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Research on structural surface feature refinement method based on fractal geometry in industrial design

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ABSTRACT
Fractal geometry is an emerging discipline that has developed rapidly in recent decades, and its study of irregular geometric shapes can be used to describe objects in nature that cannot be described by traditional geometry, and it has a broad space for development and application prospects. In this paper, the theory of fractal geometry is applied to industrial design to realize the refinement and analysis of surface features. The study includes an in-depth analysis of the theory of fractal geometry, the Koch curve as an example to illustrate the principle of fractal geometry. The study also investigates different dimension calculation methods, such as Hausdorff dimension, box dimension, correlation dimension, information dimension, generalized dimension, and self-similarity dimension of fractal geometry, and proposes a dimension calculation method for the refinement of structural surface features for industrial design. After the fractal geometry surface feature refinement simulation analysis, the porosity of the fractal map based on this paper's method ranges from 16% to 38%, and the comparison with the Serpinski method proves that the presently selected fractal model is more effective in the refinement of structural surface features for industrial design. As shown by the surface feature simulation results, there is indeed a certain degree of similarity between the roughness topography of the real structural surface of the two surface processing methods in industrial design and the roughness topography simulated by the fractal function. The above study proves that the method of refining the structural surface features of industrial design based on fractal geometry in this paper is scientific and feasible.

Keywords: fractal geometry, structural surface features, Koch curve, industrial design

1. Introduction

Industrial design refers to the process of overall planning, design and optimization of products, systems and processes using design thinking based on people, technology, culture and market [1-2]. It

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is one of the important links in industrial production and consumption, aiming to improve the performance, aesthetics and market competitiveness of products to meet the growing demand for products [3-5]. Industrial design requires designers to have profound technical, cultural and market knowledge, need to deeply understand the use of the product environment, user needs and competitors, according to this information for product aesthetics, human-computer interaction, material selection, manufacturing process and other aspects of the design [6-9], in order to ensure that the product is practical at the same time, but also to pursue the perfection of aesthetics. At the same time, it is also necessary to seek a balance between market competition and cost optimization, and constantly improve the performance and quality of products to achieve a win-win situation for users and enterprises [10-12]. With the acceleration of industrialization and the progress of science and technology, industrial design plays an increasingly important role in modern society [13-14].

Fractal geometry is a mathematical tool for studying self-similarity forms, which refers to a class of geometric objects with self-similarity and self-isomorphism properties formed by infinitely subdividing shapes by some rules [15-16]. Fractal geometry is the study of objects with complex structures, but there is a certain “infinite repetition” of the characteristics of the object. Fractal geometry has a wide range of applications in art, design, natural science, economics, sociology, and many other fields [17-20].

In industrial design, structural surface features contain a lot of valuable information that can be studied, and the study of surface microscopic features can provide theoretical reference and design basis for industrial design. Therefore, this paper is based on fractal geometry to study the structural surface features in industrial design. Through the in-depth study of the theory of fractal geometry, this paper proposes a fractal dimension calculation method for the refinement of structural surface features in industrial design on the basis of the box dimension. Then the effect of this paper's method is examined through simulation analysis and comparison with the Serpinski method. At the same time, the performance of this method in surface feature refinement is further investigated by simulating the structural surface features under two processing methods in industrial design.

2. Structural surface characterization based on fractal geometry

2.1. Fractal Geometry Theory

Fractal geometry was introduced in 1975, and although many of its concepts had appeared and been developed before then, it had not developed into a discipline before then. Classical geometry (Euclidean geometry, analytic geometry, differential geometry, topology, etc.) utilizes regular, smooth (or differentiable) spatial forms as its object of study, and provides an effective tool for studying the spatial relationships and properties of regular shapes. However, classical geometry cannot describe the shapes of clouds, mountains, coastlines, or flowers, plants, and trees, which are irregular and fragmented and cannot be described and expressed by classical fractal geometry.

Fractal geometry, also known as “nature's geometry”, is the study of objects whose geometry is irregular. Irregular geometric shapes are those that appear to be jagged, but have a specific inner pattern, such as curved coastlines and mountains with varying heights. While traditional geometry studies integer dimensions, such as zero-dimensional points, one-dimensional lines, two-dimensional planes, and three-dimensional solids, fractal geometry studies non-negative real dimensions.

