

A study on captive domination in graphs

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ABSTRACT

A subset $S \subset V(G)$ is called a captive dominating set of a graph G if S is a total dominating set and every vertex $v \in S$ is adjacent to at least one vertex which is not in S . Furthermore, a captive dominating set S is termed a minimal captive dominating set if no proper subset $S' \subset S$ qualifies as a captive dominating set. The minimum size of such captive dominating set in G is referred to as the captive domination number of G , denoted by $\gamma_{ca}(G)$. This paper investigates the relationship between the captive domination number and the order of a graph. We establish bounds on the captive domination number and present results for specific graph families obtained through various graph operations.

Keywords: domination, total domination, captive domination, inverse captive domination

1. Introduction

We consider finite, non-trivial, connected, and undirected graphs without loops or multiple edges, denoted by G , with vertex set $V(G)$ and edge set $E(G)$. For a vertex $v \in V(G)$, the open neighborhood is defined as $N_G(v) = \{u \in V(G) \mid uv \in E(G)\}$, while the closed neighborhood is $N_G[v] = \{v\} \cup N_G(v)$. The degree of v , denoted by $d_G(v)$, is the size of its open neighborhood, $|N_G(v)|$. The maximum and minimum degrees of the vertices in G are represented by $\Delta(G)$ and $\delta(G)$, respectively. When the graph G is clear from context, we simplify the notation to $N(v)$, $N[v]$, and $d(v)$ instead of $N_G(v)$, $N_G[v]$, and $d_G(v)$. The subgraph induced by a subset of vertices $D \subseteq V(G)$ is written as $G[D]$. The complement of G , denoted $\overline{G}$, is a graph with the same vertex set $V(G)$, in which two vertices are adjacent in $\overline{G}$ if and only if they are not adjacent in G .

The study of dominating sets and related concepts in graphs has been extensively explored due to

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their wide-ranging applications in addressing real-world problem-solving. Comprehensive discussions on dominating sets can be found in [2, 4, 7, 8, 10, 9]. For undefined terms and notations in graph theory, refer to [1, 17]. A subset $S \subseteq V(G)$ is a dominating set if every vertex v not in S is adjacent to at least one vertex in S . A dominating set S is minimal if no proper subset $S' \subset S$ is a dominating set of G . The minimum size of such dominating set is called the domination number and is denoted by $\gamma(G)$.

The concept of total domination was introduced by Cockayne et al. [3], which has since become a central topic in graph theory. A subset $S \subseteq V(G)$ is a total dominating set, abbreviated a TD-set if every vertex in $V(G)$ is adjacent to at least one vertex in S . Since $S = V(G)$ is always a TD-set for a graph without isolated vertices, every such graph has at least one TD-set. The total domination number, denoted $\gamma_t(G)$, is the minimum size of a TD-set in G . Extensive literature on total domination is available, including surveys in [13] and further discussions in [11].

In 2020, Al-Harere et al. [6] introduced the concepts of captive domination and inverse captive domination, which were further investigated by Vaidya and Vadhel [16]. A subset S of $V(G)$ is a captive dominating set if $G[S]$ has no isolated vertex (i.e., S is a TD-set) and every vertex $v \in S$ is adjacent to at least one vertex which is not in S . A captive dominating set S is minimal if no proper subset $S' \subset S$ is a captive dominating set. The minimum size of such captive dominating set is the captive domination number of G , denoted by $\gamma_{ca}(G)$. If S is a minimal captive dominating set and $V(G) - S$ contains a captive dominating set S' , then S' is called an inverse captive dominating set. The minimum size of a minimal inverse captive dominating set is the inverse captive domination number, denoted by $\gamma'_{ca}(G)$.

In this paper, we analyze the captive domination number in relation to the order of a graph. We establish bounds for this parameter and derive results for specific graph families obtained via various graph operations. Additionally, we compute domination-related parameters for the graph and its shadow graph, the join of two graphs, the corona of graphs, and the Mycielski graph of a given graph.

Definition 1.1. The join of two graphs G and H is denoted by $G + H$ and is the graph with vertex set $V(G + H) = V(G) \cup V(H)$ and edge set $E(G + H) = E(G) \cup E(H) \cup \{uv : u \in V(G), v \in V(H)\}$.

