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Research on optimization method of cross-border e-commerce supply chain resource allocation based on Hamiltonian loop algorithm in digital economy environment

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ABSTRACT

In recent years, foreign research on the optimal allocation of supply chain resources and operational coordination in the digital economy environment has also made more and more progress, and the current status of domestic research is in the development stage, and supply chain management in the context of the digital economy has become another research hotspot. In this regard, this paper studies the application of Hamiltonian circuit in supply chain resource allocation from three aspects, such as product pricing strategy, supply chain management optimization and consumer behavior, and constructs the optimal resource allocation model according to the steps of resource allocation establishment. Through the Hamiltonian loop algorithm, the revenue function of cross-border e-commerce supply chain services is obtained. Using the Hamiltonian loop algorithm, the optimal price model of supply chain logistics is simulated and simulated experiments are carried out, and the optimal logistics and distribution services will be priced at about 225 under the high competition situation with competition coefficient $k = 0.6$. Under the optimal allocation of supply chain resources, analyzing the user's satisfaction, retail customers' satisfaction with the supply of goods in 2021 is 81.806 points, higher than the province's 0.913 points, and the experimental results show that the resource allocation model can meet the needs of the customers, making the supply and demand of the product resources more balanced, which argues the scientific nature and reliability of this study.

Keywords: optimal resource allocation, Hamiltonian loop algorithm, cross-border e-commerce supply chain, revenue function, supply chain logistics optimal price

1. Introduction

With the rapid development of global e-commerce, cross-border e-commerce has become an important form of international trade. Cross-border e-commerce refers to the transaction and sale of

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goods and services between different countries through the Internet [1-3]. It breaks the traditional spatial limitations and national boundaries, providing enterprises and consumers with direct contact and interaction in the global market. However, cross-border e-commerce faces many challenges, and one of the most prominent problems is the optimization of supply chain resource allocation [4-7].

First, the logistics cost of cross-border e-commerce is high, and the logistics cycle is long and inefficient due to the multiple links involved in international transportation, customs clearance, and distribution, resulting in poor consumer experience. Second, there are difficulties in inventory management of cross-border e-commerce, due to the variety of commodities, inventory management is complicated, and it is easy to have problems such as inventory backlog and out-of-stock [8-11]. Supply chain is one of the core competitiveness of cross-border e-commerce, and the optimization of resource allocation in the supply chain can significantly improve the operational efficiency of enterprises, reduce costs, and improve customer satisfaction. In cross-border e-commerce, the optimization of supply chain is even more important due to the many links involved [12-15]. Through the optimization of the supply chain, enterprises can reduce inventory, improve logistics efficiency, reduce risk, and improve service quality, thus gaining a greater competitive advantage [16-17].

Gong, Z. proposed a method to optimize cross-border e-commerce (CBEC) supply chain management, which can effectively improve the efficiency and profitability of cross-border e-commerce as well as the efficiency of CBEC supply chain, and provides a solution to the challenges faced by e-commerce enterprises [18]. Xia,L., and Liu,S. proposed a method for optimizing management and coordination to improve the performance of cross-border e-commerce supply chain based on loT tracking technique and multi-objective decision making, and verified that the method has high internal consistency [19]. Wang,W. proposed an intelligent CBEC framework based on IoT technology, which is able to optimize supply chain performance through demand forecasting by deploying a machine learning model within it and proved the effectiveness of the proposed machine learning model [20]. Ye,W. proposed a deep neural network and cold supply chain optimization forecasting method based on short-term demand and selected a suitable forecasting technique based on the consideration of various influencing factors, revealing that the method has a very high accuracy [21]. Wang,D. and Li,W. constructed a dual-channel supply chain consisting of suppliers, enterprises, retailers, and consumers, and examined the benefits and cost sharing of the supply chain under centralized and decentralized decision-making, revealing that cross-border e-commerce enterprises can choose online and offline channels to optimize decentralized decision-making and achieve win-win situation among supply chain subjects [22]. Zhao,Y. and Gong,B. introduced the SARIMA-CNN-BiLSTM prediction model, which can effectively capture the seasonal and nonlinear characteristics of cross-border e-commerce supply chains, and provide support for cross-border e-commerce enterprises to reduce costs and improve operational efficiency [23]. Dong,Q. et al. examined the operation optimization problem of cross-border e-commerce supply chain network and constructed a resource scheduling model for enterprise supply chain, revealing that time delay and cost adjustment strategies can improve the comprehensive capability of cross-border e-commerce supply chain [24]. Yan, R. examined the impact of cross-border e-commerce on the supply chain of aquatic products and optimization strategies, and based on the current situation of aquatic products supply chain proposed a supply chain management strategy based on the role of big data and cross-border e-commerce in optimizing the supply chain of aquatic products and verified it [25]. Zhu,L. built a linear and nonlinear optimization model of e-commerce supply chain based on IoT, which can effectively predict the product information at each stage of e-commerce supply chain, revealing the optimization of e-commerce supply chain based on IoT, which positively affects the enterprises [26]. Zhou,F. and Liu,Y. reviewed blockchain-enabled cross-border e-commerce supply chain management using bibliometric data-driven analysis, pointing out that blockchain technology has a strong potential in the field of cross-border e-commerce supply chain, which is mainly focused on cross-border e-commerce platforms, data governance and other aspects [27]. The above study examined the supply chain optimization of cross-border e-commerce from the aspects of IoT, etc., and verified that the proposed optimization method has certain advantages in optimizing the supply chain to promote the development of cross-border e-commerce enterprises, however, as of now, no scholars have investigated the optimization of the supply chain resource allocation in cross-border e-commerce from the angle of the Hamiltonian loop algorithm.

In this paper, the resource allocation of cross-border e-commerce supply chain is represented by a discrete system, and the optimal control model of the resource allocation system is established. The gradient method is used to calculate the relevant parameters of the optimal control, and the iterative algorithm is used to satisfy the model to meet the necessary conditions according to the direction of the gradient reduction. Based on the relevant utility function of cross-border e-commerce supply chain, set the number of matching times through the form of Hamilton loop algorithm to get the cross-border e-commerce logistics and distribution revenue function model. Simulation example experiments are designed to test the optimal allocation of resources of the model constructed in this paper. At the same time, through the method of empirical investigation, the user satisfaction under the optimized allocation of supply chain resources is analyzed to further verify the scientificity and reliability of the model.