Fractal dimension, also known as fractional dimension or fractional dimension, as a basic parameter and metric in fractal theory, more closely resembles the complex attributes and states in the real natural world, qualitatively and quantitatively assessing the complexity of the pattern as well as its spatial occupation of force. The richer and more irregular the details of the pattern, the larger the value of its fractal dimension.

A line is one-dimensional, only length, when the unit length of a straight line is doubled the length becomes 2, so the fractal dimension of a straight line is $\dim(\dim ension) = \frac{\log 2}{\log 2} = 1$. A surface is two-dimensional, has both length and width, when the unit area of the surface of the length and width of the unit area is doubled the area becomes 4, so the fractal dimension of a flat surface is $\dim(\dim ension) = \frac{\log 4}{\log 2} = 2$. A body is three-dimensional, has both length, width, and height, when the unit volume of the length, width, and height of the unit volume is doubled the volume changes to 8, so the volume of the fractal dimension is 3. When the length, width and height of a unit volume are each doubled, the volume becomes 8, so the fractal dimension of the volume is $\dim(\dim ension) = \frac{\log 8}{\log 2} = 3$.

Different fractal curves have different fractal dimensions, take the Koch curve as an example, the Koch curve is shown in Fig. 1, the Koch curve whose unit length is $\dim(\dim ension) = \frac{\log 4}{\log 3} = 1.262$ becomes 1 after one transformation.

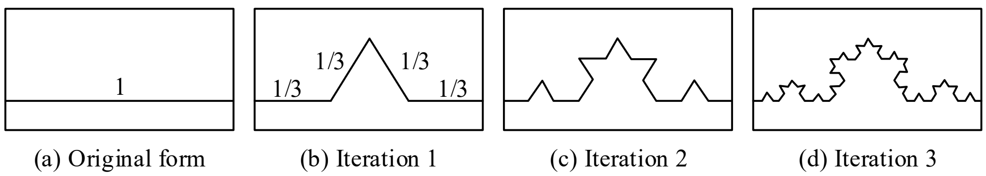


Fig. 1. Koch curve

The concept of fractal geometry has been developed in response to the fact that there are many complex shapes in nature that cannot be explained by traditional geometry. The theory of fractal geometry has effectively enhanced the understanding of these complex shapes; with the continuous development and self-improvement of fractal geometry, fractal geometry has been successfully applied to a wide range of disciplines and a large number of research results have been achieved. Simply put, fractal geometry is the application of self-similarity and scale invariance to simplify complex patterns, making it a new way to recognize complex patterns. Fractal theory, together with chaos theory and soliton theory, is the current cutting-edge theory of nonlinearity, and has become a very active field in system science research. It has demonstrated brand new properties and concepts such as scale invariance and self-similarity of complex patterns, and fractional dimension numbers, which have had a profound impact on both modern science and philosophy.

2.2. Fractal geometry based surface structure refinement

2.2.1. Fractal Geometry Based Dimension Calculation Methods. Various kinds of dimensioning have become the main tools of fractal geometry, some dimensions are more scientifically defined, while others may be less theoretical but perhaps relatively more convenient in application, common dimensions are Hausdorff dimension, box dimension, associative dimension, informative dimension, generalized dimension, self-similarity dimension and so on.

1) Hausdorff dimension. Hausdorff and box dimensions can be defined on all sets, and their role in the theory and applications of fractal geometry and its importance.

Let $\{U_i\}$ be a subset class on a metric space (X, ρ) of finitely many diameters less than or equal to δ, and A be a subset of X if:

$$A \subset \bigcup_{i=1}^{\infty} U_i \tag{1}$$

Then the subset class $\{U_i\}$ is called a δ-cover of subset A.

Let A be any bounded subset over a Borel set class $\left(R^d, \rho_E\right)$, $s >> 0$ and for any $\delta > 0$, define:

$$H_\delta^s\left(A\right) = \inf\left\{\sum_{i=1}^{\infty}\left|U_i\right|' : \{U_i\} \ is\ a\ \delta\text{-}cover\ of\ A\right\} \tag{2}$$

where $\left|U_i\right|$ denotes the diameter of the set U_i. As the value of δ becomes smaller, the -covering class covering A becomes smaller, so the value of $H_\delta^s\left(A\right)$ is not decreasing, and so the limit value:

$$H^s\left(A\right) = \lim_{\delta\to\infty} H_\delta^s\left(A\right) \tag{3}$$

Exists, easily verifiable, and $H^s\left(\cdot\right)$ defines a measure on the class $\left(R^d, \rho_E\right)$ of Borel sets. The smallest class set that satisfies the following conditions is a Borel set:

(1) Every open and closed set is a Borel set.