Definition 1.2. The corona of two graphs G and H is denoted by $G \circ H$ and is the graph obtained by taking one copy of G of order n and n copies of H , and then joining the i^{th} vertex of G to every vertex in the i^{th} copy of H .

Definition 1.3. [15] The m -shadow graph $D_m(G)$ of a connected graph G is obtained by taking m copies of G , say $G_1, G_2, \dots, G_m$, then join each vertex v in G_i to the neighbours of the corresponding vertex u in G_j , $1 \leq i, j \leq m$.

Definition 1.4. [14] For a graph G with $V(G) = \{v_1, v_2, \dots, v_n\}$, let $U = \{u_1, u_2, \dots, u_n\}$ be a disjoint copy of $V(G)$ and let w be a new vertex. Then the Mycielski graph $\mu(G)$ of G is the graph with vertex set $V(\mu(G)) = V(G) \cup U \cup \{w\}$ and edge set $E(\mu(G)) = E(G) \cup \{v_i u_j : v_i v_j \in E(G)\} \cup \{w u_i / 1 \leq i, j \leq n\}$.

Theorem 1.5. [6] Let G be a graph of order n with captive dominating set D and captive domination number $\gamma_{ca}(G)$, then

- a) The order of G is $n \geq 3$,
- b) For every $v \in D$, $d(v) \geq 2$,
- c) For every $v \in D$, $N(v) \cap D \neq \phi$ and $N(v) \cap (V(G) - D) \neq \phi$,
- d) $\gamma(G) \leq \gamma_t(G) \leq \gamma_{ca}(G)$.

Theorem 1.6. [16] A TD-set D in a graph G is a captive dominating set if either of the following conditions hold:

- a) $|epn(v, D)| \geq 1$, for every $v \in D$.
- b) $|ipn(v, D)| = 1$, for every $v \in D$ and $\delta \geq 2$.
- c) $V(G) - D$ is a dominating set in G .

Theorem 1.7. [16] Let D and $V(G) - D$ both are TD-set of G if and only if D and $V(G) - D$ both are captive dominating set of G .

Theorem 1.8. [5] Let G be a connected graph then $\gamma(Sp(G)) = \gamma_t(G)$.

Theorem 1.9. [12] Let G be a graph with minimum degree at least 2 with no C_5 -component, then $V(G)$ can be partitioned into a dominating set D and a TD-set T .

2. Main results

In the following Theorem, we explore the intriguing connection between Hamiltonian graphs and captive dominating sets. We demonstrate the existence of a captive dominating set in Hamiltonian graphs under certain conditions on their order. Specifically, we establish that for Hamiltonian graphs with an order that is divisible by either 3 or 4, a captive dominating set can always be found.

Theorem 2.1. Let G be a Hamiltonian graph of order $n \equiv 0(\text{mod } 3)$ or $n \equiv 0(\text{mod } 4)$, then G admits a captive dominating set. Moreover if $n \equiv 0(\text{mod } 3)$, then $\gamma_{ca}(G) \leq \frac{2n}{3}$ and if $n \equiv 0(\text{mod } 4)$, then $\gamma_{ca}(G) \leq \frac{n}{2}$.

Proof. Let G be a Hamiltonian graph with order $n \equiv 0(\text{mod } 3)$. Let $W = v_1 v_2 \dots v_n v_1$ be a Hamiltonian cycle in a graph G .

Consider $T = \{v_1, v_2, v_4, v_5, \dots, v_{n-2}, v_{n-1}\}$ with $|T| = \frac{2n}{3}$, then T is a TD-set as for any $v_i \in T$, $i = 1, 2, 4, 5, \dots, n-1$. Now, if $i \equiv 1(\text{mod } 3)$, then v_i is adjacent to $v_{i+1} \in T$ and if $i \equiv 2(\text{mod } 3)$, then v_i is adjacent to $v_{i-1} \in T$.

Consider $D = V(G) - T$, then $D = \{v_3, v_6, v_9, \dots, v_n\}$ with $|D| = \frac{n}{3}$. Moreover, D is a dominating set as each $v_i \in V(G) - D = T$ is adjacent to either v_{i-1} or v_{i+1} .

Thus, $V(G) = D \cup T$, where D is a dominating set and T is a TD-set.

Hence, from Theorem 1.6, T is a captive dominating set.

