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Research on optimization of vocal music teaching mode and design of personalized learning path based on AI algorithm

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ABSTRACT
Vocal music is an art about the perception and expression of sound. Successful vocal music teaching is to cultivate students' unique singing personality, so this paper constructs a personalized vocal music teaching mode with the help of AI algorithm. Subsequently, it describes the problem of service object learners under personalized learning path recommendation, proposes a personalized learning path recommendation strategy based on ant colony optimization algorithm, and verifies the recommendation effect of personalized path through simulation experiments. Then the cognitive diagnosis model based on KM-VDINA is proposed to diagnose students' vocal music knowledge under personalized learning path. The article concludes through experiments that the personalized vocal music teaching model based on AI algorithms requires the integration of online and offline teaching, while focusing on the integration of teaching inside and outside the classroom. The vocal music learning path of most students can be expressed as (000000)→(100001)→(101001)→(101100)→(111100)→(101110)→(111111). Students have multiple trajectories to master the attributes of vocal music knowledge, so teachers can explain the attributes of knowledge that are easier to master according to the actual situation, and then explain the attributes of knowledge that are difficult for students to master.

Keywords: AI algorithm, ant colony algorithm, km-vdina, cognitive diagnosis, vocal music teaching mode

1. Introduction

In recent years, the rapid development of Artificial Intelligence (AI) has brought great changes to various fields, and vocal music teaching is no exception [1-2].The application of AI technology optimizes the traditional teaching mode and provides more personalized and efficient learning

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solutions for vocal music learners, which makes the development of the field of vocal music education step into a new stage [3-5].

The development of AI technology provides a broad application space for vocal music education. Nowadays, major technology companies have invested in research and development resources to explore the application of AI technology in the field of vocal music education, through machine learning and deep learning and other technical means, AI can play multiple roles in the process of vocal music education [6-9]. The important point is that AI technology can provide personalized learning solutions, traditional vocal education model often focuses on large-scale standardized training, ignoring the individual differences of each learner [10-11]. With the help of AI technology, the learner's learning style, interests, weaknesses and other personalized information can be accurately accessed and analyzed, and based on this, the AI system can be tailored to the characteristics of each learner's learning plan and teaching materials, making the vocal learning process closer to the needs of students [12-15]. In addition, AI technology also plays an important role in vocal performance evaluation, creation and innovation, which can give students more help [16-17].

The article firstly constructs a personalized vocal music teaching model based on AI algorithm from three levels: teaching concept, combination of online and offline teaching, and combination of teaching inside and outside the classroom. Secondly, it defines and describes the learner, learning object and learning path, applies the ant colony algorithm in the personalized learning path recommendation model, defines the relevant parameter information, gives the final algorithm steps, and verifies the feasibility and effectiveness of the algorithm through simulation experiments. Then, a cognitive diagnostic model based on KM-VDINA is proposed, and the improved VDINA cognitive diagnostic model is solved using the EM algorithm to calculate the learners' mastery of all the knowledge points, and to obtain the learners' personalized weak knowledge points for a certain vocal music course. Finally, the vocal music teaching class in a school is used as an empirical research object to experimentally verify and analyze the method proposed in this paper.

2. Construction of personalized vocal music teaching mode based on AI algorithm

2.1. Influence of Personalized Teaching Concept on Personalized Vocal Music Teaching

In today's society, there is a renewed emphasis on the idea of personalized teaching because of the requirement to cultivate creative talents and the idea that "everyone can be cultivated into an appropriate talent". Personalized teaching is adaptive teaching, that is, it requires the teaching arrangement to adapt to the environmental conditions of personality differences, create the corresponding situation, construct the corresponding curriculum knowledge, and establish the corresponding evaluation system. Personalized vocal music teaching is based on respecting students' individuality and differences, through the creation of humane learning situations, so that students can learn in their own different bases, along their own suitable directions, in their own suitable ways of learning, and can get efficient development of the teaching process.

2.2. Personalized vocal teaching needs the integration of online and offline teaching

The vocal teaching resources based on AI algorithms are the basis for realizing the concept of online and offline integrated teaching, and the relationship between teacher-student interaction and resource invocation in vocal teaching is shown in Figure 1.

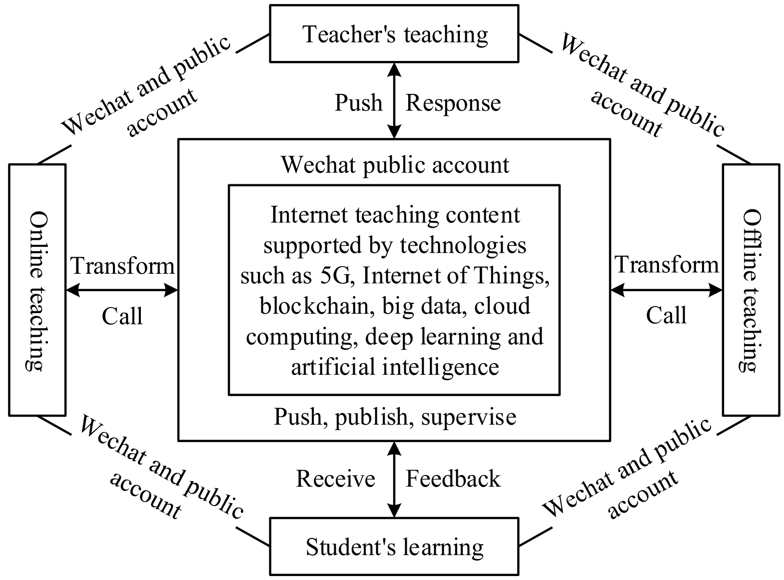


Fig. 1. Schematic diagram of integrated teaching mode

1) Create a humane learning situation, optimize the environment of students' independent learning centered on students' learning, it is necessary to pay attention to students' independent learning. Vocal teachers have the responsibility to help students do a good job of professional learning personal planning, and guide students to gradually form the habit of independent learning. Secondly, the right to choose the singing repertoire is given to the students, and the right to play the leading role of the teacher in teaching is reflected in the setting of the repertoire range. Students are guided to choose repertoire according to their own preferences, to understand the relevant background information of the works, and to anticipate the problems they may encounter in learning.

2) Introducing selected and organized online music resources. Integrate all kinds of resources and information related to vocal music teaching such as graphics, PPT, music scores, singing audio and video, etc., and if necessary, you can quickly compile them into a combination of information and push them to students' WeChat. You can also save your regular classroom video, online classroom video, and students' singing videos, etc., and push them to students at any time for reference.

3) Online and offline teaching should be connected with each other. Online teaching can make full use of online music resources, and can cross the time and space limitations, but there are also imperfections. With the help of AI-based algorithms to batch process interactive matters, you can even put the whole semester's teaching plan and teaching content online for students to access and learn on their own. Online and offline teaching should not be opposed to each other, and the choice of which method depends on the teaching content and teaching effect. As long as the students can solve the problems through independent study, the teacher can let the students solve the problems before or after the class through the online way. Offline vocal lessons, on the other hand, are used to solve problems that can only be solved by face-to-face teaching, so as to significantly improve teaching efficiency.

4) Laying the foundation for students' fragmented learning Smartphones have become a necessity in the information age, and vocal music teaching can start from its positive side, utilizing this feature of students to promote fragmented vocal music learning.

2.3. Personalized vocal teaching needs to pay attention to the integration of teaching inside and outside the classroom.

Personalized vocal teaching is a teaching system in itself, and in-class and out-of-class are also just two different teaching stages. The conceptual model of the integrated vocal music teaching model based on AI algorithm for in-class and out-of-class is shown in Figure 2. The main tasks in the pre-class phase are to familiarize with the background of the work, make personal plans for further study, and ask the teacher any problems encountered in learning through interaction. The mid-lesson stage is the traditional teaching stage, where face-to-face vocal mini-lessons are still preferred when conditions are favorable, and students should be guided to develop the habit of actively identifying problems. Singing works of the key, the teacher should be word by word and phrase by phrase to point out, individual words and phrases can be sung in the demonstration of guiding students to repeatedly chew, experience, specific teaching content can be supplemented with micro-lesson video or PPT and other online vocal resources. Singing works after the initial formation, can be exchanged in various ways, verification, encourage students to be open-minded to accept everyone's comments. The post-course stage should create conditions for students to participate in vocal practice to verify the results. The main channel of vocal practice should also be concerts, if you can go on stage to show, there will certainly be a better effect of exercise. After the work is perfected, students should be guided to summarize in time and further expand the breadth and depth of knowledge with the help of various online resources.

