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On Symmetrical Convex Polytopes and their Edge Resolvability

Sunny Kumar Sharma¹ and Vijay Kumar Bhat^{2,*}

¹ Department of Mathematics, Manipal Institute of Technology Bengaluru, Manipal Academy of Higher Education, Manipal, Karnataka, India

² School of Mathematics, Faculty of Sciences, Shri Mata Vaishno Devi University, Katra-182320, J& K, India

* **Correspondence:** vijaykumarbhat2000@yahoo.com

Abstract: The *metric dimension* of a graph $\Gamma = (V, E)$, denoted by $dim(\Gamma)$, is the least cardinality of a set of vertices in Γ such that each vertex in Γ is determined uniquely by its vector of distance to the vertices of the chosen set. The topological distance between an edge $\varepsilon = yz \in E$ and a vertex $k \in V$ is defined as $d(\varepsilon, k) = \min\{d(z, k), d(y, k)\}$. A subset of vertices R_Γ in V is called an edge resolving set for Γ if for each pair of different edges e_1 and e_2 in E , there is a vertex $j \in R_\Gamma$ implying $d(e_1, j) \neq d(e_2, j)$. An edge resolving set with minimum cardinality is called the edge metric basis for Γ and this cardinality is the edge metric dimension of Γ , denoted by $dim_E(\Gamma)$. In this article, we show that the cardinality of minimum edge resolving set is three or four, for two classes of convex polytopes (S_n and T_n , exist in the literature).

Keywords: Planar graph, Metric dimension, Convex polytope, Edge metric dimension, Resolving set

1. Introduction

Suppose $\Gamma = \Gamma(V, E)$ be an undirected, connected, simple, and non-trivial graph with order $|V(\Gamma)|$ and size $|E(\Gamma)|$. For each pair of distinct vertices $j, k \in V$, the distance between them, denoted by $d(j, k)$, is the number of edges in the shortest $j - k$ path. A vertex $p \in V(\Gamma)$ resolves (or recognize) a pair of distinct vertices j, k if $d(j, p) \neq d(k, p)$. A subset of vertices $R \subseteq V(\Gamma)$ recognizes Γ if every possible pair of different vertices in Γ are resolved by some vertex in R , then R is known as the resolving set (or metric generator) for Γ . For an ordered set $R = \{h_1, h_2, h_3, \dots, h_y\} \subseteq V$ of different vertices, the metric code (or co-ordinate) of $v \in V$ with respect to R is the y -vector $\gamma(v|R) = (d(v, h_1), d(v, h_2), d(v, h_3), \dots, d(v, h_y))$. The *metric dimension* of Γ is denoted by $dim(\Gamma)$, where $dim(\Gamma) = \min\{|R| : R \text{ is resolving in } \Gamma\}$.

The notion of resolving set for a connected simple graph was proposed by Slater [1], under the name *locating set*; he referred to a minimum resolving set as a *reference set*, and the minimum cardinality of such a generator for a given connected graph as the location number of a graph. These concepts were independently studied by Harary and Melter [2] under the name *metric dimension*. It has been extensively investigated in a number of research articles, see Sebo and Tannier [3], Sharma and Bhat [4, 5], Khuller et al. [6], Chartrand et al. [7], Imran

et al. [8], Javid et al. [9], Wei et al. [10], etc. It has appeared in several areas including robot navigation [6], pattern recognition and image processing [11], combinatorial optimization [3], and chemical science [12, 13].

An edge metric dimension is a newly introduced (by Kelenc et al. [14]) variant of the metric dimension. Then it was further investigated by Peterin and Yero [15], Zhu et al. [16], Zubrilina [17], etc. The distance between a vertex $k \in V$ and an edge $e = yz \in E$ is given by $d(e, k) = \min\{d(z, k), d(y, k)\}$. For an ordered set, $R_\Gamma = \{r_1, r_2, r_3, \dots, r_m\}$ of distinct vertices in Γ , the edge metric code (or edge code) of an edge $e_1 \in E$ with respect to R_Γ is the m -vector $\gamma_E(e_1|R) = (d(e_1, r_1), d(e_1, r_2), d(e_1, r_3), \dots, d(e_1, r_m))$. Then, R_Γ is called the edge resolving set (ERS) for Γ iff for every pair of distinct edges e_1, e_2 of Γ , we have $\gamma_E(e_1|R) \neq \gamma_E(e_2|R)$. This R_Γ with minimum number vertices is called an edge metric basis of Γ . The edge metric dimension (EMD) of Γ is denoted by $\dim_E(\Gamma)$, where $\dim_E(\Gamma) = \min\{|R_\Gamma| : R_\Gamma \text{ is an EMG of } \Gamma\}$.

In [14], Kelenc et al. initiated and introduced the study of the EMD in a non-trivial connected graphs with respect to identifying uniquely the edges of the graph. They have given some comparison between the EMD and the metric dimension. They particularly stated that it is possible to find graphs where the EMD equals the metric dimension, as well as other graphs Γ where $\dim_E(\Gamma) < \dim(\Gamma)$ or $\dim_E(\Gamma) > \dim(\Gamma)$. In this paper, we consider two convex polytopes graph families, namely, S_n and T_n , already exists in the literature [8], and analyze such situations further by comparing the value of $\dim_E(\Gamma)$ and $\dim(\Gamma)$, where Γ represents one of the convex polytopes graph S_n and T_n .

The main contribution of the paper are as follows:

- The cardinality of minimum edge resolving set for S_n and T_n is three or four.
- Edge metric dimension (S_n and T_n) \geq Metric dimension (S_n and T_n).
- The edge resolving set for S_n and T_n are independent.

These notions of metric dimension as well as its variants have been investigated for distinctively significant graph families, for instance; planar graphs: path graph, kayak paddle graph, several ladder graphs, (ladders of pentagons, heptags, nonagons, etc), antiprism graph, mobious ladder graph, wheel graph, cycle graph, various convex polytopes, tadpole graph, and many more; chemical graphs: one-pentagonal, one heptagonal, one nonagonal carbon nanocone structures, linear heptagons structures, polycyclic aromatic compound, and linear phenylene structure, for these one can refer [17–22]. The list is long but is still incomplete, i.e., there are infinite number of distinct graph families for which the notions of resolving sets and its variants have not been discussed yet. So, to address this partially, in this paper, we consider two interesting planar graph families of convex polytopes, denoted by S_n & T_n , and determine their edge metric basis as well as edge metric dimension.

The rest of this paper is organized in the following manner: Section 2 introduces some basic concepts related to the resolving and the edge resolving sets of graphs. In addition, some of the established results regarding the metric dimension of S_n and T_n are also discussed. In sections 3 and 4, we investigate the edge metric dimension of the convex polytope graphs S_n and T_n , respectively. Finally, Sect. 5 discusses the conclusion and comparison of the metric dimension and the EMD of these two graphs.

2. Preliminaries

In the present section, we define some necessary terminology and give some preliminary results about the metric dimension and the EMD.

Definition 1 (Independent resolving set (IRS)). [23] *A subset R of distinct vertices in Γ , is said to be an IRS for Γ , if R is both resolving and independent set.*

Definition 2 (Independent edge resolving set (IERS)). [4] *A subset R_E of distinct vertices in Γ , is said to be an IERS for Γ , if R_E is both edge resolving and independent set.*

Following are the some significant known results concerning the metric dimension and the EMD:

Proposition 1. [14] *For every natural $n \geq 3$,*

- 1) *Graph P_n is a path iff $1 = \dim(P_n) = \dim_E(P_n)$.*
- 2) *$2 = \dim(C_n) = \dim_E(C_n)$, where C_n is the cycle graph of order n .*
- 3) *$n - 1 = \dim(K_n) = \dim_E(K_n)$, where K_n is the complete graph of order n .*

Next, suppose \mathbb{Y} be a family of simple non-trivial graphs $\Gamma_n : \mathbb{Y} = (\Gamma_n)_{n \geq 1}$ relying on n as follows: the order $\varphi(n) = |V(\Gamma)|$ and $\lim_{n \rightarrow \infty} \varphi(n) = \infty$. If there is a constant $U > 0$ such that $\dim_E(\Gamma_n) > U$ for every $n \geq 1$ then we say that \mathbb{Y} is the family of graphs with unbounded EMD, otherwise \mathbb{Y} has a bounded EMD. If all the graphs in \mathbb{Y} have the equal EMD, then \mathbb{Y} is known as the constant EMD graphs family. In recent years, several authors have investigated the bounded and unbounded EMD for various graph families [24]. The maximum (minimum) degree in Γ is denoted by $\Delta(\Gamma)$ ($\delta(\Gamma)$). Now, for a connected graph Γ , the lower bounds with respect to $\Delta(\Gamma)$ and $\delta(\Gamma)$ for the EMD are given in the following propositions.