Since, $|T| = \frac{2n}{3}$, we can conclude that $\gamma_{ca}(G) \leq \frac{2n}{3}$.

Let G be a Hamiltonian graph with order $n \equiv 0(\text{mod } 4)$. Let $W = v_1 v_2 \dots v_n v_1$ be a Hamiltonian cycle in a graph G .

Consider $T_1 = \{v_1, v_2, v_5, v_6, \dots, v_{n-3}, v_{n-2}\}$ and $T_2 = \{v_3, v_4, v_7, v_8, \dots, v_{n-1}, v_n\}$ with $|T_1| = |T_2| = \frac{n}{2}$. Then T_1 and T_2 both are TD-set and $V = T_1 \cup T_2$ as for each $v_i \in T_1$, if $i \equiv 1(\text{mod } 4)$, then v_i is

adjacent to $v_{i+1} \in T_1$, and if $i \equiv 2(\text{mod } 4)$, then v_i is adjacent to $v_{i-1} \in T_2$.

Similarly for each $v_j \in T_2$, if $j \equiv 3(\text{mod } 4)$, then v_j is adjacent to $v_{j+1} \in T_1$, and if $j \equiv 0(\text{mod } 4)$, then v_j is adjacent to $v_{j-1} \in T_2$.

Moreover, each vertex in T_1 is adjacent to at least one vertex of T_2 and vice versa. Thus, $V = T_1 \cup T_2$, where T_1 and T_2 both are TD-set of graph G . Hence, from Theorem 1.7, T_1 and T_2 both are captive dominating set of graph G .

Since, $|T_1| = |T_2| = \frac{n}{2}$, we conclude that $\gamma_{ca}(G) \leq \frac{n}{2}$. $\square$

Note that in the case of non-Hamiltonian graphs with order $n \equiv 0(\text{mod } 3)$ or $n \equiv 0(\text{mod } 4)$, such graphs can still admit a captive dominating set (as illustrated in Figure 1(a) and 1(b)). Similarly, for Hamiltonian graphs with orders not divisible by 3 or 4, they can also have a captive dominating set (as shown as solid vertices in Figure 1(c)). This highlights the fact that the presence or absence of Hamiltonian cycles does not solely determine the existence of a captive dominating set in a graph.

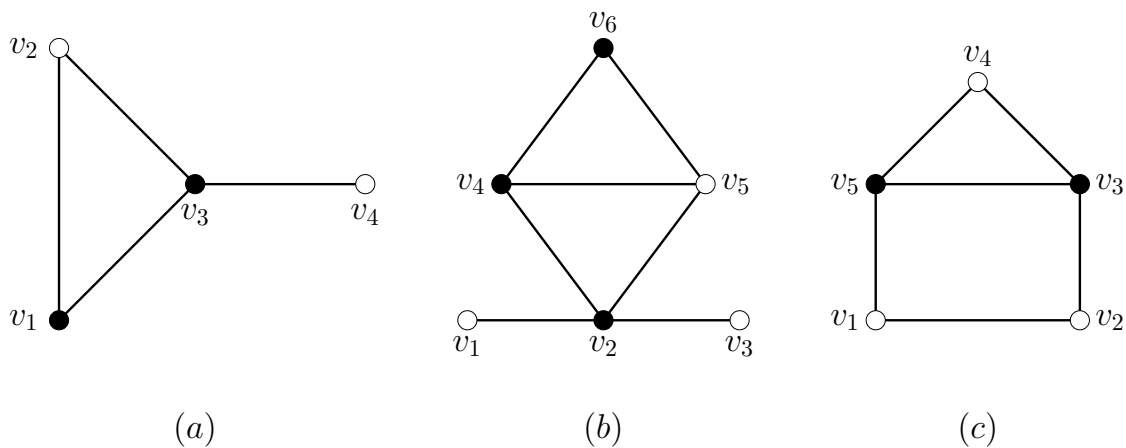


Fig. 1. Graphs with its captive dominating set

Corollary 2.2. Let G be a Hamiltonian graph with order $n \equiv 0(\text{mod } 4)$, then G admits an inverse captive dominating set.

Theorem 2.3. Let G be a graph with $\delta(G) \geq 2$ with no C_5 -component, then G always have a captive dominating set.

