thus for any vertex $v \in V(G_2)$, the set $V(G_1) \cup \{v\}$ will be a γ_{tt}−set in $G_1 \vee G_2$. Therefore, $\gamma_{tt}(G_1 \vee G_2) = n_1 + 1$. □

Proposition 3.5. *Let G be any graph order n such that $G \cong P_p + H$, where H is any graph and $2 \leq p \leq n-2$. Then*

$$\gamma_{tt}(G) = \begin{cases} p+2, & \text{if } H \cong C_4; \\ p+1, & \text{if } \gamma_t(H) = 2 \text{ and } H \ncong C_4; \\ p & \text{otherwise.} \end{cases}$$

Proof. Suppose $G \cong P_p + H$. Let $S = \{u, v\}$ be a γ_{tt}−set of H. Then from theorem *** $S' = V(P_p) \cup \{u\}$ will be the minimum transversal total dominating set in G and so $\gamma_{tt}(G) = p+1$. If $\gamma_t(H) > 2$, then $V(P_p)$ itself a transversal total dominating set in G and so $\gamma_{tt}(G) = p$. □

Converse of the above Theorem is not true as the graph $G \cong C_5$ has the transversal total domination value 3 but there is no graph H and an integer p such that $G \cong K_p + H$.

Proposition 3.6. *Let $G \cong W_n$ be a wheel on $n \geq 4$ vertices. Then*

$$\gamma_{tt}(W_n) = \begin{cases} 3, & \text{if } n = 4,\ 5; \\ 2 & \text{otherwise.} \end{cases}$$

Proof. Let $G \cong W_n$ be a wheel with n vertices. Then $W_n = K_1 \vee C_{n-1}$ where C_{n-1} is a cycle with $n-1$ vertices and K_1 is a complete graph of order one. If $n = 4$, then W_4 is a complete graph and so $\gamma_{tt}(W_n) = 3$. Suppose $n = 5$, then from the above Proposition $\gamma_{tt}(W_n) = 3$. Suppose $n \geq 6$. Then $\gamma_t(C_{n-1}) \geq 3$ and hence from Theorem 3.3 it follows that $\gamma_{tt}(W_n) = 2$. □

Theorem 3.4. *Let G_1 and G_2 be two connected graphs with n_1 and n_2 vertices respectively. Then*

$$\gamma_{tt}(G_1 \vee G_2) = \begin{cases} n_1 + 1, & \text{if } n_1 < n_2 \text{ and } \gamma_{tt}(G_2) = 2; \\ n_2 + 1, & \text{if } n_2 < n_1 \text{ and } \gamma_{tt}(G_1) = 2. \end{cases}$$

Proof. Let G_1 and G_2 be two connected graphs with n_1 and n_2 vertices respectively. We first assume that $n_1 < n_2$ and $\gamma_{tt}(G_2) = 2$. For any γ_{tt}−set S of G_2, we have the set $V(G_1) \cup \{v\}$ where $v \in S$ will be a transversal total dominating in $G_1 \vee G_2$. But $V(G_1)$ itself is not a transversal total dominating set in $G_1 \vee G_2$ since S is a γ_t-set in $G_1 \vee G_2$ disjoint from $V(G_1)$. Therefore, $V(G_1) \cup \{v\}$ will be a minimum transversal total dominating set in $G_1 \vee G_2$ proving that $\gamma_{tt}(G_1 \vee G_2) = n_1 + 1$. □

Proposition 3.7. *Let G be a connected graph of order n. Then $\gamma_{tt}(G \circ H) = n$, for any graph H.*

Proof. Let G be a graph of order n. Since $V(G)$ itself a dominating set and G is connected it follows that $V(G)$ is a minimum transversal total dominating set in $G \circ H$. Therefore, $\gamma_{tt}(G) = \gamma_t(G) = n$. □

Proposition 3.8. *For any graph G, $\gamma_{tt}(G) = \gamma_t(G)$ holds if $\cap_{i=k}^{k} S_i \neq \phi$ for any sequence $\{S_i\}_{i=1}^{k}$ of γ_t-sets in G.*

Proof. Let G be any graph. If $\cap_{i=k}^{k} S_i \neq \phi$ then any minimum transversal total dominating set intersects every γ_t-set in G and so $\gamma_{tt}(G) = \gamma_t(G)$. Conversely, suppose $\gamma_{tt}(G) = \gamma_t(G)$. Then there exists a vertex u that lies in every γ_t-set in G. Hence $u \in \cap_{i=k}^{k} S_i$ proving that $\cap_{i=k}^{k} S_i \neq \phi$. □

Corollary 3.2. *Let G be any graph with unique γ_t-set. Then $\gamma_{tt}(G) = \gamma_t(G)$.*

Proposition 3.9. *Let G be any graph. Then $\gamma_{td}(G) \leq \gamma_{tt}(G)$. Equality holds if G is claw-free.*