Proposition 2. [14] *If $\Delta(\Gamma)$ is the maximum degree in Γ , then $\lceil \log_2 \Delta(\Gamma) \rceil \leq \dim_E(\Gamma)$.*

Proposition 3. [25] *If $\delta(\Gamma)$ is the minimum degree in Γ , then $\lceil \log_2 \delta(\Gamma) \rceil + 1 \leq \dim_E(\Gamma)$.*

In this work, we consider two graphs of convex polytopes S_n & T_n [8], and we obtain their EMD. Recently, the metric dimension of these two convex polytope graphs was computed. For the metric dimension of these two graphs we have the following results:

Theorem 1. [8] *$\dim(S_n) = 3$, where $n \geq 6$ is a positive integer.*

Theorem 2. [8] *$\dim(T_n) = 3$, where $n \geq 6$ is a positive integer.*

In the next section, we consider a family of convex polytope graph S_n for which we have $E(S_n) = \{j_{\bar{q}}j_{\bar{q}+1}, j_{\bar{q}}k_{\bar{q}}, k_{\bar{q}}j_{\bar{q}+1}, k_{\bar{q}}l_{\bar{q}}, l_{\bar{q}}m_{\bar{q}}, m_{\bar{q}}l_{\bar{q}+1}, m_{\bar{q}}o_{\bar{q}}, o_{\bar{q}}o_{\bar{q}+1} : 1 \leq \bar{q} \leq n\}$ (see Figure 1). We denote the sets of edge metric co-ordinates for the edges of S_n by $A_1, A_2, A_3, A_4, A_5, A_6, A_7$, and A_8 , where $A_1 = \{\gamma_E(j_{\bar{q}}j_{\bar{q}+1}|R_E) : 1 \leq \bar{q} \leq n\}$, $A_2 = \{\gamma_E(j_{\bar{q}}k_{\bar{q}}|R_E) : 1 \leq \bar{q} \leq n\}$, $A_3 = \{\gamma_E(k_{\bar{q}}j_{\bar{q}+1}|R_E) : 1 \leq \bar{q} \leq n\}$, $A_4 = \{\gamma_E(k_{\bar{q}}l_{\bar{q}}|R_E) : 1 \leq \bar{q} \leq n\}$, $A_5 = \{\gamma_E(l_{\bar{q}}m_{\bar{q}}|R_E) : 1 \leq \bar{q} \leq n\}$, $A_6 = \{\gamma_E(m_{\bar{q}}l_{\bar{q}+1}|R_E) : 1 \leq \bar{q} \leq n\}$, $A_7 = \{\gamma_E(m_{\bar{q}}o_{\bar{q}}|R_E) : 1 \leq \bar{q} \leq n\}$, and $A_8 = \{\gamma_E(o_{\bar{q}}o_{\bar{q}+1}|R_E) : 1 \leq \bar{q} \leq n\}$.

3. Edge Metric Dimension of S_n

In this section, we study some of the basic properties and the EMD of S_n .

The Graph S_n

The convex polytope S_n [8] consists of $5n$ and $8n$ number of vertices and edges respectively (see Figure 1). It has n cycles with 3-sides, n cycles with 6-sides, n cycles with 5-sides, and two cycles with n -sides. The set of edges and vertices of S_n are denoted separately by $E(S_n)$ and $V(S_n)$, where $E(S_n) = \{j_{\bar{q}}j_{\bar{q}+1}, j_{\bar{q}}k_{\bar{q}}, k_{\bar{q}}j_{\bar{q}+1}, k_{\bar{q}}l_{\bar{q}}, l_{\bar{q}}m_{\bar{q}}, m_{\bar{q}}l_{\bar{q}+1}, m_{\bar{q}}o_{\bar{q}}, o_{\bar{q}}o_{\bar{q}+1} : 1 \leq \bar{q} \leq n\}$ and $V(S_n) = \{j_{\bar{q}}, k_{\bar{q}}, l_{\bar{q}}, m_{\bar{q}}, o_{\bar{q}} : 1 \leq \bar{q} \leq n\}$.

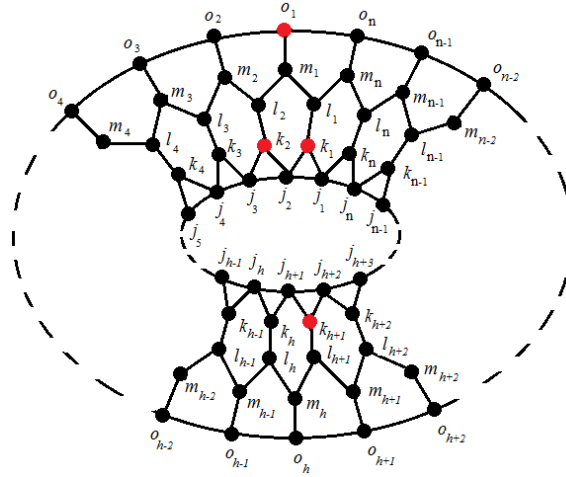


Figure 1. The Graph S_n

We name the vertices $\{j_{\bar{q}} : 1 \leq \bar{q} \leq n\}$ in the planar graph, S_n as the j -cycle vertices, the vertices $\{k_{\bar{q}} : 1 \leq \bar{q} \leq n\}$ in the planar graph, S_n as the k -vertices, the vertices $\{l_{\bar{q}} : 1 \leq \bar{q} \leq n\}$ in the planar graph, S_n as the l -vertices, the vertices $\{m_{\bar{q}} : 1 \leq \bar{q} \leq n\}$ in the planar graph, S_n as the m -vertices, and the vertices $\{o_{\bar{q}} : 1 \leq \bar{q} \leq n\}$ in the planar graph, S_n as the o -cycle vertices. For our purpose, we can write $j_1 = j_{n+1}$, $k_1 = k_{n+1}$, $l_1 = l_{n+1}$, $m_1 = m_{n+1}$, and $o_1 = o_{n+1}$. In the following result, we investigate the EMD of S_n .

Theorem 3. $dim_E(S_n) = 4$, where $n \geq 6$ is a positive integer.

Proof. It is easy to check for $6 \leq n \leq 11$ that the EMD of S_n is 4. The position of the edge basis vertices (color in red) can be found in Figure 1, for all $n \geq 6$. Now, for $n \geq 12$, we divide our proof into two cases i.e., when n is even ($n \equiv 0 \pmod{2}$) and when it is odd ($n \equiv 1 \pmod{2}$).

Case (slowromancapi@) $n \equiv 0 \pmod{2}$

Then, for the natural n , we have $n = 2h$, where $h \in \mathbb{N}$ and $h \geq 6$. Suppose $R_E = \{k_1, k_2, k_{h+1}, o_1\} \subset V(S_n)$. Next, we give edge metric representation to every edge of S_n concerning the set R_E .

For the edges of j -cycle $\{e_{\bar{q}} = j_{\bar{q}}j_{\bar{q}+1} | \bar{q} = 1, 2, 3, \dots, n\}$, the edge codes are

$\gamma_E(e_{\bar{q}} = j_{\bar{q}}j_{\bar{q}+1} R_E)$	$R_E = \{k_1, k_2, k_{h+1}, o_1\}$
$\gamma_E(e_{\bar{q}} R_E) : (\bar{q} = 1)$	$(1, 1, h - \bar{q} + 1, 4)$
$\gamma_E(e_{\bar{q}} R_E) : (\bar{q} = 2)$	$(\bar{q} - 1, 1, h - \bar{q} + 1, 4)$
$\gamma_E(e_{\bar{q}} R_E) : (3 \leq \bar{q} \leq h)$	$(\bar{q} - 1, \bar{q} - 2, h - \bar{q} + 1, \bar{q} + 1)$
$\gamma_E(e_{\bar{q}} R_E) : (\bar{q} = h + 1)$	$(\bar{q} - 1, \bar{q} - 2, 1, \bar{q} + 1)$
$\gamma_E(e_{\bar{q}} R_E) : (h + 2 \leq \bar{q} \leq 2h)$	$(2h - \bar{q} + 1, 2h - \bar{q} + 2, \bar{q} - h - 1, 2h - \bar{q} + 4)$

For the edges $\{e_{\bar{q}} = j_{\bar{q}}k_{\bar{q}}, k_{\bar{q}}j_{\bar{q}+1}, k_{\bar{q}}l_{\bar{q}}, l_{\bar{q}}m_{\bar{q}}, m_{\bar{q}}l_{\bar{q}+1}, m_{\bar{q}}o_{\bar{q}} | \bar{q} = 1, 2, 3, \dots, n\}$, the edge codes are

$\gamma_E(e_{\bar{q}} = j_{\bar{q}}k_{\bar{q}} R_E)$	$R_E = \{k_1, k_2, k_{h+1}, o_1\}$
$\gamma_E(e_{\bar{q}} R_E) : (\bar{q} = 1)$	$(0, 2, h, 3)$
$\gamma_E(e_{\bar{q}} R_E) : (\bar{q} = 2)$	$(\bar{q} - 1, 0, h - \bar{q} + 2, 3)$
$\gamma_E(e_{\bar{q}} R_E) : (3 \leq \bar{q} \leq h)$	$(\bar{q} - 1, \bar{q} - 2, h - \bar{q} + 2, \bar{q} + 1)$
$\gamma_E(e_{\bar{q}} R_E) : (\bar{q} = h + 1)$	$(\bar{q} - 1, \bar{q} - 2, 0, \bar{q} + 1)$
$\gamma_E(e_{\bar{q}} R_E) : (\bar{q} = h + 2)$	$(2h - \bar{q} + 2, \bar{q} - 2, \bar{q} - h - 1, 2h - \bar{q} + 4)$
$\gamma_E(e_{\bar{q}} R_E) : (h + 3 \leq \bar{q} \leq 2h)$	$(2h - \bar{q} + 2, 2h - \bar{q} + 3, \bar{q} - h - 1, 2h - \bar{q} + 4)$

$\gamma_E(e_{\bar{q}} = k_{\bar{q}}j_{\bar{q}+1} R_E)$	$R_E = \{k_1, k_2, k_{h+1}, o_1\}$
$\gamma_E(e_{\bar{q}} R_E):(\bar{q} = 1)$	$(0, 1, h - \bar{q} + 1, 3)$
$\gamma_E(e_{\bar{q}} R_E):(\bar{q} = 2)$	$(\bar{q}, 0, h - \bar{q} + 1, 3)$
$\gamma_E(e_{\bar{q}} R_E):(3 \leq \bar{q} \leq h)$	$(\bar{q}, \bar{q} - 1, h - \bar{q} + 1, \bar{q} + 1)$
$\gamma_E(e_{\bar{q}} R_E):(\bar{q} = h + 1)$	$(2h - \bar{q} + 1, \bar{q} - 1, 0, \bar{q} + 1)$
$\gamma_E(e_{\bar{q}} R_E):(h + 2 \leq \bar{q} \leq 2h)$	$(2h - \bar{q} + 1, 2h - \bar{q} + 2, \bar{q} - h, 2h - \bar{q} + 4)$

