The 3-extra diagnosability of alternating group graphs

Yunxia Ren, Shiying Wang
Henan Engineering Laboratory
for Big Data Statistical Analysis and Optimal Control
School of Mathematics and Information Science
Henan Normal University, Xinxiang, Henan 453007 PR China
wangshiying@htu.edu.cn

Abstract

The diagnosability of a multiprocessor system is one important study topic. In 2016, Zhang et al. proposed a new measure for fault diagnosis of the system, namely, the g-extra diagnosability, which restrains that every fault-free component has at least $(g+1)$ fault-free nodes. As a favorable topology structure of interconnection networks, the n-dimensional alternating group graph AG_n has many good properties. In this paper, we prove that the 3-extra diagnosability of AG_n is $8n-25$ for $n \geq 5$ under the PMC model and for $n \geq 7$ MM* model.

1 Introduction

Many multiprocessor systems have interconnection networks (networks for short) as underlying topologies and a network is usually represented by a graph where nodes represent processors and links represent communication links between processors. We use graphs and networks interchangeably. For a multiprocessor system, study of the topological properties of its network is important. Furthermore, some processors may fail in the system, so processor fault identification plays an important role for reliable computing. The first step to deal with faults is to identify the faulty processors from the fault-free ones. The identification process is called the diagnosis of the system. A system is said to be t-diagnosable if all faulty processors can be identified without replacement, provided that the number of faults presented does not exceed t. The diagnosability $t(G)$ of a system G is the maximum value of t such that G is t-diagnosable [2, 3, 8]. For a t-diagnosable system, Dahbura and Masson [2] proposed an algorithm with time complex $O(n^{2.5})$, which can effectively identify the set of faulty processors.

Several diagnosis models were proposed to identify the faulty processors. One major approach is the Preparata, Metze, and Chien's (PMC) diagnosis model introduced by Preparata et al. [12]. The diagnosis of the system is achieved through two linked processors testing each other. Another major approach, namely, the comparison diagnosis model (MM model), was proposed by Maeng and Malek [10]. In the MM model, to diagnose a system, a node sends the same task to two of its neighbors, and then compares their responses. In 2005, Lai et al. [8] introduced a restricted diagnosability of multiprocessor systems called conditional diagnosability. They consider the situation that no fault set can contain all the neighbors of any vertex in a system. In 2012, Peng et al. [11] proposed a measure for fault diagnosis of the system, namely, g-good-neighbor diagnosability (which is also called the g-good-neighbor conditional diagnosability), which requires that every fault-free node has at least g fault-free neighbors. In [11], they studied the g-good-neighbor diagnosability of the n-dimensional hypercube under the PMC model. In [18], Wang and Han studied the g-good-neighbor diagnosability of the n-dimensional hypercube under the MM* model. Numerous studies have been investigated under the PMC and MM model or MM* model for the condition: g-good-neighbor, see [15, 16, 21, 24, 28, 29, 30]. In 2016, Zhang et al. [31] proposed a new measure for fault diagnosis of the system, namely, the g-extra diagnosability, which restrains that every fault-free component has at least $(g+1)$ fault-free nodes. In [31], they studied the g-extra diagnosability of the n-dimensional hypercube under the PMC model and MM* model. Numerous studies have been investigated under the PMC and MM* model for the condition: g-extra, see [14, 19, 20, 25, 26, 34].

As a favorable topology structure of interconnection networks, the n-dimensional alternating group graph AG_n has many good properties. In this paper, we prove that the 3-extra diagnosability of AG_n is $8n-25$ for $n \geq 5$ under the PMC model and for $n \geq 7$ MM* model.

2 Preliminaries

A multiprocessor system is modeled as an undirected simple graph $G = (V, E)$, whose vertices (nodes) represent processors and edges (links) represent communication links. Given a nonempty vertex subset V' of V, the induced subgraph by V' in G, denoted by $G[V']$, is a graph, whose vertex set is V' and the edge set is the set of all the edges of G with both endpoints in V'. The degree $d_G(v)$ of a vertex v is the number of edges incident with v. The minimum degree of a vertex in G is denoted by $\delta(G)$. For any vertex v, we define the neighborhood $N_G(v)$ of v in G to be the set of vertices adjacent to v. u is called a neighbor vertex or a neighbor of v for $u \in N_G(v)$. Let $S \subseteq V$. We use $N_G(S)$ to denote the set $\cup_{v \in S} N_G(v) \setminus S$. For neighborhoods and degrees, we will usually omit the subscript for the graph when no confusion arises. A graph G is said to be k-regular if for any vertex v, $d_G(v) = k$. The connectivity $\kappa(G)$ of a graph G is the minimum number of vertices whose removal results in a disconnected graph or only one vertex left when G is complete. A path of length l in G is denoted by a l-path. Let F_1 and F_2 be two distinct subsets of V, and let the symmetric difference $F_1 \triangle F_2 = (F_1 \setminus F_2) \cup (F_2 \setminus F_1)$. Let $F \subseteq V(G)$ and $B_1, \ldots, B_k$ $(k \geq 2)$ be the components of $G - F$. If $|V(B_1)| \leq \cdots \leq |V(B_k)|$ $(k \geq 2)$, then B_k is called the maximum component of $G - F$. For graph-theoretical terminology and notation not defined here we follow [1].

Let $G = (V, E)$ be a connected graph. A fault set $F \subseteq V$ is called a g-good-neighbor faulty set if $|N(v) \cap (V \setminus F)| \geq g$ for every vertex v in $V \setminus F$. A g-good-neighbor cut of G is a g-good-neighbor faulty set F such that $G - F$ is disconnected. The minimum cardinality of g-good-neighbor cuts is said to be the g-good-neighbor connectivity of G, denoted by $\kappa^{(g)}(G)$. A fault set $F \subseteq V$ is called a g-extra faulty set if every component of $G - F$ has at least $(g+1)$ vertices. A g-extra cut of G is a g-extra faulty set F such that $G - F$ is disconnected. The minimum cardinality of g-extra cuts is said to be the g-extra connectivity of G, denoted by $\tilde{\kappa}^{(g)}(G)$.