(2) Every intersection or concatenation of a finite number of Borel sets and every intersection or concatenation of a countable number of Borel sets is a Borel set.

Let set A be a subset of $\left(R^d, \rho_E\right)$. For any $s \in (0, \infty)$, call $H^s\left(A\right)$ defined by equation (3) the $S-$-dimensional Hausdorff measure of set A.

For fixed A, $H^s\left(A\right)$ is taken as a function of $s \in (0, \infty)$ and is special. Its domain of values consists of only two kinds, one with two values, zero and infinity, and the other with three values, zero, a finite value, and an infinite value.

Let the set A be a finite subset of $\left(R^d, \rho_E\right)$, then there exists a unique real value $s_0 \in [0, d]$ such that:

$$H^s\left(A\right) = \begin{cases} \infty, s < s_0 \\ 0, s > s_0 \end{cases} \tag{4}$$

Let set A be a subset of $\left(R^d, \rho_E\right)$, and the unique real value s_0 determined by Theorem 2 is called the Hausdorff dimension of set A, denoted $\dim H^A = s_0$.

2) Box dimension. Hausdorff dimension is very theoretical but less practical background, in fact, often used is the box dimension, the box dimension can be calculated by experimental approximation, in some of the more “regular” set, the box dimension and Hausdorff dimension is equal.

Let set A be a bounded subset on $\left(R^d, \rho_E\right)$, and for each $\delta > 0$, denote by $N_\delta(A)$ the least number of hemispheres covering A if:

$$\lim_{\delta\to\infty} \frac{\ln N_\delta(A)}{-\ln \delta} \tag{5}$$

Exists, then this limit set is called the box dimension of set A, denoted D.

3) Information dimension. The information dimension differs from the Hausdorff dimension and the box dimension in that it takes into account the number of elements of the fractal set contained in each covering U,. Let P, denote the probability that an element of the fractal set belongs to cover U, then the information dimension is:

$$\dim I = -\lim_{\delta \to 0} \frac{-\sum_{i=1}^{N} P_i \ln P_i}{\ln \delta} \tag{6}$$

Or write it as:

$$\dim I \neq \lim_{\delta \to 0} \frac{\sum_{i=1}^{N} P_i \ln P_i}{\ln \delta} \tag{7}$$

It is known from the definition of information dimension $\dim I$:

$$S(\delta) = -\sum_{i=1}^{N} P_i \ln P_i \tag{8}$$

It's the Shannon entropy of the system, which can also be written as:

$$\dim I = -\lim_{\delta \to 0} \frac{S(\delta)}{\ln \delta} \tag{9}$$

That is, the information dimension describes the system entropy in terms of another metric, yet the Hausdorff dimension happens to be equal probability:

$$P_i = \frac{1}{N(\delta)} \tag{10}$$

The information entropy of the situation, in fact, makes:

$$P_i = \frac{1}{N} \tag{11}$$

There is:

$$\dim I = -\lim_{\delta \to 0} \frac{-\sum_{i=1}^{N} \frac{1}{N} \ln \frac{1}{N}}{\ln \delta} = -\lim_{\delta \to \infty} \frac{\ln N(\delta)}{\ln \delta} = \dim H \tag{12}$$

4) Correlation dimension. The algorithm of correlation digit is easy to extract the fractal dimension from the one-dimensional observation volume, and the correlation digit is widely used as a measurement parameter of chaotic behavior. On the basis of phase space reconstruction of the one-dimensional observation volume, the integral of the phase space vector is calculated:

$$C(r) = \frac{1}{M^2} \sum_{\substack{i,j=1 \\ i \neq j}}^{M} H\left(r - \left|X_i - X_j\right|\right) \tag{13}$$

Any vector whose distance is less than a given positive number r is called an associated vector. Here the phase space vectors:

$$X_i = \left\{x_i, x_{i+,}, \ldots, x_{i+(m-1)r}\right\}, i = 1, 2, \cdots, M, X_i \in R^m \tag{14}$$

H is the Heaviside function:

$$H\left(X\right) = \begin{cases} 0, x \leq 0 \\ 1, x > 0 \end{cases} \tag{15}$$

The value of R is obtained appropriately, and $C(r)$ increases exponentially and rapidly with r. The association dimension is defined as:

$$\dim C = \lim_{\rightarrow\rightarrow 0} \frac{\ln C(r)}{\ln r} \tag{16}$$

Sometimes $\dim C$ is also commonly represented by D_2.