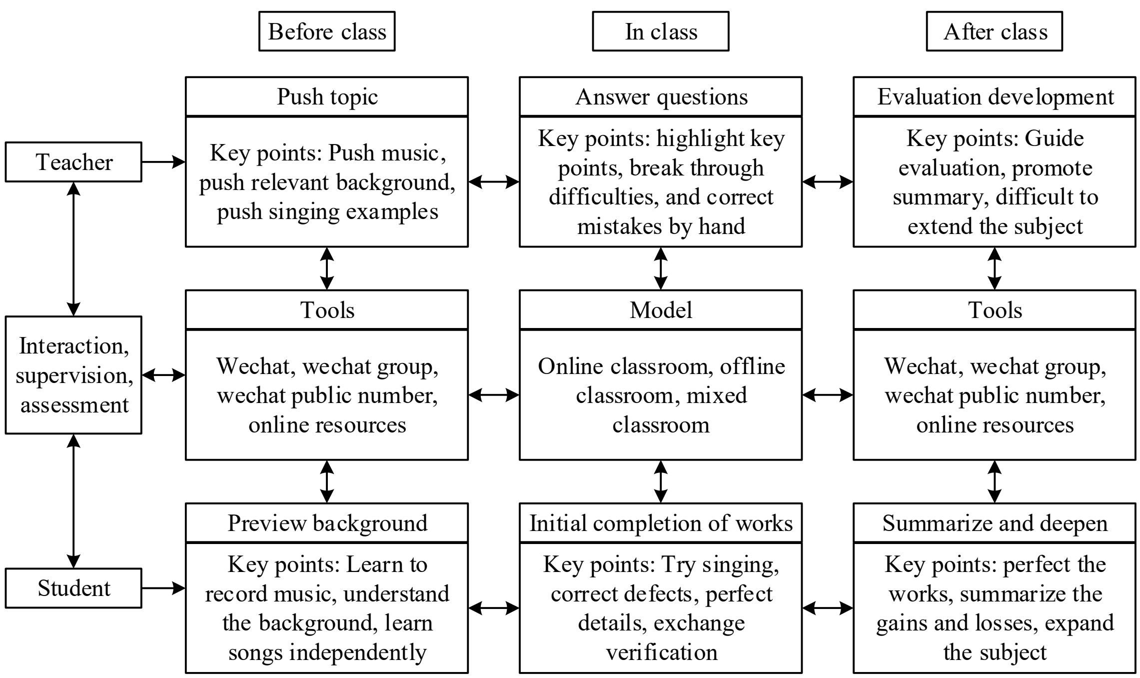


Fig. 2. The idea model of the integrated vocal music teaching model in the course of class

3. Recommendation of personalized learning path under the optimization of vocal music teaching mode

3.1. Description of the problem

3.1.1. Learner characterization:

1) Characterization of learners. Learning style is the learner's continuous and consistent way of learning with personality characteristics, which is a synthesis of learning strategies and learning

tendencies. Learning styles are divided into four categories: ① Convergent. This type of learner is good at finding the practical value of theories and can effectively solve practical problems, and this type of learner prefers text-based learning objects. ② Divergent. This type of learners are good at multi-perspective examination of specific situations or situations, often using the observation method to find answers to problems from a variety of points of view, this type of learners prefer learning objects rich in graphs, tables, animation and other vivid symbols. ③ Assimilative. This type of learner is usually interested in theories and abstract concepts, needs explanatory explanations rather than practical interactions, and prefers learning objects rich in verbal explanations such as audio and video. ④ Regulated. This type of learner is hands-on, enjoys implementing challenging programs, and they prefer to learn through experience.

The same learner will show a tendency towards multiple learning types, and different learners have different degrees of tendency towards various learning styles.

To this end, the vector $s=(s_1,s_2,s_3,s_4)$ describes the learner's stylistic tendency, where s_1,s_2,s_3,s_4 denote the degree to which the learner belongs to the aggregative, divergent, assimilative, and moderating types, respectively degree, and there are $0\le s_i \le 1(i=1,2,3,4), s_1+s_2+s_3+s_4=1$.

2) Learner's knowledge level. In order to accomplish the learning objectives, the learner needs to interact with the original concepts in the learner's mind by continuously acquiring the information expressed in the new concepts and then integrating them into the conceptual structure that the learner already has. According to the breadth and depth of knowledge that the learner has already mastered, the e-learning system can analyze the learner's knowledge level through a simple test and quantitatively describe it with $l(0\le l\le 1)$, the closer l is to 0 means that the learner's knowledge level is closer to that of a beginner, and the closer it is to 1 means that the learner is closer to that of an expert.

3.1.2. Learning Object Characterization:

1) Knowledge expression characteristics of learning objects. A learning object is any digital resource that can be reused to support learning, and it constitutes the basic learning resource of an online learning system. Learning objects can be used to express knowledge in a variety of ways. The vector $c=(c_1,c_2,c_3,c_4)$ quantitatively describes the extent to which a learning object adopts the various ways of knowledge expression, where c_1,c_2,c_3,c_1 denote that a learning object adopts text, respectively, symbols, videos, and interactive software to express the proportion of the knowledge content of a learning object to the total knowledge content of that learning object, and there are $0\le c_i \le 1(i=1,2,3,4), c_1+c_2+c_3+c_4=1$.

2) Difficulty coefficient of the learning object. The designer of a learning object can give a difficulty coefficient $d(0\le l\le 1)$ based on the breadth and depth of knowledge expressed by the learning object. The closer d is to 0, the more suitable the learning object is for beginners: the closer it is to 1, the more suitable the learning object is for experts.

3.1.3. Metadata-based learning path definition. The repository of an online learning system not only stores learning objects, but also metadata (LOM) describing the attributes of learning objects. Using the metadata, the system can build a directed graph $G(O,A)$ that describes the association relationship of each learning object. In the graph G, $O=\{o_1,o_2,...,o_n\}$ represents the set of learning objects oriented to a particular learning task. $A=[a_{ij}]$ denotes the adjacency matrix between learning objects. $a_{ij}=1$ indicates that o_j is learned before o_i is learned. $a_{ij}=0$ means that there is no connection between the objects o_i and o_j. The so-called learning path is to select a set of learning object combinations $J = \langle o_i, o_j, \cdots, o_k \rangle$ with a backward and forward order in the directed graph G. In order to complete the learning task, the learner needs to complete the learning of all the learning objects on the learning path in turn.

3.1.4. Recommendation problem for learning paths. An efficient learning path enables learners to complete the learning task with minimum time cost. However, it is often difficult for learners to determine a reasonable learning path in advance because: ① Each learner has his/her own unique attributes, and each learning object has its own unique attributes, and different types of learning objects are suitable for different types of learners. ② The online learning system contains a large number of learning objects, due to information overload, it is impossible for learners to carefully analyze the attribute characteristics of each learning object, thus it is difficult to judge the most suitable learning object for them. ③ The quality of completing the learning task depends on the overall optimization of various combinations of learning objects on the learning path, and it is difficult for the learner to estimate in advance whether a learning object conforms to the optimization on the whole learning path.

In real life, when a person encounters the problem of learning path selection, he has to resort to two aspects of information: ① The degree of conformity between the characteristics of learning materials and his own characteristics. ② What learning paths learners similar to themselves tend to choose.