Proposition 1 *([13]) Let G be a connected graph. Then $\tilde{\kappa}^{(g)}(G) \leq \kappa^{(g)}(G)$.*

Proposition 2 *([13]) Let G be a connected graph. Then $\kappa^{(1)}(G) = \tilde{\kappa}^{(1)}(G)$.*

Under the PMC model [10, 29], to diagnose a system $G = (V(G), E(G))$, two adjacent nodes in G are capable to perform tests on each other. For two adjacent nodes u and v in $V(G)$, the test performed by u on v is represented by the ordered pair (u, v). The outcome of a test (u, v) is 1 (resp. 0) if u evaluate v as faulty (resp. fault-free). We assume that the testing result is reliable (resp. unreliable) if the node u is fault-free (resp. faulty). A test assignment T for G is a collection of tests for every adjacent pair of vertices. It can be modeled as a directed testing graph $T = (V(G), L)$, where $(u, v) \in L$ implies that u and v are adjacent in G. The collection of all test results for a test assignment T is called a syndrome. Formally, a syndrome is a function $\sigma : L \mapsto \{0, 1\}$. The set of all faulty processors in G is called a faulty set. This can be any subset of $V(G)$. For a given syndrome σ, a subset of vertices $F \subseteq V(G)$ is said to be consistent with σ if syndrome

σ can be produced from the situation that, for any $(u, v) \in L$ such that $u \in V \setminus F$, $\sigma(u, v) = 1$ if and only if $v \in F$. This means that F is a possible set of faulty processors. Since a test outcome produced by a faulty processor is unreliable, a given set F of faulty vertices may produce a lot of different syndromes. On the other hand, different faulty sets may produce the same syndrome. Let $\sigma(F)$ denote the set of all syndromes which F is consistent with. Under the PMC model, two distinct sets F_1 and F_2 in $V(G)$ are said to be indistinguishable if $\sigma(F_1) \cap \sigma(F_2) \neq \emptyset$, otherwise, F_1 and F_2 are said to be distinguishable. Besides, we say (F_1, F_2) is an indistinguishable pair if $\sigma(F_1) \cap \sigma(F_2) \neq \emptyset$; else, (F_1, F_2) is a distinguishable pair.

Using the MM model, the diagnosis is carried out by sending the same testing task to a pair of processors and comparing their responses. We always assume the output of a comparison performed by a faulty processor is unreliable. The comparison scheme of a system $G = (V, E)$ is modeled as a multigraph, denoted by $M = (V(G), L)$, where L is the labeled-edge set. A labeled edge $(u, v)_w \in L$ represents a comparison in which two vertices u and v are compared by a vertex w, which implies $uw, vw \in E(G)$. The collection of all comparison results in $M = (V(G), L)$ is called the syndrome, denoted by σ^*, of the diagnosis. If the comparison $(u, v)_w$ disagrees, then $\sigma^*((u, v)_w) = 1$, otherwise, $\sigma^*((u, v)_w) = 0$. Hence, a syndrome is a function from L to $\{0, 1\}$. The MM* model is a special case of the MM model. In the MM* model, all comparisons of G are in the comparison scheme of G, i.e., if $uw, vw \in E(G)$, then $(u, v)_w \in L$. Similarly to the PMC model, we can define a subset of vertices $F \subseteq V(G)$ is consistent with a given syndrome $\sigma*$ and two distinct sets F_1 and F_2 in $V(G)$ are indistinguishable (resp. distinguishable) under the MM* model.

A system $G = (V, E)$ is g-good-neighbor t-diagnosable if F_1 and F_2 are distinguishable for each distinct pair of g-good-neighbor faulty subsets F_1 and F_2 of V with $|F_1| \leq t$ and $|F_2| \leq t$. The g-good-neighbor diagnosability $t_g(G)$ of G is the maximum value of t such that G is g-good-neighbor t-diagnosable.

Proposition 3 *([11]) For any given system G, $t_g(G) \leq t_{g'}(G)$ if $g \leq g'$.*

In a system $G = (V, E)$, a faulty set $F \subseteq V$ is called a conditional faulty set if it does not contain all the neighbor vertices of any vertex in G. A system G is conditional t-diagnosable if every two distinct conditional faulty subsets $F_1, F_2 \subseteq V$ with $|F_1| \leq t, |F_2| \leq t$, are distinguishable. The conditional diagnosability $t_c(G)$ of G is the maximum number of t such that G is conditional t-diagnosable. By [6], $t_c(G) \geq t(G)$.

Theorem 4 *[15] For a system $G = (V, E)$, $t(G) = t_0(G) \leq t_1(G) \leq t_c(G)$.*

In [15], Wang et al. proved that the 1-good-neighbor diagnosability of the Bubble-sort graph B_n under the PMC model is $2n-3$ for $n \geq 4$. In [33], Zhou et al. proved the conditional diagnosability of B_n is $4n-11$ for $n \geq 4$ under the PMC model. Therefore, $t_1(B_n) < t_c(B_n)$ when $n \geq 5$ and $t_1(B_n) = t_c(B_n)$ when $n = 4$.

In a system $G = (V, E)$, a faulty set $F \subseteq V$ is called a g-extra faulty set if every component of $G - F$ has more than g nodes. G is g-extra t-diagnosable if and only if for each pair of distinct faulty g-extra vertex subsets $F_1, F_2 \subseteq V(G)$ such that $|F_i| \leq t$, F_1 and F_2 are distinguishable. The g-extra diagnosability of G, denoted by $\tilde{t}_g(G)$, is the maximum value of t such that G is g-extra t-diagnosable.