5) Generalized Entropy and Generalized Dimension. Henchel et al. define the generalized dimension as:

$$\dim K = -\lim_{\delta\to 0} \frac{S_K(\delta)}{\ln \delta} \tag{17}$$

In the formula:

$$S_K(\delta) = \frac{1}{1-K} \ln\left(\sum_{\delta\to 0}^{N(\delta)} P_i^K \right) \tag{18}$$

Is the K st order Renyi information, and $\dim K$ is called the K rd order generalized dimension, also known as the Renyi information dimension.

K in Eq. (17) and Eq. (18) takes positive and negative integers. When $K = 0$, it is provable:

$$\dim K\big|_{K=0} = \dim H \tag{19}$$

When $K = 1$, the limit is obtained by taking the limit using Lobida's law:

$$\dim I = \lim_{K=1} \dim K \tag{20}$$

And:

$$\dim K\big|_{K=2} = \dim C \tag{21}$$

That is, at $K = 2$, the generalized dimension $\dim K$ is the associated dimension $\dim C$.

6) Self-similarity dimension. Let A be a bounded subset on $\left(R^d, \rho_E\right)$. If A can be divided into $N(N > 1)$ equal and similar parts to A, then A is said to be a similar set if the similarity ratio of each part to A is:

$$r = \left(\frac{1}{N}\right)^{\frac{1}{D}} \tag{22}$$

Then D is said to be the self-similar dimension of the self-similar set A, denoted $\dim_s A = D$:

$$D = dim_s A = \frac{\ln N}{l\left(\frac{1}{r}\right)} = -\frac{\ln N}{\ln r} \tag{23}$$

The self-similarity dimension is only meaningful for a small class of sets that are strictly self-similar, and Eq. is introduced by the angle at which the graphs are reduced; if a graph is enlarged by a factor of n in linearity and the graphs are enlarged by a factor of K, the similarity dimension is obtained from $K = n^D$:

$$D = \dim_s A = \frac{\ln N}{\ln n} \tag{24}$$

It has been proven:

$$\dim H = \sup\{\dim k\}, D = \dim_s = \inf\left\{\dim k = \frac{\ln p_i}{|\ln \delta|}\right\} \tag{25}$$

2.2.2. Calculation of Fractal Dimension for Refinement of Structural Surface Features. In this paper, we utilize the characteristics of box dimension to calculate the fractal dimension of industrial design images, in this study, we use the box dimension method under Image J software to calculate the surface fractal dimension.

The formula for calculating the box dimension number of Imaga J software is:

$$D = -\lim_{r \to 0} \frac{\lg N(r)}{\lg r} \tag{26}$$

where D is the box dimension of the desired graph; r is the box with side length r; $N(r)$ is the number of boxes needed to cover the whole graph with the box with side length r.

The steps to calculate the fractal dimension using ImageJ software are as follows:

1) The captured image is binarized using Make Binary in the Process menu to obtain an analyzable binarized data matrix.

2) Divide the binarized data matrix sequentially into blocks such that the number of rows and columns in each block is k, noting the number of all those blocks containing 1 (or 0) as $N(r)$, usually taken to be $r = 1, 2, 4, \cdots\cdots 2^i$, to the box number $N(1), N(2), N(4), \cdots\cdots, N(2^i)$.

3) Perform a double logarithmic linear regression fit, $(\lg r, \lg N(r))$, $r = 1, 2, 4, \cdots\cdots 2^i$, and the absolute value of the slope of the resulting line is the fractal dimension of the image D.