2. Resource allocation in cross-border e-commerce supply chain in digital economy environment

2.1. Application of Hamiltonian Circuit in Supply Chain Resource Allocation

2.1.1. Product Pricing Strategy. The Hamiltonian circuit plays a significant role in product pricing, especially in meeting the needs of different user groups and maximizing revenue [28]. Price sensitivity analysis helps enterprises to understand the differences in user preferences for price and service [29]. For example, enterprises promote cost-effective traffic packages for young users and emphasize service stability and after-sales service for middle-aged and elderly users, and this differentiated pricing enhances user satisfaction and loyalty. At the same time, during holidays or events, in response to the big data prediction of the event during the surge in traffic demand, communications companies can flexibly adjust the price, the introduction of time-limited traffic packages, which can not only meet the user's large traffic demand, but also to increase revenue. It can be seen that big data-driven pricing strategy can enable enterprises to more accurately meet the market demand, improve pricing flexibility and revenue.

2.1.2. Supply chain management optimization. Hamilton Loop has injected new vitality into supply chain management and improved the operational efficiency and cost control ability of enterprises. By utilizing historical sales data, seasonal demand changes and in-depth analysis of the effectiveness of promotional activities, the company was able to build an accurate inventory forecasting model and scientifically set the initial purchase quantity, effectively avoiding the risk of inventory backlogs or out-of-stocks. At the same time, it also helps to improve the efficiency of logistics. Combined with data analysis, enterprises can instantly optimize distribution paths, cut down on ineffective trips and waiting time, reasonably dispatch logistics vehicles and optimize distribution routes, significantly improving logistics efficiency and accuracy. Through continuous data monitoring, enterprises can dynamically adjust logistics resources, reduce logistics costs, speed up supply chain response and improve service quality. In short, Hamilton Loop revolutionizes inventory management and logistics distribution, and provides strong support for the improvement of overall supply chain efficiency, which helps enterprises to stand out in the fierce competition.

2.1.3. Consumer behavior. Consumer behavior analysis is a key application of big data in the allocation of market resources. Through in-depth mining of user behavior, enterprises can accurately target groups and develop personalized marketing strategies. By integrating multi-dimensional user information, such as age, gender, occupation and consumption habits, a detailed user profile can be constructed. This kind of customized service based on profiles greatly improves product competitiveness and user satisfaction. At the same time, the analysis of consumption habits helps companies to explore market potential, such as the discovery of the user's night traffic demand, the company launched a night traffic packages, not only to meet specific needs, but also to enhance user loyalty. Big data analysis can also reveal the shortcomings of the service in a timely manner, and urge enterprises to make precise improvements. In short, big data-driven analysis of consumer behavior can not only enhance the competitiveness of enterprises, but also optimize the user experience.

2.2. *Model for optimal allocation of resources*

The resource allocation can be represented as a discrete system as in Figure 1 [30].

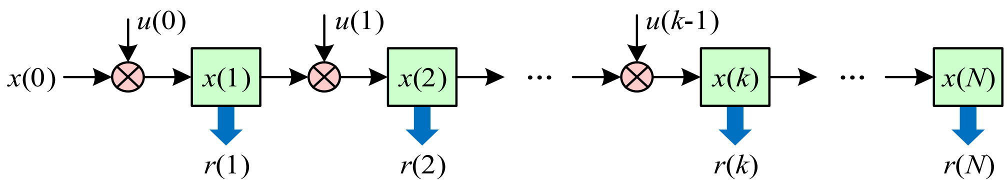


Fig. 1. Resource configuration model

Let $x(k)$ denote the number of resources that the system has in stage k to serve the project. This gives the sequence of state vectors $x(1), x(2), \cdots, x(k), x(N)$, $k = 1, 2, \cdots, N$. $x(0)$ denotes the amount of resources in the initial state of the system and $x(N)$ denotes the amount of resources in the end state of the system.

Let $u(k)$ denote the rate of change relative to the supply of resources in stage $k+1$, i.e., the amount of change in resources in stage $k+1$ compared to month k. There are both positive and negative values, with positive values indicating that resource supply needs to increase relative to the previous month, and negative values indicating that resource supply needs to decrease relative to the previous month. Thus there are control vector sequences $u(0), u(1), \cdots, u(k), \cdots u(N-1)$ and $k = 0, 1, 2, \cdots, N-1$.

Let $r(k)$ denote the demand for resources by the project in phase k. There are known state sequences $r(1), r(2), \cdots, r(k), r(N)$ and $k = 1, 2, \cdots, N$, and the resource requirements are generally obtained from the project plan.

Based on the above model, the optimal control model of the resource allocation system is developed step by step.

1) The state of the system at each stage obeys the following equation with difference:

$$x(k+1) = x(k) + u(k); k = 0, 1, 2, \cdots, N-1 \tag{1}$$

The equation shows that given state $x(0)$, the state sequence $x(k)$ can be obtained by adjusting the control sequence $u(k)$.

2) In a practical engineering problem, $u(k)$ should satisfy the following constraints:

$$x(k) + u(k) \geq 0; k = 0, 1, 2, \cdots, N-1 \tag{2}$$

That is, the number of resources of the actual system cannot be negative.

3) Setting the performance index function of the system

Measured from the project progress indicators, take the first indicator $[x(k) + u(k) - r(k)]^2$, which indicates that each stage of the planned arrangement of resources and project demand should be as consistent as possible, so as to meet the resources to ensure the progress of the project. In reality, the planned resources are less than the demand, and the square is guaranteed to be positive.

In terms of project quality indicators, the second indicator is $[x(k)+u(k)-\overline{r}]^2$. It means that the number of resources planned for each month should be maintained as close as possible to the project's average demand for resources, so as to avoid the impact of huge fluctuations in resources on the project. In particular, when a large increase in resource allocation is required, there is a risk that the demand will not be met and the quality of the resources will be difficult to ensure in a timely manner.

Where $\overline{r}=\frac{\sum_{i=1}^{N} r(k)}{N}$, denotes the average demand of resources for the project at each stage.

The final performance metric function of the system is defined as:

$$
\begin{aligned}
J &= \varphi[x(N),N] \\
&+ \sum_{k=0}^{k=N-1} \left\{ A[x(k)+u(k)-r(k+1)]^2 + B[x(k)+u(k)-\overline{r}]^2 \right\} / \overline{r}^2
\end{aligned}
\tag{3}
$$

where $\varphi[x(N),N]$ denotes a constraint on the terminal. Eq. (3) does not explicitly give the constraints, i.e., the formula is not needed if the system has no constraints on the terminal state. A and B are the adjustment factors for the weights of the two performance metrics, and $\overline{r}^2$ is the adjustment factor for the square of the performance metric.