E-learning systems record a wealth of information about learning objects and learners, and in addition, many of them require learners to evaluate the objects they have learned. Aiming at this feature of online learning systems, this paper investigates how to make intelligent agents utilize the various information of online learning systems to determine the learning path preferences of target users and actively make learning path recommendations to learners, so as to improve the personalized service level of online learning systems.

3.2. Personalized learning path recommendation model construction

3.2.1. Introduction to the Ant Colony Algorithm. Ant Colony Optimization (ACO) algorithm is a bionic optimization algorithm that simulates the intelligence of an ant colony.The ACO algorithm searches for the optimal solution through the process of colony evolution composed of candidate solutions. Let m be the total number of ants in the colony, $d_{ij}(i, j = 1, 2, \ldots.., n)$ denotes the distance between city i and j, $B_i(t)$ denotes the number of ants located in element i at the moment of t, and $\Gamma = \{f_{ij}(t) \mid c_i, c_j \subset C\}$ is the number of ants in the set C at the moment t is the set of the amount of information remaining on the two-by-two connection of elements in l_{ij}. With equal amount of information on each path at the initial moment, let $\tau_{ij}(0) = C$ (C is a constant) as a simulation of the secretion of actual ants [18]. The ants $K\,(K = 1, 2 - \ldots, m)$ determines the transfer direction $p_{ij}^k(t)$ according to the amount of information on each path during the movement process, and $p_{ij}^k(t)$ denotes the probability of the ant k transferring from the city i to the city j at the moment of t, then:

$$P_{ij}^k(t) = \begin{cases} \dfrac{\left[\tau_{ij}(t)\right]^{\alpha} \cdot \left[\eta_{ik}(t)\right]^{\beta}}{\sum_{s \subset allowed_k} \left[\tau_{is}(t)\right]^{\alpha} \cdot \left[\eta_{is}(t)\right]^{\beta}} & \text{if } j \in allowed_k \\ 0 & \text{Or else} \end{cases} \tag{1}$$

where: $allowed_k = (0,1,2,...,n-1)$, $allowed_k$ denotes the set of cities that ants k are allowed to pass through next, this artificial mosquito swarm system and has some memory function, $tabu_k(k=1,2,...,m)$ is used to record the list of cities that ants k have currently traveled through, and the set $tabu_k$ is dynamically adjusted with the evolutionary process. α indicates the amount of information on the path plays a large role in the ant's choice of path, η_{ij} is the expected degree of transfer from city i to city j. And $\beta = 0$ at that time, it becomes a heuristic algorithm with pure positive feedback. After n moments, the ants can walk through all the cities and complete the cycle once. The path traveled by each ant is a solution. At this point, the amount of information on each path is to be updated according to the following equation:

$$\tau_{ij}(t+n) = (1-\rho)\cdot\tau_{ij}(t) + \Delta\tau_{ij}(t) \tag{2}$$

$$\Delta\tau_{ij}(t) = \sum_{k=1}^{m} \Delta\tau_{ij}^{k}(t) \tag{3}$$

where ρ denotes the pheromone volatilization coefficient, then $1-\rho$ denotes the pheromone residual factor, in order to prevent infinite accumulation of information, the value range of ρ is $\rho \subset [0,1)$, and $\Delta\tau_{ij}(t)$ denotes the pheromone increment on the path (i, j) in the current loop, the The initial moment $\Delta\tau_{ij}(t)=0$, and $\Delta\tau_{ij}^{k}(t)$ denotes the amount of pheromone left on the path by the kth ant in this loop.

3.2.2. Definition of relevant parameters. Relevant parameters including pheromone, pheromone update rule, heuristic information, constraints and other parameters need to be defined before the learning path recommendation algorithm is constructed:

1) Pheromone. Evaluation information is equivalent to the pheromone left on the path passed by the ants during foraging, and the intensity of the pheromone is used as the basis for the later learners to choose the learning object. The pheromone on the learner's path from knowledge point i to knowledge point j, i.e., the evaluation of the learning object by the previous learner k is $\tau_{ij}(0 \le \tau_{ij} \le 1)$, and at the initial moment, the amount of information on the learning path is 0.

2) Pheromone update rule. Historical evaluation information evaporates over time, and new evaluation information plays a more important role in the path selection process. Therefore, the pheromone update rule on the path (i, j) at the $t+n$ moment can be adjusted as follows:

$$\tau_{ij}(t+n) = (1-\rho)\cdot\tau_{ij}(t) + \Delta\tau_{ij}(t) \tag{4}$$

The learner evaluates each knowledge point as it is completed during the learning process, and when the learner k completes $q_i \to q_j$, the pheromone's local update rule is:

$$\Delta\tau_{ij}^{k}(t) = \begin{cases} P & \text{If the learner completes learning } q_j \text{ between } t \text{ and } t+1 \\ 0 & \text{Or else} \end{cases} \tag{5}$$

The global updating rule for pheromone is to evaluate the effect of the overall learning path after the learner has completed the given learning task and update the pheromone on the learning path with the updating formula:

$$\Delta\tau_{ij}^{k} = \begin{cases} \dfrac{P}{M_k} & \text{If you are learning } k\text{, you have learned } q_j \text{ in this learning task} \\ 0 & \text{Or else} \end{cases} \tag{6}$$

where P denotes the evaluation information of the learner, which affects the convergence speed of the algorithm to a certain extent, and M_k denotes the number of knowledge points learned by the learner in this learning process [19].

3) Inspired information. Let $\overline{q^e}$ denote the average value of the expression feature of the knowledge point that the learner K has already learned, and $tabu_k$ is the set of knowledge points that the learner has already learned, and the expression feature of the current knowledge point i q_i^e and the expression feature of the knowledge point that will be learned j, q_j^e. The matching degree $sim_e(i,j)$ is:

$$sim_e(i,j) = \frac{\sum_{i \in tabu_k} (q_i^e - \overline{q^e})(q_j^e - \overline{q^e})}{\sqrt{\sum_{i \in tabu_k} (q_i^e - \overline{q^e})^2} \sqrt{(q_j^e - \overline{q^e})^2}} \tag{7}$$

Let $\overline{q^l}$ denote the average of the difficulty coefficients of the knowledge points that have been learned by learner K. The difficulty coefficient q_i^l of the user's learned historical knowledge points i matches the difficulty coefficients of the knowledge points that will be learned j q_j^l to the extent $sim_l(i,j)$:

$$sim_l(i,j) = \frac{\sum_{i \in tabu_k} (q_i^l - \overline{q^l})(q_j^l - \overline{q^l})}{\sqrt{\sum_{i \in tabu_k} (q_i^l - \overline{q^l})^2} \sqrt{(q_j^l - \overline{q^l})^2}} \tag{8}$$

4) Constraints. It is necessary to find learners with similar learning levels and learning styles as the current learner k. Let $K=(k_1,k_2,...,k_n)$ denotes the set of learners who have completed the learning objectives, $K_j(j=1,2,3,...,m)$ denotes the knowledge level of the jth learner in the set, and the degree of similarity between the cognitive level of the current learner k and that of k_j can be expressed by the parameter $\delta_1(0 \le \delta_1 \le 1)$.

Then the set of learners who are similar to k in cognitive level can be defined as:

$$K_{similar}^{l} = \{K_j \,||\, K_j^l - k^l \,| \le \delta_1, K_j \in K\} \tag{9}$$

The learning style vector of a learner is $K^s = \{k_i^s \mid k_i^s = (s_i^1, s_i^2, s_i^3, s_i^4), i = 1,2,...,n\}$, then the learning style of learner j in set K is $k_j^s = (s_j^1, s_j^2, s_j^3, s_j^4), j = 1,2,...,n\}$, the degree of similarity between the learning style of the current learner k and K_j can be expressed by the parameter $\delta_2(0 \le \delta_2 \le 1)$, then the set of learners who are similar to k in terms of learning style can be defined as:

$$K_{similar}^{s} = \{K_j \,||\, K_j^s - k^s \,| \le \delta_2, K_j \in K\} \tag{10}$$

The learning path recommended by the teaching system is the evaluation information provided as pheromone by learners who have similar learning paths and knowledge levels with the current learner and have completed the learning task, which is located in the set of:

$$K_{similar} = K_{similar}^{s} \cap K_{similar}^{l} \tag{11}$$

3.2.3. Personalized Learning Path Algorithm Construction:

1) Learning path recommendation goal. Teaching system in the learning path recommendation process, to control the learning path, and find the optimal learning path problem of the specific tasks can be described as follows:

(1) traversing the knowledge points on the effective learning path once and recommend a higher probability of learning path recommended to the learner.