3. Simulation analysis of surface structure refinement based on fractal geometry

3.1. Comparison of surface structure refinement in industrial design

In this paper, two fractal geometry functions are utilized to obtain the surface texture design results of 40 geometric micro-units in industrial design, due to the different fractal geometry functions which have different arrangement results for the same micro-geometric unit. In order to better compare and analyze the effect of the fractal geometry functions on the weaving design results, the design results are compared and analyzed here.

It is found that with the increase of the number of fractal iterations, the fractal pattern becomes more and more complex, and the porosity of the surface textile increases with the increase of the number of micro-pits, so this paper chooses the model with a porosity of about 26.5%, and deduces its pore area and porosity based on the method of this paper and the fractal characteristics of the Serbinsky theory (denoted as ST), respectively. The pore area increases with the number of iterations, i.e., among the two fractal functions, the pore area of the fourth iteration of the five types (triangular, circular, square, pentagonal, pentagram) microgeometric fractal weaving is the largest, and in order to reduce the computational effort 10 fractal weaving of the four iterations can be directly selected for comparison. Therefore, assuming that the cross-sectional area of the two types of weaves is dimensionless number 9, the pore areas of the 10 fractal weaves can be derived. Since the fractal dimensions of these two methods are different, the comparison is made on the premise that the fractal dimensions are the same, and the results of the fractal refinement comparison are shown in Fig. 2. The results of the comparative analysis show that the porosity of the five fractal graphs based on Scherbinski is between 30% and 70%. While the porosity of the five fractal graphs based on the method of this paper is between 16% and 38%. The magnitude of the interfacial roughness factor is not only related to the porosity, but also affected by the fractal dimension of the graphs, the larger the fractal dimension, the smoother the surface of the structure. By analyzing and comparing the five fractal graphs, it is proved that the fractal model proposed in this paper is more effective in refining the surface features of industrial design structures.

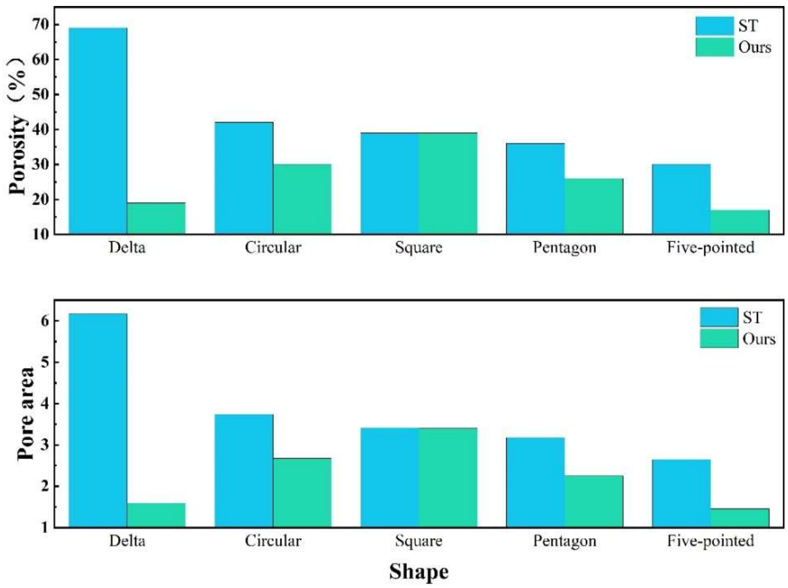
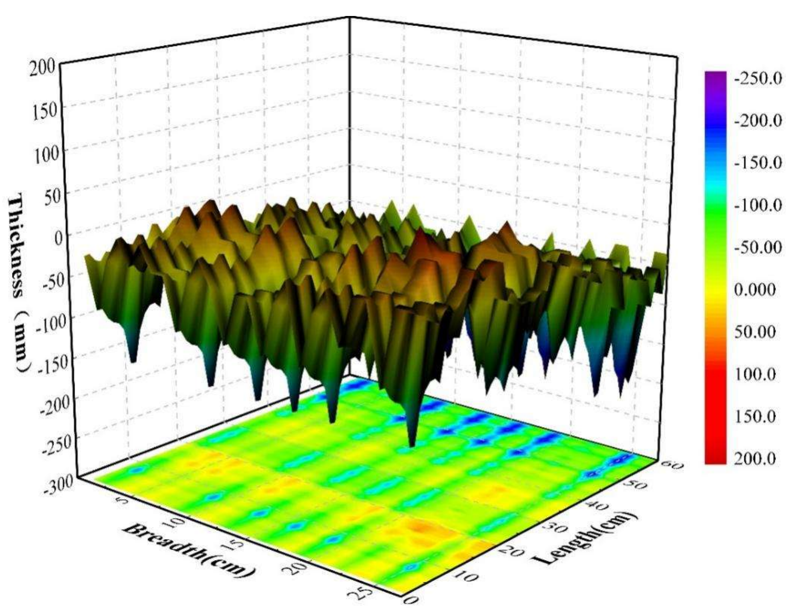