Proof. Let S be any $\gamma_{tt}-$set in G. Then S is a dominating set in G intersecting every γ_t-set. Since every total dominating set is also dominating set, it follows that S intersects every $\gamma-$set in G. Therefore $\gamma_{td}(G) \leq |S|$, proving that $\gamma_{tt}(G) = \gamma_t(G)$. Suppose G is a claw-free graph and S' be a $\gamma_{td}-$set in G. Since any dominating set in G is same as the total dominating set it follows that S' is a transversal total dominating set in G. Therefore $\gamma_{tt}(G) \leq \gamma_{td}(G)$ and so $\gamma_{td}(G) = \gamma_{tt}(G)$. □

Theorem 3.5. *If G is a disconnected graph with components $G_1, G_2, ..., G_m$ then $\gamma_{tt}(G) = min_{1 \leq i \leq m}\{\gamma_{tt}(G_i) + \Sigma_{j=1, j\neq i}^{m} \gamma(G_j)\}$.*

Proof. We prove this result by using mathematical induction. The result is trivially true for $m = 1$. Suppose $m = 2$. Then $G = G_1 \cup G_2$. Let D_1, D_2 be the $\gamma_{tt}-$sets of G_1 and G_2 respectively. Then, clearly $D_1 \cup S_2$ and $D_2 \cup S_1$ are transversal total dominating sets in G, where S_1 and S_2 are γ_t-sets of G_1 and G_2 respectively. Therefore $\gamma_{tt}(G) \leq min\{\gamma_{tt}(G_1) + \gamma_t(G_2), \gamma_{tt}(G_2) + \gamma_t(G_1)\}$.

Let S be any transversal total dominating set in G. Then S should intersect the vertex set of both G_1 and G_2 and also S contains the total dominating set of G_1 and G_2. Moreover, S must contain the transversal dominating set of G_1 or G_2. For otherwise, there exists minimum total dominating sets S_1 and S_2 of G_1 and G_2 respectively such that $S_1 \cup S_2$ will be the minimum total dominating set of G which is not intersected by S, a contradiction. This contradiction shows that $|S| \geq min\{\gamma_{tt}(G_1) + \gamma_t(G_2), \gamma_{tt}(G_2) + \gamma_t(G_1)\}$. Therefore the result holds for $m = 2$.

Next,assume that $m > 2$ and the result for $m = k-1$. Suppose G is a graph with components $G_1, G_2, ..., G_{k-1}, G_k$. Let G' be the graph whose components are $G_1, G_2, ..., G_{k-1}$. By induction hypothesis we have $\gamma_{tt}(G') = min_{1 \leq i \leq k-1}\{\gamma_{tt}(G_i) + \Sigma_{j=1,j \neq i}^{k-1} \gamma_t(G_j)\}$. Now, we have $G = G' \cup G_m$. That is, G is a graph having two components G' and G_m. Then, by the case $m = 2$, we may conclude that $\gamma_{tt}(G) = min_{1 \leq i \leq k}\{\gamma_{tt}(G_i) + \Sigma_{j=1,j \neq i}^{m} \gamma(G_j)\}$. Therefore the result is true for $m = k$ and hence true for any positive integer m. □

4 Some Bounds for γ_{tt}

Proposition 4.1. *Let G be any connected graph of order n. Then $2 \leq \gamma_{tt}(G) \leq n$. Further for $n \geq 2$, $\gamma_{tt}(G) = n$ if and only if $G \cong P_2$ and $\gamma_{tt}(G) = 2$ if and only if $G \cong K_1 + H$, for some graph H.*

Proof. The inequalities are trivial. Now, suppose $n \geq 2$ and $\gamma_{tt}(G) = n$. Assume that $n \geq 3$. Then, there exists a vertex $v \in V(G)$ such that the set $V(G) - \{v\}$ is itself a transversal total dominating set. Therefore, $\gamma_{tt}(G) \leq n - 1$ which is a contradiction. This contradiction shows that $n = 2$ and so $G \cong P_2$.

Finally, suppose $\gamma_{tt}(G) = 2$. Then there exists a vertex v in G that lies in every total dominating set in G. Taking $H = G - \{v\}$, we get $G \cong K_1 + H$. Conversely, if $V(K_1) = \{u\}$ then for any vertex $v \in V(H)$, the set $\{u, v\}$ will be a $\gamma_{tt}-$set in G and so $\gamma_{tt}(G) = 2$. □

Corollary 4.1. *Let G be any disconnected graph of order $n = 2k$. Then $\gamma_{tt}(G) = n$ if and only if $G = \cup_{i=1}^{k} P_2$.*

Corollary 4.2. *Let G be any disconnected graph of order $n = 2k+1$. Then $\gamma_{tt}(G) \leq n - 1$. Equality holds if $G = \cup_{i=1}^{k-1} P_2 \cup P_3$.*

Let $\mathcal{G}$ be the collection of complete graph K_n, Cycle C_4, path P_3 and $C_4 + e$.