(2) The learning path to be recommended is influenced by historical learner evaluation information and follows the knowledge structure rules.

(3) The expression characteristics and difficulty coefficients of the knowledge points on the path should be adapted to the learners.

2) Learning object selection probability. The teaching system decides the next knowledge point it will recommend for the current learner in the process of path recommendation based on the amount of information on the learning path. The table $allowed_k$ is used to record the knowledge points that are allowed to be selected by the current learner k in the next step, which abides by the organizational and structural relationships of the knowledge structure, and the set is dynamically adjusted with the learning process. During the search process, the system calculates the state transfer probability based on the pheromone and heuristic information on each path to derive the next knowledge point that will be recommended.

$p_{ij}^{k}(t)$ denotes the state transfer probability of j as the next recommended knowledge point based on the current learner's learning of the knowledge point of i at the moment of t:

$$P_{ij}^{k}(t) = \begin{cases} \dfrac{\left[\tau_{ij}(t)\right]^{\alpha} \cdot \left[sim_e\left(i,j\right)\right]^{\beta_1} \cdot \left[sim_l(i,s)\right]^{\beta_2}}{\sum\limits_{s \subset allowed_k} \left[\tau_{is}(t)\right]^{\alpha} \cdot \left[sim_e\left(i,s\right)\right]^{\beta_1} \cdot \left[sim_l(i,s)\right]^{\beta_2}} & \text{If } j \in allowed_k \\ 0 & \text{Or else} \end{cases} \tag{12}$$

where: α denotes the size of the role played by the evaluation information on the path on the system recommended knowledge points, β_1 and β_2 for the expected degree of shifting from the knowledge point i to the knowledge point j, respectively, denotes the relative importance of the expression characteristics of the knowledge points and the difficulty coefficients in the process of path recommendation. Then the recommendation probability of the final learning path is the product of the probability that the knowledge point on the path is recommended.

3) Algorithm steps. According to the above principle, the following learning path recommendation algorithm is recommended:

(1) Construct a directed acyclic graph $G(O, A)$ based on learning object metadata and association relations.

(2) Derive all possible learning paths based on the directed acyclic graph.

(3) Parameter initialization.

(4) Construct the set of users $K_{similar}$ that are similar to the current learner's learning style and cognitive level and to complete the learning task according to Eq. Complete the corresponding pheromone set.

(5) Based on the learning objects that the current learner has completed, calculate the probability that he/she may choose the next learning object according to the state transfer probability formula.

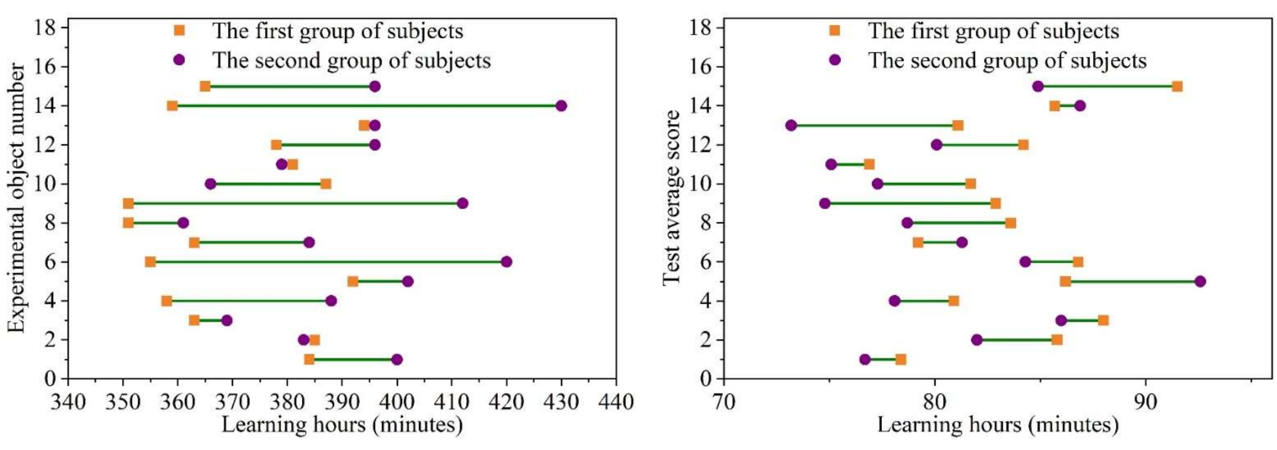
(6) Pheromone update.

(7) According to the knowledge structure relationship on the learning path, select the next learning object for the learner in accordance with the inspirational information and pheromone, in each iteration, the system has to make M selections in order to discover a suitable learning path for the learner, and a complete solution establishment is completed.

(8) For the objective function, the algorithm terminates when it reaches the maximum number of iterations, and recommends the first N paths in the solution set with higher probability to be recommended to the current learner.

3.3. Simulation experiments

3.3.1. Algorithm effectiveness analysis. The effectiveness of the algorithm examines whether the recommended path can effectively improve students' vocal performance and shorten their study time. As known from the experimental process, 30 experimental subjects were divided into two groups to learn, 15 people in each group. The experimental subjects in the first group learn according to the path recommended by the algorithm, and the experimental subjects in the second group learn according to the default path. The experiment and analysis are shown in Fig. 3 (Fig. a is the comparison of the experimental subjects' learning time, and Fig. b is the comparison of the experimental subjects' average test scores). From the figure, it can be seen that the average learning time of the experimental subjects in the first group is 373.2 minutes, and the average learning time of the experimental subjects in the second group is 392.1 minutes. The average test mean score of the experimental subjects in the first group was 83.3 and the average test mean score of the experimental subjects in the second group was 81.6. The learning duration of the experimental subjects in the first group is overall greater than that of the experimental subjects in the second group, and the average test score of the experimental subjects in the first group is overall greater than that of the experimental subjects in the second group. From the results, it can be seen that the average score of the experimental subjects who learn according to the recommended path is better than the average score of those who learn according to the default path, and the time spent on learning is also shorter. It can be seen that the algorithm proposed in this paper has good effectiveness.



(a) The study length of the subjects was compared (b) Test average score contrast

Fig. 3. Experiment and analysis

3.3.2. Algorithm accuracy analysis. Algorithm accuracy analysis mainly examines whether the learning path given by the recommendation algorithm matches the user's needs. This experiment analyzes the accuracy of the algorithm from two perspectives: recommendation matching degree and F1 value.

1) Recommendation Matching Degree. Recommended Matching Degree (RMD) refers to the ratio of the number of repeated sections between the recommended learning path and the user's favorite learning path to the number of all sections in the path. The recommended matching degree of the experimental subjects is shown in Fig. 4. From the figure, it can be intuitively seen that the minimum value of the recommended matching degree of the experimental objects is 0.4721, the maximum value is 0.8936, and most of the values are between 0.68 and 0.90, indicating that there is a high degree of similarity between the learning paths recommended by the algorithm in this paper and the students' favorite learning paths. Therefore, the learning path recommendation algorithm proposed in this paper has good accuracy.