$\gamma_E(e_{\bar{q}} = k_{\bar{q}}l_{\bar{q}} R_E)$	$R_E = \{k_1, k_2, k_{h+1}, o_1\}$
$\gamma_E(e_{\bar{q}} R_E):(\bar{q} = 1)$	$(0, 2, h - \bar{q} + 2, 2)$
$\gamma_E(e_{\bar{q}} R_E):(\bar{q} = 2)$	$(\bar{q}, 0, h - \bar{q} + 2, 2)$
$\gamma_E(e_{\bar{q}} R_E):(3 \leq \bar{q} \leq h)$	$(\bar{q}, \bar{q} - 1, h - \bar{q} + 2, \bar{q})$
$\gamma_E(e_{\bar{q}} R_E):(\bar{q} = h + 1)$	$(\bar{q}, \bar{q} - 1, 0, \bar{q})$
$\gamma_E(e_{\bar{q}} R_E):(\bar{q} = h + 2)$	$(2h - \bar{q} + 2, \bar{q} - 1, \bar{q} - h, 2h - \bar{q} + 3)$
$\gamma_E(e_{\bar{q}} R_E):(h + 3 \leq \bar{q} \leq 2h)$	$(2h - \bar{q} + 2, 2h - \bar{q} + 3, \bar{q} - h, 2h - \bar{q} + 3)$

$\gamma_E(e_{\bar{q}} = l_{\bar{q}}m_{\bar{q}} R_E)$	$R_E = \{k_1, k_2, k_{h+1}, o_1\}$
$\gamma_E(e_{\bar{q}} R_E):(\bar{q} = 1)$	$(1, 2, h - \bar{q} + 3, \bar{q})$
$\gamma_E(e_{\bar{q}} R_E):(\bar{q} = 2)$	$(\bar{q} + 1, 1, h - \bar{q} + 3, \bar{q})$
$\gamma_E(e_{\bar{q}} R_E):(3 \leq \bar{q} \leq h - 1)$	$(\bar{q} + 1, \bar{q}, h - \bar{q} + 3, \bar{q})$
$\gamma_E(e_{\bar{q}} R_E):(\bar{q} = h)$	$(\bar{q} + 1, \bar{q}, 2, \bar{q})$
$\gamma_E(e_{\bar{q}} R_E):(\bar{q} = h + 1)$	$(\bar{q} + 1, \bar{q}, 1, \bar{q})$
$\gamma_E(e_{\bar{q}} R_E):(h + 2 \leq \bar{q} \leq 2h - 1)$	$(2h - \bar{q} + 3, 2h - \bar{q} + 4, \bar{q} - h + 1, 2h - \bar{q} + 2)$
$\gamma_E(e_{\bar{q}} R_E):(\bar{q} = 2h)$	$(2, 2h - \bar{q} + 4, \bar{q} - h + 1, 2h - \bar{q} + 2)$

$\gamma_E(e_{\bar{q}} = m_{\bar{q}}l_{\bar{q}+1} R_E)$	$R_E = \{k_1, k_2, k_{h+1}, o_1\}$
$\gamma_E(e_{\bar{q}} R_E):(\bar{q} = 1)$	$(2, 1, h - \bar{q} + 2, \bar{q})$
$\gamma_E(e_{\bar{q}} R_E):(\bar{q} = 2)$	$(\bar{q} + 2, 2, h - \bar{q} + 2, \bar{q})$
$\gamma_E(e_{\bar{q}} R_E):(3 \leq \bar{q} \leq h - 1)$	$(\bar{q} + 2, \bar{q} + 1, h - \bar{q} + 2, \bar{q})$
$\gamma_E(e_{\bar{q}} R_E):(\bar{q} = h)$	$(\bar{q} + 2, \bar{q} + 1, 1, \bar{q})$
$\gamma_E(e_{\bar{q}} R_E):(\bar{q} = h + 1)$	$(2h - \bar{q} + 2, 2h - \bar{q} + 3, 2, \bar{q})$
$\gamma_E(e_{\bar{q}} R_E):(h + 2 \leq \bar{q} \leq 2h - 1)$	$(2h - \bar{q} + 2, 2h - \bar{q} + 3, \bar{q} - h + 2, 2h - \bar{q} + 2)$
$\gamma_E(e_{\bar{q}} R_E):(\bar{q} = 2h)$	$(1, 2h - \bar{q} + 3, \bar{q} - h + 2, 2h - \bar{q} + 2)$

and

$\gamma_E(e_{\bar{q}} = m_{\bar{q}}o_{\bar{q}} R_E)$	$R_E = \{k_1, k_2, k_{h+1}, o_1\}$
$\gamma_E(e_{\bar{q}} R_E):(\bar{q} = 1)$	$(2, 2, h - \bar{q} + 3, \bar{q} - 1)$
$\gamma_E(e_{\bar{q}} R_E):(\bar{q} = 2)$	$(\bar{q} + 2, 2, h - \bar{q} + 3, \bar{q} - 1)$
$\gamma_E(e_{\bar{q}} R_E):(3 \leq \bar{q} \leq h - 1)$	$(\bar{q} + 2, \bar{q} + 1, h - \bar{q} + 3, \bar{q} - 1)$
$\gamma_E(e_{\bar{q}} R_E):(\bar{q} = h)$	$(\bar{q} + 2, \bar{q} + 1, 2, \bar{q} - 1)$
$\gamma_E(e_{\bar{q}} R_E):(\bar{q} = h + 1)$	$(2h - \bar{q} + 3, \bar{q} + 1, 2, \bar{q} - 1)$
$\gamma_E(e_{\bar{q}} R_E):(h + 2 \leq \bar{q} \leq 2h - 1)$	$(2h - \bar{q} + 3, 2h - \bar{q} + 4, \bar{q} - h + 2, 2h - \bar{q} + 1)$
$\gamma_E(e_{\bar{q}} R_E):(\bar{q} = 2h)$	$(2, 2h - \bar{q} + 4, \bar{q} - h + 2, 2h - \bar{q} + 1)$

Finally, for the edges of o -cycle $\{e_{\bar{q}} = o_{\bar{q}}o_{\bar{q}+1}|\bar{q} = 1, 2, 3, \dots, n\}$, the edge codes are

$\gamma_E(e_{\bar{q}} = o_{\bar{q}}o_{\bar{q}+1} R_E)$	$R_E = \{k_1, k_2, k_{h+1}, o_1\}$
$\gamma_E(e_{\bar{q}} R_E):(\bar{q} = 1)$	$(\bar{q} + 2, 3, h - \bar{q} + 2, \bar{q} - 1)$
$\gamma_E(e_{\bar{q}} R_E):(2 \leq \bar{q} \leq h - 1)$	$(\bar{q} + 2, \bar{q} + 1, h - \bar{q} + 2, \bar{q} - 1)$
$\gamma_E(e_{\bar{q}} R_E):(\bar{q} = h)$	$(\bar{q} + 2, \bar{q} + 1, 3, \bar{q} - 1)$
$\gamma_E(e_{\bar{q}} R_E):(h + 1 \leq \bar{q} \leq 2h - 1)$	$(2h - \bar{q} + 2, 2h - \bar{q} + 3, \bar{q} - h + 2, 2h - \bar{q})$
$\gamma_E(e_{\bar{q}} R_E):(\bar{q} = 2h)$	$(3, 2h - \bar{q} + 3, \bar{q} - h + 2, 2h - \bar{q})$

From these edge codes in S_n , we find that $|A_i| = n$ ($1 \leq i \leq 8$) and the sum of all of these cardinalities is equal to $|E(S_n)|$ and which is $8n$. Moreover, all of these sets are pairwise disjoint, and so we find that no pair of two distinct edges in S_n are having the same edge metric coordinates, which implies that $dim_E(S_n) \leq 4$. Now, to finish the proof for this case, we prove that $dim(S_n) \geq 4$ by proving that there is no set R_E with $|R_E| = 3$, which can resolve every pair of edges in S_n . On the contrary, suppose $dim_E(S_n) = 3$. Then, we have the following to be considered:

