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Research on the optimization of piano technique training based on Monte Carlo algorithm under the innovation framework of music education in colleges and universities

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ABSTRACT

The article is based on the need for music education innovation in colleges and universities to optimize the traditional piano skill training through Monte Carlo algorithm. Taking the finger as the research entry point, based on the physiological structure of the hand, the reduced-density Monte Carlo method is used to carry out the mechanical design of the finger trainer and plan the finger training movement mode. Through kinematic simulation experiments to understand the feasibility of the piano finger training device in this paper. Analyze the error sensitivity of position and posture on the finger training device. Finally, the teaching experiment method is utilized to explore the training effectiveness of the Monte Carlo-based piano finger training device in this paper. This paper has good usability. When the position error of the mechanism varies in the range of -40mm~40mm, the position error gradually decreases in the X-axis and Z-axis, and the position error in the direction of Y-axis remains stable. The attitude error of the mechanism gradually increases with the increase of the X-axis rotation angle. The output accuracy gradually increases during the rotation from -5° to 5° around the Y-axis. The angular attitude around the Z-axis has no significant effect on the output accuracy. The two groups did not have significant differences in the four dimensions of piano playing skills before the experiment. After the teaching experiment, the experimental group was much better than the control group, and the post-test results of the two groups produced significant differences, and the pre-test and post-test results of the experimental group possessed very significant differences. The Monte Carlo optimization-based piano finger training device has a significant effect on the improvement of students' piano skills.

Keywords: Monte Carlo method, finger training optimization, error sensitivity, training effect

1. Introduction

Nowadays, with the rapid development of social economy and social progress, music education in colleges and universities, as a discipline, is more and more concerned and emphasized by the society.

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Piano teaching as an important part of music education, its piano playing skills are crucial in the interpretation of musical works, concerning the author's emotional expression and piano level [1]. Piano technique training is usually divided into three stages, and the focus of training in each stage is different. The first stage is the preparatory stage, this stage is mainly to learn to read music quickly and form an intuitive artistic perception, can quickly mobilize the fingers to touch the keys according to the music score, and at the same time, train the sitting posture, control the breathing rhythm [2-4]. The second stage is the practical training stage, to grasp the sense of rhythm, to find the appropriate strength of the keys through repeated training, and to reasonably adjust the mood of the performance according to the emotion of the musical work, so that the personal artistic experience and the performance skills are reasonably integrated, according to the process of the performance of the sound quality of the timbre changes in the performance of the timely adjustments to the way of playing [5-7]. The third stage is the stage of summarizing and improving, this stage of playing skills tend to be skilled, by constantly summarizing the experience of playing, at the same time, listen to more famous performers, artistic perception and artistic imagination, constant reflection, and further enhancement [8]. In the first and second stage of teaching, the collective teaching of playing skills with teacher experience, repeated practice of skills training, teacher feedback, to a certain extent, such a training method is effective. However, with the needs of students and educational innovations, the disadvantages of this traditional method are highlighted. First, the subjective experience of teaching, which is easy to cause students to be exposed to one-sided knowledge and skills and solidified thinking. Secondly, under the collective teaching, the teacher's unified teaching method and teaching difficulty exacerbate the students' individual differences, and the teacher's feedback is not timely, and it is difficult to correct the deviation of the students' skill movements [9-10]. In addition, in the process of entering the third stage of the second phase, which is often the bottleneck period for most students, it takes months or even a dozen months to make technical breakthroughs, and the elongated front is easy to wear down students' interest and patience [11]. However, in addition to the learning of music theory knowledge, the cultivation of piano playing skills also relies on scientific and systematic methods [12].

Under the development of artificial intelligence and data analysis and other technologies, music education has stepped into scientific, digital and intelligent, gradually changing the traditional teaching mode. Monte Carlo algorithm by capturing the geometric quantities and geometric features of the movement of things, the use of mathematical methods to simulate, that is, to carry out a kind of digital simulation experiments [13]. It is based on a probabilistic model and follows the process depicted in this model as an approximate solution to the problem by simulating the results of the experiment. It is because of this stochastic and numerical simulation properties that Monte Carlo algorithms have been applied to the computational analysis of a wide range of complex behaviors by quantifying different complex scenarios and combining them with artificial intelligence [14].