Fig. 2. Fractal contrast

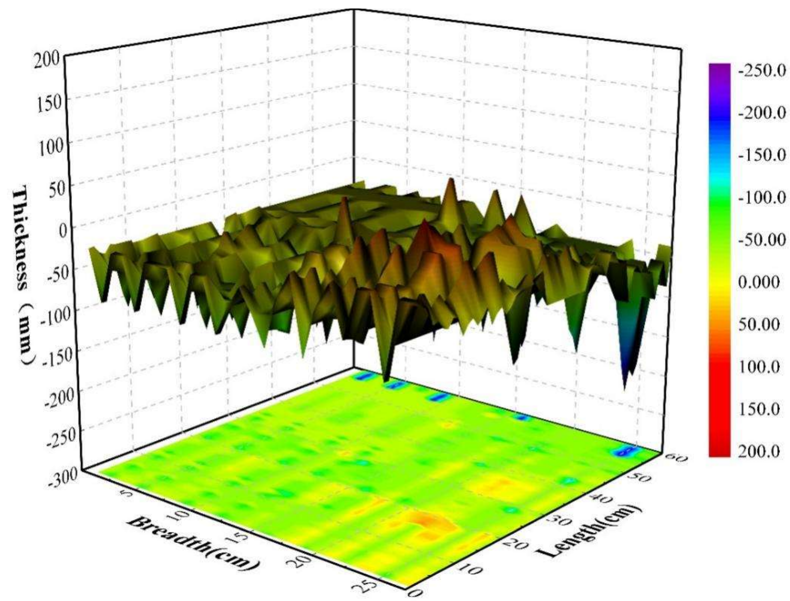
3.2. Fractal geometry based surface simulation of industrially designed structures

3.2.1. Fractal parameter D solution. Fractal dimension is a quantitative parameter that describes the fractal characteristics, which is a magical quantity that can represent the proportion of similarity, a parameter used to represent the geometric shape, structure, object irregularity and complexity.

In this paper, the experiments use the spectrum to find the dimension D, and related fractal parameters, first, the experimental measurement object for the industrial design of grinding processing and electrochemical finishing processing two kinds of processing structure surface mode, the measured three-dimensional surface micro-geometry is shown in Figure 3, (a), (b) are grinding processing and electrochemical finishing processing, respectively. From the figure, it is concluded that the surface microgeometric profile of the grinding process is not the same as that of the electrochemical finishing process, and its micro-convex features are also different, with the grinding process showing a rougher surface with obvious concavities and convexities, while the electrochemical finishing process produces a relatively flat surface profile.



(a)Grinding surface



(b)Electrochemical photofinishing surface

Fig. 3. Three-dimensional surface microgeometry

According to the theory of fractal geometry, the fractal dimension D and the characteristic scale factor G are evaluation parameters that do not depend on the resolution of the measuring instrument and the length scale of the surface contour. The fractal dimension is an important parameter to characterize the surface fractal geometry, according to the measured roughness experimental data, the discrete data of the contour of the fast Fourier transform to obtain the power spectrum of the contour, with the least squares method of linear fitting to obtain the average slope of the power spectrum K. Contour height distribution of the variance and the height distribution of the variance of the W-M function, to obtain the characteristic scale G. And to determine the two kinds of surface contour of the fractal parameter based on the identified Based on the determined fractal parameters, a two-dimensional W-M function fractal mathematical model is used to simulate the actual machined surface contour. The two ways of processing surface geometry profile simulation is shown in Figure 4, the figure of the two surface processing methods under the profile fluctuation is not flat, grinding processing surface geometry profile simulation and the actual value of the difference between ± 7mm, electrochemical finishing processing surface geometry profile simulation and the actual value of the difference between ± 3mm, indicating that the experimental measurement of the industrial design species, the roughness of the surface of the real structure of the topographic features and after the fractal This indicates that there is a certain degree of similarity between the roughness profile of the real structural surface and that simulated by the fractal function of the industrial design species measured in the test.