Proposition 5 *[20] For any given system G, $\tilde{t}_g(G) \leq \tilde{t}_{g'}(G)$ if $g \leq g'$.*

Theorem 6 *[20] For a system $G = (V, E)$, $t(G) = \tilde{t}_0(G) \leq \tilde{t}_g(G) \leq t_g(G)$.*

Theorem 7 *[20] For a system $G = (V, E)$, $\tilde{t}_1(G) = t_1(G)$.*

In the permutation $\left\{ \begin{array}{cccc} 1 & 2 & \dots & n \\ p_1 & p_2 & \dots & p_n \end{array} \right\}$, $i \longrightarrow p_i$. For the convenience, we denote the permutation $\left\{ \begin{array}{cccc} 1 & 2 & \dots & n \\ p_1 & p_2 & \dots & p_n \end{array} \right\}$ by $p_1\ p_2\ \dots\ p_n$. Every permutation can be denoted by a product of cycles [7]. For example, $\left\{ \begin{array}{ccc} 1 & 2 & 3 \\ 3 & 1 & 2 \end{array} \right\} = (132)$. Specially, $\left\{ \begin{array}{cccc} 1 & 2 & \dots & n \\ 1 & 2 & \dots & n \end{array} \right\} = (1)$. The product $\sigma\tau$ of two permutations is the composition of function τ followed by σ, that is, $(12)(13) = (132)$. For terminology and notation not defined here we follow [7].

Let $[n] = \{1, 2, \cdots, n\}$, and let S_n be the symmetric group on $[n]$ containing all permutations $p = p_1 p_2 \cdots p_n$ of $[n]$. The alternating group A_n is the subgroup of S_n containing all even permutations. It is well known that $\{(12i), (1i2), 3 \leq i \leq n\}$ is a generating set for A_n. The n-dimensional alternating group graph AG_n is the graph with vertex set $V(AG_n) = A_n$ in which two vertices u, v are adjacent if and only if $u = v(12i)$ or $u = v(1i2)$, $3 \leq i \leq n$. The identity element of A_n is (1). The graphs AG_3 and AG_4 are depicted in Fig. 1. It is easy to see from the definition that AG_n is a $2(n-2)$-regular graph on $n!/2$ vertices. We decompose AG_n along the last position, denoted by $AG_n^1, AG_n^2, \dots, AG_n^n$. It is obvious that AG_n^i is isomorphic to AG_{n-1} for $i \in [n]$. The edges joining vertices in the distinct subgraphs AG_n^i and AG_n^j are called external edges (or cross-edges), and the edges joining vertices in the same subgraph AG_n^i are called internal edges. There are many researches about alternating group graphs. See [4, 5, 9, 22, 23, 25, 32].

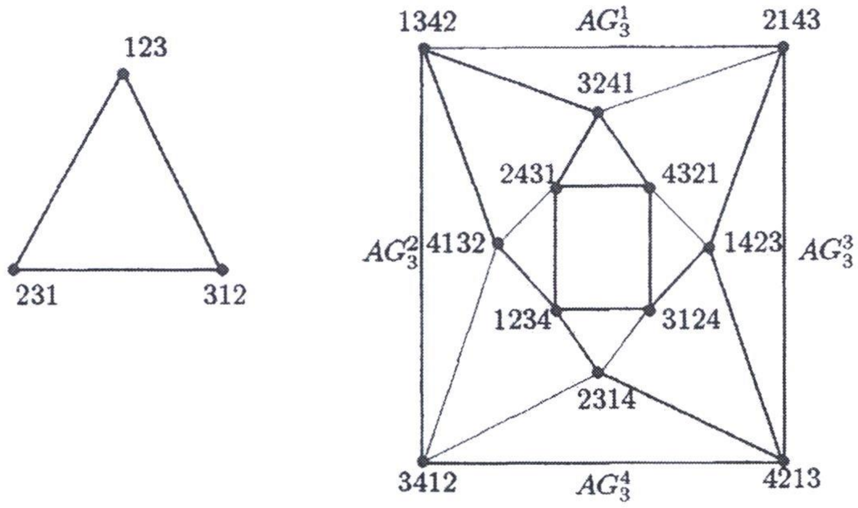

Fig 1. AG_3 and AG_4

As a favorable topology structure of interconnection networks, alternating group graphs have been shown to have many desirable properties such as strong hierarchy, high connectivity, small diameter and average distance, etc.. For details, see [5] for a comparison of the hypercube, the star graph and the alternating group graph.

Proposition 8 *[9] Let AG_n^i be defined as above. Then there are $(n-2)!$ independent cross-edges between two different AG_n^i's.*

Proposition 9 *[9] Let u, v be any two vertices of AG_n. Then (1) If $uv \in E(AG_n)$, then $|N(u) \cap N(v)| = 1$. (2) If $uv \notin E(AG_n)$, then $|N(u) \cap N(v)| \leq 2$.*

Proposition 10 *[9] $\kappa(AG_n) = \delta(AG_n) = 2n - 4$ for $n \geq 3$.*

Proposition 11 *[27] For $u \in V(AG_n^r)$, $u^+ \in V(AG_n^i)$, $u^- \in V(AG_n^j)$ and $i \neq j$.*

Theorem 12 *([9]) The 3-extra connectivity of AG_n, $\tilde{\kappa}^{(3)}(AG_n) = 8n - 28$ for $n \geq 5$.*

A connected graph G is super g-extra connected if every minimum g-extra cut F of G isolates one connected subgraph of order $g+1$. If, in addition, $G-F$ has two components, one of which is the connected subgraph of order $g+1$, then G is tightly $|F|$ super g-extra connected.

Theorem 13 *[27] For $n \geq 4$, the alternating group graph AG_n is tightly $(8n-28)$ super 3-extra connected.*

Proposition 14 *For $n \geq 4$, there is no induced subgraph of the alternating group graph AG_n which is a 5-cycle.*

Proof. For an edge $\{v, v(12i)\}$, $AG_n[\{v(1i2), v, v(12i)\}]$ is a triangle. Therefore, every edge is in a triangle. It is easy to verify that there is no induced subgraph of AG_4 which is a 5-cycle. We proceed by induction on n. Assume $n \geq 5$ and the result holds for AG_{n-1}. Since AG_n is a Cayley graph, AG_n is vertex transitive. Therefore, we find a path P from the identity element $(1) = v_1$ in AG_n such that $AG_n[V(P)]$ a path. Without loss of generality, let $v_2 = (123)$. We consider $v_3 = (123)(12n) = (13)(2n)$ or $v_3 = (123)(1n2) = (1n3)$. We consider a possible 5-cycle in AG_n. Then $v_3v_4v_5$ in AG_n^2. Note the cross-edges incident with $v_1 = (1)$ are $\{(12n), v_1\}$ and $\{(1n2), v_1\}$. If $v_3 = (123)(1n2) = (1n3)$, then there is no induced subgraph of the alternating group graph AG_n which is a 5-cycle. Suppose $v_3 = (123)(12n) = (13)(2n)$. Let $v_4 = (123)(12n)(12i)$ and $v_5 = (123)(12n)(12i)(12k)$ and $i, k \neq n$. Then $i \neq k$. In $(123)(12n)(12i)(12k)$, if $i \neq 3$, then $i \to 3$ and $v_5 = (1n2)$. In $(123)(12n)(12i)(12k)$, if $i = 3$, then $k \to n$ and $v_5 = (1n2)$. So, in this case, there is no induced subgraph of AG_n which is a 5-cycle.