Edge Resolving Set	Contradictions
$R_E = \{p_1, p_2, p_j\}; (3 \leq j \leq n)$	$\gamma_E(\bar{q}_1 p_1 R_E) = \gamma_E(p_1 \bar{q}_n R_E)$, for $3 \leq j \leq n$, a contradiction.
$R_E = \{\bar{q}_1, \bar{q}_2, \bar{q}_j\}; (3 \leq j \leq n)$	$\gamma_E(\bar{q}_1 r_1 R_E) = \gamma_E(p_1 \bar{q}_1 R_E)$, for $3 \leq j \leq h - 1$, and $\gamma_E(\bar{q}_4 p_4 R_E) = \gamma_E(p_4 \bar{q}_3 R_E)$, for $h \leq j \leq n$, a contradiction.
$R_E = \{r_1, r_2, r_j\}; (3 \leq j \leq n)$	$\gamma_E(p_1 p_n R_E) = \gamma_E(p_1 \bar{q}_n R_E)$, for $3 \leq j \leq h - 1$, and $\gamma_E(\bar{q}_3 p_4 R_E) = \gamma_E(p_4 \bar{q}_4 R_E)$, for $h \leq j \leq n$, a contradiction.
$R_E = \{s_1, s_2, s_j\}; (3 \leq j \leq n)$	$\gamma_E(p_1 p_n R_E) = \gamma_E(p_1 \bar{q}_n R_E)$, for $3 \leq j \leq h - 2$, and $\gamma_E(\bar{q}_4 p_5 R_E) = \gamma_E(p_5 \bar{q}_5 R_E)$, for $h - 1 \leq j \leq n$, a contradiction.
$R_E = \{t_1, t_2, t_j\}; (3 \leq j \leq n)$	$\gamma_E(r_1 s_1 R_E) = \gamma_E(r_2 s_2 R_E)$, for $3 \leq j \leq n$, a contradiction.
$R_E = \{p_1, p_j, \bar{q}_1\}; (2 \leq j \leq n)$	$\gamma_E(r_1 s_1 R_E) = \gamma_E(r_1 s_n R_E)$, for $2 \leq j \leq n$, a contradiction.
$R_E = \{p_1, p_j, r_1\}; (2 \leq j \leq n)$	$\gamma_E(r_1 s_1 R_E) = \gamma_E(r_1 s_n R_E)$, for $2 \leq j \leq n$, a contradiction.
$R_E = \{p_1, p_j, s_1\}; (2 \leq j \leq n)$	$\gamma_E(p_1 p_n R_E) = \gamma_E(p_1 \bar{q}_n R_E)$, for $2 \leq j \leq h - 1$, and $\gamma_E(\bar{q}_4 p_4 R_E) = \gamma_E(p_4 \bar{q}_3 R_E)$, for $h \leq j \leq n$, a contradiction.
$R_E = \{p_1, p_j, t_1\}; (2 \leq j \leq n)$	$\gamma_E(p_2 \bar{q}_2 R_E) = \gamma_E(\bar{q}_1 p_2 R_E)$, for $2 \leq j \leq n$, a contradiction.
$R_E = \{\bar{q}_1, \bar{q}_j, r_1\}; (2 \leq j \leq n)$	$\gamma_E(p_1 p_n R_E) = \gamma_E(p_1 \bar{q}_n R_E)$, for $2 \leq j \leq h - 1$, and $\gamma_E(\bar{q}_3 p_4 R_E) = \gamma_E(p_4 \bar{q}_4 R_E)$, for $h \leq j \leq n$, a contradiction.
$R_E = \{\bar{q}_1, \bar{q}_j, s_1\}; (2 \leq j \leq n)$	$\gamma_E(p_1 p_n R_E) = \gamma_E(p_1 \bar{q}_n R_E)$, for $2 \leq j \leq h - 1$, and $\gamma_E(\bar{q}_3 p_4 R_E) = \gamma_E(p_4 \bar{q}_4 R_E)$, for $h \leq j \leq n$, a contradiction.
$R_E = \{\bar{q}_1, \bar{q}_j, t_1\}; (2 \leq j \leq n)$	$\gamma_E(p_1 p_n R_E) = \gamma_E(p_1 \bar{q}_n R_E)$, for $2 \leq j \leq h$; $\gamma_E(p_2 p_3 R_E) = \gamma_E(p_1 \bar{q}_n R_E)$, for $j = h + 1$; $\gamma_E(p_2 p_3 R_E) = \gamma_E(p_1 p_2 R_E)$, for $j = h + 2$, and $\gamma_E(p_{h+2} p_{h+3} R_E) = \gamma_E(p_{h+2} \bar{q}_{h+2} R_E)$, for $h + 3 \leq j \leq n$, a contradiction.
$R_E = \{r_1, r_j, s_1\}; (2 \leq j \leq n)$	$\gamma_E(\bar{q}_{n-1} p_n R_E) = \gamma_E(p_n \bar{q}_n R_E)$, for $2 \leq j \leq n - 2$, and $\gamma_E(\bar{q}_4 p_4 R_E) = \gamma_E(p_4 \bar{q}_3 R_E)$, for $n - 1 \leq j \leq n$, a contradiction.
$R_E = \{r_1, r_j, t_1\}; (2 \leq j \leq n)$	$\gamma_E(p_1 p_n R_E) = \gamma_E(p_1 \bar{q}_n R_E)$, for $2 \leq j \leq h$; $\gamma_E(p_2 p_3 R_E) = \gamma_E(p_1 p_n R_E)$, for $j = h + 1$; $\gamma_E(p_2 p_3 R_E) = \gamma_E(p_1 p_2 R_E)$, for $j = h + 2$, and $\gamma_E(p_3 \bar{q}_3 R_E) = \gamma_E(p_3 p_4 R_E)$, for $h + 3 \leq j \leq n$, a contradiction.
$R_E = \{s_1, s_j, t_1\}; (2 \leq j \leq n)$	$\gamma_E(\bar{q}_1 r_1 R_E) = \gamma_E(r_1 s_n R_E)$, for $2 \leq j \leq h$; $\gamma_E(t_1 t_n R_E) = \gamma_E(t_1 t_2 R_E)$, for $j = h + 1$; $\gamma_E(\bar{q}_2 r_2 R_E) = \gamma_E(r_2 s_2 R_E)$, for $h + 2 \leq j \leq n$, a contradiction.

Edge Resolving Set	Contradictions
$R_E = \{p_1, \bar{q}_1, r_{h+j}\}; (1 \leq j \leq n)$	$\gamma_E(\bar{q}_3 p_3 R_E) = \gamma_E(\bar{q}_2 p_3 R_E)$, for $j = 1$; $\gamma_E(\bar{q}_2 p_2 R_E) = \gamma_E(p_2 p_3 R_E)$, for $2 \leq j \leq h + 1$; $\gamma_E(p_1 p_n R_E) = \gamma_E(p_1 \bar{q}_n R_E)$, for $h + 2 \leq j \leq n$, a contradiction.
$R_E = \{p_1, \bar{q}_1, s_{h+j}\}; (1 \leq j \leq n)$	$\gamma_E(p_n \bar{q}_n R_E) = \gamma_E(\bar{q}_{n-1} p_n R_E)$, for $j = 1, 2, \dots, h - 3, h - 1, h + 1, h + 2, \dots, n$, and $\gamma_E(\bar{q}_4 p_4 R_E) = \gamma_E(p_4 \bar{q}_3 R_E)$, for $j = h$ and $h - 2$, a contradiction.
$R_E = \{p_1, \bar{q}_1, t_{h+j}\}; (1 \leq j \leq n)$	$\gamma_E(p_{h+2} \bar{q}_{h+1} R_E) = \gamma_E(\bar{q}_{h+2} p_{h+2} R_E)$, for $j = 1$; $\gamma_E(p_2 p_3 R_E) = \gamma_E(\bar{q}_2 p_2 R_E)$, for $2 \leq j \leq h$; $\gamma_E(p_1 p_n R_E) = \gamma_E(p_1 \bar{q}_n R_E)$, for $h + 1 \leq j \leq n - 1$; $\gamma_E(\bar{q}_h p_{h+1} R_E) = \gamma_E(p_{h+1} \bar{q}_{h+1} R_E)$, for $j = n$, a contradiction.
$R_E = \{\bar{q}_1, r_1, s_{h+j}\}; (1 \leq j \leq n)$	$\gamma_E(p_4 \bar{q}_4 R_E) = \gamma_E(\bar{q}_3 p_4 R_E)$, for $j = 1, 2, \dots, h + 1, h + 3, h + 5, \dots, n$, and $\gamma_E(\bar{q}_n p_n R_E) = \gamma_E(p_n \bar{q}_{n-1} R_E)$, for $j = h + 2$ and $h + 4$, a contradiction.
$R_E = \{\bar{q}_1, r_1, t_{h+j}\}; (1 \leq j \leq n)$	$\gamma_E(p_1 \bar{q}_1 R_E) = \gamma_E(\bar{q}_1 p_2 R_E)$, for $1 \leq j \leq n$, a contradiction.

By the symmetry S_n , other relations can be considered, and they will produce the same kind of contradictions. As a result, it can be found that $dim_E(S_n) = 4$, when n is even.

Case (slowromancapii@) $n \equiv 1 \pmod{2}$

Then, for the natural n , we have $n = 2h + 1$, where $h \in \mathbb{N}$ and $h \geq 6$. Suppose $R_E = \{k_1, k_2, k_{h+1}, o_1\} \subset V(S_n)$. Next, we give edge metric representation to every edge of S_n concerning the set R_E . For the edges of j -cycle $\{e_{\bar{q}} = j_{\bar{q}}j_{\bar{q}+1} | \bar{q} = 1, 2, 3, \dots, n\}$, the edge codes are

$\gamma_E(e_{\bar{q}} = j_{\bar{q}}j_{\bar{q}+1} R_E)$	$R_E = \{k_1, k_2, k_{h+1}, o_1\}$
$\gamma_E(e_{\bar{q}} R_E): (\bar{q} = 1)$	$(1, 1, h - \bar{q} + 1, 4)$
$\gamma_E(e_{\bar{q}} R_E): (\bar{q} = 2)$	$(\bar{q} - 1, 1, h - \bar{q} + 1, 4)$
$\gamma_E(e_{\bar{q}} R_E): (3 \leq \bar{q} \leq h)$	$(\bar{q} - 1, \bar{q} - 2, h - \bar{q} + 1, \bar{q} + 1)$
$\gamma_E(e_{\bar{q}} R_E): (\bar{q} = h + 1)$	$(\bar{q} - 1, \bar{q} - 2, 1, \bar{q} + 1)$
$\gamma_E(e_{\bar{q}} R_E): (\bar{q} = h + 2)$	$(2h - \bar{q} + 2, \bar{q} - 2, \bar{q} - h - 1, 2h - \bar{q} + 5)$
$\gamma_E(e_{\bar{q}} R_E): (h + 3 \leq \bar{q} \leq 2h + 1)$	$(2h - \bar{q} + 2, 2h - \bar{q} + 3, \bar{q} - h - 1, 2h - \bar{q} + 5)$

