J. COMBIN. MATH. COMBIN. COMPUT. 127b (2025) 5711--5730

Journal of Combinatorial Mathematics
and Combinatorial Computing
www.combinatorialpress.com/jcmcc

A Study on the Construction of a Predictive Model for Physical Education Teachers' Professional Development Trajectories Supported by ARIMA Algorithm Driven by Physical Education Reforms

Xiao Zhou[1, ✉]

[1] *Department of Physical Education, Chengdu Technological University, Chengdu, Sichuan, 611730, China*

ABSTRACT
The professional development of physical education teachers is the direction and basic requirement of modern education development, which is better promoted by strengthening the tracking and prediction of the trajectory of the professional development of physical education teachers. In this paper, a combined ARIMA-LSTM model is established to visualize the PE teachers' professional development trajectory by predicting their professional development scores, using the advantages of ARIMA model in handling linear time series data, while combining the powerful ability of LSTM network in capturing the long-term dependency of data. Three physical education teachers were randomly selected as research subjects to predict their PE teacher professional development trajectories. The root mean square error (RMSE) and mean absolute percentage error (MAPE) values were used as the assessment indexes of the model, and the MAPE and RMSE of the ARIMA-LSTM model were less than those of the ARIMA model and the LSTM model for the physical education teachers in No. 1 and No. 3. And on the prediction of physical education teacher No. 2, the MAPE comparison of ARIMA-LSTM model and LSTM model increased by 1.12%, but the RMSE decreased by 73.4563, and the prediction curve of the professional development score was close to the original sequence, and the ARIMA-LSTM model still showed better prediction effect.

Keywords: ARIMA model, LSTM network, ARIMA-LSTM model, professional development of physical education teachers

1. Introduction

With the steady advancement of quality education and basic education curriculum reform, teacher professionalization has become an important part of China's education reform and development, and

✉ Corresponding author.
E-mail address: zhouxiao@cdtu.edu.cn (X. Zhou).
Received 03 February 2024; Revised 12 April 2024; Accepted 10 November 2024; Published Online 16 April 2025.
DOI: 10.61091/jcmcc127b-315

an inevitable choice for teacher education reform and innovation [1-2]. Physical education teachers are an important part of students' physical training and education work, and they are responsible for the important tasks of cultivating students' healthy body, improving students' motor skills and shaping students' masculinity, while the professional development of physical education teachers is of great significance to improve the level of teaching and to enhance the quality of the school sports career [3-6].

The professional development of physical education teachers refers to the professional growth process in which physical education teachers gradually acquire professional knowledge and skills in physical education through long-term and systematic specialized training (including physical fitness, skills, etc.) and lifelong learning throughout their careers, and continuously improve their professional quality and professionalism in teaching practice to achieve professional autonomy and become an excellent professional worker in physical education [7-10]. In this process, the professional development from “ordinary people” to “physical education workers” has been realized, and the professional development of physical education teachers essentially emphasizes the professional growth and development of individual physical education teachers [11-13]. On the one hand, this process is long, that is, the individual development and growth of physical education teachers is ongoing, throughout the career of individual physical education teachers [14-16]. On the other hand, it shows the growth nature, i.e., the professional development of physical education teachers has a cumulative and continuous character, and it is a process of professional growth based on what has been learned in the past [17-18].

Tannehill,D. et al. examined Continuing Professional Development (CPD) in physical education in European countries, describing current international CPD in general education and physical education, identifying current trends and issues in physical education, aiming to guide teacher learning, and providing insights for teacher education programs and CPD providers [19]. Cardina,C.E. and DeNysschen ,C. describe professional development (PD) for physical education teachers and compare teachers in other subjects, noting that physical education teachers report less PD activity compared to teachers in other subjects, and that newly recruited physical education teachers report higher levels of professional support compared to other newly recruited teachers, with recommendations [20]. Ward,P. and vanderMars,H. aim to draw attention to the importance of attention to the educational practices of physical education teachers, highlighting that physical education teachers have a responsibility to engage in CPD and that PD should be viewed as an active process as it is positive and ongoing, and that national organizations have a responsibility to provide teachers with opportunities for PD through events [21]. Braga,L. et al. aimed to explore the perceptions of physical education teachers about their experiences of a research-informed CPD program, and the impact it had on teachers' implementation of innovative content professional preparation, demonstrating the need for research-informed CPD efforts in physical education [22]. Leeder,T.M. and Beaumont,L.C. examined the opportunities and challenges encountered by physical education teachers in the United Kingdom when delivering lifestyle sport in order to understand their professional development needs, concluding that teachers have different conceptualizations of lifestyle sport and that factors such as environmental and personal are influencing their practice [23]. Sum,K.W.R. et al. analyzed the effects of CPD on teachers' physical literacy and self-efficacy as well as students' physical literacy and participation in physical activity (PA) using a randomized controlled trial methodology revealing that teachers' physical literacy and self-efficacy influence students' physical literacy and PA participation [24]. Beni,. S. designed a PD program by introducing teachers to innovative and effective methods of physical education and understanding their experiences in the PD process, and the results showed that teachers valued communities of practice and modeling when learning to implement meaningful physical education, and that there was some tension between the ideal and realistic forms of PD [25]. The status of CPD for physical education teachers was analyzed to examine the characteristics, influences on effectiveness, and effects of teachers' perceptions of continuing professional education, and the results indicated that physical education CPD has characteristics such as an emphasis on motor skill development and revealed that the traditional model of continuing professional development is limited in promoting continuous learning among teachers [26]. Karaiskos, L., et al. explored the relationship between physical education teacher training, professional development, and self efficacy by finding a positive correlation between training and physical education teachers' self-efficacy in all aspects of teaching and learning, while self-efficacy, classroom management, and student engagement were all positively correlated [27]. Durden-Myers,E.J. and Keegan,S. examined the role of physical education teaching practices of the multifaceted classroom teacher and the professional development of the specialized physical education teacher in fostering physical literacy role in the development of active students, emphasizing the premise of providing teachers with effective and responsive professional development [28].

In order to achieve the prediction of the professional development trajectory of physical education teachers, this paper introduces the theory of ARIMA model, and adopts a hybrid model ARIMA-LSTM based on modeling linear and nonlinear components of the time series, respectively. Firstly, the linear component is calculated by ARIMA model and then the nonlinear component of the time series is predicted with the help of LSTM model and the sum of the error values between the ARIMA model and LSTM model is calculated. The weights of the model are calculated based on the error values, and finally the weight values of the model and the final ARIMA-LSTM hybrid model for each prediction are obtained. Taking the employment development data of physical education teachers in Shanghai, China, from 2018 to 2024 as the research data, the professional development score of physical education teachers is used as an indicator to measure the changes in the professional development trajectory of physical education teachers, and the predictive effect of the ARIMA-LSTM hybrid model on the professional development trajectory of physical education teachers is analyzed.

2. Research Objects and Methods

2.1. Objects of study

In view of the relative lack of research on the professional development of physical education teachers, this paper takes the professional development trajectory of physical education teachers as the object of research to predict the professional development trajectory of physical education teachers. Since it is difficult to determine the professional development trajectory of physical education teachers through specific figures, this paper takes the professional development score of physical education teachers as an indicator to measure the changes in the professional development trajectory of physical education teachers. The professional development score of physical education teachers covers the following scoring items: scientific research achievements of physical education teachers (number of papers published, number of projects), participation rate of physical education teachers in training, promotion of physical education teachers' titles, and number of innovative practices of physical education teachers.

2.2. Data sources

The data source for this experiment is the employment development data of physical education teachers in Shanghai, China from 2018-2024.

2.3. Research methodology

In this paper, the ARIMA-LSTM combination model will be constructed based on the ARIMA model theory to provide a methodology for the prediction of physical education teachers' professional development trajectory.