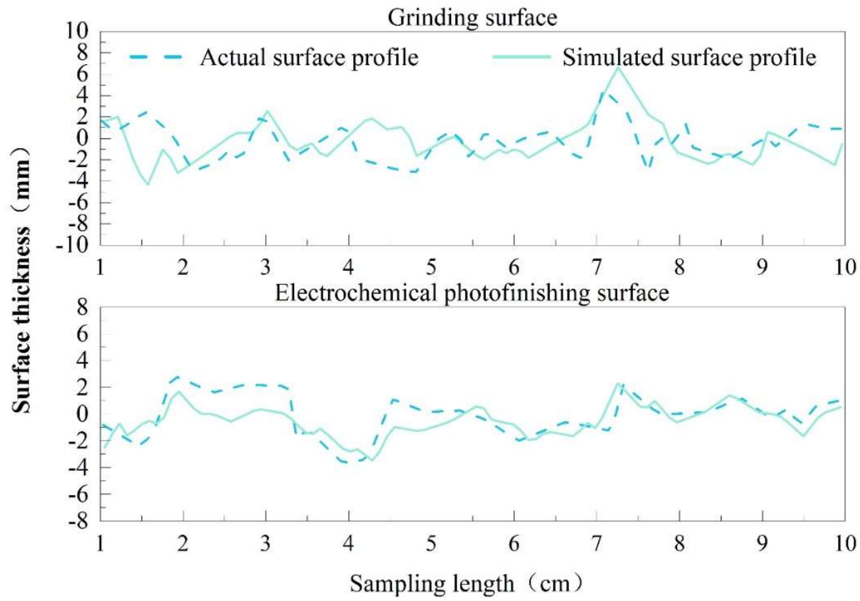


Fig. 4. Two methods of machining surface geometry simulation

3.2.2. Comparative analysis of surface simulation of industrial design structures. The processing methods in industrial design can change the fractal parameters of the surface micro-geometry, compared with grinding processing, electrochemical finishing processing can make the surface micro-geometry less irregular, or rather, electrochemical finishing processing can make the surface micro-geometry more regular. For further verification, combined with the three-dimensional W-M fractal function, the three-dimensional surface geometries of different processing methods were established using MATLAB, and the structural surface feature data statistics are shown in Table 1. From the data, it can be seen that the actual surface crag and skewness of electrochemical finishing processing are 1.95 and -1.35, respectively, indicating that it makes the dispersion of the surface micro-geometry reduced, and the increase in the degree of deviation from the mean value can make the dispersion of the surface micro-geometry reduced. Compared to the grinding processing method, the electrochemical finishing surface is characterized by flatter, larger support length ratio and deeper valleys. In view of the different surface features of grinding and surface features of finishing, and analyze the effect of each fractal parameter on the distribution of their microscopic features. Each fractal parameter will act on the surface unevenness feature distribution, or the size of the high and low amplitude, and produce different microscopic surfaces.

It is proved that the method of this paper can simulate the structural surface features in line with the industrial design by changing the frequency correlation constants, and it also further verifies the reasonableness and scientificity of the surface feature refinement method of this paper.

Table 1. Data statistics on structural surface characteristics

Processing method sheer	Grinding surface			Electrochemical photofinishing surface		
	Face surface	Simulated surface	Difference value	Face surface	Simulated surface	Difference value
Deflection	6.51	2.35	4.16	1.95	2.07	0.12
Processing method	-0.25	-0.09	0.16	-1.35	-0.14	1.21