For the edges $\{e_{\bar{q}} = j_{\bar{q}}k_{\bar{q}}, k_{\bar{q}}j_{\bar{q}+1}, k_{\bar{q}}l_{\bar{q}}, l_{\bar{q}}m_{\bar{q}}, m_{\bar{q}}l_{\bar{q}+1}, m_{\bar{q}}o_{\bar{q}} | \bar{q} = 1, 2, 3, \dots, n\}$, the edge codes are

$\gamma_E(e_{\bar{q}} = j_{\bar{q}}k_{\bar{q}} R_E)$	$R_E = \{k_1, k_2, k_{h+1}, o_1\}$
$\gamma_E(e_{\bar{q}} R_E): (\bar{q} = 1)$	$(0, 2, h - \bar{q} + 2, 3)$
$\gamma_E(e_{\bar{q}} R_E): (\bar{q} = 2)$	$(\bar{q} - 1, 0, h - \bar{q} + 2, 3)$
$\gamma_E(e_{\bar{q}} R_E): (3 \leq \bar{q} \leq h)$	$(\bar{q} - 1, \bar{q} - 2, h - \bar{q} + 2, \bar{q} + 1)$
$\gamma_E(e_{\bar{q}} R_E): (\bar{q} = h + 1)$	$(\bar{q} - 1, \bar{q} - 2, 0, \bar{q} + 1)$
$\gamma_E(e_{\bar{q}} R_E): (\bar{q} = h + 2)$	$(2h - \bar{q} + 3, \bar{q} - 2, \bar{q} - h - 1, 2h - \bar{q} + 5)$
$\gamma_E(e_{\bar{q}} R_E): (h + 3 \leq \bar{q} \leq 2h + 1)$	$(2h - \bar{q} + 3, 2h - \bar{q} + 4, \bar{q} - h - 1, 2h - \bar{q} + 5)$

$\gamma_E(e_{\bar{q}} = k_{\bar{q}}j_{\bar{q}+1} R_E)$	$R_E = \{k_1, k_2, k_{h+1}, o_1\}$
$\gamma_E(e_{\bar{q}} R_E): (\bar{q} = 1)$	$(0, 1, h - \bar{q} + 1, 3)$
$\gamma_E(e_{\bar{q}} R_E): (\bar{q} = 2)$	$(\bar{q}, 0, h - \bar{q} + 1, 3)$
$\gamma_E(e_{\bar{q}} R_E): (3 \leq \bar{q} \leq h)$	$(\bar{q}, \bar{q} - 1, h - \bar{q} + 1, \bar{q} + 1)$
$\gamma_E(e_{\bar{q}} R_E): (\bar{q} = h + 1)$	$(\bar{q}, \bar{q} - 1, 0, \bar{q} + 1)$
$\gamma_E(e_{\bar{q}} R_E): (h + 2 \leq \bar{q} \leq 2h + 1)$	$(2h - \bar{q} + 2, 2h - \bar{q} + 3, \bar{q} - h, 2h - \bar{q} + 5)$

$\gamma_E(e_{\bar{q}} = k_{\bar{q}}l_{\bar{q}} R_E)$	$R_E = \{k_1, k_2, k_{h+1}, o_1\}$
$\gamma_E(e_{\bar{q}} R_E): (\bar{q} = 1)$	$(0, 2, h - \bar{q} + 2, 2)$
$\gamma_E(e_{\bar{q}} R_E): (\bar{q} = 2)$	$(\bar{q}, 0, h - \bar{q} + 2, 2)$
$\gamma_E(e_{\bar{q}} R_E): (3 \leq \bar{q} \leq h)$	$(\bar{q}, \bar{q} - 1, h - \bar{q} + 2, \bar{q})$
$\gamma_E(e_{\bar{q}} R_E): (\bar{q} = h + 1)$	$(\bar{q}, \bar{q} - 1, 0, \bar{q})$
$\gamma_E(e_{\bar{q}} R_E): (\bar{q} = h + 2)$	$(2h - \bar{q} + 3, \bar{q} - 1, \bar{q} - h, 2h - \bar{q} + 4)$
$\gamma_E(e_{\bar{q}} R_E): (h + 3 \leq \bar{q} \leq 2h + 1)$	$(2h - \bar{q} + 3, 2h - \bar{q} + 4, \bar{q} - h, 2h - \bar{q} + 4)$

$\gamma_E(e_{\bar{q}} = l_{\bar{q}}m_{\bar{q}} R_E)$	$R_E = \{k_1, k_2, k_{h+1}, o_1\}$
$\gamma_E(e_{\bar{q}} R_E): (\bar{q} = 1)$	$(1, 2, h - \bar{q} + 3, \bar{q})$
$\gamma_E(e_{\bar{q}} R_E): (\bar{q} = 2)$	$(\bar{q} + 1, 1, h - \bar{q} + 3, \bar{q})$
$\gamma_E(e_{\bar{q}} R_E): (3 \leq \bar{q} \leq h - 1)$	$(\bar{q} + 1, \bar{q}, h - \bar{q} + 3, \bar{q})$
$\gamma_E(e_{\bar{q}} R_E): (\bar{q} = h)$	$(\bar{q} + 1, \bar{q}, 2, \bar{q})$
$\gamma_E(e_{\bar{q}} R_E): (\bar{q} = h + 1)$	$(\bar{q} + 1, \bar{q}, 1, \bar{q})$
$\gamma_E(e_{\bar{q}} R_E): (\bar{q} = h + 2)$	$(2h - \bar{q} + 4, \bar{q}, \bar{q} - h + 1, 2h - \bar{q} + 3)$
$\gamma_E(e_{\bar{q}} R_E): (h + 2 \leq \bar{q} \leq 2h)$	$(2h - \bar{q} + 4, 2h - \bar{q} + 5, \bar{q} - h + 1, 2h - \bar{q} + 3)$
$\gamma_E(e_{\bar{q}} R_E): (\bar{q} = 2h + 1)$	$(2, 2h - \bar{q} + 4, \bar{q} - h + 1, 2h - \bar{q} + 3)$

$\gamma_E(e_{\bar{q}} = m_{\bar{q}}l_{\bar{q}+1} R_E)$	$R_E = \{k_1, k_2, k_{h+1}, o_1\}$
$\gamma_E(e_{\bar{q}} R_E):(\bar{q} = 1)$	$(2, 1, h - \bar{q} + 2, \bar{q})$
$\gamma_E(e_{\bar{q}} R_E):(\bar{q} = 2)$	$(\bar{q} + 2, 2, h - \bar{q} + 2, \bar{q})$
$\gamma_E(e_{\bar{q}} R_E):(3 \leq \bar{q} \leq h - 1)$	$(\bar{q} + 2, \bar{q} + 1, h - \bar{q} + 2, \bar{q})$
$\gamma_E(e_{\bar{q}} R_E):(\bar{q} = h)$	$(\bar{q} + 2, \bar{q} + 1, 1, \bar{q})$
$\gamma_E(e_{\bar{q}} R_E):(\bar{q} = h + 1)$	$(2h - \bar{q} + 3, \bar{q} + 1, 2, \bar{q})$
$\gamma_E(e_{\bar{q}} R_E):(h + 2 \leq \bar{q} \leq 2h)$	$(2h - \bar{q} + 3, 2h - \bar{q} + 4, \bar{q} - h + 2, 2h - \bar{q} + 3)$
$\gamma_E(e_{\bar{q}} R_E):(\bar{q} = 2h + 1)$	$(1, 2h - \bar{q} + 4, h + 2, 2h - \bar{q} + 3)$

and

$\gamma_E(e_{\bar{q}} = m_{\bar{q}}o_{\bar{q}} R_E)$	$R_E = \{k_1, k_2, k_{h+1}, o_1\}$
$\gamma_E(e_{\bar{q}} R_E):(\bar{q} = 1)$	$(2, 2, h - \bar{q} + 3, \bar{q} - 1)$
$\gamma_E(e_{\bar{q}} R_E):(\bar{q} = 2)$	$(\bar{q} + 2, 2, h - \bar{q} + 3, \bar{q} - 1)$
$\gamma_E(e_{\bar{q}} R_E):(3 \leq \bar{q} \leq h - 1)$	$(\bar{q} + 2, \bar{q} + 1, h - \bar{q} + 3, \bar{q} - 1)$
$\gamma_E(e_{\bar{q}} R_E):(\bar{q} = h)$	$(\bar{q} + 2, \bar{q} + 1, 2, \bar{q} - 1)$
$\gamma_E(e_{\bar{q}} R_E):(\bar{q} = h + 1)$	$(\bar{q} + 2, \bar{q} + 1, 2, \bar{q} - 1)$
$\gamma_E(e_{\bar{q}} R_E):(h + 2 \leq \bar{q} \leq 2h)$	$(2h - \bar{q} + 4, 2h - \bar{q} + 5, \bar{q} - h + 2, 2h - \bar{q} + 2)$
$\gamma_E(e_{\bar{q}} R_E):(\bar{q} = 2h + 1)$	$(2, 2h - \bar{q} + 5, \bar{q} - h + 2, 2h - \bar{q} + 2)$

Finally, for the edges of o -cycle $\{e_{\bar{q}} = o_{\bar{q}}o_{\bar{q}+1}|\bar{q} = 1, 2, 3, \dots, n\}$, the edge codes are