2.3.1. ARIMA modeling theory:

1) AR model. The AR model is an all-pole model and the most common time series smoothing model that predicts the values of variables at subsequent points in time from the values of variables at earlier points in time in the series. Mathematically, for a time series X_t with zero expectation, residuals conforming to a white noise distribution $\varepsilon_t \sim WN(0,\sigma^2)$ and a regression order of p, the mathematical expression for its $AR(p)$ model can be written:

$$X_t = a_1 X_{t-1} + a_2 X_{t-2} + \cdots + a_p X_{t-p} + \varepsilon_t \tag{1}$$

where $a=(a_1,a_2,...,a_p)^T a=(a_1,a_2,...,a_p)^T$ denotes the autoregressive coefficient vector.

Multiplying both sides of equation (1) simultaneously by X_{t-k} and taking the expectation, the autocovariance function of the time series can be obtained as:

$$\gamma_k = E(X_t X_{t-k}) = \sum_{i=1}^{p} a_i E(X_{t-i} X_{t-k}) + E(\varepsilon_t X_{t-k}) \tag{2}$$

where γ_k is the self-covariance function, and since ε_t is a white noise sequence, it can be shown that $E(\varepsilon_t X_{t-k})=0$. When $t \ge k$, the Yule-Walker equation for the time series X_t can be further derived based on the nature of the self-covariance function and the independence of the white noise [29].The Yule-Walker equation utilizes the self-covariance or autocorrelation function of the time series to estimate autoregressive coefficients as:

$$\begin{cases} \gamma_1 = a_1\gamma_0 + a_2\gamma_1 + \cdots + a_t\gamma_{t-1} \\ \gamma_2 = a_1\gamma_1 + a_2\gamma_2 + \cdots + a_t\gamma_{t-2} \\ \quad\cdots\cdots\cdots \\ \gamma_t = a_1\gamma_{t-1} + a_2\gamma_{t-2} + \cdots + a_t\gamma_0 \end{cases} \tag{3}$$

Noting that $\gamma_t = (\gamma_1,\gamma_2,...,\gamma_t)^T$, Γ_t denotes the self-covariance matrix, and:

$$\Gamma_t = \begin{bmatrix} \gamma_0 & \gamma_1 & \cdots & \gamma_{t-1} \\ \gamma_1 & \gamma_0 & \cdots & \gamma_{t-2} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{t-1} & \gamma_{t-2} & \cdots & \gamma_0 \end{bmatrix} \tag{4}$$

$$a_t = (a_1,a_2,...,a_p,0,...,0)^T \tag{5}$$

Then equation (3) can be expressed as $\gamma_t = \Gamma_t a_t$.

Based on the self-covariance function of the time series, the relationship between the time series X_t at the current moment and its observations at the previous k moments can be derived, and this relationship is expressed in terms of the autocorrelation coefficient (ACF), which is defined by the expression:

$$\rho_k = \frac{E(X_t X_{t-k})}{E(X_t)^2} = \frac{\gamma_k}{\gamma_0} \tag{6}$$

where ρ_k represents the kth order autocorrelation coefficient, and γ_k and γ_o are the kth order autocovariance and the 0th order autocovariance (i.e., the overall autocovariance), respectively. When $k=0$, the autocorrelation coefficient $\rho_0 = 1$, indicating that observations at the same moment are perfectly correlated with themselves.

Consider the time series X_{t-k} for which the kth moment is known, and the correlation between X_{t-k} and X_t can be calculated directly. In this case, the relationship between the two is described by the partial autocorrelation function (PACF), whose measure is the partial autocorrelation coefficient, expressed as:

$$\rho_{X_t,X_{t-k}} = \frac{E(X_t X_{t-k})}{E(X_{t-k})^2} \tag{7}$$

According to equation (4), the self-covariance matrix Γ_t of the time series has a positive definite nature and the coefficients are all zero when $t > p$, indicating that the biased autocorrelation coefficients of the time series exhibit truncated tails.

2) MA model. The stochastic character of time series makes its intrinsic correlation not easy to observe and analyze directly. The moving average (MA) model aims to capture the random fluctuations and white noise components in the time series. For a time series with mean value μ, its q-order MA model expression can be expressed as:

$$X_t = \mu + \varepsilon_t - b_1\varepsilon_{t-1} - \cdots - b_q\varepsilon_{t-q} \tag{8}$$

where $b=(b_1,b_2,...,b_q)^T$ are the coefficients of the moving average model, and $\{\varepsilon_t\}$ is the white noise sequence satisfying $\varepsilon_t \sim WN(0,\sigma^2)$. From Eq. It can be seen that the MA model is considered as a smooth sequence with a mean value of μ. When $\mu=0$, the model is called a centered $MA(q)$ model.

Let $B(\mathrm{B}) = 1 - b_1\mathrm{B} - b_2\mathrm{B}^2 - \cdots - b_q\mathrm{B}^q, B(\mathrm{B}) \neq 0$. When $|\mathrm{B}| \leq 1$ i.e. $B(\mathrm{B}) = 1 - b_1\mathrm{B} - b_2\mathrm{B}^2 - \cdots - b_q\mathrm{B}^q, B(\mathrm{B}) \neq 0$, and the $MA(q)$ model has invertibility when $|\mathrm{B}| \leq 1$, i.e., when all the zeros of $B(\mathrm{B})$ lie outside the unit circle. Further, when $\mu=0$, it can be expressed as:

$$X_t = B(\mathrm{B})\varepsilon_t \tag{9}$$

Eq. (9) is obtained by dividing both sides of $B(\mathrm{B})$ simultaneously:

$$\varepsilon_t = \frac{X_t}{B(\mathrm{B})} = X_t B(\mathrm{B})^{-1} \tag{10}$$

By expanding $B(\mathrm{B})^{-1}$ using Taylor's formula, we get:

$$B^{-1}(\mathrm{B}) = \sum_{j=0}^{\infty} \varphi_j \mathrm{B}^j \tag{11}$$

Let $I(\mathrm{B}) = B^{-1}(\mathrm{B})$, then we have:

$$\varepsilon_t = I(\mathrm{B})X_t \tag{12}$$

Substituting Eq. (10) into Eq. (9) gives:

$$X_t = B(\mathrm{B})I(\mathrm{B})X_t \tag{13}$$

Substituting $B(\mathrm{B}) = 1 - b_1\mathrm{B} - b_2\mathrm{B}^2 - \cdots - b_q\mathrm{B}^q$ into Eq. (11) and expanding it, and utilizing the method of determining coefficients, we get :

$$\begin{cases} \varphi_0 = 1 \\ \varphi_j = \sum_{k=1}^{j} \theta_k \varphi_{j-k}, j \geq 1 \end{cases} \tag{14}$$

Among them:

$$\theta_k = \begin{cases} b_k & k \leq q \\ 0 & k > q \end{cases} \tag{15}$$

Given that the MA model focuses on modeling the white noise component of the time series, finding the variance yields the variance of the time series X_t:

$$\begin{aligned} Var(X_t) &= Var(\mu + \varepsilon_t - b_1\varepsilon_{t-1} - \cdots - b_q\varepsilon_{t-q}) \\ &= (1 + b_1^2 + \cdots + b_q^2)\sigma^2 \end{aligned} \tag{16}$$

Thereby deriving its self-covariance function:

$$\gamma_k = E((X_t - \mu)(X_{t-k} - \mu)) = \begin{cases} \sigma^2 \sum_{j=0}^{q} b_j^2 & k = 0 \\ \sigma^2(-b_k + \sum_{j=1}^{q-k} b_j b_{j+k}) & 1 \le k \le q \\ 0 & k > q \end{cases} \tag{17}$$

This equation illustrates that the variance of the time series depends not only on the variance of the white noise σ^2, but is also affected by the moving average coefficients $b_1, b_2, \ldots, b_q$, reflecting the contribution of each white noise component of the MA model to the overall variance.