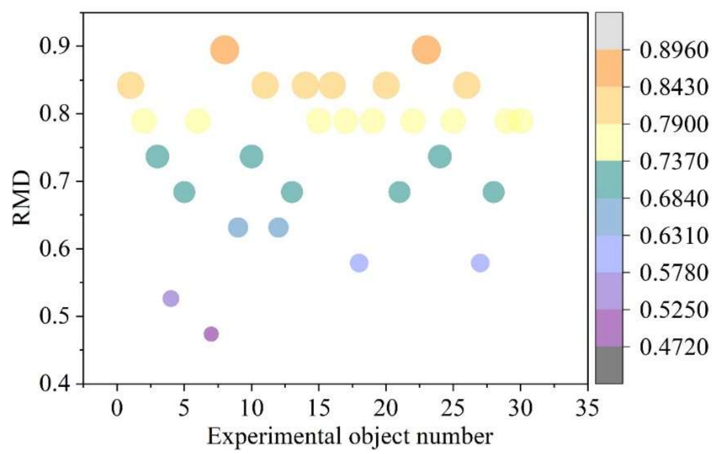


Fig. 4. Recommended matching of subjects

2) Fl value. Precision rate refers to the proportion of all correctly retrieved items to all items that were actually retrieved. Recall refers to the ratio of all correctly retrieved items to all items that should have been retrieved. In this experiment, let N represent the total number of learning paths chosen by the experimental subjects themselves and the learning paths recommended by the algorithm, the evaluation questionnaire of the recommended learning paths is rated by the users on a 5-point scale, and in this paper, we believe that 1~2 points indicate that the users do not like it, and 3~5 points indicate that the users like it, and according to the statistical information of the experimental subjects' evaluations of the recommended paths on the three dimensions obtained from the questionnaire, the evaluation information of the experimental subjects on the recommended path evaluation information is shown in Table 1.

Table 1. The evaluation information of the experimental object on the recommended path

Evaluation dimension	Statistics of various levels of evaluation					number of Dislike	number of like
	1	2	3	4	5		
The learning path meets my needs	0	1	2	5	26	1	33
I accept the difference between the recommended path and the path I choose	0	2	1	20	8	2	29
I think the recommended path is more in line with my needs	0	2	6	15	5	2	26

The precision and recall data are shown in Table 2 (1, 2 and 3 in the table denote the three dimensions for evaluating the recommendation path, respectively).

Table 2. Accuracy and recall data

	Recommended learning path number			No recommended number of learning paths		
	1	2	3	1	2	3
The number of paths that the experimental object likes	29	28	28	30	30	30
The number of paths that the subjects don't like	1	2	2	0	0	0
Total path	30	30	30	30	30	30

Using the data in the table to calculate the F1 value of the three dimensions, the F1 value of the three evaluation dimensions of the recommended path is shown in Figure 5. From the figure, it can be seen that the F1 values of the three evaluation dimensions of the recommended path are all above 0.62, indicating that the research subjects are more satisfied with the path recommended by the algorithm, which further proves that the learning path recommendation algorithm proposed in this paper has good accuracy.

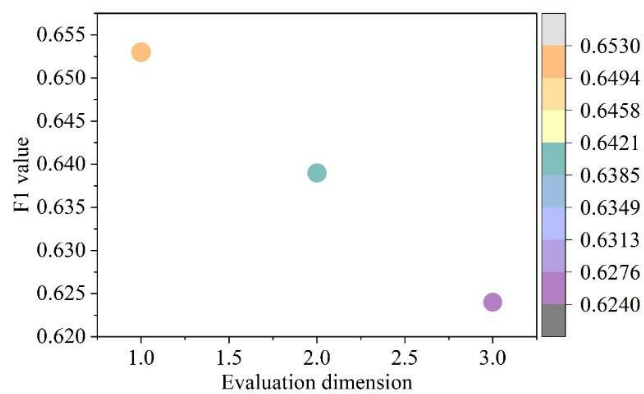


Fig. 5. The recommendation path three F1 values for the dimension of the evaluation

3.3.3. Algorithm adaptation analysis. The algorithm adaptation analysis mainly examines whether the features of the learning objects on the learning path given by the recommendation algorithm match the learning features of the user. The results of the recommended path adaptation are shown in Figure 6. From the figure, it can be visualized that the adaptations of the recommended paths are all below 0.33, and most of them are between 0.1 and 0.25. It can be seen that the learning objects on the recommended learning paths are more in line with the user's learning characteristics, so the learning paths recommended to the user by the algorithm proposed in this paper have high adaptability.

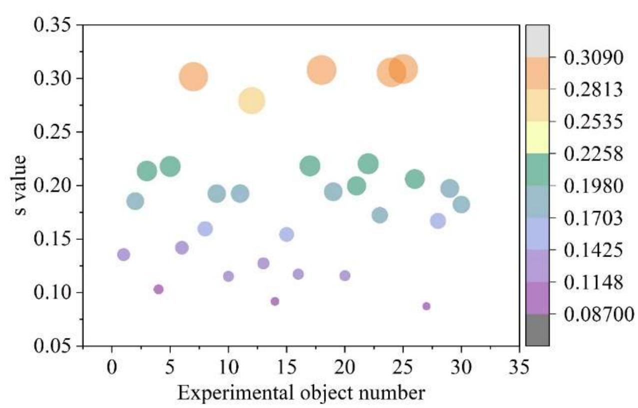


Fig. 6. Recommended path fitness results

4. Vocal knowledge diagnosis based on KM-VDINA

4.1. Basic Overview of Cognitive Diagnostics

4.1.1. Cognitive Diagnostic Theory. Cognitive Diagnostic Theory (CDT) is constructed on Item Response Theory. Cognitive Diagnostic Theory has gradually become a hot topic of research in the fields of smart education and psychology, promoting the personalized development of smart education.

4.1.2. Cognitive Diagnostic Model. Cognitive diagnostic model is a measurement model to achieve cognitive diagnosis through the learner's test answer data and the knowledge point association of the test questions.DINA model can model each learner as a vector of knowledge point mastery by using the score information of the test questions that the learner has done with him/her, and the association information of the test questions with the knowledge points [20]. There are U learners in the model $P=\{P_1,P_2,........,P_U\}$, V trials $J=\{J_1,J_2,.......,J_V\}$ and K knowledge points $S=\{S_1,S_2,......,S_K\}$. The learner-test question information can be represented as a score matrix $R_{U\times V}$, which represents each learner's score for each test question, and r_{uv}, which represents the learner's P_u scores for the test question J_v. The test-knowledge point information can be represented as an association matrix $Q_{V\times K}$, where $q_{vk}=1$ indicates that the test question J_v contains the knowledge point S_k and $q_{vk}=0$ indicates that the test question J_v does not contain the knowledge point S_k.

Each learner P_u is described as a knowledge point mastery vector $\vec{\alpha}_u=\{\alpha_{u1},\alpha_{u2},.......,\alpha_{uK}\}$, where each item indicates whether the learner P_u has mastered the knowledge point S_k: when $\alpha_{uk}=1$ means that learner P_u has mastered knowledge points S_k, and conversely, when $\alpha_{uk}=0$ means learner P_u has not mastered knowledge points S_u. Then the potential response of the learner P_u to the test question J_v, η_{uv}, is calculated as in Eq. $\eta_{uv}=0$ indicating that the learner P_u cannot answer the test question J_v correctly. $\eta_{uv}=1$ indicates that the learner P_u can answer the test question J_v correctly.

$$\eta_{uv}=\prod_{k=1}^{K}\alpha_{uk}^{q_{uk}} \tag{13}$$

Two parameters, the miss rate s and the guess rate g, are introduced to model the learner's answering of a question in the real state. s_v denotes the probability that the learner has not answered the question correctly even though the learner has fully grasped all the knowledge points contained in the test question J_v. g_v denotes the probability that the learner has not mastered all the knowledge points contained in the test question J_v, but has answered the question correctly. Then the probability that a student P_u answers the test question J_v correctly in the real state is:

$$P_v(\alpha_u)=P(r_{uv}=1\mid\alpha_u)=g_v^{1-\eta_{uv}}(1-s_v)^{\eta_{uv}} \tag{14}$$

The marginal maximum likelihood estimation of the solution formula was solved using the EM algorithm to obtain parameter estimates for each trial question $\{\hat{s}_1,\hat{s}_2,\cdots,\hat{s}_V\},\{\hat{g}_1,\hat{g}_2,\cdots,\hat{g}_V\}$. And by maximizing the posterior probability of the test score of the learner P_u, the knowledge level of this learner α_u is calculated as shown in Equation (15).