By the inductive hypothesis, there is no induced subgraph of AG_n^n which is a 5-cycle. Therefore, there is no induced subgraph of AG_n which is a 5-cycle. □

3 The 3-extra diagnosability of alternating group graphs under the PMC model

In this section, we will give 3-extra diagnosability of alternating group graphs under the PMC model.

Theorem 15 *([29]) A system $G = (V, E)$ is g-extra t-diagnosable under the PMC model if and only if there is an edge $uv \in E$ with $u \in V \setminus (F_1 \cup F_2)$ and $v \in F_1 \triangle F_2$ for each distinct pair of g-extra faulty subsets F_1 and F_2 of V with $|F_1| \leq t$ and $|F_2| \leq t$.*

Theorem 16 *[17] Let $G = (V(G), E(G))$ be a g-extra connected graph, and let $V(G) \neq F_1 \cup F_2$ for each distinct pair of g-extra faulty subsets F_1 and F_2 of G with $|F_1| \leq \tilde{\kappa}^{(g)}(G) + g$ and $|F_2| \leq \tilde{\kappa}^{(g)}(G) + g$. If there is connected subgraph H of G with $|V(H)| = g + 1$ such that $N(V(H))$ is a minimum g-extra cut of G, then the g-extra diagnosability of G is $\tilde{\kappa}^{(g)}(G) + g$ under the PMC model.*

Lemma 17 *Let $A = \{(1), (123), (13)(24), (142)\}$, $n \geq 4$ and let $F_1 = N_{AG_n}(A)$, $F_2 = A \cup N_{AG_n}(A)$. Then $|F_1| = 8n - 28$, $|F_2| = 8n - 24$, F_1 is a 3-extra cut of AG_n, and $AG_n - F_1$ has two components $AG_n - F_2$ and $AG_n[A]$.*

Proof. By $A = \{(1), (123), (13)(24), (142)\}$, we have that $AG_n[A]$ is a 4-cycle. Suppose $n = 4$. Then $N(A) = \{3124, 2431, 1342, 4213\}$ in AG_4 (see Fig. 1). We decompose AG_n into n sub-alternating group graph, $AG_n^1, AG_n^2, \ldots, AG_n^n$, where each AG_n^i has a fixed i in the last position of the label strings which represents the vertices and is isomorphic to AG_{n-1}. Let $F_1 = N_{AG_n}(A)$ and let $F_i^* = F_1 \cap V(AG_n^i)$ for $i \in \{1, 2, \ldots, n\}$. Then $A \subseteq V(AG_n^n)$. Suppose $n = 5$. Then $A \subseteq V(AG_5^5)$ and $\{31245, 24315, 13425, 42135\} \subseteq V(AG_5^5)$. By Proposition 11, $|N(A) \cap (V(AG_5^1) \cup \cdots \cup V(AG_5^4))| = 8$. Thus, $|F_1| = 4 + 8 = 12 = 8 \times 5 - 28$. We prove this lemma (part) by induction on n. The result holds for $n = 5$. Assume that $n \geq 6$ and the result holds for $AG_n^n \cong AG_{n-1}$, i.e., $|F_n^*| = 8(n-1) - 28 = 8n - 36$. Note that $A \subseteq V(AG_n^n)$. By Proposition 11, $|N(A) \cap (V(AG_n^1) \cup \cdots \cup V(AG_n^{n-1}))| = 8$. Thus, $|F_1| = 8n - 36 + 8 = 8n - 28$ and $|F_2| = 8n - 24$.

Note that $AG_4 - F_1$ is two 4-cycles. We prove this lemma (part) by induction on n. The result holds for $n = 4$. Assume $n \geq 5$ and the result holds for $AG_n^n \cong AG_{n-1}$, i.e., F_n^* is a 3-extra cut of AG_n^n, and $AG_n^n - F_n^*$ has two components $AG_n^n - F_2^*$ and $AG_n^n[A]$, where $F_2^* = F_n^* \cup A$. Note that $N(A) \cap (V(AG_n^1) \cup \cdots \cup V(AG_n^{n-1})) = \{(12n), (1n2), (13)(2n), (1n3), (2n4), (1n)(24), (142n3), (1n423)\}$. Therefore, $|N(A) \cap V(AG_n^1)| = \cdots = |N(A) \cap V(AG_n^4)| = 2$ and $|N(A) \cap V(AG_n^i)| = 0$ for $i = 5, \ldots, n-1$. Thus, $|F_1^*| = \cdots = |F_4^*| = 2$ and $|F_5^*| = \cdots = |F_{n-1}^*| = 0$. By Proposition 10, $AG_n^i - F_i^*$ is connected for $i = 1, 2, \ldots, n-1$. By Proposition 8, $AG_n[V(AG_n^1 - F_1^*) \cup V(AG_n^2 - F_2^*) \cup \cdots \cup V(AG_n^{n-1} - F_{n-1}^*)]$ is connected for $n \geq 5$. By inductive hypothesis, $AG_n^n - F_n^*$ has two components $AG_n^n - (F_n^* \cup A)$ and $AG_n^n[A]$. By Proposition 11, $AG_n[V(AG_n^n - F_2^*) \cup V(AG_n^1 - F_1^*) \cup V(AG_n^2 - F_2^*) \cup \cdots \cup V(AG_n^{n-1} - F_{n-1}^*)]$ is connected. Therefore, $AG_n - F_2$ is connected. Note that $|V(AG_n - F_2)| \geq 4$ and $|V(AG_n[A])| = 4$. Therefore, F_1 is a 3-extra cut of AG_n, and $AG_n - F_1$ has two components $AG_n - F_2$ and $AG_n[A]$. □