Proposition 4.2. *Let G be any graph of order n. Then $\gamma_{tt}(G) = n - 1$ if and only if $G \in \mathcal{G}$.*

Proof. Let G be any graph and assume that $\gamma_{tt}(G) = n - 1$. Let u, v be any two vertices in G. Assume, u and v are non-adjacent. Suppose

$n \geq 5$. Then $V-\{u,v\}$ itself a transversal total dominating set in G and so $\gamma_{tt}(G) \leq n-2$, a contradiction. This contradiction shows that G is either a complete graph K_n or G is a graph of order at most 4. Assume G is not a complete graph. If $n=3$, then clearly $G \cong P_3$. Finally suppose $n=4$, then $S = G-\{u,v\}$ is either K_2 or $\overline{K_2}$. Suppose $V-\{u,v\} = \overline{K_2}$. If at least one vertex has only one neighbor in S, then $\gamma_{tt}(G) = 2$, a contradiction. Thus, both u,v have two neighbors in S and so $G \cong C_4$. Further, if the neighbors of u and v are adjacent then $G \cong C_4 + e$. Converse is obvious. □

For any graph G, as the dominating set is contained in a total dominating set, every transversal dominating set in G intersects all the minimum total dominating set.

Observation 4.1. *For any graph G, we have $\gamma(G) \leq \gamma_t(G)$ but it is not true that $\gamma_{td}(G) \leq \gamma_{tt}(G)$ always. For instance, if G is a complete graph, then $\gamma_{tt}(G) < \gamma_{td}(G)$. Further, $\gamma_{td}(G) = \gamma_{tt}(G)$ holds if G is a claw-free graph.*

Proposition 4.3. *Let G be a cycle or a path of order $n \geq 3$. Then $\gamma_{tt}(G) = 2\gamma_{td}(G)$ if and only if $n \equiv 0 \,(mod\ 3)$.*

Proposition 4.4. *For any graph G, $\gamma_{tt}(G) < \gamma_{td}(G)$ if one of the following conditions holds.*

1. *Every pair of vertices gives a dominating set in G.*
2. *$\delta(G) \geq \frac{n-k}{2}$ and $N(u) \cap N(v) = 2-k$ for every edge $e = uv \in E(G)$, where $k \in \{1,2\}$.*

Proof. Let G be any graph of order n. Suppose every pair of vertices gives a dominating set in G. Then $\gamma_{td}(G) \geq n-1$. Suppose every pair of vertices are adjacent in G. Then G is a complete graph and hence $\gamma_{tt}(G) = n-1$, we are dome. Suppose (u,v) be a non-adjacent pair of vertices in G. Then $V(G) - \{u,v\}$ will be a transversal total dominating set in G. Therefore $\gamma_{tt}(G) \leq n-2$, proving that $\gamma_{tt}(G) < \gamma_{td}(G)$. □

Proposition 4.5. *If every pair of vertices is a dominating set in G, then $\gamma_{tt}(G) \leq \gamma_{td}(G)$.*

Proposition 4.6. *Let G be a complete graphofordern. Then for any edge $e \in E(G)$, we have $\gamma_{tt}(G-e) = \gamma_{tt}(G-2e) = n-2$.*

Theorem 4.1. *Let G be any graph without isolated vertices and having no vertex of degree $n-1$. Then $4 \leq \gamma_{tt}(G) + \gamma_{tt}(\overline{G}) \leq 2n-2$.*

5 Conclusion

In recent year the theory of domination is a very important area in graph theory in which tremendous research is going on. Many researchers are working in this area and number of new domination parameters have been introduced. In this direction, we have introduced a new domination invariant called minimum transversal domination. In this paper we have just initiated the study of this parameter. We have calculated the exact values for some standard graphs also we have established some important properties of this parameter. Some bounds for this parameter are established and we have attempted to find the relation with other domination invariants.

References

[1] J.A.Bondy, U.S.R Murty, *Graph theory with application*,Elsevier science Publishing Co, Sixth printing, 1984.

[2] T.W.Haynes, S.T.Hedetniemi, P.J.Slater ,*Fundamentals of Domination in Graphs*, Marcel Dekker, New york, 1998.

[3] T.W. Haynes, S.T. Hedetniemi and P.J. Slater, *Domination in Graphs: Advanced Topics* (Marcel Dekker, New York, 1998).

[4] S.T. Hedetniemi and R.C. Laskar, *Topics on Domination*, Discrete Math. 86 (1990).

[5] Ismail Sahul Hamid, *Independant transversal Domination in Graphs*, Discussiones Mathematicae, Graph Theory 32 (2012) 5-17.

Authors :

Nayaka S.R.
Research scholar
Department of Mathematics,
PES College of Engineering,
Mandya-571 401, INDIA.
nayaka.abhi11@gmail.com.

Puttaswamy
Associate Professor,
Department of Mathematics,
PES College of Engineering,
Mandya-571 401, INDIA.
prof.puttaswamy@gmail.com.

Anwer Alwardi
Post Doctoral Researcher,
DoS in Mathematics,
University of Mysore,
Mysore.
a_wardi@hotmail.com.