4) introduces the Hamiltonian equation [31]:

$$
\begin{aligned}
H(x,u,\lambda,k) &= \left\{ A[x(k)+u(k)-r(k+1)]^2 + B[x(k)+u(k)-\overline{r}]^2 \right\} / \overline{r}^2 \\
&+ \lambda(k+1)[x(k)+u(k)]
\end{aligned}
\tag{4}
$$

5) Using the great value principle for discrete systems to solve, the necessary conditions for J to take an extremely small value are as follows:

$$
\begin{aligned}
\lambda(k) = \frac{\partial H(k)}{\partial x(k)} &= \frac{2A}{\overline{r}^2}[x(k)+u(k)-r(k)] \\
&+ \frac{2B}{\overline{r}^2}[x(k)+u(k)-\overline{r}] + \lambda(k+1)
\end{aligned}
\tag{5}
$$

$$
\begin{aligned}
\frac{\partial H(k)}{\partial u(k)} &= \frac{2A}{r^2}[x(k)+u(k)-r(k+1)] \\
&+ \frac{2B}{r^2}[x(k)+u(k)-\overline{r}] + \lambda(k+1)
\end{aligned}
\tag{6}
$$

If the control vectors are constrained, the Hamiltonian function takes an extreme value on the optimal control sequence according to the principle of great values, i.e., the Hamiltonian function takes an extreme value on the optimal trajectory extremum $x^*(t)$ and on the optimal control $u^*(t)$ (the case of taking a minimal value is indicated below).

$$
H\left[x^*(k),u^*(k),\lambda^*(k+1),k\right] = \min_{U\in\Omega} H\left[x^*(k),u^*(k),\lambda^*(k+1),k\right] \tag{7}
$$

$$
U = [u(0),u(1),\cdots,u(N-1)]^T \tag{8}
$$

U belongs to a bounded closed set Ω, i.e. $U \in \Omega$.

2.3. *Optimal Control Gradient Calculation Methods*

The computational methods of optimal control are usually divided into two categories: direct and indirect methods. The gradient method is a direct method that is widely used. It is characterized by first guessing any control sequence, which may not satisfy the necessary condition that H is taken to be extremely small, and then using an iterative algorithm to improve U according to the direction of the H gradient reduction so that it satisfies the necessary condition. The computational steps are as follows.

1) Guess a control sequence $u^u(k)=u^0(k)$, $k=0,1,\cdots,k,\cdots,N-1$. M is iterated steps, initially when $M=0$. u^0 by project experience.

2) In step M, solve the state sequence $x^M(k)$, $k=1,2,\cdots,k,\cdots,N$ sequentially with the estimated value $u^M(k)$ and the given initial conditions $x(0)$.

3) Compute the gradient vector of the Hamiltonian function H for U:

$$G^M=\left(\frac{\partial H}{\partial U}\right)_M \tag{9}$$

where $G^M=[g(0),g(1),\cdots,g(N-1)]^T$, substituting into equation (9) yields:

$$g(k)_M=\left(\frac{2A}{r^2}[x(k)+u(k)-r(k+1)]+\frac{2B}{\overline{r}^2}[x(k)+u(k)-\overline{r}]+\lambda(k+1)\right)_M \tag{10}$$

4) Correct the control vector:

$$U^{M+1}=U^M-\alpha G^M \tag{11}$$

Substituting into equation (10) gives:

$$\begin{aligned}&u^{M+1}(k)=u^M(k)\\&-\alpha\left(\frac{2A}{\overline{r}^2}[x(k)+u(k)-r(k+1)]+\frac{2B}{r^2}[x(k)+u(k)-\overline{r}]+\lambda(k+1)\right)_M\end{aligned} \tag{12}$$

where α is the step factor, obtained from project experience.

5) Calculate the following performance metrics:

$$\left|\frac{J\left(U^{M+1}\right)-J\left(U^M\right)}{J\left(U^M\right)}\right|<\varepsilon \tag{13}$$

Among them:

$$J\left(U^k\right)=\sum_{k=0}^{k=N-1}\left\{A[x(k)+u(k)-r(k+1)]^2+B[x(k)+u(k)-\overline{r}]^2\right\}/\overline{r}^2 \tag{14}$$

ε is the specified small amount. Stop the computation if it is satisfied, otherwise make $M=M+1$ and go to step 2.

Note the necessary condition for computing the Hamiltonian function H to take a very small amount $\frac{\partial H}{\partial U}=0$, as seen in $\lambda(k)=0$, $k=0,1,\cdots,k,\cdots,N-1$. If there is no constraint on the terminals, there is no transversal condition constraint, i.e., $\lambda(N)=0$.

2.4. *Resource allocation of cross-border e-commerce supply chain based on Hamiltonian loop*

Based on the previous description of the cross-border e-commerce supply chain platform problem and the research of existing models, the utility functions of suppliers and distributors of the cross-border e-commerce supply chain are proposed, which are represented by equation (15):

$$\begin{aligned}u_b&=n_c\alpha_b-dp+r\gamma+n_b\phi_b+c_b-\theta_b\omega_b\\u_c&=n_b\alpha_c-M\delta+dp\beta\lambda+n_c\phi_c+c_c-\theta_c\omega_c\end{aligned} \tag{15}$$

where u_b and u_c are functional expressions of the utility obtained by suppliers and delivery agents through the platform, n_b and n_c are the number of cross-border e-commerce suppliers and delivery

agents, respectively. a_b and a_c are the strengths of cross-network externalities, p is the average payment of suppliers in a delivery, and λ is the proportion of the total payment earned by delivery staff in a single transaction. In this paper, we assume that the proportion of the deliveryman's commission is the same across different platforms, which is consistent with the reality. w_b and w_c are the average waiting time of suppliers and delivery workers, θ_b, θ_c are the time value parameters of suppliers and delivery workers, c_b, c_c are the fixed goodwill utility of suppliers and delivery workers to the platform.
where β is the matching rate of suppliers and delivery workers. This paper further assumes that suppliers are waiting time and price sensitive, while distributors are revenue sensitive, i.e., suppliers are more sensitive to the effect of time on utility compared to distributors.

In this chapter, the number of matches is set by the form of Hamiltonian loop algorithm. Then, the revenue function of the cross-border e-commerce delivery platform can be obtained, as shown in equation (16):

$$\pi = dn_b p\beta\left(1-\lambda\right) \tag{16}$$

The revenue of cross-border e-commerce delivery platform is each transaction completion platform to carry out a proportional draw, and cross-border e-commerce delivery platform operation of the marginal cost is low, so this chapter assumes that the marginal cost of cross-border e-commerce delivery platform is zero.