Proof. Let G be a graph with minimum degree at least 2 with no C_5 -component, then by Proposition 1.9, $V(G)$ can be partitioned into a dominating set D and a TD-set T . Thus by Theorem 1.6, G admits a captive dominating set. $\square$

Corollary 2.4. Let G be any graph that admits an inverse captive dominating set, then $n \geq 4$ and $\delta(G) \geq 2$.

Theorem 2.5. Let G admits captive and inverse captive dominating sets, then $\gamma_{ca}^{-1}(G) \leq \frac{n \cdot \Delta(G)}{1 + \Delta(G)}$.

Proof. Suppose that G admits captive and inverse captive dominating sets.

Now for any non-trivial graph G which admits a captive dominating set, we have,

$$\left\lceil \frac{n}{1 + \Delta(G)} \right\rceil \leq \gamma(G) \leq \gamma_t(G) \leq \gamma_{ca}(G) \Rightarrow \frac{n}{1 + \Delta(G)} \leq \left\lceil \frac{n}{1 + \Delta(G)} \right\rceil \leq \gamma_{ca}(G).$$

Now,

$$\gamma_{ca}(G) + \gamma_{ca}^{-1}(G) \leq n \Rightarrow \gamma_{ca}^{-1}(G) \leq n - \gamma_{ca}(G) \Rightarrow \gamma_{ca}^{-1}(G) \leq n - \frac{n}{1 + \Delta(G)} \Rightarrow \gamma_{ca}^{-1}(G) \leq \frac{n \cdot \Delta(G)}{1 + \Delta(G)}.$$

□

Corollary 2.6. *Let G be any graph with $\text{diam}(G) \geq 3$, then $\gamma_{ca}(\overline{G}) = 2$.*

Theorem 2.7. *Let G and H be any graph, then $\gamma_{ca}(G + H) = 2$.*

Proof. Let G be a non-trivial graph with $V(G) = \{v_1, v_2, \dots, v_n\}$ and H be any graph with $V(H) = \{u_1, u_2, \dots, u_m\}$. Now consider the graph $G + H$, then each vertex v_i is adjacent to every vertex of $V(H)$ and each vertex u_j is adjacent to every vertex of $V(G)$; $1 \leq i \leq n$, $1 \leq j \leq m$.

Consider $D = \{u, v\}$ with $|D| = 2$, where $u \in V(H)$ and $v \in V(G)$, then D is minimal TD-set of graph $G + H$. Moreover each vertex $u \in V(H) - D$ is adjacent to v and every vertex $v \in V(G) - D$ is adjacent to u . Thus, D is a minimal captive dominating set of $G + H$. Hence, $\gamma_{ca}(G + H) = 2$. □

Theorem 2.8. *Let $\mathcal{X}$ be the family of TD-set of graph G and $\mathcal{Y}$ be the family of TD-set of graph H . Then $\mathcal{X} \cup \mathcal{Y}$ is the family of captive dominating set of $G + H$.*

Proof. Let $\mathcal{X}$ be the family of TD-set of graph G . Let $D \in \mathcal{X}$ be a total dominating of graph G and $S \in \mathcal{Y}$ be a total dominating of graph H . Note that $V(H) \subseteq V(G + H) - D$ and $V(G) \subseteq V(G + H) - S$.

Since, D is a TD-set of G and for each $v \in D$, there exists $u \in V(H)$ such that $u \in N(v) \cap (V(G + H) - D)$ and so $N(v) \cap (V(G + H) - D) \neq \emptyset$. Similarly S is a TD-set of H and for each $u \in S$, there exists $v \in V(G)$ such that $v \in N(u) \cap (V(G + H) - S)$ and so $N(u) \cap (V(G + H) - S) \neq \emptyset$. Hence, D and S both are minimal captive dominating sets of $G \cup H$. Then $\mathcal{X} \cup \mathcal{Y}$ is the family of captive dominating set of $G + H$. □

Lemma 2.9. *Let G be any graph without isolated vertices, then $D_m(G)$ always contains a captive dominating set.*

Proof. Consider a graph G without isolated vertices. Let D be a TD-set of G . Then D serves as a TD-set of $D_m(G)$. Since $D \subseteq V(G)$, every vertex v in D is adjacent to at least one vertex in $V(D_m(G)) - V(G)$. Therefore, D qualifies as a captive dominating set of $D_m(G)$. □