$$\begin{aligned}\hat{\alpha}_u &= \arg\max_{\alpha}(\alpha \mid R_u) \\ &= \arg\max_{\alpha} L(R_u \mid \alpha, \hat{s}_v, \hat{g}_v)P(\alpha) \\ &= \arg\max_{\alpha} L(R_u \mid \alpha, \hat{s}_v, \hat{g}_v) \\ &= \arg\max_{\alpha} \prod_{v=1}^{v} P(R_u \mid \alpha, \hat{s}_v, \hat{g}_v)\end{aligned} \tag{15}$$

4.2. Overall framework of the KM-VDINA algorithm

In this section, the KM-VDINA personalized learning path recommendation algorithm is proposed by combining knowledge graph, cognitive diagnosis and collaborative filtering.The overall framework of the KM-VDINA algorithm is shown in Fig. 7. Firstly, based on the improved VDINA cognitive diagnosis model, the learner's mastery of knowledge points is diagnosed based on learner-video-knowledge point related data, and the learner's personalized weak knowledge points are identified. Then in order to obtain more accurate knowledge point similarity information, the semantic information in the knowledge graph and the user's past learning records are comprehensively considered to obtain the final inter-knowledge point similarity.

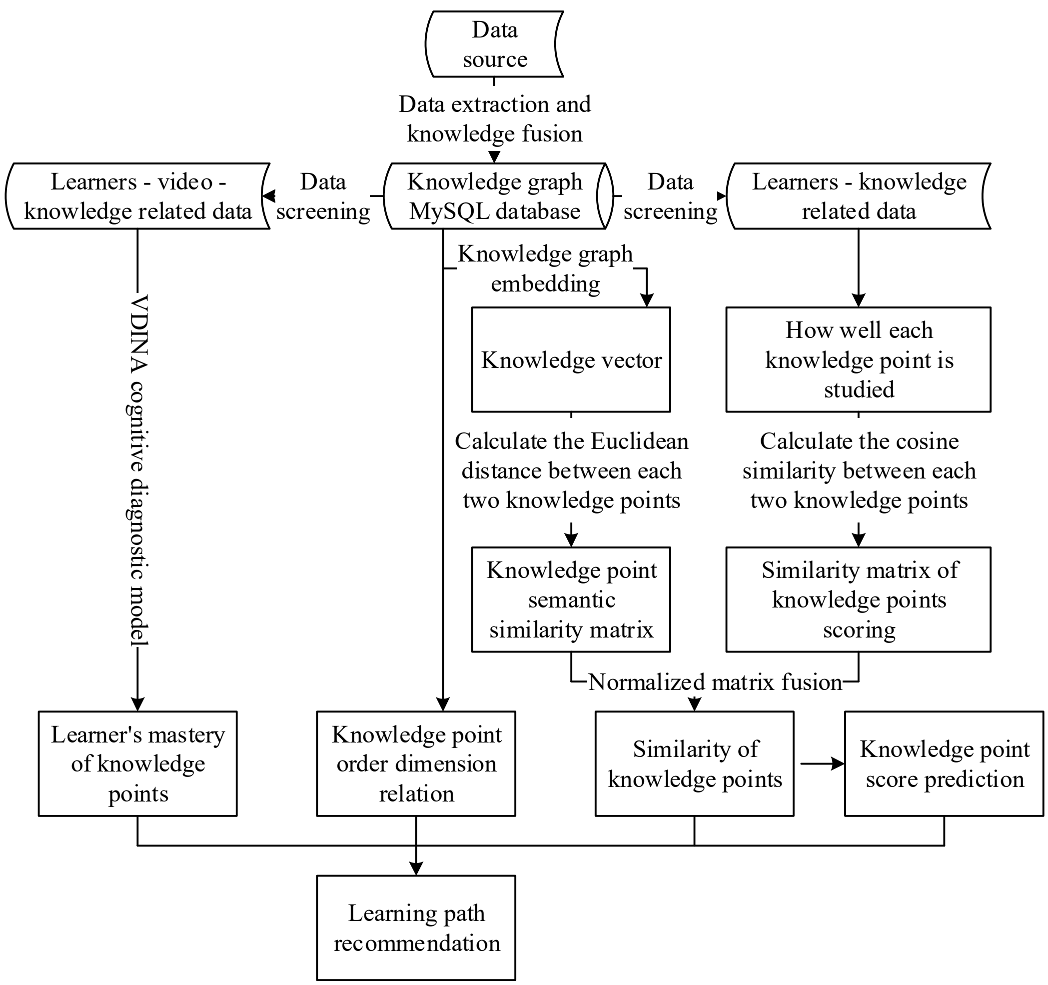


Fig. 7. KM-VDINA algorithm framework

4.3. Diagnosis of learners' cognitive knowledge of vocal music

This paper improves the traditional DNA model by calculating the a posteriori probability of all the ways of combining the mastery degree of knowledge points, and taking the probability of mastering a certain knowledge point as the mastery degree of that knowledge point, so as to model the mastery degree of a knowledge point from a discrete value to a continuous one between 0 and 1, and to derive a continuous type of mastery degree of each knowledge point by the learner.

Firstly, the roles included in the VDINA model are introduced.The VDNA model has U learners $P=\{P_1,P_2,.......,P_U\}$, V videos $J=\{J_1,J_2,.......,J_V\}$ and K knowledge points $S=\{S_1,S_2,......,S_K\}$. The learner-video learning situation can be represented as a matrix $R_{U\times V}$, which represents U learners watching the video, which is quantified by r_{av} = the number of hours the learner P_u has watched the video J_v/total number of hours of the video J_v, and quantifies the number of hours that the learner P_u of the video J_v. The video-knowledge point association information can be represented as a matrix $Q_{V\times K}$, which illustrates which knowledge points are contained in each video respectively, one row of the matrix represents a video and one column represents a knowledge point, $q_{vk}=1$ indicates that the video J_v contains the knowledge point S_k, $q_{vk}=0$ means that the video J_v does not contain the knowledge point S_k.

The cognitive level of each learner P_u is described as a knowledge point mastery vector $\vec{\alpha}_u=\{\alpha_{u1},\alpha_{u2},\ldots\ldots,\alpha_{uK}\}$, each dimension of the vector corresponds to a knowledge point and is a continuous value from 0 to 1. α_{uk} denotes the probability that the learner P_u has mastered the knowledge point S_k, with $\alpha_{uk}=1$ indicating that P_u has fully mastered S_k and $\alpha_{uk}=0$ indicating that P_u has not mastered S_k at all. Then the learner P_u is calculated as shown in the equation, which uses the proportion of the knowledge points of the learner P_u mastering the video J_v to all the knowledge points of the video J_v, as the learner P_u after watching the video P_u J_v.

$$\eta_{av}=\frac{\alpha_u^{'}\cdot q_v}{q_v^{'}\cdot q_v} \tag{16}$$

where $\alpha_u^{'}$ denotes the transpose of the learner's P_u knowledge point mastery vector, q_v denotes the vth row of the video-knowledge point correlation matrix $Q_{V\times K}$, i.e., the knowledge points contained in the video J_v, and $q_v^{'}$ denotes the transpose of q_v.

Two parameters, probability s and probability g, are introduced to model the learner's viewing of the video in the real state, and the probability s_v denotes the probability that the learner grasps all the knowledge points but does not finish watching the video J_v. The probability g_v denotes the probability that the learner has not mastered all the knowledge points, but has watched the video J_v. Then the probability that learner P_u in the real state has watched the video J_v is shown in Eq. The probability of $r_{uv}=1$ in the learner-video learning situation matrix $R_{U\times V}$ for learner P_u with cognitive level $\bar{\alpha}_u$.

$$P_v(\vec{\alpha}_u)=P(r_{uv}=1\,|\,\vec{\alpha}_u)=g_v^{1-\eta_{uv}}(1-s_v)^{\eta_{uv}} \tag{17}$$

The main task of the VDINA model is: based on the known learner-video learning situation matrix $R_{U\times V}$ and the video knowledge point correlation information matrix $Q_{V\times K}$, firstly, the parameter estimation of the probability s and probability g of each video is calculated, and then based on this, each learner P_u 's knowledge mastery level $\vec{\alpha}_u = \{\alpha_u 1, \alpha_u 2, \cdots \alpha_{uK}\}$. The following section describes the exact calculation method.