3. Simulation and analysis of algorithms

3.1. Analysis of dynamic optimization allocation of cross-border e-commerce supply chain resources

3.1.1. Dynamic optimization of supply chain configuration optimal solution. Next, the validation of the model constructed in Subsection 2.4 is carried out through an arithmetic example and the relationship between the influence of each parameter on the solution results is investigated. Assume that there are 7 enterprises (i.e., N = 7) that are resourcing a certain model of pallet through the cross-border e-commerce platform P within 3 years (i.e., T = 3), among which there are 3 demand-side (i.e., $n=3$, $i=1,2,3$), and 4 supply-side (i.e., $N-n=4$, $j=4,5,6,7$). Set the discount factor $\rho=0.9$, a unit of pallets of cross-border e-commerce platform revenue conversion factor and matching cost coefficients of $s_p=80$, $m_p=16$ (to 10 pallets as a unit for calculation). The initial number of pallet units for each firm is $X_1(0)=X_{10}=20$, $X_2(0)=X_{20}=30$, $X_3(0)=X_{30}=24$, $X_4(0)=X_{40}=18$, $X_5(0)=X_{50}=22$, $X_6(0)=X_{60}=30$, $X_7(0)=X_{70}=23$. The number of pallet units that are idle on each supply side each year, i.e., the number of pallet units that can be used for sharing, and the number of resources that can be supplied are shown in Fig. 2. The number of resources that can be supplied by the four suppliers varies with respect to the three demand-side supply resources, and the second supplier ($j=5$) has the highest number of pallets supplied to the outside world, with the total number of units being 32.

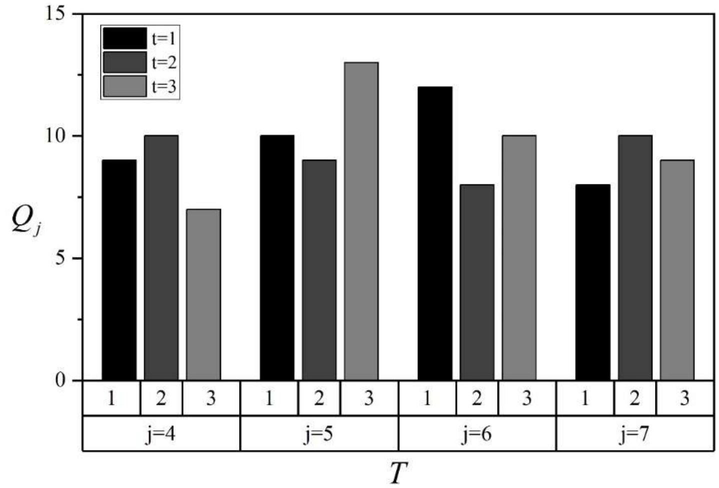


Fig. 2. Quantity of resources available

Considering that pallets are not a consumable resource, and in order to simplify the calculation, it is assumed here that the attenuation coefficient per unit of pallet is the same, i.e., $r_k = 6\%(k=1,2,\ldots,7)$, and the revenue conversion factor per unit of pallet is the same, i.e., = $s_k = 200(k=1,2,\ldots,7)$. The other relevant parameters are shown in Table 1, with the value of m_k on the demand side of the resource ranging from 16 to 19, and the value of m_k on the supply side ranging from 13 to 15.5.

Table 1. Parameter value

/	Resource demand side(n=3)			Resource provider(N-n=4)			
Argument	$i=1$	$i=2$	$i=3$	$j=4$	$j=5$	$j=6$	$j=7$
m_k	19	17	16	15.5	15.5	14	13
ϕ_k	5%	4%	5%	5%	7%	2%	6%
v_k	1800	1800	2000	2000	1800	2600	1700

Based on the data in Fig. 2, each intermediate variable is calculated and the optimal proportion of dynamic optimal allocation of supply chain resources based on cross-border e-commerce platform and the approximate number of units of each enterprise's pallet in 3 years are obtained, as shown in Table 2 and Table 3, and Table 4. being able to monitor the proportion of resources that the supply side of the supply chain's idle resources is sharing to the demand side in each time period, the optimal proportion of the supply of external resources of Supplier $j=4$ at the moment of $t=3$ is the highest, which is respectively 26.98%, 28.56%, and 28.56%. The peak supply moment of supplier $j=7$ is at moment $t=1$, and the optimal proportion is 26.58%, 36.85%, and 25.42%, respectively, which verifies the feasibility of the Hamiltonian loop model.

Table 2. The optimal proportion of dynamic optimal configuration a_{ij}^*

a_{ij}^*	$i=1$			$i=2$			$i=3$		
	$t=1$	$t=2$	$t=3$	$t=1$	$t=2$	$t=3$	$t=1$	$t=2$	$t=3$
$j=4$	19.21%	15.63%	26.98%	27.56%	20.58%	28.56%	18.23%	15.23%	28.56%
$j=5$	23.42%	24.89%	14.23%	31.35%	31.59%	14.78%	22.88%	24.68%	15.26%
$j=6$	14.56%	23.48%	18.96%	21.26%	30.97%	19.85%	13.89%	22.87%	20.25%
$j=7$	26.58%	16.59%	20.85%	36.85%	21.19%	21.36%	25.42%	16.24%	22.69%

Table 3. The number of the desired square pallets X_i^* (integral)

Demand side	$i=1$		
	$t=1$	$t=2$	$t=3$
X_i^*	28	30	34
$X_i(0)$	28		
Demand side	$i=2$		
	$t=1$	$t=2$	$t=3$
X_i^*	40	45	45
$X_i(0)$	32		
Demand side	$i=3$		
	$t=1$	$t=2$	$t=3$
X_i^*	30	35	40
$X_i(0)$	25		

Table 4. The supplier's pallet approximation unit number X_j^* (integral)

Supplier	$j=4$		
	$t=1$	$t=2$	$t=3$
X_j^*	14	8	2
$X_j(0)$	20		
Supplier	$j=5$		
	$t=1$	$t=2$	$t=3$
X_j^*	15	8	5
$X_j(0)$	24		
Supplier	$j=6$		
	$t=1$	$t=2$	$t=3$
X_j^*	24	15	10
$X_j(0)$	31		
Supplier	$j=7$		
	$t=1$	$t=2$	$t=3$
X_j^*	17	10	6
$X_j(0)$	25		