4. Conclusion

The development and application of fractal geometry theory provides a new basis for the study of surface structural features in industrial design. In this paper, under the demand of industrial design, the study on the refinement of structural surface features is carried out based on fractal geometry, and this paper proposes a dimension calculation method for the refinement of structural surface features based on the number of box dimensions, and compares it with the Serpinski method. The comparative analysis of the results shows that the porosity of the five fractal graphs based on this paper's method is between 16% and 38%, which is smaller compared to the porosity of the five fractal graphs of Scherbinski's method (30%-70%), which proves that the fractal method proposed in this paper has a better performance in the refinement of the structural surface features of industrial design. In addition, this paper is oriented to industrial design, and two industrial design processing methods are simulated and analyzed, and the contour fluctuates unevenly under the two surface processing methods. Among them, the difference between the surface simulation and the actual value of the grinding processing method and electrochemical finishing processing is within ±7mm and ±3mm respectively, and there is a similarity between the roughness morphology characteristics of the real structural surface and the fractal function simulation in this paper. In addition, the actual surface roughness and skewness of electrochemical finishing are 1.95 and -1.35, respectively, and the simulated and actual values are lower than those of the grinding processing method. The degree of dispersion of the surface microgeometry with electrochemical finishing processing is reduced, and its structural surface has a flatter.

After the above study, it is proved that the structural surface feature refinement method based on fractal geometry in this paper can simulate the structural surface features that meet the needs of industrial design, which provides a design basis for industrial design.

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References

[1] Baxter, M. (2018). Product design. CRC press.

[2] Ulrich, K. T., & Eppinger, S. D. (2016). Product design and development. McGraw-hill.

[3] Cross, N. (2021). Engineering design methods: strategies for product design. John Wiley & Sons.

[4] Berg, L. P., & Vance, J. M. (2017). Industry use of virtual reality in product design and manufacturing: a survey. Virtual reality, 21, 1-17.

[5] Muminovic, A. J., Saric, I., Mesic, E., Pervan, N., & Delic, M. (2019). Research about characteristics of designs from industrial designers and product designers. Periodicals of Engineering and Natural Sciences (PEN), 7(2), 860-869.

[6] Zhu, W., & He, Y. (2017). Green product design in supply chains under competition. European Journal of Operational Research, 258(1), 165-180.

[7] Seider, W. D., Lewin, D. R., Seader, J. D., Widagdo, S., Gani, R., & Ng, K. M. (2017). Product and process design principles: synthesis, analysis, and evaluation. John Wiley & Sons.

[8] Den Hollander, M. C., Bakker, C. A., & Hultink, E. J. (2017). Product design in a circular economy: Development of a typology of key concepts and terms. Journal of Industrial Ecology, 21(3), 517-525.

[9] Bocken, N. M., De Pauw, I., Bakker, C., & Van Der Grinten, B. (2016). Product design and business model strategies for a circular economy. Journal of industrial and production engineering, 33(5), 308-320.

[10] Towler, G., & Sinnott, R. (2021). Chemical engineering design: principles, practice and economics of plant and process design. Butterworth-Heinemann.

[11] Tao, F., Sui, F., Liu, A., Qi, Q., Zhang, M., Song, B., ... & Nee, A. Y. (2019). Digital twin-driven product design framework. International Journal of Production Research, 57(12), 3935-3953.

[12] Tao, F., Cheng, J., Qi, Q., Zhang, M., Zhang, H., & Sui, F. (2018). Digital twin-driven product design, manufacturing and service with big data. The International Journal of Advanced Manufacturing Technology, 94, 3563-3576.

[13] Akao, Y. (2024). Quality function deployment: integrating customer requirements into product design. CRC Press.

[14] Bell, S., & Bala, K. (2015). Learning visual similarity for product design with convolutional neural networks. ACM transactions on graphics (TOG), 34(4), 1-10.

[15] Fraser, J. M. (2020). Assouad dimension and fractal geometry (Vol. 222). Cambridge University Press.

[16] Cohen, R., & Eldar, Y. C. (2020). Sparse array design via fractal geometries. IEEE transactions on signal processing, 68, 4797-4812.

[17] Dwivedi, R. P., & Kommuri, U. K. (2019). Compact high gain UWB antenna using fractal geometry and UWB-AMC. Microwave and Optical Technology Letters, 61(3), 787-793.

[18] Pal, B., & Saha, K. (2018). Flat bands in fractal-like geometry. Physical Review B, 97(19), 195101.

[19] Zhou, Z., Su, Y., Wang, W., & Yan, Y. (2017). Application of the fractal geometry theory on fracture network simulation. Journal of Petroleum Exploration and Production Technology, 7, 487-496.

[20] Jahanmiri, F., & Parker, D. C. (2022). An overview of fractal geometry applied to urban planning. Land, 11(4), 475.