The EM algorithm is first used to compute an edge maximum likelihood estimate of the formula, which leads to parameter estimates $\{\hat{s}_1, \hat{s}_2, \cdots, \hat{s}_Y\}$, $\{\hat{g}_1, \hat{g}_2, \cdots, \hat{g}_V\}$. The EM algorithm is divided into the following four steps:

1) Randomly initialize each video probability s and probability g with parameters $\{s_1, s_2, \cdots, s_V\}, \{g_1, g_2, \cdots, g_V\}$

2) Perform step E. Based on $\{s_1, s_2, \cdots, s_V\}, \{g_1, g_2, \cdots, g_V\}$ and the learner-video learning situation matrix $R_{U\times V}$, for each video J_v, calculate $E1, E2, E3, E4$ separately, where $E1$ = the expectation of the number of learners who have not mastered the video J_v containing all the knowledge points. $E2$ =Expectation of the number of learners who have mastered video J_v containing all knowledge points. $E3$=Expectation of the number of learners who have not mastered all the points contained in video J_v, but have watched the video. $E4$ = Expectation of the number of learners who have mastered all the points in Video J_v and have watched the video.

3) Perform step M. For each video J_v: the probability of updating it s_v is the ratio of the expectation of the number of learners who have mastered all the knowledge points contained in the video J_v but have not watched the video, to the expectation of the number of learners who have mastered all the knowledge points contained in the video J_v. The probability of updating it g_v is the ratio of the expectation of the number of learners who have not mastered all the knowledge points contained in the video J_v but have watched the video, to the expectation of the number of learners who have not mastered all the knowledge points contained in the video J_v. This is shown in equation (18).

$$\hat{s}_v = \frac{E2 - E4}{E2}, \hat{g}_v = \frac{E3}{E1} \tag{18}$$

4) Iterate steps 2 and 3 repeatedly until the algorithm converges by obtaining parameter estimates $\{\hat{s}_1, \hat{s}_2, \cdots, \hat{s}_V\}, \{\hat{g}_1, \hat{g}_2, \cdots, \hat{g}_V\}$ for each video probability s and probability g.

Then diagnose each learner's P_u knowledge point mastery $\vec{\alpha}_u = \{\alpha_{u1}, \alpha_{u2}, \cdots \alpha_{uK}\}$. Calculate the posterior probability of all knowledge points mastery degree combination way, the probability of mastering a knowledge point as the mastery degree of that knowledge point. It is known that there are K knowledge points, and each knowledge point has two cases of mastery and non-mastery, then there are 2^K cases in the vector of mastery degree of all knowledge points $\vec{\alpha}$. For learner P_u, the learner's P_u mastery of knowledge point S_k can be expressed as a continuous value between 0 and 1 by dividing the sum of probabilities of the various cases in which the kth digit of $\vec{\alpha}$ is equal to 1 by the sum of probabilities of all the $\vec{\alpha}$ as Eq. (19) shown.

$$\begin{aligned}\tilde{\alpha}_{uk} = P(\alpha_{uk}=1|R_a) &= \frac{\sum_{aa_{uk}=1} P(\vec{\alpha}_x|R_a)}{\sum_{x=1}^{2^K} P(\vec{\alpha}_x|R_a)} \\ &= \frac{\sum_{a_{uk}=1} L(R_a|\vec{\alpha}_x,\hat{s}_V,\hat{g}_V)P(\vec{\alpha}_x)}{\sum_{x=1}^{2^K} P(\vec{\alpha}_x|R_u)} \\ &= \frac{\sum_{a_{uk}=1}\prod_{v=1}^{V} L(R_{uv}|\vec{\alpha}_x,\hat{s}_V,\hat{g}_V)P(\vec{\alpha}_x)}{\sum_{x=1}^{2^K} P(\vec{\alpha}_x|R_u)}\end{aligned} \tag{19}$$

5. Cognitive Diagnostic Model Application Analysis

In order to verify the validity of the methodology proposed in this paper, this section was tested in a school vocal music teaching class as a research subject. The study investigated 8 classes with 352 students, of which 173 were male and 179 were female. The self-developed "Cognitive Diagnostic Assessment Test Questions" consisted of 10 multiple-choice questions, 4 fill-in-the-blanks questions, and 4 answer questions, totaling 18 questions (A1~A6 cognitive attributes are represented as auditory perception, theoretical cognition, physiological cognition, skill application, artistic expression, and metacognition, respectively). Data entry in the cognitive diagnostic model yielded a matrix and an ideal measurement model. Among them, there are 9 ideal mastery patterns, which are (100000) (110000) (101000) (101100) (101110) (111000) (111100) (111110) (111111).

1) Analysis of the probability of students of different genders in mastering attributes in vocal music teaching. In order to examine whether there is an effect of gender in vocal music learning, we choose the parameter estimation in the platform and enter the data of male and female students' responses respectively, and the attribute mastery probability of students of different genders is shown in Figure 8. In the figure, it can be visualized that the attribute mastery probability of male students in A1, A3, A6 and A5 is slightly higher than that of female students, and only in the two attributes of A2 and A4 is lower than that of female students. In general, there is a clear difference between male and female students in learning knowledge. Therefore, teachers need to instruct girls in A5 and A6.

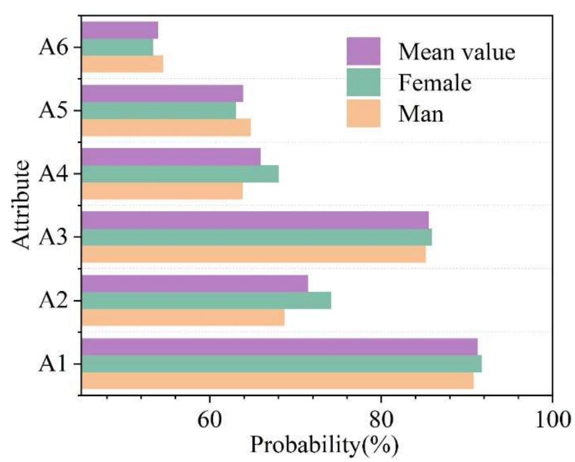


Fig. 8. The properties of different gender students have the probability

2) Analysis of the probability of students in different classes in mastering the content of vocal music teaching. According to the response data of 352 subjects, selecting the parameter estimation in the platform, the run can produce eight classes in each attribute mastery probability, the attribute mastery probability of students in different classes is shown in Figure 9. In the figure, the classes from high to low mastery probability are class 1, class 5, class 6, class 7, class 8, class 3, class 4, class 2, and the mean value of the attribute mastery probability of the first four classes is higher than 68%, which indicates that these four classes have a better mastery of the four operations of complex numbers, and they can be continued to carry out the subsequent teaching. The mean value of the attribute mastery probability of class 8 and class 3 is close to 70%, which indicates that the two classes have a good mastery of this knowledge point is good and remedial teaching is needed for the mastery of attributes A5 and A6. The mean value of the probability of mastery of attributes for classes 5 and 2 is close to 70% and remedial teaching is needed for the mastery of attributes A4-A6. This shows that there are some differences in the mastery of each attribute of this knowledge across the classes. At the same time, it can be observed that the average value of the probability of mastery of attributes in each class is relatively clustered, between 67% and 80%, which indicates that the students have a better mastery of the content of this section, and that the teacher only needs to spend a certain amount of time on remedial teaching of the difficult attributes (A6, A4).