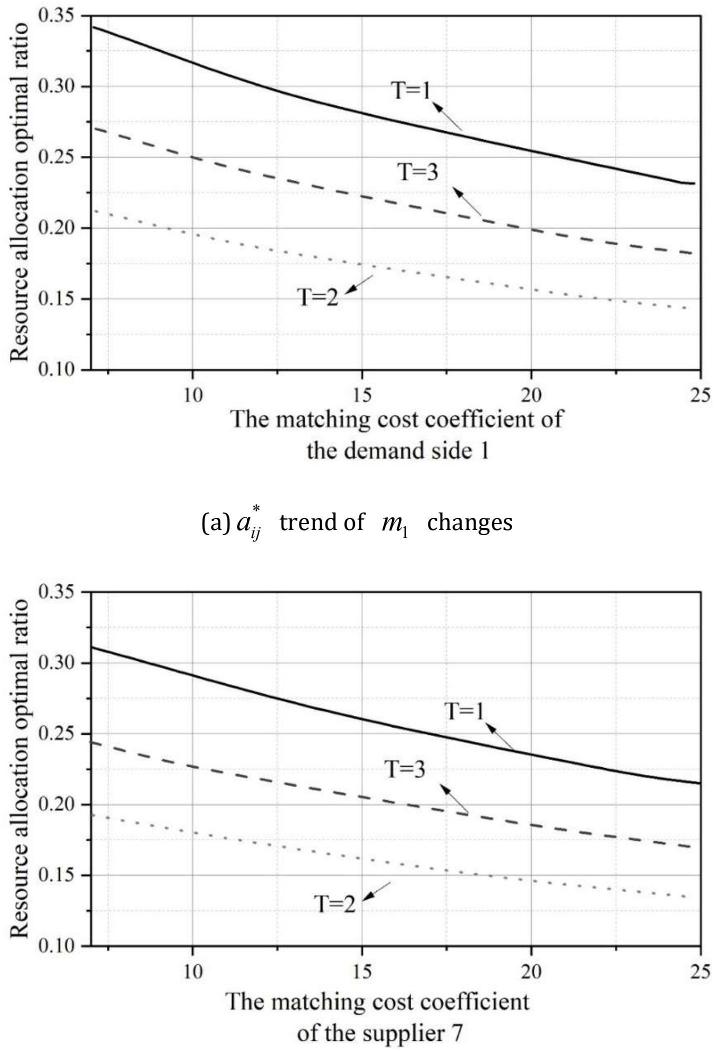
3.1.2. Parametric sensitivity analysis. Next, taking resource demander 1 and resource supplier 7 as examples, the sensitivity analysis of relevant parameters is conducted to specifically analyze the impact of unit resource matching cost coefficient m_1 on the demand side of supply chain resources, unit resource matching cost coefficient m_7 on the supply side, and conversion coefficient s_p on the cross-border e-commerce platform to convert unit resources into revenues on the matching results.

Because the total amount of resources that the supply side can use to trade on the cross-border e-commerce platform at each time is different, the proportion of resources shared by the supply side to the demand side a_{ij}^* does not represent the amount of resource allocation, but the level of the proportion of resource allocation can represent the willingness of the supply and demand sides to trade on shared resources under the premise of maximizing the profit of the system. It is worth noting that the following understanding of the willingness or motivation of the demand side of resources, cross-border e-commerce, and the supply side of resources is based on the goal of maximizing the profit of the system.

1) Resource matching cost coefficient impact $(m_1、m_7)$. Figure 3 shows the trend of matching cost coefficient, Figure (a), (b) for the other parameters remain unchanged, respectively, the supply chain idle resource demand side, supply side unit resource matching cost coefficients m_1, m_7, respectively, from 7 gradually change to 25, the resource matching results of the change situation.

(1) With the increase of the matching cost coefficient, the proportion of resource allocation all shows a gradual decline. Figure (a) shows that as the matching cost coefficient increases, the profit of the demand side will be reduced, on the one hand, affecting the total profit of the supply chain resource sharing system, thus negatively affecting the supply side of the same system, leading to the supply side is more inclined to provide resources to the demand side of the lower matching cost, reducing the proportion of the supply of this demand side, in order to protect their own interests, the proportion of the supply of the resource of the moment $t = 2$ is the lowest from 0.21 to 0.15 or less. On the other hand, the demand side can increase the capacity of its own resources to meet the demand and will reduce the willingness to share resources. Figure (b) illustrates that with the increase of the matching cost coefficient of the supply side, its cost incurred through the sharing platform transaction increases, and the profit decreases, on the one hand, the supply side will reduce the willingness to provide resources in the face of the pressure to reduce the profit, and similarly, moment $t = 2$ has the lowest proportion of resource supply, and the proportion of resource supply is only 0.143 in the case of the matching cost coefficient of 25. on the other hand, the impact of the supply chain profit of the idle resource sharing system, make the profit of the demand side lower, lead to the demand side more inclined to deal with the low-cost supply side, reduce the proportion of transactions with this supply side, to ensure that the system profit maximization.

(2) The elasticity of the supply side's ability to acquire resources to the change of its matching cost coefficient is smaller than that of the resource demand side, and the optimal ratio of resource supply in three moments ranges from 0.192 to 0.313. When there is a resource surplus on the supply side, if it is not traded on the cross-border e-commerce platform, it will result in a waste of resources and will not make any profit, so the supply side is willing to accept a certain degree of increase in the matching cost as long as it can ensure its profitability. As for the demand side of resources, when the cost of matching through the platform is larger, and the cost of increasing one's own resource capacity through investment is smaller than the cost of matching, the willingness of the demand side to acquire resource capacity through the sharing platform decreases.



(a) a_{ij}^{*} trend of m_1 changes

(b) a_{ij}^{*} trend of m_7 changes

Fig. 3. Matching cost coefficient change trend

2) The impact of the conversion factor of the cross-border e-commerce platform unit resource revenue (s_p). Figure 4 shows the trend of change in revenue, and the trend of change in resource allocation strategy when changing the conversion factor of cross-border e-commerce platform's revenue per unit of resources s_p from 10 to 28, while other parameters remain unchanged. The following conclusions can be obtained:

(1) The proportion of resource sharing from the supply side to the demand side always increases with the increase of the cross-border e-commerce platform unit resource conversion factor. The revenue conversion coefficient of the cross-border e-commerce platform per unit of resource represents the revenue it can get from the unit resource allocation, when s_p keeps getting bigger, it indicates that the cross-border e-commerce platform gains more profit through the transaction of the unit of resource, at this time, the platform will have an incentive to pay more cost for the matching to promote the supply and demand sides to reach more transactions, so that the proportion of resource allocation increases. On the other hand, the increase in platform revenue promotes the increase in the profit of the sharing system, supply and demand sides at this time are more willing to choose the sharing platform with better system profit, and share resources on the platform with good revenue to promote the conclusion of more transactions, which makes the proportion of resource matching increase, and among the three moments, the optimal proportion of resource allocation at the moment

of $t=1$ contributes the most, and when the platform revenue conversion coefficient is 28, the optimal ratio reaches 0.112.