Corollary 17.1 *Let $n \geq 5$. Then the 3-extra diagnosability of the n-dimensional alternating group graph AG_n under the PMC model is $8n-25$.*

Proof. Let F_1 and F_2 be two distinct 3-extra faulty subsets of AG_n with $|F_1| \leq 8n-25$ and $|F_2| \leq 8n-25$. Assume $V(AG_n) = F_1 \cup F_2$. By the definition of A_n, $|F_1 \cup F_2| = |A_n| = n!/2$. We claim that $n!/2 > 16n-50$ for $n \geq 5$, i.e., $n! > 32n-100$ for $n \geq 5$. When $n = 5$, $n! = 120, 32n-100 = 60$. So $n! > 32n-100$ for $n = 5$. Assume that $n! > 32n-100$ for $n \geq 5$. $(n+1)! = n!(n+1) > (n+1)(32n-100) = n(32n-100)+(32n-68)-32 = [32(n+1)-100]+n(32n-100)-32 = [32(n+1)-100]+4(8n^2-25n-8)$. It is sufficient to show that $8n^2-25n-8 \geq 0$ for $n \geq 5$. Let $y = 8x^2-25x-8$. Then $y = 8x^2-25x-8$ is a quadratic function. When $x \geq 5$, $y = 8x^2-25x-8 \geq 0$. Since $n \geq 5$, we have that $n!/2 = |V(AG_n)| = |F_1 \cup F_2| = |F_1|+|F_2|-|F_1 \cap F_2| \leq |F_1|+|F_2| \leq 2(8n-25) = 16n-50$, a contradiction to $n!/2 > 16n-50$. Therefore, $V(AG_n) \neq F_1 \cup F_2$.

Let A be defined in Lemma 17. By Lemma 17, $|A| = 3+1 = 4$ such that $N(A)$ is a minimum 3-extra cut of AG_n. By Theorem 16, the 3-extra diagnosability of AG_n is $\tilde{\kappa}^{(3)}(AG_n)+3 = 8n-28+3 = 8n-25$ under the PMC model. □

4 The 3-extra diagnosability of alternating group graphs under the MM* model

It is a difficult problem to prove the g-extra diagnosability of a network. Before discussing the 3-extra diagnosability of the n-dimensional alternating group graph AG_n under the MM* model, we first give a theorem.

Theorem 18 *([2, 29]) A system $G = (V, E)$ is g-extra t-diagnosable under the MM^* model if and only if for each distinct pair of g-extra faulty subsets F_1 and F_2 of V with $|F_1| \leq t$ and $|F_2| \leq t$ satisfies one of the following conditions. (1) There are two vertices $u, w \in V \setminus (F_1 \cup F_2)$ and there is a vertex $v \in F_1 \triangle F_2$ such that $uw \in E$ and $vw \in E$. (2) There are two vertices $u, v \in F_1 \setminus F_2$ and there is a vertex $w \in V \setminus (F_1 \cup F_2)$ such that $uw \in E$ and $vw \in E$. (3) There are two vertices $u, v \in F_2 \setminus F_1$ and there is a vertex $w \in V \setminus (F_1 \cup F_2)$ such that $uw \in E$ and $vw \in E$.*

Lemma 19 *Let $n \geq 5$. Then the 3-extra diagnosability of the n-dimensional alternating group graph AG_n under the MM* model is less than or equal to $8n-25$, i.e., $\tilde{t}_3(AG_n) \leq 8n-25$.*

Proof. Let $A = \{(1), (123), (13)(24), (142)\}$, and let $F_1 = N_{AG_n}(A)$, $F_2 = A \cup N_{AG_n}(A)$. By Lemma 17, $|F_1| = 8n-28$, $|F_2| = 8n-24$, F_1 is a 3-extra cut of AG_n, and $AG_n - F_1$ has two components $AG_n - F_2$ and $AG_n[A]$. Therefore, F_1 and F_2 are both 3-extra faulty sets of AG_n with $|F_1| = 8n-28$ and $|F_2| = 8n-24$. By the definitions of F_1 and F_2, $F_1 \triangle F_2 = A$. Note $F_1 \setminus F_2 = \emptyset$, $F_2 \setminus F_1 = A$ and $(V(AG_n) \setminus (F_1 \cup F_2)) \cap A = \emptyset$. Therefore, both F_1 and F_2 are not satisfied with any one condition in Theorem 18, and AG_n is not 3-extra $(8n-24)$-diagnosable under MM* model. Hence, $\tilde{t}_3(AG_n) \leq 8n-25$. Thus, the proof is complete. □

A component of a graph G is odd according as it has an odd number of vertices. We denote by $o(G)$ the number of add components of G.

Lemma 20 *([1] Tutte's Theorem) A graph $G = (V, E)$ has a perfect matching if and only if $o(G-S) \leq |S|$ for all $S \subseteq V$.*

Lemma 21 *Let $n \geq 7$. Then the 3-extra diagnosability of the n-dimensional alternating group graph AG_n under the MM* model is more than or equal to $8n-25$, i.e., $\tilde{t}_3(AG_n) \geq 8n-25$.*

Proof. By the definition of 3-extra diagnosability, it is sufficient to show that AG_n is 3-extra $(8n-25)$-diagnosable.

Suppose, on the contrary, that there are two distinct 3-extra faulty subsets F_1 and F_2 of AG_n with $|F_1| \leq 8n-25$ and $|F_2| \leq 8n-25$, but the vertex set pair (F_1, F_2) is not satisfied with any one condition in Theorem 18. Without loss of generality, suppose that $F_2 \setminus F_1 \neq \emptyset$. Assume $V(AG_n) = F_1 \cup F_2$. By the definition of A_n, $|F_1 \cup F_2| = |A_n| = n!/2$. Similarly to the discussion on $V(AG_n) \neq F_1 \cup F_2$ in Corollary 17.1, we can deduce $V(AG_n) \neq F_1 \cup F_2$.