The autocorrelation coefficient of the $MA(q)$ model is:

$$\rho_k = \frac{\gamma_k}{\gamma_0} = \begin{cases} 1 & k = 0 \\ \dfrac{-b_k + \sum_{j=1}^{q-k} b_j b_{j+k}}{1 + b_1^2 + \cdots + b_q^2} & 1 \le k \le q \\ 0 & k > q \end{cases} \tag{18}$$

3) ARIMA model [30]. In the AR model, the residual series ε_t are required to be uncorrelated with each other, whereas in the MA model it is assumed that the sequence value at the current moment is only correlated with the residual term at the previous q moments. In reality the time series at the current moment is not only correlated with the previous p time series values, but there is also a significant correlation in its residual term. In view of this, the autoregressive moving average model (ARMA) is proposed, which has the mathematical expression:

$$X_t = a_1 X_{t-1} + \cdots + a_p X_{t-p} + \varepsilon_t - b_1\varepsilon_{t-1} - \cdots - b_q\varepsilon_{t-q} \tag{19}$$

Subsequently, further to the ARMA model, the difference operation was introduced and the summated autoregressive sliding average model (ARIMA model) was proposed in order to deal with non-stationary time series data.The expression of the ARIMA model can be expressed as:

$$(1 - \sum_{i=1}^{p} a_i L^i)(1 - L)^d X_t = \sum_{i=0}^{q} b_i L^i \varepsilon_t \tag{20}$$

Where L denotes the lag operator, $L^k X_t = X_{t-k}$; and $(1-L)^d$ denotes the smoothing of the time series by performing a d -times difference. The model not only combines the advantages of AR model and MA model, but also effectively handles the challenge of non-smooth time series through the difference operation, which greatly expands the application scope of time series analysis and forecasting.

The steps of ARIMA model construction are:

(1) Smoothness test. Since the time series to which the ARIMA model applies is a smooth time series, it is first necessary to carry out a smoothness test on the time series. The ADF test corresponds to the model as:

$$\nabla X_t = \beta^T D_t + \varphi X_{t-1} + \sum_{j=1}^{p} \phi_j \nabla X_{t-j} + \varepsilon_t \tag{21}$$

where $\beta^T D_t$ is the deterministic part, e.g., the constant term; the difference $\nabla X_{t-j} = X_{t-j} - X_{t-j-1}$ is used to model the error term of the ARIMA model, and ϕ_j satisfies $\phi(Z) = 1 - \sum_{i=1}^{\infty} \phi_i Z^i$. According to the principle of hypothesis testing, the following original and alternative hypotheses are proposed:

$H_0 : \rho = 0$ and the time series is non-stationary.

$H_1 : \rho < 0$, time series is smooth

The KPSS test differs from the ADF test in that the original assumption is that the time series is smooth.The KPSS test is modeled as:

$$X_t = \mu + \tau_t + \varepsilon_t \tag{22}$$

where X_t denotes the time series, μ is the mean of the time series, τ_t is the stochastic trend term, and ε_t is the stochastic error term.The statistic for the KPSS test is based on the estimation of the variance of the cumulative sum of the series, with the following formula:

$$KPSS = \frac{\sum_{t=1}^{T} S_t^2}{T^2 \hat{\sigma}^2} \tag{23}$$

where $S_t = \sum_{i=1}^{t} \varepsilon_t$ is the cumulative sum of errors up to period t, $\hat{\sigma}^2$ is the long-run variance estimate of the error term ε_t, and T is the the number of observations in the time series.

For a time series of length N, when the upper bounds of the autoregressive order p and the moving average order q are p_0 and q_0, respectively, notate that $\mathrm{H} = \{(k,l) \mid 0 \le k \le p_0, 0 \le 1 \le q_0\}, \forall (k,l) \in \mathrm{H}$, then the AIC information criterion and the BIC information criterion can be obtained for each pair k,l in this parameter space:

$$AIC(k,l) = \ln(\hat{\sigma}^2(k,l)) + \frac{2(k+l+1)}{N} \tag{24}$$

$$BIC(k,l) = \ln(\hat{\sigma}^2(k,l)) + \frac{\ln(N)(k+l+1)}{N} \tag{25}$$

where $\hat{\sigma}^2(k,l)$ is the estimate of the error variance when $p=k, q=l$.

(2) In time series analysis, common parameter estimation methods include moment estimation, least squares estimation (OLS) and maximum likelihood estimation (MLE) [31]. The method of moments estimation estimates the parameters based on the principle that the sample moments are equal to the overall moments; the method of least squares estimation determines the parameter values by minimizing the sum of squares of errors; and the method of maximum likelihood estimation searches for the parameter values that make the probability of the data appearing (the likelihood function) the largest, given the observed data.

(3) Model testing. For the white noise test of the residuals, different statistics are generally used according to the length of the time series: when the series is long ($N \ge 50$), the Q statistic is generally used with the formula:

$$Q(N,m) = N \sum_{k=1}^{m} \hat{\rho}_k^2 \sim \chi^2(m) \tag{26}$$

where m is the specified number of delay periods, $\chi^2(m)$ denotes the chi-square distribution with m degrees of freedom, $\hat{\rho}_k$ is the sample autocorrelation coefficient, and N denotes the number of observation periods. When the series are short, the Ljung-Box (LB) statistic is generally used with the formula:

$$LB(N,m)=N(N+2)\sum_{k=1}^{m}\frac{\hat{\rho}_k^2}{N-k}\sim\chi^2(m) \tag{27}$$

Consider the original hypothesis H_0 and the alternative hypothesis H_1:

$$H_0:\rho_0=\rho_1=\cdots=\rho_m=0,\forall m\geq 1 \tag{28}$$

$$H_1:\exists m\geq k\geq 1,\rho_k\neq 0,\forall m\geq 1 \tag{29}$$

Under the original hypothesis H_0, set the test level to $\alpha(\alpha=0.05)$ and check the table of the chi-square distribution with degrees of freedom m to obtain the critical value χ_α, which satisfies $P(\chi^2(m)>\chi_\alpha)=\alpha$, when $Q(N,m)>\chi_\alpha(LB(N,m)>\chi_\alpha)$, it is reasonable to reject the original hypothesis and accept the alternative hypothesis, that is, the sequence is considered as a non-white noise sequence; otherwise, H_0 is accepted as a white noise sequence.

(4) Model prediction. If the model test is passed, the model can be used to complete the prediction of future values.

2.3.2. ARIMA-LSTM modeling:

1) LSTM neural network model. Neural networks are known to handle nonlinear tasks well. Due to the large dimensionality and generality of their parameters and the use of nonlinear activation functions in each layer, the models are able to adapt to nonlinear trends in the data. However, empirical studies on financial data have shown that the performance of neural networks is quite complex. For example, in the literature by D. Nelson et al. the accuracy of LSTM neural networks for stock price prediction is typically higher than other non-neural network models [32]. However, there is an overlap within each model's accuracy, which means that the model does not always outperform the other models. This provides a basis for using hybrid ARIMA-LSTM models that incorporate both linear and nonlinear to produce more accurate results than a single LSTM neural network model.

This paper draws on a standard LSTM cell with forgetting gates. This LSTM model consists of four cross-able neural networks representing the forgetting gate, the input gate, the input candidate gate, and the output gate. The forgetting gate outputs a vector whose element values are between 0 and 1. It acts as a forgetter and this output vector is multiplied by the cell state c_{t-1} of the previous time step to remove unwanted values and retain the values needed for prediction:

$$f_t=\sigma(W_f\times[h_{t-1},x_t]+b_f) \tag{30}$$

The σ function, denoted by the same symbol, is the activation function, often referred to as the sigmoid activation function, used to enable the nonlinearity of the model:

$$\sigma(X)=\frac{i}{1+e^{-X}} \tag{31}$$

In the next stage, the input gate and the input candidate gate act together to generate a new cell state c_t, which will be passed to the next time step as the updated cell state. The input gate uses the sigmond function as the activation function, while the input candidate gate chooses the tanh function as the function and outputs i_t and $\tilde{C}_t$. i_t is used to select which feature in Ct should be reflected into the new cell state $\tilde{C}_t$. Then:

$$i_t=\sigma(W_i\times[h_{t-1},x_t]+b_i) \tag{32}$$

$$\tilde{C}_t=\tanh(W_c\times[h_{t-1},x_t]+b_C) \tag{33}$$

The tanh function, is a hyperbolic tangent function. Unlike the sigmond function, which outputs a value between 0 and 1, the tanh output is between -1 and 1.