Theorem 2.10. *For any connected, non-trivial graph G , $\gamma_{ca}(D_m(G)) = \gamma_t(G)$.*

Proof. Let S be a TD-set of G such that $|S| = \gamma_t(G)$. Let v^i represent the corresponding vertices of v in $D_m(G)$ for $1 \leq i \leq m$, where $N_G(v) = N_G(v^i) \cap V(G)$ for each i . Since S is a TD-set of G ,

it follows that for every $v \in V(G)$, $N(v) \cap S \neq \emptyset$. Let $u \in N(v) \cap S$; then, for each i , the vertex v^i is dominated by u . As a result, S is a TD-set of $D_m(G)$. Moreover, since every vertex $v \in S$ is adjacent to at least one vertex in $V(D_m(G)) - V(G)$, we conclude that S is a captive dominating set of $D_m(G)$. Hence, $\gamma_t(G) \leq \gamma_{ca}(D_m(G))$.

Now, let T be a captive dominating set of $D_m(G)$ such that $|T| = \gamma_{ca}(D_m(G))$. Suppose $T = T_1 \cup T_2$, where $T_1 \subseteq V(G)$ and $T_2 \subseteq V(D_m(G)) - V(G)$. If $T_2 = \emptyset$, then $T = T_1$ forms a TD-set of G . Assume instead that $|T_2| \neq \emptyset$. For any vertex $v' \in T_2$, consider the set $T' = (T - \{v'\}) \cup \{x\}$, where x is any vertex in $N[v] - T$. Repeating this process produces a dominating set S of $D_m(G)$ such that $|T| = |S|$, where $S \subseteq V(G)$ and $G[S]$ contains no isolated vertices. Consequently, $\gamma_{ca}(D_m(G)) = |T| = |S| \geq \gamma_t(G)$. Thus, $\gamma_{ca}(D_m(G)) = \gamma_t(G)$. $\square$

Theorem 2.11. *Let G be any connected non-trivial graph then $\gamma(D_m(G)) = \gamma_t(G)$.*

Proof. Let S be any dominating set of $D_m(G)$ with $|S| = \gamma(D_m(G))$. If $S \subseteq V(G_i)$ in $D_m(G)$ for some $i > 1$. Then by replacing each element of S by its corresponding vertex in $V(G)$, we get, S a dominating set of G . Since, $Spl_m(G) \subset D_m(G)$ with $V(Spl_m(G)) = V(D_m(G))$, by Theorem 1.8, S is a TD-set of G . Therefore, we assume that $S \not\subseteq V(G_i)$ and $S \subseteq D_m(G)$, for some $i > 1$. Let $S = S_1 \cup S_2$, where $S_1 \subseteq V(G)$ and $S_2 \subseteq V(D_m(G)) - V(G)$. If $S_2 = \emptyset$ then $S = S_1$ is a TD-set of G . Hence, we may assume that $|S_2| \neq \emptyset$. Let $u \in S_2$ be a corresponding vertex of v in $D_m(G)$. Consider $S' = (S - \{u\}) \cup \{x\}$, where x is any vertex in $N[v] - S$. Clearly S' is a dominating set of $D_m(G)$. By repeating this process we obtain a dominating set D of $D_m(G)$ such that $|S| = |D|$, where $D \subseteq V(G)$ and $G[D]$ has no isolated vertex. Hence, $\gamma(D_m(G)) = |A| = |D| \geq \gamma_t(G)$.

From Theorem 2.10, $\gamma(D_m(G)) = \gamma_t(G)$ and $\gamma(D_m(G)) \leq \gamma_{ca}(D_m(G))$. Hence, $\gamma(D_m(G)) \leq \gamma_t(G)$. Thus, $\gamma(D_m(G)) = \gamma_t(G)$. $\square$

Corollary 2.12. *For any connected graph G , the following equality holds:*

$$\gamma_{ca}(D_m(G)) = \gamma_t(G) = \gamma(D_m(G)) = \gamma_t(D_m(G)).$$

Corollary 2.13. *Let G be a connected, non-trivial graph. Then, $\gamma_{ca}(D_m(G)) = \gamma(G)$ if and only if $\gamma(G) = \gamma_t(G)$.*

Corollary 2.14. *For any connected, non-trivial graph G , $\gamma_{ca}(D_m(G)) = \gamma_{ca}(G)$ if and only if $\gamma_{ca}(G) = \gamma_t(G)$.*

The proof of above corollaries follows from Theorem 2.10 and 2.11.