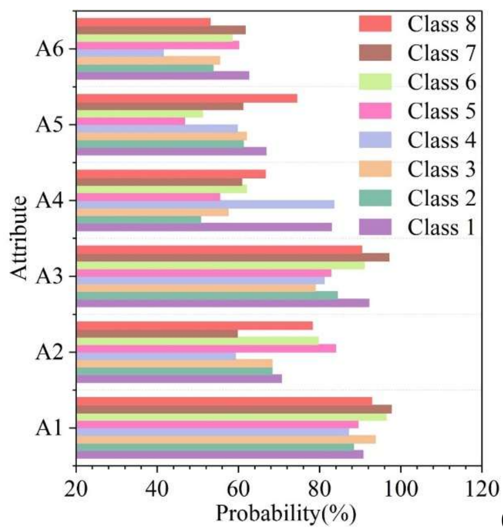


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Fig. 9. The properties of different class students have the probability

3) Analysis of cognitive attribute mastery in subjects with the same total score. According to the cognitive diagnosis theory, it is possible that the attribute mastery patterns of students with the same scores in the quiz are different, while students with different total scores may have the same attribute mastery patterns. The response data of 6 students were recorded separately, and the probability of attribute mastery of subjects with the same total score is shown in Figure 10. Through comparison, it was found that all six students answered 11 questions correctly in questions 1-14, and all of them obtained 78 points, but the probability of cognitive attributes mastered by these six subjects showed a very obvious difference, i.e., there were different knowledge structures among them. As shown in the figure, No.3 students need to remediate the learning of each knowledge point, especially the learning of attributes A2, A4 and A5, for this kind of students, the teacher should strengthen the students' counseling and teaching of the cognition of vocal theory, technique application and artistic expression.Students No.2, No.4 and No.5 did not fully master the attributes A4, A5 and A6, and the teacher must teach the students' technique application, artistic expression and metacognition with the tutorial instruction accordingly. Therefore, when the total scores are the same, the cognitive diagnostic assessment is more conducive to evaluating the students' individualized level.

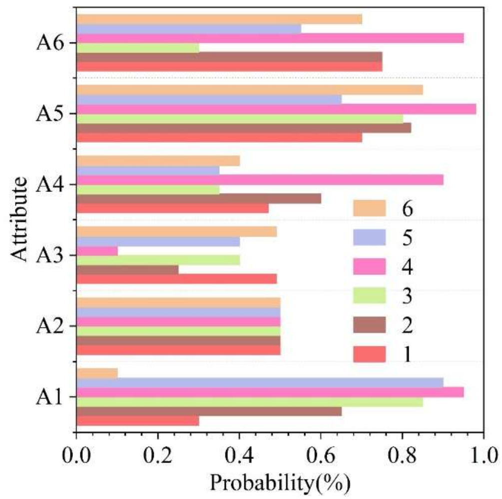

Fig. 10. The same thing with the same test property

4) Learning path analysis. The students' vocal music learning path represents the hierarchical characteristics of the knowledge states and portrays the partial order relationship existing between the knowledge states, which is formed strictly according to the cognitive laws of the subjects. The learning path of this study is shown in Figure 11. In the figure, the trajectory represents (000000) → (100001) → (101001) → (101100) → (111100) → (101110) → (111111). The trajectory represents the order of attribute mastery for most subjects, which can clearly portray the process of subjects' acquisition of knowledge attributes during the teaching and learning process, and points out a clear path and direction for subjects to develop from a low level of learning to a high level of learning. There are multiple trajectories for students to master all attributes, rather than mastering all attributes in the order in which they are defined, i.e., after mastering attribute A1, the majority of students mastered attributes A3 and A4 before mastering attribute A2, and then mastering attributes A5 and A6. Although some students master the attributes according to the defined order, but only a small number of students in the order of mastery, so according to the logical structure of the students, in the vocal music teaching prioritize the knowledge attributes that are easier for the students to master, and then explain the knowledge attributes that are difficult for the students to master, so as to make the teaching and learning more efficient and in line with the student's logical thinking.

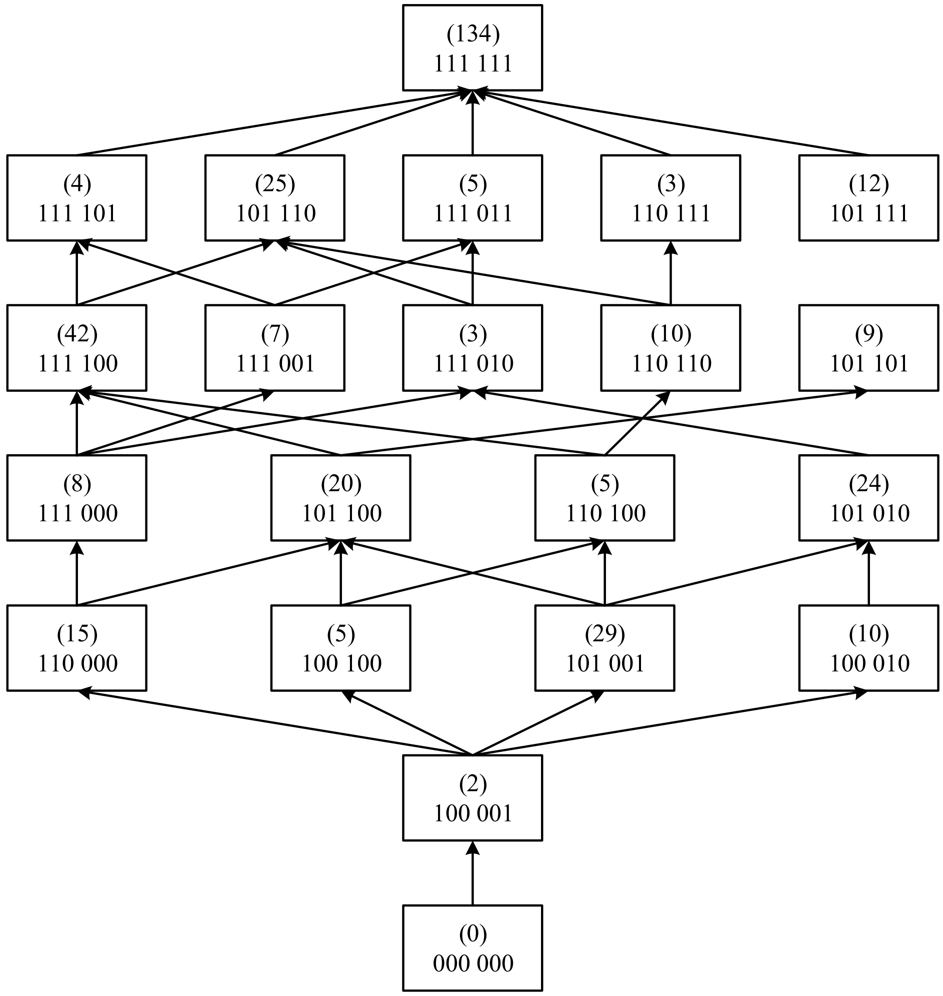

Fig. 11. Learning path

6. Conclusion

In this paper, we carry out the exploration of vocal personalized teaching mode based on AI algorithm, meanwhile, we propose to optimize the path recommendation problem in personalized knowledge service by adopting ant colony algorithm, and we propose the cognitive diagnosis model of vocal music based on KM-VDINA. The article draws the following conclusions:

1) The average test mean scores of the personalized learning path and comparison method groups are 83.3 and 81.6 respectively, and the test mean scores of the experimental subjects in the first group are larger than those of the experimental subjects in the second group as a whole. The average score of the experimental subjects who learn according to the recommended path is better than the average score of those who learn according to the default path, i.e. the algorithm proposed in this paper has good effectiveness.

2) The trajectory of the order of mastery of vocal knowledge attributes for most of the subject students is represented as The trajectory is represented as (000000) → (100001) → (101001) → (101100) → (111100) → (101110) → (111111). Students have multiple trajectories to master all the attributes, so teachers can prioritize the knowledge attributes that are easier for students to master and then the knowledge attributes that are difficult for students to master in vocal music teaching according to the logical structure of the students to make teaching and learning more efficient and in line with the logical thinking of the students.

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