Eventually, the output gate decides to combine O_t with the cell state C_t activated with tanh to the value chosen by the output gate h_t. The final updated cell state C_t is the combination of the previous cell state C_{t-1} with the forgetting gate applied and the cell state $\tilde{C}_t$ activated by the updated tanh function:

$$\sigma_t = \sigma(W_o \times [h_{t-1}, x_t] + b_o) \tag{34}$$

$$C_t = f_t \times C_{t-1} + i_t \times \tilde{C}_t \tag{35}$$

$$h_t = o_t \times \tanh(C_t) \tag{36}$$

The resulting cell state C_t and the output h_t will be passed to the next time step and will undergo the same process. Depending on the task, further activation functions such as the Softmax activation function or the hyperbolic tangent tanh activation function can be used to activate the output gate h_t. In the experiments of this paper, it is a regression task with output values between -1 and 1, so the tanh function is chosen to activate the output of the last element of the data vector X.

2) ARIMA-LSTM hybrid forecasting models. Many time series models contain both linear and nonlinear relationships. Although ARIMA models are good at modeling linear relationships in time series, they are not good enough to model nonlinear relationships.LSTM models can model both linear and nonlinear relationships, but they cannot provide the same results for each dataset. Therefore, in order to obtain the best forecasting results, hybrid models based on modeling the linear and nonlinear components of the time series separately were used. These models have achieved better results in predictive time series analysis. The use of multiple learning algorithms results in better estimation performance compared to constructive learning algorithms. These hybrid models are based on supervised machine learning algorithms, so they can be used for training and forecasting purposes. In addition, hybrid models increase the diversity of models and enable better prediction results.

The results obtained by using hybrid models and the results obtained by using the models alone, even if they are uncorrelated with each other, are observed to reduce the general variance or error. Therefore, hybrid models are considered to be the most successful models for prediction tasks. In order to make predictions, many hybrid models consisting of linear and nonlinear models have been used by different researchers. In this paper, future forecasting is done by using historical data from time series.

The time series forecasting equation of the generated model can usually be expressed as the sum of linear and nonlinear components as shown in Equation (37).

$$y_t = L_t + N_t \tag{37}$$

where L_t denotes the linear component of the time series and N_t denotes the nonlinear component. In the hybrid model, the linear component L_t is first calculated by the ARIMA model, and then the nonlinear component N_t of the time series is predicted using the LSTM model. The sum of the error values of the two models is then calculated. The formulas for the computation of L_t and N_t are given in Eqs. (38) and (39):

$$LSTM_{error} = LSTM\_mean[error] \tag{38}$$

$$ARIMA_{error} = ARIMA\_mean[error] \tag{39}$$

The weights of the model are calculated by using the error values obtained in equations (40) and (41):

$$LSTM_{weight} = \left(1 - \left(\frac{LSTM_{error}}{LSTM_{error} + ARIMA_{error}}\right)\right) \times 2 \tag{40}$$

$$ARIMA_{weight} = 2 - LSTM_{error} \tag{41}$$

The weight values of the model and each of the predicted values of the final hybrid model are obtained using the given equations:

$$Hybrid_{predict}[i] = \frac{LSTM_{weight}[i] \times LSTM_{error}[i] + ARIMA_{weight}[i] \times ARIMA_{error}[i]}{2} \tag{42}$$

3. ARIMA-LSTM modeling of physical education teachers' professional development scores

3.1. Data processing

Time in "months" as a unit, the initial sequence of "professional development scores" is plotted, as shown in Figure 1. It can be seen that the initial series of "professional development score" vibration amplitude is large and "annual" cyclical changes.

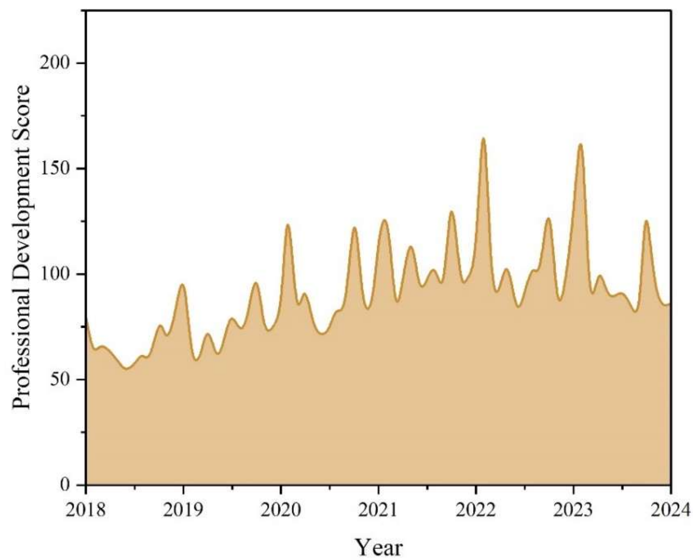


Fig. 1. Initial sequence distribution

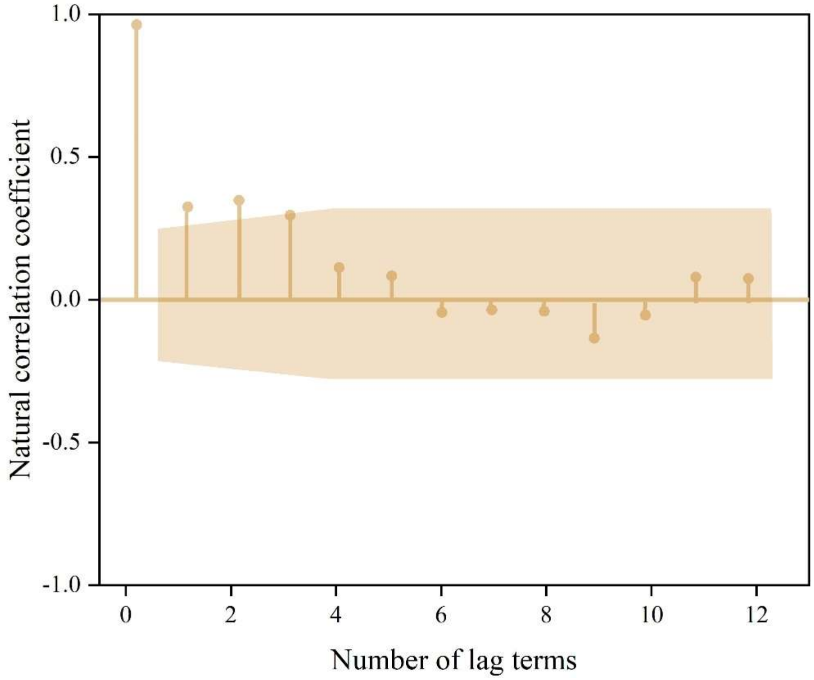
The ARIMA time series model requires the data to be smooth, so before modeling, the smoothness of the data is checked first. The ADF method was used to test the "professional development score" of physical education teachers, as shown in Table 1. As can be seen from the table, the P value of the initial series of "professional development score" is <0.05, so the series of "professional development score" is a smooth time series.

Table 1. Initial sequence ADF test

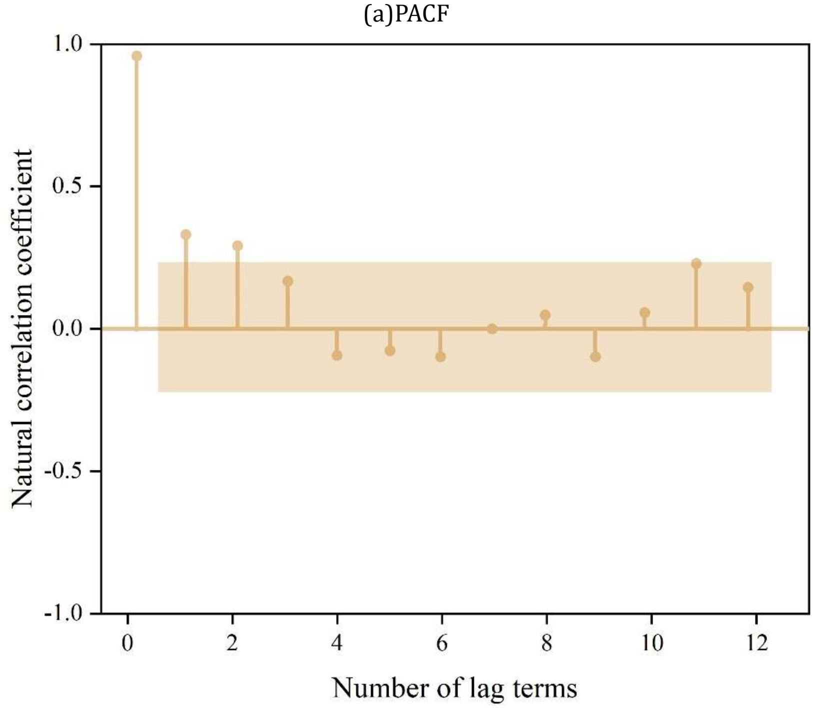
Sequence type	T	Confidence interval			P
		1%	5%	10%	
Professional Development Score	-3.422	-3.518	-2.912	-2.587	0.006