Theorem 2.15. *For any connected nontrivial graph G , $\gamma_{ca}(\mu(G)) = \gamma_t(G) + 1$.*

Proof. Let D be a minimal TD-set of graph G with $\gamma_t(G) = |D|$. Then $D \cup \{x\}$ is a TD-set of $\mu(G)$ as $G[V(\mu(G)) - \{x\}] = Sp(G)$, where $x \in V(\mu(G))$ with $d_{\mu(G)}(x) = |V(G)| + 1$. Note that $d_{\mu(G)}(v) = 2d_G(v)$ and so for each $v \in D \cap V(G)$, there exist $u \in V(G') - D$ such that $uv \in E(\mu(G))$. Thus, $D \cup \{x\}$ is a captive dominating set of $\mu(G)$. Hence, $\gamma_t(G) + 1 \geq \gamma_{ca}(\mu(G))$.

Let A be a captive dominating set of $\mu(G)$ with $\gamma_{ca}(\mu(G)) = |A|$. Let $z \in A$ then there exist at least one $x' \in V(\mu(G))$ such that $x' \in A$. Consider $A_1 = (A - \{z\}) \cup \{y\}$, where $y \in N(x') \cap V(G)$.

Here, $N(x') \cap V(G) \cap A \neq \phi$ as A is a captive dominating set. If $v' \in A_1 \cap V(\mu(G))$ then consider $A_2 = (A_1 - \{v'\}) \cup \{u\}$, where u is any vertex in $(N[v] \cap V(G)) - A_1$. By successively applying this approach, we derive a TD-set D of G such that $|A| = |D|$, where $D \subseteq V(G)$ and $G[D]$ contains no isolated vertex. Therefore, we have $\gamma_t(G) + 1 \leq \gamma_{ca}(\mu(G))$. Thus, $\gamma_{ca}(\mu(G)) = \gamma_t(G) + 1$. $\square$

Theorem 2.16. *For any connected non-trivial graph G and H with order n and m respectively, $\gamma_{ca}(G \circ H) = n$.*

Proof. Let G and H be two graph with order n and m , respectively. Consider $D = V(G)$, then D is a TD-set of graph $G \circ H$ and $\gamma_t(G \circ H) = n$. Since each vertex $v \in D$ is adjacent to atleast one vertex in $V(G \circ H) - V(G)$, we get, $N(v) \cap [V(G \circ H) - V(G)] \neq \phi$. Thus, D is a captive dominating set of $G \circ H$. Hence, $\gamma_{ca}(G \circ H) = n$. $\square$

3. Benefits of using social media - an application of captive dominating sets

Over the last two decades, both the world at large and India, in particular, have experienced rapid growth in the field of Information Technology. Today in India, the usage of Social Media Platforms (SMPs) has significantly increased, allowing users to chat, share information, and create web content. Teenagers are among the most active users of SMPs and are considered particularly vulnerable. In many cases, they become addicted, which affects their careers, as they are unable to distinguish between information that should or should not be shared. This has had far-reaching impacts on the academic performance of students.

By using any SMP, the user grants access to the platform's applications, allowing it to access their name, phone contacts, email, place of work, geographical location, education details, and data from their search history, views, opinions, interests, attitudes, behaviors, access patterns, time spent on the platform, as well as personal and professional life details, family and relationship information, and many other activities. In short, the user's personal information becomes publicly available, leading to privacy violations.

When used thoughtfully, however, social media can be a blessing. Various SMPs can be linked together to share data such as interests, skills, academic and career opportunities, and cultural information, all of which can foster personal growth. In this context, SMPs can be modeled as vertices of a captive dominating set, denoted by D , while users represent the vertices of $V(G) - D$.

For example, if a user utilizes both SMP A and SMP B, information about that user can be fetched by A from B and vice versa. This means that useful information can be obtained from either SMP. Using the captive domination model allows information to be accessed more quickly and efficiently, benefiting users by offering integrated services that understand their preferences across multiple platforms.

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