Claim 1. $AG_n - F_1 - F_2$ has no isolated vertex.

Suppose, on the contrary, that $AG_n - F_1 - F_2$ has at least one isolated vertex w_1. Since F_1 is one 3-extra faulty set, there is a vertex $u \in F_2 \setminus F_1$ such that u is adjacent to w_1. Meanwhile, since the vertex set pair (F_1, F_2) is not satisfied with any one condition in Theorem 18, by the condition (3) of Theorem 18, there is at most one vertex $u \in F_2 \setminus F_1$ such that u is adjacent to w_1. Thus, there is just a vertex $u \in F_2 \setminus F_1$ such that u is adjacent to w_1. If $F_1 \setminus F_2 = \emptyset$, then $F_1 \subseteq F_2$. Since F_2 is a 3-extra faulty set, every component G_i of $AG_n - F_1 - F_2 = AG_n - F_2$ has $|V(G_i)| \geq 4$. Therefore, $AG_n - F_1 - F_2$ has no isolated vertex. Thus, $F_1 \setminus F_2 \neq \emptyset$. Similarly, we can deduce that there is just a vertex $a \in F_1 \setminus F_2$ such that a is adjacent to w_1. Let $W \subseteq A_n \setminus (F_1 \cup F_2)$ be the set of isolated vertices in $AG_n[A_n \setminus (F_1 \cup F_2)]$, and let H be the induced subgraph by the vertex set $A_n \setminus (F_1 \cup F_2 \cup W)$. Then for any $w \in W$, there are $(2n-6)$ neighbors in $F_1 \cap F_2$. By Lemmas 20, $|W| \leq o(AG_n - (F_1 \cup F_2)) \leq |F_1 \cup F_2| = |F_1| + |F_2| - |F_1 \cap F_2| \leq 2(8n-25) - (2n-6) = 14n - 44$. Since $n \geq 5$, $n!/4 > 14n - 44$ holds. Therefore, $|W| < n!/4$. Assume $V(H) = \emptyset$. Then $n!/2 = |V(AG_n)| = |F_1 \cup F_2| + |W| = |F_1| + |F_2| - |F_1 \cap F_2| + |W| \leq 2(8n-25) - (2n-6) + |W| = 14n - 44 + |W| < n!/4 + 14n - 44$ and hence $n!/4 < 14n - 44$, a contradiction to that $n \geq 5$. So $V(H) \neq \emptyset$.

Since the vertex set pair (F_1, F_2) is not satisfied with the condition (1) of Theorem 18, and any vertex of $V(H)$ is not isolated in H, we induce that there is no edge between $V(H)$ and $F_1 \triangle F_2$. Note $F_2 \setminus F_1 \neq \emptyset$. Since $|F_1 \cap F_2 \geq 2n-6$, then $F_1 \cap F_2 \neq \emptyset$. Thus, $F_1 \cap F_2$ is a cut of AG_n. Since F_1 is a 3-extra faulty set of AG_n, we have that every component H_i of H has $|V(H_i)| \geq 4$ and every component B_i of $AG_n[W \cup (F_2 \setminus F_1)]$ has $|V(B_i)| \geq 4$. Since F_2 is a 3-extra faulty set of AG_n, we have that every component B_i' of $AG_n[W \cup (F_1 \setminus F_2)]$ has $|V(B_i')| \geq 4$. Note that $AG_n - (F_1 \cap F_2)$ has two parts (for convenience): H and $AG_n[W \cup (F_1 \setminus F_2) \cup (F_2 \setminus F_1)]$. Let $\mathcal{B}_i$ be a component of $AG_n[W \cup (F_1 \setminus F_2) \cup (F_2 \setminus F_1)]$ and let $b_i \in V(\mathcal{B}_i)$. If $b_i \in W$, then there is a component B_i of $AG_n[(F_2 \setminus F_1) \cup W]$ ($|V(B_i)| \geq 4$) and a component B_i' of $AG_n[(F_1 \setminus F_2) \cup W]$ ($|V(B_i')| \geq 4$) such that $b_i \in V(B_i)$ and $b_i \in V(B_i')$. It follows that $B_i \cup B_i'$ is connected in $AG_n[W \cup (F_1 \setminus F_2) \cup (F_2 \setminus F_1)]$ and $b_i \in V(B_i \cup B_i')$. Since connection is an equivalence relation on the vertex set $W \cup (F_1 \setminus F_2) \cup (F_2 \setminus F_1)$, $\mathcal{B}_i = B_i \cup B_i'$ holds. Therefore, $|V(\mathcal{B}_i)| \geq 4$. If $b_i \in (F_2 \setminus F_1)$, then there is a component G_i of $AG_n([F_2 \setminus F_1])$ ($|V(G_i)| \geq 4$) such that $b_i \in V(G_i)$. It follows that G_i is connected in $AG_n[W \cup (F_1 \setminus F_2) \cup (F_2 \setminus F_1)]$ and $b_i \in V(G_i)$. Since connection is an equivalence relation on the vertex set $W \cup (F_1 \setminus F_2) \cup (F_2 \setminus F_1)$, we have that G_i is a subgraph of $\mathcal{B}_i$. Therefore, $|V(\mathcal{B}_i)| \geq 4$. Similarly, if $b_i \in (F_1 \setminus F_2)$, then $|V(\mathcal{B}_i)| \geq 4$. Therefore, $F_1 \cap F_2$ is a 3-extra cut of AG_n. By Theorem 12, $|F_1 \cap F_2| \geq 8n - 28$.

Since $|F_1| \leq 8n-25$, $|F_2| \leq 8n-25$ and $|F_1 \cap F_2| \geq 8n-28$, we have $|F_2 \setminus F_1| = |F_2| - |F_1 \cap F_2| \leq 8n - 25 - (8n-28) = 3$ and $|F_1 \setminus F_2| \leq 3$.