$\gamma_E(e_{\bar{q}} = o_{\bar{q}}o_{\bar{q}+1} R_E)$	$R_E = \{k_1, k_2, k_{h+1}, o_1\}$
$\gamma_E(e_{\bar{q}} R_E):(\bar{q} = 1)$	$(\bar{q} + 2, 3, h - \bar{q} + 2, \bar{q} - 1)$
$\gamma_E(e_{\bar{q}} R_E):(2 \leq \bar{q} \leq h - 1)$	$(\bar{q} + 2, \bar{q} + 1, h - \bar{q} + 2, \bar{q} - 1)$
$\gamma_E(e_{\bar{q}} R_E):(\bar{q} = h)$	$(\bar{q} + 2, \bar{q} + 1, 3, \bar{q} - 1)$
$\gamma_E(e_{\bar{q}} R_E):(\bar{q} = h + 1)$	$(2h - \bar{q} + 3, \bar{q} + 1, \bar{q} - h + 2, 2h - \bar{q} + 1)$
$\gamma_E(e_{\bar{q}} R_E):(h + 2 \leq \bar{q} \leq 2h)$	$(2h - \bar{q} + 3, 2h - \bar{q} + 4, \bar{q} - h + 2, 2h - \bar{q} + 1)$
$\gamma_E(e_{\bar{q}} R_E):(\bar{q} = 2h + 1)$	$(3, 2h - \bar{q} + 4, h + 2, 2h - \bar{q} + 1)$

From these edge codes in S_n , we find that $|A_i| = n$ ($1 \leq i \leq 8$) and the sum of all of these cardinalities is equal to $|E(S_n)|$ and which is $8n$. Moreover, all of these sets are pairwise disjoint, and so we find that no pair of two distinct edges in S_n are having the same edge metric coordinates, which implies that $dim_E(S_n) \leq 4$ as well in this case.

Now, on assuming that $dim_E(S_n) = 3$, there are the same possibilities as discussed in Case (slowromancapi@) and a contradiction can be deduced similarly. Therefore, we have $dim_E(S_n) = 4$ as well in this case, which proves the theorem. \square

Corollary 1. *The minimum independent edge resolving set for S_n has cardinality four, for every $n \geq 6$.*

Remark 1. $dim(S_n) < dim_E(S_n)$, for every $n \geq 6$.

In the next section, we consider second family of convex polytope graph T_n for which we have $E(T_n) = \{j_{\bar{q}}j_{\bar{q}+1}, j_{\bar{q}}k_{\bar{q}}, k_{\bar{q}}j_{\bar{q}+1}, k_{\bar{q}}l_{\bar{q}}, l_{\bar{q}}l_{\bar{q}+1} : 1 \leq \bar{q} \leq n\}$ (see Figure 2). We denote the sets of edge metric co-ordinates for the edges of T_n by A_1, A_2, A_3, A_4 , and A_5 , where $A_1 = \{\gamma_E(j_{\bar{q}}j_{\bar{q}+1}|R_E) : 1 \leq \bar{q} \leq n\}$, $A_2 = \{\gamma_E(j_{\bar{q}}k_{\bar{q}}|R_E) : 1 \leq \bar{q} \leq n\}$, $A_3 = \{\gamma_E(k_{\bar{q}}j_{\bar{q}+1}|R_E) : 1 \leq \bar{q} \leq n\}$, $A_4 = \{\gamma_E(k_{\bar{q}}l_{\bar{q}}|R_E) : 1 \leq \bar{q} \leq n\}$, and $A_5 = \{\gamma_E(l_{\bar{q}}l_{\bar{q}+1}|R_E) : 1 \leq \bar{q} \leq n\}$.

4. Edge Metric Dimension of T_n

In this section, we study some of the basic properties and the EMD of T_n .

The Graph T_n

The convex polytope T_n [8] consists of $3n$ and n number of vertices and edges respectively. It has n cycles with 3-sides, n cycles with 5-sides, and two cycles with n -sides (see Figure 2). The set of edges and vertices of T_n are denoted separately by $E(T_n)$ and $V(T_n)$, where $E(T_n) = \{j_{\bar{q}}j_{\bar{q}+1}, j_{\bar{q}}k_{\bar{q}}, k_{\bar{q}}j_{\bar{q}+1}, k_{\bar{q}}l_{\bar{q}}, l_{\bar{q}}l_{\bar{q}+1} : 1 \leq \bar{q} \leq n\}$ and $V(T_n) = \{j_{\bar{q}}, k_{\bar{q}}, l_{\bar{q}} : 1 \leq \bar{q} \leq n\}$.

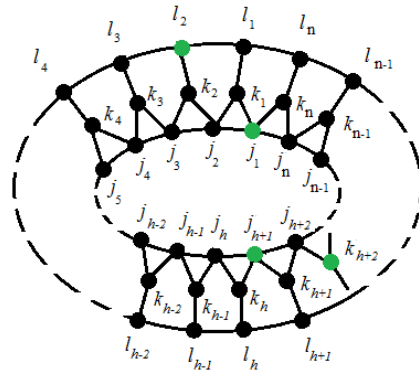


Figure 2. The Graph T_n

We name the vertices $\{j_{\bar{q}} : 1 \leq \bar{q} \leq n\}$ in the planar graph, T_n as the j -cycle vertices, the vertices $\{k_{\bar{q}} : 1 \leq \bar{q} \leq n\}$ in the planar graph, T_n as the k -vertices, and the vertices $\{l_{\bar{q}} : 1 \leq \bar{q} \leq n\}$ in the planar graph, T_n as the l -cycle vertices. For our purpose, we can write $j_1 = j_{n+1}$, $k_1 = k_{n+1}$, and $l_1 = l_{n+1}$. In the present section, we consider a family of convex polytope graph T_n for which we have $E(T_n) = \{j_{\bar{q}}j_{\bar{q}+1}, j_{\bar{q}}k_{\bar{q}}, k_{\bar{q}}j_{\bar{q}+1}, k_{\bar{q}}l_{\bar{q}}, l_{\bar{q}}l_{\bar{q}+1} : 1 \leq \bar{q} \leq n\}$. We denote the sets of edge metric co-ordinates for the edges of T_n by \mathbb{A} , \mathbb{B} , and \mathbb{C} , where $\mathbb{A} = \{\gamma_E(j_{\bar{q}}j_{\bar{q}+1}|R_E), \gamma_E(j_{\bar{q}}k_{\bar{q}}|R_E) : 1 \leq \bar{q} \leq n\}$, $\mathbb{B} = \{\gamma_E(k_{\bar{q}}j_{\bar{q}+1}|R_E), \gamma_E(k_{\bar{q}}l_{\bar{q}}|R_E) : 1 \leq \bar{q} \leq n\}$, and $\mathbb{C} = \{\gamma_E(l_{\bar{q}}l_{\bar{q}+1}|R_E) : 1 \leq \bar{q} \leq n\}$. In the following result, we investigate the EMD of T_n .

Theorem 4. For $n \geq 6$, we have

$$\dim_E(T_n) = \begin{cases} 3, & \text{if } n = 2h \ h \in \mathbb{N}; \\ 4, & \text{if } n = 2h + 1 \ h \in \mathbb{N}. \end{cases}$$

Proof. It is easy to check for $6 \leq n \leq 11$ that the EMD of T_n is 3 and 4, when the natural n is even and odd respectively. The position of the edge basis vertices (color in green) can be found in Figure 2, for all $n \geq 6$. Now, for $n \geq 12$, we divide our proof into two cases i.e., when n is even ($n \equiv 0 \pmod{2}$) and when it is odd ($n \equiv 1 \pmod{2}$).

Case (slowromancapi@) $n \equiv 0 \pmod{2}$

Then, for the natural n , we have $n = 2h$, where $h \in \mathbb{N}$ and $h \geq 6$. Suppose $R_E = \{j_1, j_{h+1}, l_2\} \subset V(T_n)$. Next, we give edge metric representation to every edge of T_n concerning the set R_E . For the edges of j -cycle $\{e_{\bar{q}} = j_{\bar{q}}j_{\bar{q}+1} | \bar{q} = 1, 2, 3, \dots, n\}$, the edge codes are

$\gamma_E(e_{\bar{q}} R_E)$	$R_E = \{j_1, j_{h+1}, l_2\}$
$\gamma_E(e_{\bar{q}} R_E):(1 \leq \bar{q} \leq 2)$	$(\bar{q} - 1, h - \bar{q}, 2)$
$\gamma_E(e_{\bar{q}} R_E):(3 \leq \bar{q} \leq h)$	$(\bar{q} - 1, h - \bar{q}, \bar{q} - 1)$
$\gamma_E(e_{\bar{q}} R_E):(\bar{q} = h + 1)$	$(2h - \bar{q}, \bar{q} - h - 1, \bar{q} - 1)$
$\gamma_E(e_{\bar{q}} R_E):(h + 2 \leq \bar{q} \leq 2h)$	$(2h - \bar{q}, \bar{q} - h - 1, 2h - \bar{q} + 3)$

For the edges $\{e_{\bar{q}} = j_{\bar{q}}k_{\bar{q}}, k_{\bar{q}}j_{\bar{q}+1}, k_{\bar{q}}l_{\bar{q}} | \bar{q} = 1, 2, 3, \dots, n\}$, the edge codes are

$\gamma_E(e_{\bar{q}} = j_{\bar{q}}k_{\bar{q}} R_E)$	$R_E = \{j_1, j_{h+1}, l_2\}$
$\gamma_E(e_{\bar{q}} R_E): (\bar{q} = 1)$	$(\bar{q} - 1, h - \bar{q} + 1, 2)$
$\gamma_E(e_{\bar{q}} R_E): (2 \leq \bar{q} \leq h + 1)$	$(\bar{q} - 1, h - \bar{q} + 1, \bar{q} - 1)$
$\gamma_E(e_{\bar{q}} R_E): (\bar{q} = h + 2)$	$(2h - \bar{q} + 1, \bar{q} - h - 1, \bar{q} - 1)$
$\gamma_E(e_{\bar{q}} R_E): (h + 3 \leq \bar{q} \leq 2h)$	$(2h - \bar{q} + 1, \bar{q} - h - 1, 2h - \bar{q} + 3)$