(2) The relationship between the size of the proportion of resources supplied by the supplier to the demand side is changing. With the increase of the platform revenue conversion coefficient, although the supply proportion is increasing, but the speed of increase is different, resulting in the cross-over of the resource allocation proportion. When the platform revenue conversion coefficient is small, the proportion of resource allocation in the second year is higher than that in the third year, but the proportion of resources in the second year increases at a slower rate, so as s_p continues to increase, after the revenue conversion coefficient is 18, the proportion of resource allocation in the third year gradually exceeds the proportion of allocation in the second year, which indicates that the size relationship of its allocation willingness over time is not static.

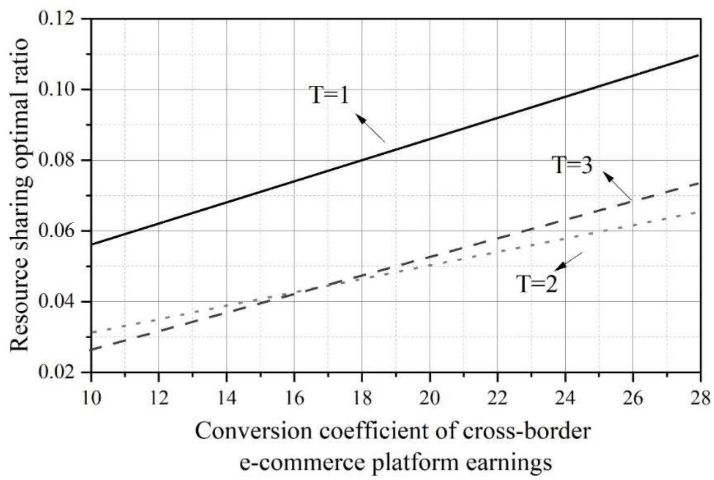


Fig. 4. a_{ij}^{*} trend of s_p changes

3.2. *Optimal price for cross-border e-commerce supply chain logistics and distribution*

In this section, the Hamiltonian loop algorithm is used to simulate the optimal price model of logistics social delivery service considering platform competition to verify the effect of the dynamic pricing strategy of logistics platform social delivery service. The effect of logistics platform competition coefficient on the dynamic price of cross-border e-commerce logistics service and the expected revenue of cross-border e-commerce logistics is analyzed. The numerical simulation parameters are selected as follows: $\sigma=1$, $\eta_i=50$, $\eta_j=50$, $\mu=0.1$, $h=1$, $\alpha=8\times10^3$, $\theta=10^3$, $C_0=1$, and the time parameter of the equilibrium point of supply and demand of logistics distribution services $T_{ba}=45$. The competition coefficient $k=(0.4,0.5,0.6)$ is taken to portray the three scenarios of low, medium, and high competition, respectively. Under different competitive situations of cross-border e-commerce logistics platform, the trajectory of the optimal logistics service price over time is different, Figure 5 shows the trajectory of the optimal price of logistics and distribution services, and the simulation results are shown. The more competitive the cross-border e-commerce logistics platform, the lower the starting price of logistics services $P(t_0)$, such as competition coefficient $k=0.6$ logistics platform social distribution service price is less than the competition coefficient $k=0.5$ and $k=0.4$ logistics platform social distribution service price, in the case of high competition, the optimal price of logistics and distribution services will be in the range of 225.

In the peak demand time [0,45] when the supply of logistics services is less than the demand, the logistics platform adopts the dynamic pricing strategy for logistics distribution services in order to incentivize the distribution personnel to join the platform to provide logistics distribution services, and the optimal price of logistics distribution services $P_i^{*}T_{ba}(t)$ is monotonically increasing with time.

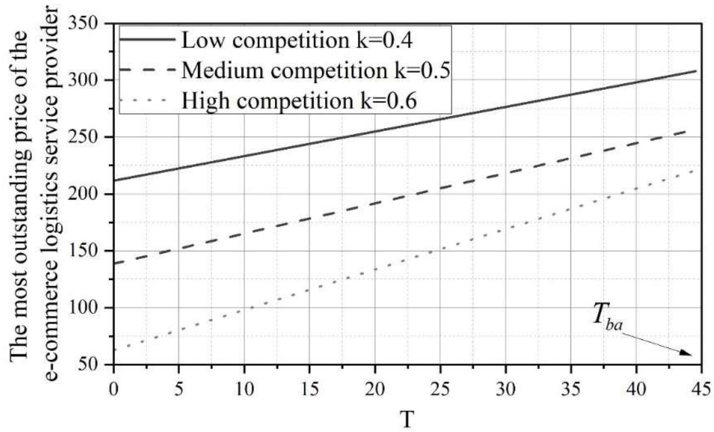


Fig. 5. Logistics distribution service is the most outstanding target

The increase of social distribution service price of logistics platform will affect the supply capacity of social distribution service and logistics order demand, Figure 6 shows the trajectory of the change of logistics distribution service supply rate and order demand, the increase of logistics service price makes the platform pay the compensation to the distribution personnel increase accordingly, which prompts the supply of logistics platform's distribution service to increase continuously over time, and the demand of logistics order will also be affected by the change of logistics distribution service price. The demand for logistics orders will also be affected by changes in the price of logistics and distribution services, and the increase in the price of logistics services will slightly reduce the demand for orders. At the equilibrium point of supply and demand $T_{ba} = 45$, the supply of logistics distribution services and the demand for logistics orders have reached a balanced state, which is equal to about 7750.

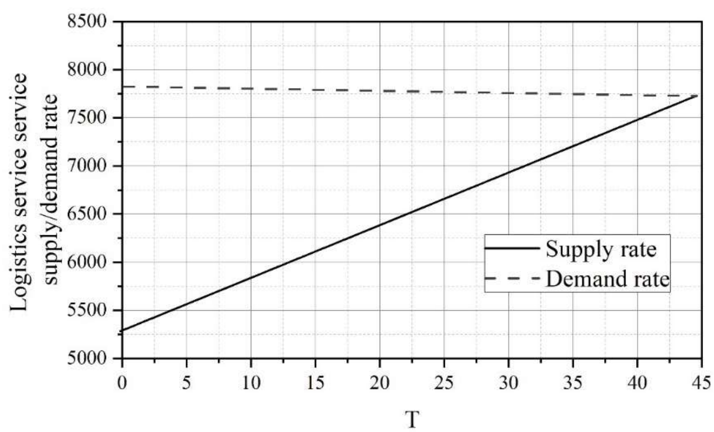


Fig. 6. Logistics distribution service supply rate and order demand change trajectory

The cross-border e-commerce platform competition coefficient k represents the degree of price competition between platforms, and the larger the competition coefficient k is, the more intense the platform competition is. Competition coefficient k will have an impact on the price of cross-border e-commerce logistics and distribution services and the platform's expected return. Figure 7 shows the relationship between the expected return of cross-border e-commerce distribution personnel and the change of platform competition coefficient k, the expected return of the logistics platform will be reduced with the increase of the competition coefficient, the expected return from the competition coefficient of 0.352×10^{8} to the competition coefficient of 0.7 of 0. That is to say, the more intense the competition between platforms, the lower the expected return of the logistics platform, the expected return of the distribution personnel does not change significantly, and the logistics operation of the actual situation is not in line with the expected return. This is in line with the actual situation of logistics operation.