Let $|F_2 \setminus F_1| = 3$. Then $|F_1 \cap F_2| = 8n - 28 = \tilde{\kappa}^{(3)}(AG_n)$ and $F_1 \cap F_2$ is a minimum 3-extra cut of AG_n. Recall $|W| \leq 14n - 44$. Therefore, $5 \leq |F_2 \setminus F_1| + |F_1 \setminus F_2| + |W| \leq 14n - 38$. By Theorem 13, AG_n is tightly $(8n-28)$ super 3-extra connected, i.e., $AG_n - (F_1 \cap F_2)$ has two components, one of which is a subgraph of order 4. Therefore, we have that $|V(AG_n - F_1 - F_2 - W)| = 4$. Thus, $n!/2 = |V(AG_n)| = |V(AG_n - F_1 - F_2 - W)| + |F_2 \setminus F_1| + |F_1 \setminus F_2| + |W| + |F_2 \cap F_1| \leq 4 + 14n - 38 + 8n - 28 = 22n - 62$, a contradiction to $n \geq 5$.

Let $|F_2 \setminus F_1| = 2$. Then $|F_1 \cap F_2| = 8n - 28 = \tilde{\kappa}^{(3)}(AG_n)$ or $|F_1 \cap F_2| = 8n - 27$. Suppose $|F_1 \cap F_2| = 8n - 28 = \tilde{\kappa}^{(3)}(AG_n)$. Similarly to above, we

Since $|F_1 \cap F_2| = 8n-26$ and $|F_1| \le 8n-25$, $|F_1 \setminus F_2| = 1$. Let $x \in F_2 \setminus F_1$. Since F_1 is a 3-extra faulty set of AG_n, we have that every component B_i of $AG_n[W \cup (F_2 \setminus F_1)])$ has $|V(B_i)| \ge 4$. Since $x \in F_2 \setminus F_1$, $AG_n[W \cup (F_2 \setminus F_1)])$ has a component B_1. Let $x \in V(B_1)$. Since $|V(B_1)| \ge 4$, $|W| \ge 3$ and there are $a, b, c \in W$ such that $xa, xb, xc \in E(AG_n)$. Let $y \in F_1 \setminus F_2$. Since F_2 is a 3-extra faulty set of AG_n, $ya, yb, yc \in E(AG_n)$, a contradiction to Proposition 9.

Case 3. $|F_1 \cap F_2| = 8n - 27$.

Let $x \in F_2 \setminus F_1$. Since F_1 is a 3-extra faulty set of AG_n, we have that every component B_i of $AG_n[W \cup (F_2 \setminus F_1)])$ has $|V(B_i)| \ge 4$. Since $x \in F_2 \setminus F_1$, $AG_n[W \cup (F_2 \setminus F_1)])$ has a component B_1. Let $x \in V(B_1)$. Since $|V(B_1)| \ge 4$, $|W| \ge 3$ and there are $a, b, c \in W$ such that $xa, xb, xc \in E(AG_n)$. Since $|F_1 \cap F_2| = 8n - 27$ and $|F_2| \le 8n - 25$, $|F_1 \setminus F_2| \le 2$. Suppose $|F_1 \setminus F_2| = 1$ and $y \in F_1 \setminus F_2$. Since F_2 is a 3-extra faulty set of AG_n, $ya, yb, yc \in E(AG_n)$, a contradiction to Proposition 9. Suppose $|F_1 \setminus F_2| = 2$ and $y, z \in F_1 \setminus F_2$. Let $yz \notin E(AG_n)$. Since there is just a vertex of $F_1 \setminus F_2$ such that it is adjacent to a vertex of W, this is a contradiction to Proposition 9. Let $yz \in E(AG_n)$. If $ya, yb, yc \in E(AG_n)$ or $za, zb, zc \in E(AG_n)$, then this case is the same to $|F_1 \setminus F_2| = 1$. If $ya, zb \in E(AG_n)$, then this case is a contradiction to Proposition 14. The proof of Claim 1 is complete.

Let $u \in V(AG_n) \setminus (F_1 \cup F_2)$. By Claim 1, u has at least one neighbor vertex in $AG_n - F_1 - F_2$. Since the vertex set pair (F_1, F_2) is not satisfied with any one condition in Theorem 18, by the condition (1) of Theorem 18, for any pair of adjacent vertices $u, w \in V(AG_n) \setminus (F_1 \cup F_2)$, there is no vertex $v \in F_1 \triangle F_2$ such that $uw \in E(AG_n)$ and $vw \in E(AG_n)$. It follows that u has no neighbor in $F_1 \triangle F_2$. By the arbitrariness of u, there is no edge between $V(AG_n) \setminus (F_1 \cup F_2)$ and $F_1 \triangle F_2$. If $F_1 \cap F_2 = \emptyset$, then this is a contradiction to that AG_n is connected. Therefore, $F_1 \cap F_2 \neq \emptyset$ and $F_1 \cap F_2$ is a cut of AG_n. Since $F_2 \setminus F_1 \neq \emptyset$ and F_1 is a 3-extra faulty set, we have that every component H_i of $AG_n - F_1 - F_2$ has $|V(H_i)| \ge 4$ and every component G_i of $AG_n([F_2 \setminus F_1])$ has $|V(G_i)| \ge 4$. Suppose that $F_1 \setminus F_2 = \emptyset$. Then $F_1 \cap F_2 = F_1$. Since F_1 is a 3-extra faulty set of AG_n, we have that $F_1 \cap F_2 = F_1$ is a 3-extra faulty set of AG_n. Since there is no edge between $V(AG_n) \setminus (F_1 \cup F_2)$ and $F_2 \setminus F_1$, we have that $F_1 \cap F_2 = F_1$ is a 3-extra cut of AG_n. Suppose that $F_1 \setminus F_2 \neq \emptyset$. Similarly, every component B_i of $AG_n([F_1 \setminus F_2])$ has $|V(B_i)| \ge 4$. Note that $AG_n - (F_1 \cap F_2)$ has three parts (for convenience): H, $AG_n[F_1 \setminus F_2]$ and $AG_n[F_2 \setminus F_1]$. Therefore, $F_1 \cap F_2$ is a 3-extra cut of AG_n. By Theorem 12, we have $|F_1 \cap F_2| \ge 8n - 28$. Therefore, $|F_2| = |F_2 \setminus F_1| + |F_1 \cap F_2| \ge 4 + (8n - 28) = 8n - 24$, which contradicts $|F_2| \le 8n - 25$. Therefore, AG_n is 3-extra $(8n-25)$-diagnosable have that $n!/2 = |V(AG_n)| = |V(AG_n - F_1 - F_2 - W)| + |F_2 \setminus F_1| + |F_1 \setminus F_2| + |W| + |F_2 \cap F_1| \leq 4 + 14n - 38 + 8n - 28 = 22n - 62$, a contradiction to $n \geq 5$. Suppose $|F_1 \cap F_2| = 8n - 27$.