3.2. ARIMA modeling

Parameter estimation of ARIMA model is performed by partial autocorrelation plot (PACF) and autocorrelation plot (ACF) generated from the processed smooth time series as shown in Figure 2. Figures (a) and (b) correspond to PACF and ACF, respectively, and the brown part is the confidence interval. It can be seen that the autocorrelation coefficient of the "Professional Development Score" series is truncated at order 2, and the partial autocorrelation plot is trailing. Therefore, the model parameters can be estimated as ARIMA(0,0,2), ARIMA(1,0,2), ARIMA(1,0,1), and the AIC values of the three models are 465.38, 461.85, and 462.17, respectively, according to the principle of the minimum information content. Based on the principle of minimum information, the best model for "professional development score" is ARIMA(1,0,2).



(a)PACF



(b)ACF

Fig. 2. ACF and PACF diagrams

The ARIMA model of "Professional Development Score" series was established, and the residual series of the model were tested for normality and autocorrelation to ensure the validity of the model. The normality test is shown in Figure 3. It can be seen that the residual series basically obey the normal distribution, at the same time, the residuals of the D - W test, “professional development scores” model of the D - W test value of 1.96, D - W test value are close to 2, the model meets the autocorrelation test.

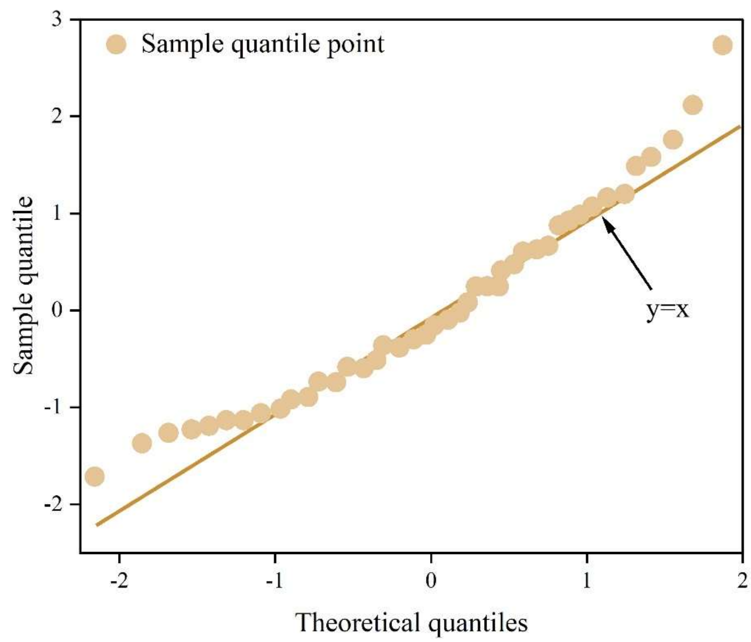


Fig. 3. Residual QQ plot

The ARIMA model predictions for the “professional development score” sequence are shown in Figure 4. The vertical dotted line in the figure is used to distinguish the sample sets, the left side of the dotted line is the training set samples, and the right side is the test set samples. As can be seen from the figure, the ARIMA model of “professional development score” has a good prediction effect, which is closer to the distribution curve of the initial sequence data, and can predict the trend of the sequence test set data.

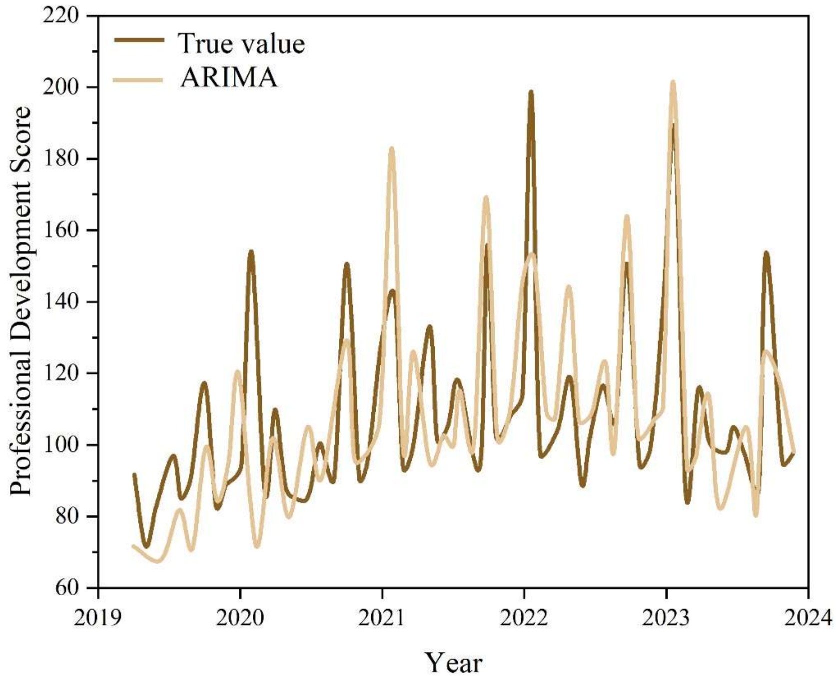


Fig. 4. ARIMA model prediction

3.3. LSTM modeling

Based on Pytorch 1.7.1, an LSTM neural network was constructed, and an LSTM prediction model was built based on the residual sequences of the ARIMA model of "Professional Development Score", aiming to predict the residuals in the test set by fitting the nonlinear time-series characteristics of the residual sequences in the training set, so as to obtain better prediction results of the combined model. The aim is to predict the residuals in the test set by fitting the nonlinear time-series characteristics of the residuals in the training set, so that the combined model can obtain better prediction results. The training set and test set of residuals are also divided by the above time nodes, and the specific modeling process is as follows:

1) Normalize the sequences and compress the data into the range of [-1,1] to ensure that the LSTM model can converge quickly.

2) Select hyperparameters for the model, after repeated comparisons, it is found that the model can well capture the nonlinear factors in the residual sequence when the time step of the residual sequence is 4. After repeated adjustments, the final hyperparameters are: Adam optimizer, learning rate of 0.001, batch_size=1, number of neural network layers is 1, number of neurons is 45, and number of iterations is 140.

3) The final model output results need to be inverse normalized to reduce the data.

The model prediction results are shown in Figure 5. It can be seen that the LSTM neural network model is able to extract certain effective timing information from the residual sequence.