$\gamma_E(e_{\bar{q}} = k_{\bar{q}}j_{\bar{q}+1} R_E)$	$R_E = \{j_1, j_{h+1}, l_2\}$
$\gamma_E(e_{\bar{q}} R_E): (\bar{q} = 1)$	$(\bar{q}, h - \bar{q}, 2)$
$\gamma_E(e_{\bar{q}} R_E): (2 \leq \bar{q} \leq h)$	$(\bar{q}, h - \bar{q}, \bar{q} - 1)$
$\gamma_E(e_{\bar{q}} R_E): (\bar{q} = h + 1)$	$(2h - \bar{q}, \bar{q} - h, \bar{q} - 1)$
$\gamma_E(e_{\bar{q}} R_E): (h + 2 \leq \bar{q} \leq 2h)$	$(2h - \bar{q}, \bar{q} - h, 2h - \bar{q} + 3)$

and

$\gamma_E(e_{\bar{q}} = k_{\bar{q}}l_{\bar{q}} R_E)$	$R_E = \{j_1, j_{h+1}, l_2\}$
$\gamma_E(e_{\bar{q}} R_E): (\bar{q} = 1)$	$(\bar{q}, h - \bar{q} + 1, 1)$
$\gamma_E(e_{\bar{q}} R_E): (2 \leq \bar{q} \leq h)$	$(\bar{q}, h - \bar{q} + 1, \bar{q} - 2)$
$\gamma_E(e_{\bar{q}} R_E): (\bar{q} = h + 1)$	$(2h - \bar{q} + 1, \bar{q} - h, \bar{q} - 2)$
$\gamma_E(e_{\bar{q}} R_E): (h + 2 \leq \bar{q} \leq 2h)$	$(2h - \bar{q} + 1, \bar{q} - h, 2h - \bar{q} + 2)$

Finally, for the edges of l -cycle $\{e_{\bar{q}} = l_{\bar{q}}l_{\bar{q}+1} | \bar{q} = 1, 2, 3, \dots, n\}$, the edge codes are

$\gamma_E(e_{\bar{q}} = l_{\bar{q}}l_{\bar{q}+1} R_E)$	$R_E = \{j_1, j_{h+1}, l_2\}$
$\gamma_E(e_{\bar{q}} R_E): (\bar{q} = 1)$	$(\bar{q} + 1, h - \bar{q} + 1, 0)$
$\gamma_E(e_{\bar{q}} R_E): (2 \leq \bar{q} \leq h - 1)$	$(\bar{q} + 1, h - \bar{q} + 1, \bar{q} - 2)$
$\gamma_E(e_{\bar{q}} R_E): (\bar{q} = h)$	$(\bar{q} + 1, 2, \bar{q} - 2)$
$\gamma_E(e_{\bar{q}} R_E): (\bar{q} = h + 1)$	$(2h - \bar{q} + 1, \bar{q} - h + 1, \bar{q} - 2)$
$\gamma_E(e_{\bar{q}} R_E): (h + 2 \leq \bar{q} \leq 2h - 1)$	$(2h - \bar{q} + 1, \bar{q} - h + 1, 2h - \bar{q} + 1)$
$\gamma_E(e_{\bar{q}} R_E): (\bar{q} = 2h)$	$(2, \bar{q} - h + 1, 2h - \bar{q} + 1)$

From these edge codes in T_n , we find that $|A_i| = n$ ($1 \leq i \leq 5$) and the sum of all of these cardinalities is equal to $|E(T_n)|$ and which is $5n$. Moreover, all of these sets are pairwise disjoint, and so we find that no pair of two distinct edges in T_n are having the same edge metric coordinates, which implies that $dim_E(T_n) \leq 3$. Using proposition 3 we obtain $dim_E(T_n) = 3$, in this case.

Case (slowromancapii@) $n \equiv 1 \pmod{2}$

From this, we have $n = 2h + 1$, where $h \in \mathbb{N}$ and $h \geq 6$. Suppose $R_E = \{j_1, j_{h+1}, l_2, k_{h+2}\} \subset V(T_n)$. Next, we give edge metric representation to every edge of T_n with respect to the set R_E .

For the edges of j -cycle $\{e_{\bar{q}} = j_{\bar{q}}j_{\bar{q}+1} | \bar{q} = 1, 2, 3, \dots, n\}$, the edge codes are

$\gamma_E(e_{\bar{q}} R_E)$	$R_E = \{j_1, j_{h+1}, l_2, k_{h+2}\}$
$\gamma_E(e_{\bar{q}} R_E): (\bar{q} = 1)$	$(\bar{q} - 1, h - \bar{q}, 2, h)$
$\gamma_E(e_{\bar{q}} R_E): (\bar{q} = 2)$	$(\bar{q} - 1, h - \bar{q}, 2, h - \bar{q} + 2)$
$\gamma_E(e_{\bar{q}} R_E): (3 \leq \bar{q} \leq h)$	$(\bar{q} - 1, h - \bar{q}, \bar{q} - 1, h - \bar{q} + 2)$
$\gamma_E(e_{\bar{q}} R_E): (\bar{q} = h + 1)$	$(\bar{q} - 1, \bar{q} - h - 1, \bar{q} - 1, h - \bar{q} + 2)$
$\gamma_E(e_{\bar{q}} R_E): (\bar{q} = h + 2)$	$(2h - \bar{q} + 1, \bar{q} - h - 1, \bar{q} - 1, 1)$
$\gamma_E(e_{\bar{q}} R_E): (h + 3 \leq \bar{q} \leq 2h + 1)$	$(2h - \bar{q} + 1, \bar{q} - h - 1, 2h - \bar{q} + 4, \bar{q} - h - 2)$

For the edges $\{e_{\bar{q}} = j_{\bar{q}}k_{\bar{q}}, k_{\bar{q}}j_{\bar{q}+1}, k_{\bar{q}}l_{\bar{q}} | \bar{q} = 1, 2, 3, \dots, n\}$, the edge codes are

$\gamma_E(e_{\bar{q}} = j_{\bar{q}}k_{\bar{q}} R_E)$	$R_E = \{j_1, j_{h+1}, l_2, k_{h+2}\}$
$\gamma_E(e_{\bar{q}} R_E): (\bar{q} = 1)$	$(\bar{q} - 1, h - \bar{q} + 1, 2, h)$
$\gamma_E(e_{\bar{q}} R_E): (2 \leq \bar{q} \leq h + 1)$	$(\bar{q} - 1, h - \bar{q} + 1, \bar{q} - 1, h - \bar{q} + 3)$
$\gamma_E(e_{\bar{q}} R_E): (\bar{q} = h + 2)$	$(2h - \bar{q} + 2, \bar{q} - h - 1, \bar{q} - 1, 0)$
$\gamma_E(e_{\bar{q}} R_E): (h + 3 \leq \bar{q} \leq 2h + 1)$	$(2h - \bar{q} + 2, \bar{q} - h - 1, 2h - \bar{q} + 4, \bar{q} - h - 2)$

$\gamma_E(e_{\bar{q}} = k_{\bar{q}}j_{\bar{q}+1} R_E)$	$R_E = \{j_1, j_{h+1}, l_2, k_{h+2}\}$
$\gamma_E(e_{\bar{q}} R_E): (\bar{q} = 1)$	$(\bar{q}, h - \bar{q}, 2, h - \bar{q} + 2)$
$\gamma_E(e_{\bar{q}} R_E): (2 \leq \bar{q} \leq h)$	$(\bar{q}, h - \bar{q}, \bar{q} - 1, h - \bar{q} + 2)$
$\gamma_E(e_{\bar{q}} R_E): (\bar{q} = h + 1)$	$(2h - \bar{q} + 1, \bar{q} - h, \bar{q} - 1, h - \bar{q} + 2)$
$\gamma_E(e_{\bar{q}} R_E): (\bar{q} = h + 2)$	$(2h - \bar{q} + 1, \bar{q} - h, \bar{q} - 1, 0)$
$\gamma_E(e_{\bar{q}} R_E): (h + 3 \leq \bar{q} \leq 2h)$	$(2h - \bar{q} + 1, \bar{q} - h, 2h - \bar{q} + 4, \bar{q} - h - 1)$
$\gamma_E(e_{\bar{q}} R_E): (\bar{q} = 2h + 1)$	$(2h - \bar{q} + 1, h, 2h - \bar{q} + 4, \bar{q} - h - 1)$

and

$\gamma_E(e_{\bar{q}} = k_{\bar{q}}l_{\bar{q}} R_E)$	$R_E = \{j_1, j_{h+1}, l_2, k_{h+2}\}$
$\gamma_E(e_{\bar{q}} R_E): (\bar{q} = 1)$	$(\bar{q}, h - \bar{q} + 1, 1, h + 1)$
$\gamma_E(e_{\bar{q}} R_E): (2 \leq \bar{q} \leq h)$	$(\bar{q}, h - \bar{q} + 1, \bar{q} - 2, h - \bar{q} + 3)$
$\gamma_E(e_{\bar{q}} R_E): (\bar{q} = h + 1)$	$(\bar{q}, \bar{q} - h, \bar{q} - 2, h - \bar{q} + 3)$
$\gamma_E(e_{\bar{q}} R_E): (\bar{q} = h + 2)$	$(2h - \bar{q} + 2, \bar{q} - h, \bar{q} - 2, 0)$
$\gamma_E(e_{\bar{q}} R_E): (h + 3 \leq \bar{q} \leq 2h + 1)$	$(2h - \bar{q} + 2, \bar{q} - h, 2h - \bar{q} + 3, \bar{q} - h - 1)$