Let $x, y \in F_2 \setminus F_1$. Suppose $xy \notin E(AG_n)$. Since F_1 is a 3-extra faulty set of AG_n, we have that every component B_i of $AG_n[W \cup (F_2 \setminus F_1)])$ has $|V(B_i)| \geq 4$. Since $xy \notin E(AG_n)$, $AG_n[W \cup (F_2 \setminus F_1)])$ has two components B_1 and B_2. Let $x \in V(B_1)$. Since $|V(B_1)| \geq 4$, $|W| \geq 3$ and there are $a, b, c \in W$ such that $xa, xb, xc \in E(AG_n)$. The same reason, there are $d, e, f \in W$ such that $yd, ye, yf \in E(AG_n)$. Since there is just a vertex of $F_2 \setminus F_1$ such that it is adjacent to a vertex of W, $|W| \geq 6$. If $|F_1 \setminus F_2| = 3$, then, by the above proof, there is a contradiction. If $|F_1 \setminus F_2| = 1$, then there is a contradiction to Proposition 9. When $F_1 \setminus F_2 = \{u, v\}$ and $uv \in E(AG_n)$, we will discuss it below. Therefore, suppose $uv \notin E(AG_n)$. Note $N(W) \setminus ((F_2 \setminus F_1) \cup (F_1 \setminus F_2)) \subseteq F_1 \cap F_2$ and $|N(W) \setminus ((F_2 \setminus F_1) \cup (F_1 \setminus F_2))| \geq 6(2n - 6) - 18 = 12n - 54 > 8n - 27$ when $n \geq 7$, a contradiction to $|F_1 \cap F_2| = 8n - 27$.

Suppose $xy \in E(AG_n)$. Let $AG_n - F_1 - F_2$ have one isolated vertex. Since there is just a vertex of $F_2 \setminus F_1$ such that it is adjacent to a vertex of W, $|W \cup (F_2 \setminus F_1)| = 3$, a contradiction to that F_1 is a 3-extra faulty set of AG_n. Therefore, $|W| \geq 2$. Without loss of generality, let $|W| = 2$ and $a, b \in W$. Since F_1 is a 3-extra faulty set of AG_n, $xa, xb \in E(AG_n)$ or $xa, yb \in E(AG_n)$. Since $|F_1 \cap F_2| = 8n - 27$ and $|F_2| \leq 8n - 25$, $|F_1 \setminus F_2| \leq 2$. Since F_2 is a 3-extra faulty set of AG_n, $|F_1 \setminus F_2| = 2$. Let $c, d \in F_1 \setminus F_2$. Since F_2 is a 3-extra faulty set of AG_n and there is just a vertex of $F_1 \setminus F_2$ such that it is adjacent to a vertex of W, we have $cd \in E(AG_n)$. Therefore, $ac, bc \in E(AG_n)$ or $ac, bd \in E(AG_n)$. If $xa, xb \in E(AG_n)$ and $ac, bc \in E(AG_n)$, then $|F_2 \cap F_1| \geq 2(2n - 6) + 2(2n - 7) + 2(2n - 5) - 12 = 12n - 48 > 8n - 27 = |F_1 \cap F_2|$ when $n \geq 7$, a contradiction. If $xa, yb \in E(AG_n)$ and $ac, bc \in E(AG_n)$, then this case is a contradiction to Proposition 14. Suppose $xa, yb \in E(AG_n)$ and $ac, bd \in E(AG_n)$. If $e \in N(a) \cap N(b)$, then $aebyxa$ is a 5-cycle, a contradiction to Proposition 14. Therefore, $N(a) \cap N(b) = \emptyset$. The same reason, $N(x) \cap N(d) = \emptyset$ and $N(y) \cap N(c) = \emptyset$. Therefore, $|F_2 \cap F_1| \geq 6(2n - 6) - 12 = 12n - 48 > 8n - 27 = |F_1 \cap F_2|$ when $n \geq 6$, a contradiction.

Let $|F_2 \setminus F_1| = 1$. Then $|F_1 \cap F_2| = 8n - 28 = \tilde{\kappa}^{(3)}(AG_n)$ or $|F_1 \cap F_2| = 8n - 27$ or $|F_1 \cap F_2| = 8n - 26$. We consider the following cases.

Case 1. $|F_1 \cap F_2| = 8n - 28$.

Similarly to above, there is a a contradiction.

Case 2. $|F_1 \cap F_2| = 8n - 26$.

and $\tilde{t}_3(AG_n) \geq 8n - 25$. The proof is complete. □

Combining Lemmas 19 and 21, we have the following theorem.

Theorem 22 *Let $n \geq 7$. Then the 3-extra diagnosability of the the n-dimensional alternating group graph AG_n under the MM* model is $8n-25$.*

5 Conclusions

The conditional diagnosability are one important metrics for fault tolerance of a multiprocessor system. In this paper, we investigate the problems of the 3-extra diagnosability of alternating group graphs. It is proved that the 3-extra diagnosability of the n-dimensional alternating group graph AG_n under the PMC model and MM* model is $8n-25$, where $n \geq 7$. The above results show that the 3-extra diagnosability is several times larger than the classical diagnosability of AG_n depending on the condition: 3-extra. The work will help engineers to develop more different measures of 3-extra diagnosability based on application under environment, network topology, network reliability, and statistics related to fault patterns.

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