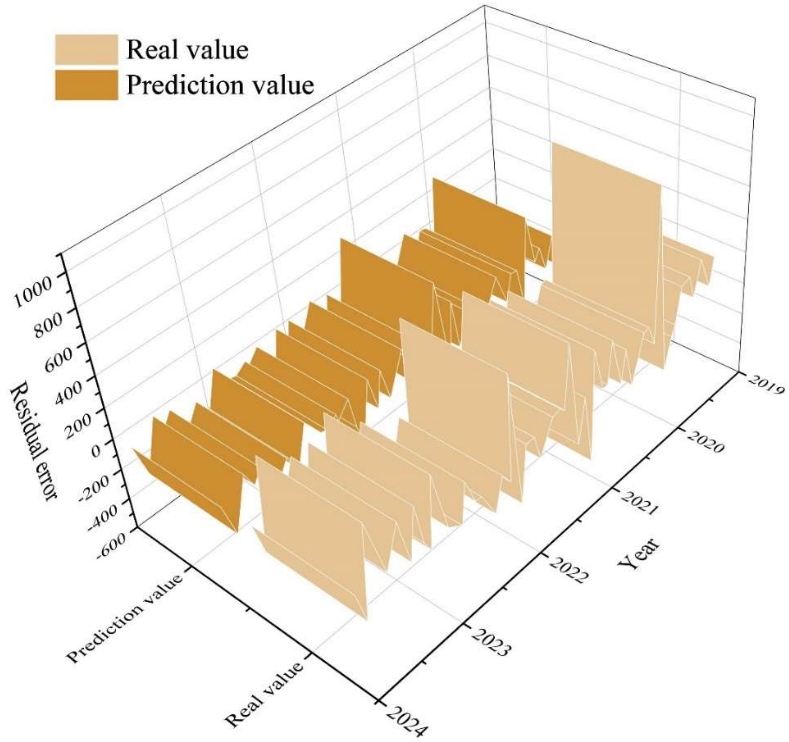


Fig. 5. LSTM residual prediction

3.4. ARIMA - LSTM combination modeling

The ARIMA predicted values were linearly combined with the residual predicted values obtained by LSTM to obtain the final combined model predicted values. Comparison of the initial series, ARIMA model predicted values and combined model predicted values of "professional development score" indicators are shown in Figure 6. The ARIMA - LSTM combined model of the "Professional Development Score" indicator in the figure has a great improvement compared with the single ARIMA model, which is closer to the original series, and the ARIMA - LSTM combined model can be better fitted to predict the trend changes in the series.

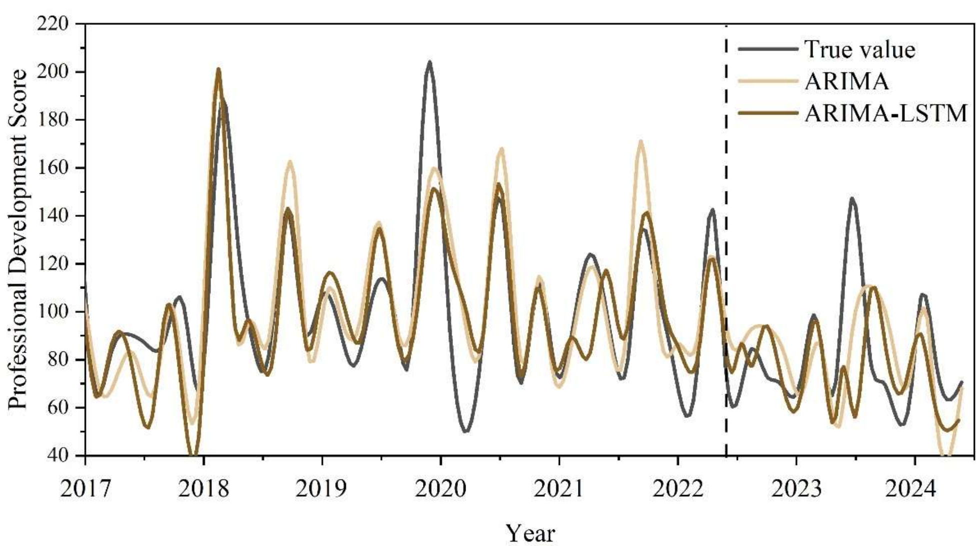


Fig. 6. Comparison of the predicted results

RMSE and MAPE are used to measure the prediction effect of the model, as shown in Table 2. It can be seen that the combination prediction model is better in each index. the ARIMA - LSTM combination model compared with the ARIMA model, the RMSE and the MAPE are reduced by 4.09 and 26.48% respectively, that is, compared with a single model, the RMSE and the MAPE of the combination model are improved by 55.65% and 52.27% respectively, and the "professional development scores "The improvement effect of the combined prediction model is remarkable.

Table 2. Model evaluation

Sequence type	Forecasting model	E_{RMSE}	E_{MAPE}
Professional Development Score	ARIMA	7.35	50.66%
	ARIMA-LSTM	3.26	24.18%

4. Analysis of the predicted effects of physical education teachers' professional development trajectories

Before testing the predictive effect of the ARIMA-LSTM model on the professional development trajectory of physical education teachers, i.e., the professional development score of physical education teachers, it is necessary to test the residual series predictive ability of the LSTM model for the ARIMA model, the effect of which is shown in Figure 7 below. As can be seen from the figure, the residual series curve of the ARIMA model has uneven ups and downs, and the value is in the range of (-0.2,0.2) tumbling transformation. The LSTM model can predict the curve changes more accurately.

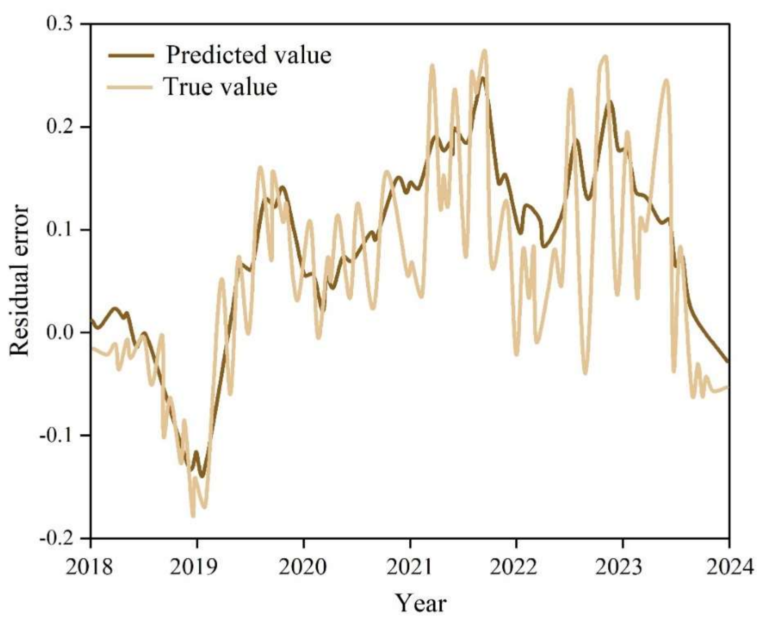
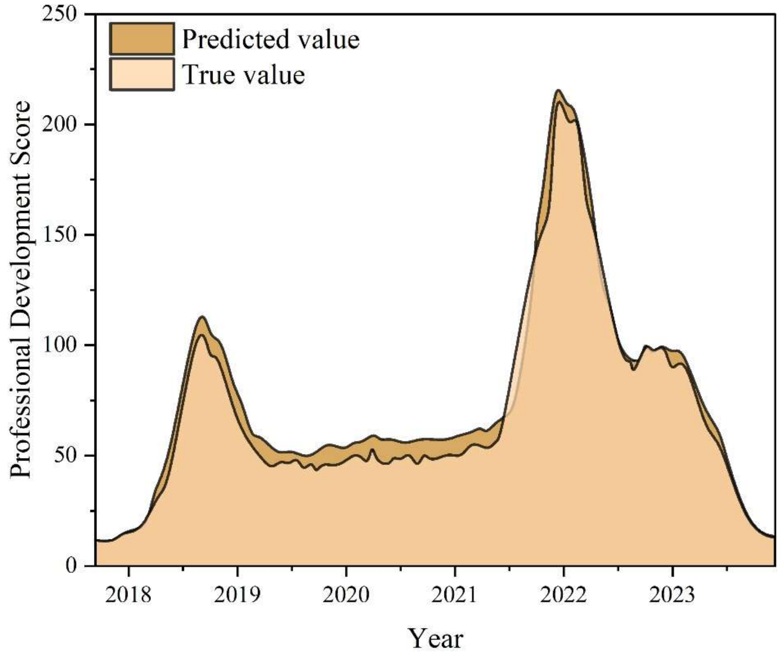
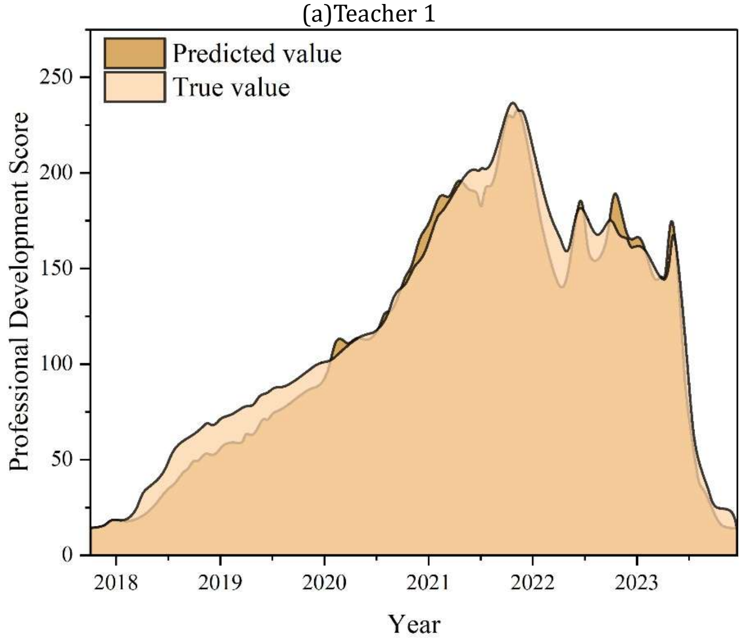