Finally, for the edges of l -cycle $\{e_{\bar{q}} = l_{\bar{q}}l_{\bar{q}+1} | \bar{q} = 1, 2, 3, \dots, n\}$, the edge codes are

$\gamma_E(e_{\bar{q}} = l_{\bar{q}}l_{\bar{q}+1} R_E)$	$R_E = \{j_1, j_{h+1}, l_2, k_{h+2}\}$
$\gamma_E(e_{\bar{q}} R_E): (\bar{q} = 1)$	$(\bar{q} + 1, h - \bar{q} + 1, 0, h - \bar{q} + 2)$
$\gamma_E(e_{\bar{q}} R_E): (2 \leq \bar{q} \leq h - 1)$	$(\bar{q} + 1, h - \bar{q} + 1, \bar{q} - 2, h - \bar{q} + 2)$
$\gamma_E(e_{\bar{q}} R_E): (\bar{q} = h)$	$(\bar{q} + 1, 2, \bar{q} - 2, h - \bar{q} + 2)$
$\gamma_E(e_{\bar{q}} R_E): (\bar{q} = h + 1)$	$(2h - \bar{q} + 2, \bar{q} - h + 1, \bar{q} - 2, h - \bar{q} + 2)$
$\gamma_E(e_{\bar{q}} R_E): (h + 2 \leq \bar{q} \leq 2h)$	$(2h - \bar{q} + 2, \bar{q} - h + 1, 2h - \bar{q} + 2, \bar{q} - h - 1)$
$\gamma_E(e_{\bar{q}} R_E): (\bar{q} = 2h + 1)$	$(2, h + 1, 2h - \bar{q} + 2, \bar{q} - h - 1)$

From these edge codes in T_n , we find that $|A_i| = n$ ($1 \leq i \leq 5$) and the sum of all of these cardinalities is equal to $|E(T_n)|$ and which is $5n$. Moreover, all of these sets are pairwise disjoint, and so we find that no pair of two distinct edges in T_n are having the same edge metric coordinates, which implies that $dim_E(T_n) \leq 4$. Now, to finish the proof for this case, we prove that $dim_E(T_n) \geq 4$ by proving that there is no set R_E with $|R_E| = 3$, which can resolve every pair of distinct edges in T_n . On the contrary, suppose $dim_E(T_n) = 3$. Then, we have the following:

Edge Resolving Set	Contradictions
$R_E = \{j_1, j_i, j_g\}; (2 \leq i \leq h+1 \text{ and } 3 \leq g \leq n)$	$\gamma_E(k_1j_2 R_E) = \varphi_E(j_2k_2 R_E)$, for $2 \leq i \leq h+1$ and $3 \leq g \leq n$, a contradiction.
$R_E = \{k_1, k_i, k_g\}; (2 \leq i \leq h+1 \text{ and } 3 \leq g \leq n)$	$\gamma_E(k_nj_n R_E) = \varphi_E(j_nk_{n-1} R_E)$, for $2 \leq i \leq h+1$ and $3 \leq g \leq n-2$; $\gamma_E(k_{h+2}j_{h+3} R_E) = \varphi_E(j_{h+3}k_{h+3} R_E)$, for $2 \leq i \leq h+1$ and $n-1 \leq g \leq n$, a contradiction.
$R_E = \{l_1, l_i, l_g\}; (2 \leq i \leq h+1 \text{ and } 3 \leq g \leq n)$	$\gamma_E(j_1k_1 R_E) = \varphi_E(k_1j_2 R_E)$, for $2 \leq i \leq h+1$ and $3 \leq g \leq n$, a contradiction.
$R_E = \{j_1, j_i, k_g\}; (2 \leq i \leq h+1 \text{ and } 1 \leq g \leq n)$	$\gamma_E(k_nj_1 R_E) = \varphi_E(j_nj_1 R_E)$, for $2 \leq i \leq h+1$ and $1 \leq g \leq h$; $\gamma_E(j_hk_h R_E) = \varphi_E(j_hk_{h-1} R_E)$, for $2 \leq i \leq h+1$ and $h+1 \leq g \leq n$, a contradiction.
$R_E = \{j_1, j_i, l_g\}; (2 \leq i \leq h+1 \text{ and } 1 \leq g \leq n)$	$\gamma_E(k_nj_1 R_E) = \varphi_E(j_nj_1 R_E)$, for $2 \leq i \leq h+1$ and $1 \leq g \leq n-1$; $\gamma_E(j_nk_{n-1} R_E) = \varphi_E(j_nj_{n-1} R_E)$, for $2 \leq i \leq h$ and $g = n$; $\gamma_E(j_{h+1}j_{h+2} R_E) = \varphi_E(j_{h+1}k_{h+1} R_E)$, for $i = h+1$ and $g = n$, a contradiction.
$R_E = \{k_1, k_i, l_g\}; (2 \leq i \leq h+1 \text{ and } 1 \leq g \leq n)$	$\gamma_E(k_nj_1 R_E) = \varphi_E(j_nj_1 R_E)$, for $2 \leq i \leq h+1$ and $1 \leq g \leq n-1$; $\gamma_E(j_nk_{n-1} R_E) = \varphi_E(j_nj_{n-1} R_E)$, for $2 \leq i \leq h-1$ and $g = n$; $\gamma_E(j_{h+1}k_{h+2} R_E) = \varphi_E(j_{h+1}k_{h+1} R_E)$, for $i = h$ and $g = n$; $\gamma_E(j_1j_2 R_E) = \varphi_E(j_1j_n R_E)$, for $i = h+1$ and $g = n$, a contradiction.
$R_E = \{l_1, l_i, j_g\}; (2 \leq i \leq h+1 \text{ and } 1 \leq g \leq n)$	$\gamma_E(j_nj_1 R_E) = \varphi_E(k_nj_1 R_E)$, for $2 \leq i \leq h$ and $1 \leq g \leq h+1$; $\gamma_E(j_{h+1}k_h R_E) = \varphi_E(k_{h+1}l_{h+1} R_E)$, for $2 \leq i \leq h$ and $h+2 \leq g \leq n$; $\gamma_E(j_2j_3 R_E) = \varphi_E(k_2j_2 R_E)$, for $i = h+1$ and $g = 1, 2, h+3, h+4, \dots, n$; $\gamma_E(j_{h+2}j_{h+3} R_E) = \varphi_E(k_{h+2}j_{h+2} R_E)$, for $i = h+1$ and $3 \leq g \leq h+2$, a contradiction.

Edge Resolving Set	Contradictions
$R_E = \{l_1, l_i, k_g\}; (2 \leq i \leq h+1 \text{ and } 1 \leq g \leq n)$	$\gamma_E(j_nj_1 R_E) = \varphi_E(k_nj_1 R_E)$, for $2 \leq i \leq h+1$ and $1 \leq g \leq h$; $\gamma_E(j_nk_n R_E) = \varphi_E(j_1j_n R_E)$, for $2 \leq i \leq h+1$ and $h+1 \leq g \leq n-1$; $\gamma_E(j_nk_{n-1} R_E) = \varphi_E(j_nj_{n-1} R_E)$, for $2 \leq i \leq h-1$ and $g = n$; $\gamma_E(j_{h+1}j_{h+2} R_E) = \varphi_E(k_{h+1}j_{h+2} R_E)$, for $i = h$ and $g = n$; $\gamma_E(j_{h+2}j_{h+3} R_E) = \varphi_E(k_{h+2}j_{h+3} R_E)$, for $i = h+1$ and $g = n$, a contradiction.
$R_E = \{j_1, k_i, l_g\}; (1 \leq i \leq h+1 \text{ and } 1 \leq g \leq n)$	$\gamma_E(j_nj_1 R_E) = \varphi_E(k_nj_1 R_E)$, for $1 \leq i \leq h$ and $1 \leq g \leq n-1$; $\gamma_E(j_nj_{n-1} R_E) = \varphi_E(j_nk_{n-1} R_E)$, for $i = 1$ and $g = n$; $\gamma_E(j_1k_1 R_E) = \varphi_E(j_1j_n R_E)$, for $2 \leq i \leq h$ and $g = n$; $\gamma_E(j_{h+1}k_{h+1} R_E) = \varphi_E(k_{h+1}j_{h+2} R_E)$, for $i = h+1$ and $1 \leq g \leq n$, a contradiction.

By symmetry of T_n , other relations can be considered, and they will produce the same kind of contradictions. As a result, it can be found that $dim_E(T_n) = 4$, when n is odd. \square

Corollary 2. *The minimum independent edge resolving set for T_n has cardinality three (for even n) and four (for odd n), for $n \geq 6$.*

Remark 2. *For odd integer $n \geq 7$, we have $dim(T_n) < dim_E(T_n)$.*

5. Conclusion and Discussion

In this article, the edge metric dimension of two graphs S_n and T_n have been studied. For these classes of convex polytopes, we proved that $dim_E(S_n) = 4$ and $dim_E(T_n) = 3$ or 4 . We also found that $dim_E(S_n) > dim(S_n)$ for all n , $dim(T_n) < dim_E(T_n)$ for odd n , and $dim(T_n) = dim_E(T_n)$ for even n (a partial response to the problem reported in [14]). We also determined that the minimum edge metric generators for all of these convex polytopes are independent. In the future, we shall try to find the values for some other metric dimension variants such as fault-tolerant edge and vertex metric dimension, mixed metric dimension, local metric dimension etc. for the convex polytope families of graphs S_n and T_n [26].

Data Availability

Data sharing is not applicable to this article as no data set were generated or analyzed during the current study.

Conflicts of Interest

The authors declare no conflict of interest.

Authors' Contributions

All the authors have equally contributed to the final manuscript.

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