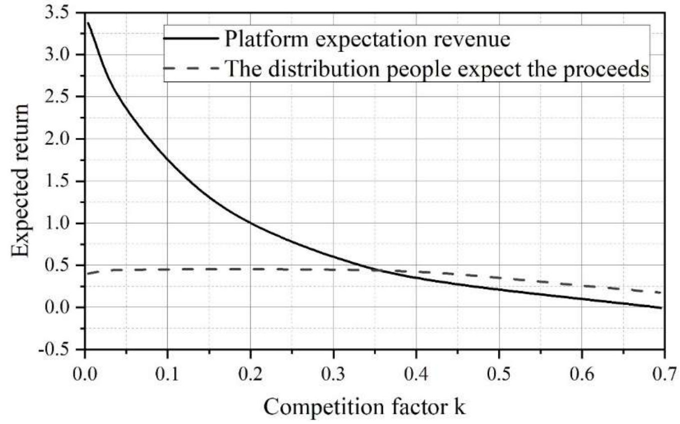


Fig. 7. Distribution personnel expect earnings and platform competition coefficient changes

Figure 8 shows the changes in the supply rate and demand rate of distribution services of cross-border e-commerce service platforms, which further validates the relationship between the supply rate and order demand rate of logistics service platforms and the platform competition coefficient k. The slopes of the trajectories of changes in the supply of distribution services of logistics platforms at the time of competition coefficient $k=0.6$ are greater than the slopes of the trajectories of changes in the supply of social distribution services of logistics platforms at the time of competition coefficients $k=0.5$ and $k=0.4$ (Slope of $k=0.6$ is 123.29> $k=0.5$ 81.227 and 54.83 for $k=0.4$). That is, when the competition of the logistics platform increases, the platform will accelerate the increase of the optimal price of the logistics and distribution services, and correspondingly accelerate the increase of the supply capacity of the social distribution services, so as to make the supply capacity of the social distribution services of the logistics platform balanced with the demand of the orders, to reduce the delay of the orders, and to maximize the expected return of the logistics platform.

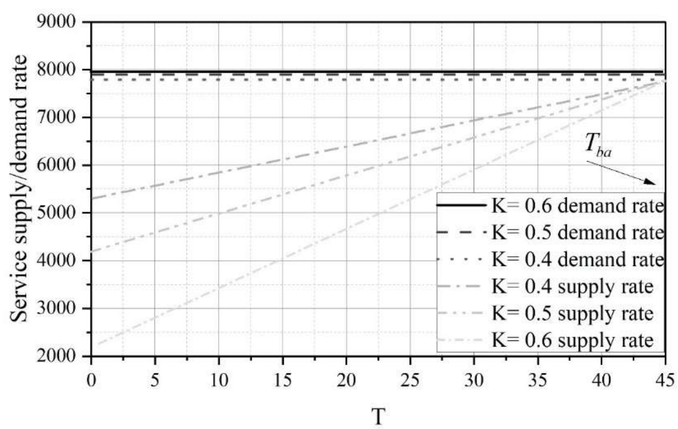


Fig. 8. The delivery service supply rate and demand rate change of cross-border e-commerce

Figure 9 shows the growth trajectory of the expected revenue of the cross-border e-commerce platform. Although the actual platform competition reduces the expected revenue of the logistics platform, the dynamic optimal pricing strategy of the logistics platform, by rapidly increasing the optimal price of the delivery service, the revenue of the logistics platform increases gradually over time, therefore, the dynamic optimal pricing strategy of the logistics platform's delivery service optimizes the logistics operation and maximizes the expected revenue of the logistics platform, and the competition coefficient is $k = 0.4$, the total expected revenue is maximized, which is 3.621×10^7.

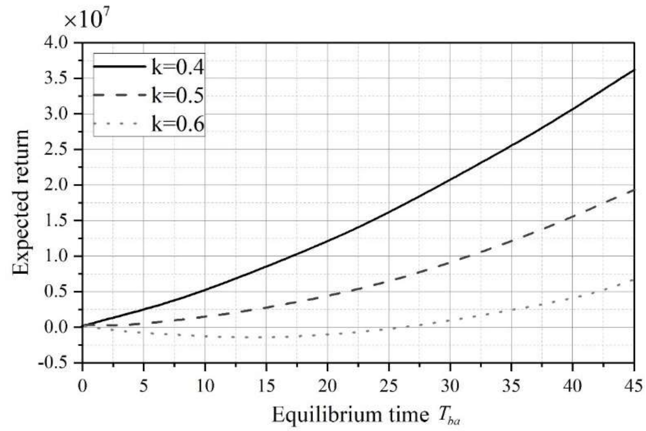


Fig. 9. Cross-border e-commerce platforms expect revenue growth

3.3. User Satisfaction under Optimized Supply Chain Resource Allocation

3.3.1. User service satisfaction. Figure 10 shows the overall satisfaction of retail customers in Company C. In 2021, the overall satisfaction of retail customers in City C was 85.989 points, and the overall satisfaction in 2021 decreased by 0.951 points compared with 2020, and the three indicators, customer service and product offerings, decreased by varying margins, and the satisfaction of market management increased from the previous year. Therefore, the focus is on analyzing customer service and product supply to find the reason for the low score.

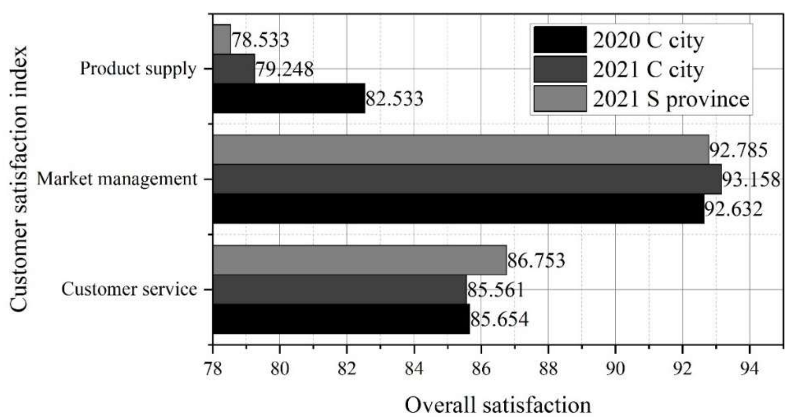


Fig. 10. C retail customer overall satisfaction

Figure 11 shows the customer service satisfaction, and in 2021 Company C's retail customers' satisfaction with customer service is 85.89 points, which is higher than the province's 0.034 points. Customer service refers to the comprehensive evaluation of retail customers on the four services of cross-border e-commerce supply chain companies, ordering, delivery, account manager and settlement, and there is no comparison of the new complaint service.