Fig. 7. LSTM model residual sequence prediction

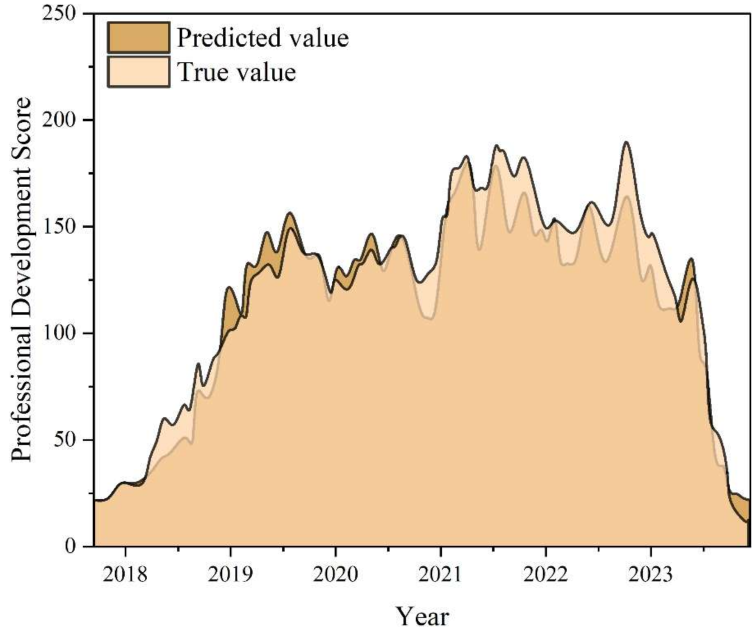
Three physical education teachers (named as Teachers 1~3 respectively) were randomly selected as the research subjects to predict their physical education teachers' professional development scores. After constructing the ARIMA-LSTM model, the model prediction results need to be tested to determine the accuracy of the model in the problem of predicting the professional development scores of physical education teachers, and the results are shown in Figure 8. Figures (a)~(c) correspond to teachers No.1~3 respectively.The ARIMA-LSTM model predicts the professional development score of physical education teacher No.1 better, and the prediction curve fits the actual value curve better. The ARIMA-LSTM model predicts the professional development scores of physical education teacher No. 2 better, the peak changes of the predicted values are closer to the actual values, and several peaks of the professional development scores can be better predicted.The ARIMA-LSTM model also predicts the professional development scores of physical education teacher No. 3 better, the model can accurately predict the peaks and valleys of the peaks and troughs of the peaks of the peaks of the model, and the peaks of the peaks of the predicted results are better. The ARIMA-LSTM model can accurately predict the change rule of the model compared with the prediction results of the ARIMA model, and at the same time, it has a better prediction effect for the trend change of the professional development scores of physical education teachers.



(a)Teacher 1



(b)Teacher 2



(c)Teacher 3

Fig. 8. The prediction results of ARIMA-LSTM model

After visualizing and analyzing the prediction results of ARIMA-LSTM model for PE teachers' professional development scores, the prediction results of ARIMA, LSTM and ARIMA-LSTM models were compared and analyzed more intuitively using MAPE and RMSE, and the results are shown in Table 3. It can be seen that the ARIMA-LSTM model was more effective in predicting the professional development score of PE teacher No.1.The ARIMA-LSTM model compared with ARIMA model and LSTM model in MAPE decreased by 9.18% and 5.14%, and in RMSE decreased by 20.548 and 3.0819.In the prediction of the professional development score of PE teacher No.2, the ARIMA-LSTM model was compared with ARIMA model, its MAPE decreased by 13.7%, and increased by 1.12% compared with LSTM model, from the MAPE, ARIMA-LSTM model prediction is not satisfactory, but from the analysis of RMSE, ARIMA-LSTM model compared with ARIMA model, LSTM model decreased respectively by 64.98 and 73.4563, and also combined with the prediction curve of the professional development score of physical education teacher No.2 above, it can also prove that the ARIMA-LSTM model has better prediction effect. Comparing the predicted professional development scores of PE teacher #3 with different models, the MAPE and RMSE of the ARIMA-LSTM model decreased by 3.33% and 46.6725, respectively, based on the better predicted LSTM model.

Table 3. Model prediction results

Data classification	Model types	MAPE	RMSE
Teacher 1	ARIMA	25.47%	68.7962
	LSTM	21.43%	51.3301
	ARIMA-LSTM	16.29%	48.2482
Teacher 2	ARIMA	35.45%	76.5799
	LSTM	22.87%	68.1036
	ARIMA-LSTM	21.75%	141.5599
Teacher 3	ARIMA	27.23%	197.1296

	LSTM	24.65%	178.8103
	ARIMA-LSTM	21.32%	132.1378

To summarize the above charts, the ARIMA model is only suitable for time series forecasting without stochastic characteristics, but it is not suitable for other cases with more stochasticity. The LSTM model is very good at dealing with time series with stochastic characteristics. The ARIMA-LSTM model established in this paper uses the LSTM model to process the residual series of the ARIMA model, and the ARIMA-LSTM model is better than the ARIMA and LSTM model in predicting the professional development scores of physical education teachers, i.e., the trajectory of professional development of physical education teachers.

5. Conclusion

In this paper, a prediction model combining linear and nonlinear, i.e., ARIMA-LSTM hybrid model, is used as the main research method to predict the professional development trajectory of physical education teachers. The employment development data of physical education teachers in Shanghai, China, from 2018 to 2024 are selected as the research sample, and the professional development scores of physical education teachers are selected as the indicators to measure the changes in the professional development trajectories of physical education teachers, so that the ARIMA-LSTM model for the prediction of the professional development scores of physical education teachers is established. Based on the principle of minimum information content, the best ARIMA model for the index of “professional development score” was determined as ARIMA(1,0,2). Based on the residual series of the ARIMA model of “professional development score”, the LSTM prediction model was established. The ARIMA predicted values and the residual predicted values obtained by LSTM were linearly combined to obtain the final combined model predicted values, and the ARIMA-LSTM combined model was established for the index of “professional development score”. Compared with the ARIMA model, the RMSE and MAPE of the combined ARIMA-LSTM model were reduced by 4.09 and 26.48%, respectively, and the prediction effect was better.

Three physical education teachers (named as Teacher No. 1~3 respectively) were randomly selected as the research subjects to analyze the predictive effect of the ARIMA-LSTM combination model on the professional development trajectory of physical education teachers. Comparing the prediction curves of ARIMA-LSTM combination model with the actual value curves, the professional development scores, peaks and peaks and valleys of the predicted values of Teachers No. 1~3 were better fitted to the actual values. MAPE and RMSE indicators were used to visually compare the prediction results of ARIMA-LSTM model, ARIMA model, and LSTM model. On the predicted results of the professional development scores of physical education teachers #1, the ARIMA-LSTM model compared the ARIMA model, LSTM model in the MAPE decreased by 9.18% and 5.14%, and in the RMSE decreased by 20.548 and 3.0819. The ARIMA-LSTM model was also better than the ARIMA model and the LSTM model in predicting the professional development scores of physical education teachers No. 3. In comparison with the LSTM model, which had relatively better predictive results, the MAPE and RMSE of the ARIMA-LSTM model decreased by 3.33% and 46.6725, respectively. In contrast, the ARIMA-LSTM model showed a 13.7% decrease in MAPE compared to the ARIMA model but a 1.12% increase compared to the LSTM model on the results of the professional development score predictions made by PE teacher #2. However, in terms of RMSE, comparing the ARIMA model and LSTM model, the ARIMA-LSTM model reduces 64.98 and 73.4563 respectively.Combined with the fit between the prediction curve and the actual value curve, it is still able to prove that the ARIMA-LSTM model has better prediction effect.