From the four indicators of customer service, the ordering method has the highest satisfaction, 86.44 points, and the satisfaction score of account manager is 85.943 points. From the evaluation of retail customers on the service of account manager, most of the customers are more satisfied with the service of account manager.

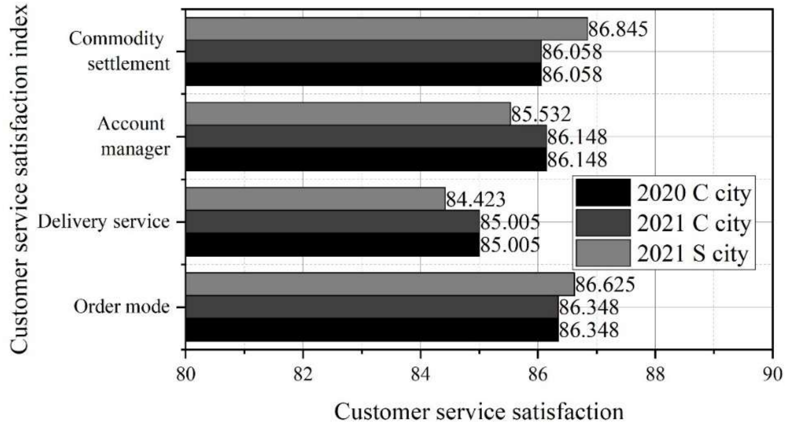


Fig. 11. Customer service satisfaction

3.3.2. Supplier satisfaction. Figure 12 shows supplier satisfaction, and retail customers' satisfaction with commodity supply in 2021 is 81.806 points, which is higher than the province's 0.913 points. Commodity supply is the retail customers' comprehensive evaluation of Company C in five aspects: the total amount of commodity supply, whether the supply information is open and transparent, the fairness of the distribution of the supply, whether there is bundled sales and the profitability. 68% of retail customers are "generally satisfied" with the current retail customer rating and believe that their rating should be improved. Commercial enterprises should have a more scientific and precise classification of retail customer grades, optimize the allocation of product resources, and at the same time inform the grading criteria, so as to improve customer satisfaction.

From the five indicators of commodity supply, retail customers' satisfaction with bundled sales is the highest at 93.783 points, followed by satisfaction with profitability at 86.834 points, and satisfaction with supply information ranks third at 81.6 points. Satisfaction with total supply and tight supply allocation was relatively low, at 72.625 and 73.618 points respectively.

Overall, Company C needs to further improve its services to enhance retail customers' sales profitability by formulating strategies to guide sales and rationally allocating commodity sources.

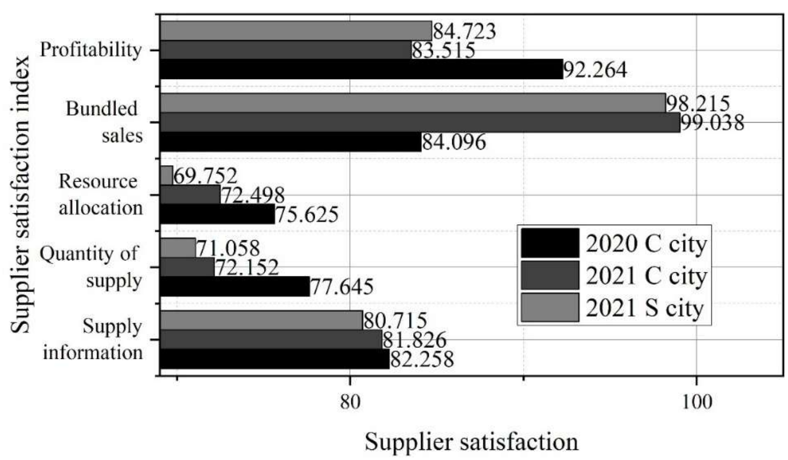


Fig. 12. Supplier satisfaction

3.4. Increase in clients

Table 5 shows the customer enhancement, the optimization of cross-border e-commerce supply chain resource allocation through the Hamiltonian loop algorithm, the empirical study in the system of Company C proves that the resource allocation model and the source allocation strategy proposed in this paper can be practically applied, in which the incremental amount of users of an order in one cycle reaches 2,656, at the same time, the resource allocation model can satisfy the customers' demand, which makes the supply and demand of product resources more balanced, and argues that the study is This demonstrates the scientific validity and reliability of this study.

Table 5. Customer enhancement

Date	The number of customers in this issue	The number of customers ordered last time	Order customer increment	The number of customers is booked	The number of customers is booked up	Subscribe to customer increment
Monday	2148	1685	463	2178	1485	693
Tuesday	2893	2142	751	2839	1824	1015
Wednesday	2788	1856	932	2785	1678	1107
Thursday	2345	2085	260	2318	1585	733
Friday	1855	1605	250	1825	1275	550
Total	12029	9373	2656	11945	7847	4098

4. Conclusion

This paper explores the application of Hamiltonian circuit in cross-border e-commerce supply chain resource allocation, constructs a model of optimal resource allocation, and proposes an optimal control gradient calculation method, which reconstructs the cross-border e-commerce supply chain resource allocation based on the description of cross-border e-commerce supply chain platform problems and the study of existing models .

1) The usability of the constructed model is verified by using an arithmetic example, in which four suppliers have different quantities of supply resources for three demand sides, and the second supplier has the largest number of external supplies, totaling 32 pieces. Supplier $j=4$ has the highest optimal percentage of external resources supplied at moment $t=3$, which are 26.98%, 28.56% and 28.56% respectively.

2) Sensitivity analysis of the parameters shows that the supply side's ability to acquire resources is less elastic to changes in its matching cost coefficient than the resource demand side, and the optimal proportion of resource supply at the three moments ranges from 0.192 to 0.313. Therefore, for the supplier, as long as it can ensure its profit, the supply side is willing to accept a certain degree of increase in matching cost.

3) Under the optimization of supply chain resource allocation, the user satisfaction situation is analyzed through empirical surveys.In 2021, the customer service satisfaction of retail customers of Company C is 85.89 points, which is higher than that of the province by 0.034 points. In 2021, Company C's retail customers' satisfaction with customer service was 85.89 points, which was higher than the province's 0.034 points, and most customers were more satisfied with supply chain services.

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