Overall, the combined ARIMA-LSTM model constructed in this paper is better than the ARIMA and LSTM models in predicting the professional development scores, i.e., the professional development trajectories of physical education teachers, and can well realize the tracking of the professional development trajectories of physical education teachers.

References

[1] Myrberg, E., Johansson, S., & Rosén, M. (2019). The relation between teacher specialization and student reading achievement. Scandinavian Journal of Educational Research, 63(5), 744-758.

[2] Zulkifli, M. A., & Kutty, F. M. (2022). Relationship between specialization of teaching subject and teaching experiences with teacher's motivational strategies. Creative Education, 13(10), 3238-3254.

[3] Zalech, M. (2021). Student perception of pe teachers and its effect on their participation in pe classes and sports: A new perspective on teacher competencies. Journal of Physical Education and Sport, 21, 1106-1111.

[4] Bertills, K., Granlund, M., Dahlström, Ö., & Augustine, L. (2018). Relationships between physical education (PE) teaching and student self-efficacy, aptitude to participate in PE and functional skills: with a special focus on students with disabilities. Physical Education and Sport Pedagogy, 23(4), 387-401.

[5] Lee, O., Griffiths, M., Goodyear, V., Jung, H., Son, H., Lee, U., & Choi, E. (2023). Influence of a professional development programme on the life skills teaching practices of secondary PE teachers. Sport, Education and Society, 28(8), 901-914.

[6] Cardina, C. E., & James, A. R. (2018). Targeting professional development for beginning physical education teachers. Journal of Physical Education, Recreation & Dance, 89(7), 41-47.

[7] Derri, V., Papamitrou, E., Vernadakis, N., Koufou, N., & Zetou, E. (2014). Early professional development of physical education teachers: Effects on lesson planning. Procedia-Social and Behavioral Sciences, 152, 778-783.

[8] Maher, A. J., & Fitzgerald, H. (2020). Initial teacher education and continuing professional development: the perspectives of special school physical education teachers. Curriculum Studies in Health and Physical Education, 11(1), 18-33.

[9] Lander, N., Lewis, S., Nahavandi, D., Amsbury, K., & Barnett, L. M. (2022). Teacher perspectives of online continuing professional development in physical education. Sport, Education and Society, 27(4), 434-448.

[10] Cox, A., Noonan, R. J., & Fairclough, S. J. (2023). PE teachers' perceived expertise and professional development requirements in the delivery of muscular fitness activity: PE Teacher EmPOWERment Survey. European Physical Education Review, 29(2), 251-267.

[11] Armour, K., Quennerstedt, M., Chambers, F., & Makopoulou, K. (2017). What is 'effective'CPD for contemporary physical education teachers? A Deweyan framework. Sport, education and society, 22(7), 799-811.

[12] Lawson, H. A., Kirk, D., & MacPhail, A. (2020). The professional development challenge: Achieving desirable outcomes for students, teachers and teacher educators. In School physical education and teacher education (pp. 141-152). Routledge.

[13] Sum, R. K. W., Morgan, K., Ma, M. M. S., & Choi, S. M. (2021). The influence of a customized continuing professional development programme on physical education teachers' perceived physical literacy and efficacy beliefs. Prospects, 50(1), 87-106.

[14] Makopoulou, K., Penney, D., Neville, R., & Thomas, G. (2022). What sort of 'inclusion'is continuing professional development promoting? An investigation of a national CPD programme for inclusive physical education. International journal of inclusive education, 26(3), 245-262.

[15] Stepanchenko, N. I., & Briskin, Y. A. (2019). Dispositional factors of personality professional development of the future teachers of physical education and sport. Physical education of students, 23(4), 202-208.

[16] Kahts-Kramer, S., & Wood, L. (2023). Professional development for physical education teachers: A participatory approach to identifying learning needs. South African Journal of Education, 43(2), 1-9.

[17] Setiawan, I., Kurniawan, W. R., & Wijayanti, D. G. S. (2024). Implementation of online teacher professional development programs: Physical education teacher perspectives. Retos: nuevas tendencias en educación física, deporte y recreación, (57), 445-454.

[18] Bjørke, L., Standal, Ø. F., & Mordal Moen, K. (2021). 'While we may lead a horse to water we cannot make him drink': three physical education teachers' professional growth through and beyond a prolonged participatory action research project. Sport, Education and Society, 26(8), 889-902.

[19] Tannehill, D., Demirhan, G., Čaplová, P., & Avsar, Z. (2021). Continuing professional development for physical education teachers in Europe. European Physical Education Review, 27(1), 150-167.

[20] Cardina, C. E., & DeNysschen, C. (2018). Professional development activities and support among physical education teachers in the United States. Physical Educator, 75(1), 138-157.

[21] Ward, P., & van der Mars, H. (2020). Confronting the challenge of continuous professional development for physical education teacher educators. Journal of Physical Education, Recreation & Dance, 91(1), 7-13.

[22] Braga, L., Jones, E., Bulger, S., & Elliott, E. (2017). Empowering teachers to implement innovative content in physical education through continuous professional development. Teacher Development, 21(2), 288-306.

[23] Leeder, T. M., & Beaumont, L. C. (2021). Lifestyle sports and physical education teachers' professional development in the United Kingdom: A qualitative survey analysis. Education Sciences, 11(10), 642.

[24] Sum, K. W. R., Wallhead, T., Ha, S. C. A., & Sit, H. P. C. (2018). Effects of physical education continuing professional development on teachers' physical literacy and self-efficacy and students' learning outcomes. International Journal of Educational Research, 88, 1-8.

[25] Beni, S., Fletcher, T., & Chróinín, D. N. (2021). Teachers' engagement with professional development to support implementation of meaningful physical education. Journal of Teaching in Physical Education, 41(4), 570-579.

[26] Lee, O., Choi, E., Griffiths, M., Goodyear, V., Armour, K., Son, H., & Jung, H. (2019). Landscape of secondary physical education teachers' professional development in South Korea. Sport, Education and Society.

[27] Karaiskos, L., Sotiras, M. N., Antonopoulou, P., & Gdonteli, K. (2024). The impact of training and professional development on physical education teacher self-efficacy. Journal of Physical Education and Sport, 24(11), 2022-2033.

[28] Durden-Myers, E. J., & Keegan, S. (2019). Physical literacy and teacher professional development. Journal of Physical Education, Recreation & Dance, 90(5), 30-35.

[29] Spezia Luigi. (2024). Bayesian prior modeling in vector autoregressions via the Yule-Walker equations. Communications in Statistics - Theory and Methods,53(14),5230-5247.

[30] Liping Wang,Longjun Chen,Shucen Jin & Chuang Li. (2025). Forecasting the green behaviour level of Chinese enterprises: A conjoined application of the autoregressive integrated moving average (ARIMA) model and multi-scenario simulation. Technology in Society,81,102825-102825.

[31] Kostyantyn Kharchenko,Oleksandr Beznosyk,Bogdan Bulakh,Bogdan Kyriusha & Vadym Yaremenko. (2024). Forecasting software development costs in scrum iterations using ordinary least squares method. Technology audit and production reserves,4(2),30-33.

[32] Yuqing Tang,Fusheng Yu,Witold Pedrycz,Fang Li & Chenxi Ouyang. (2024). Oriented to a multi-learning mode: Establishing trend-fuzzy-granule-based LSTM neural networks for time series forecasting. Applied Soft Computing,166,112195-112195.