This paper tries to optimize piano skill training in the context of music education reform in colleges and universities, starting with the most important fingers of piano playing, and analyzing the finger skills of piano playing in depth. The Monte Carlo algorithm is applied to piano finger training, and the Monte Carlo-based piano finger training mechanical device is designed through the reduced-density Monte Carlo method to plan the students' finger training movement. Then the kinematic simulation of piano finger training is carried out to explore the usability of the piano finger training device designed in this paper. The error sensitivity of the piano finger training device is analyzed in terms of both positional and attitude parameters. Finally, the training effect of the Monte Carlo-based piano finger training device designed in this paper is tested by teaching experiments.

2. Comprehensive training of finger skills for piano players

The foundation of piano playing technique lies in the fingers, and the even control of the speed and strength of the fingers is the basic skill of playing, so it can be seen that the training of the function of the fingers is of vital importance.

2.1. *Finger pivot training*

A good hand shape guarantees hand support, and good hand support determines the stability of the movement structure during the playing process, and a stable movement structure leads to better hand function during the playing process.
1) Single-tone finger drop exercise: touch the keys with one finger and use the fingertip to skip notes in a relaxed manner, starting with the 3rd finger, then practicing the 2nd finger, 4th finger, 1st finger, 5th finger, and so on. You can also start by repeating each of the five fingertips on the same key several times to experience the strength of each finger. It is also possible to strengthen the weak finger exercises individually, so that it is easier to find the feeling of standing. It is required to have a standardized hand shape, fixed fingers, relaxed arms, and force to the fingertips.
2) Diatonic finger drop exercises: choose a combination of diatonic fingers, such as 13 fingers, 15 fingers, 14 fingers, 24 fingers, 25 fingers, 35 fingers, 45 fingers, etc., or choose a diatonic practice piece or piece of music. Diatonic falling finger exercises, in order to step method to do parallel to the swing requires that each two fingers fall after the strength of uniform, clear, clean sound, the rest of the hand to remain immobile or stick to the keys.
3) Three chords falling finger exercises: on the basis of two-tone falling finger further training on finger support ability, such as 135 fingers, 145 fingers, 125 fingers, the training should control the strength, so that the sound is pleasant to the ear.

2.2. *Finger exercises*

1) Finger strength training: Slow practice is the key, using slow finger lifting and fast key laying. The speed of the keys should be fast, the fingers should be perpendicular to the keys, the distal phalanges should be the support point, the fingertips should touch the keys, and the finger strength should be strengthened by skipping and vibrating exercises, and the weak fingers can be strengthened by conscious finger splitting exercises. Finger strength is the most basic, but also the guarantee of other qualities.
2) Finger speed exercises: speed is an important form of musical expression. Therefore, to improve the speed of fingers, we should first improve the precision and strength of fingers. Can also take the rhythmic practice method to achieve the purpose. In accelerating should be gradual, for example, want to play high notes, scales, need to start from the low scale, and gradually improve.
3) Finger flexibility exercises: flexibility is the maximum range of motion that can be achieved by the finger joints, affected by the hand muscles, tendons, ligaments, but also related to the excitability of the central nervous system. Training methods can be through the contraction and relaxation of the relevant muscles to increase the range of motion of the joints. It can also be carried out by pulling exercises with external force (passive training).
4) Finger endurance exercises: In the process of playing, the prolonged activity of the fingers will cause finger fatigue. When training, the stronger the finger's ability to overcome fatigue, the longer it can last. Beginners should consciously control the fingers to reach fatigue time, each practice can adhere to a longer period of time, and gradually enhance the endurance of finger muscles.
5) Finger sensitivity exercises: sensitivity depends on the flexibility and plasticity of the nervous system, need to be built on the strength, speed, rhythm and other qualities and skills. You can help train the flexibility of the nervous system by tapping the computer or finger training games. Finger vitality can be developed by lightly snapping the fingers against the palm of the hand, finger snapping, and dribbling and shooting. When playing, finger running dexterity can also be improved through techniques such as finger rotation and arpeggios.

2.3. *Intensive training of fingers*

“Reinforcement” is a neutral term, defined simply as the effect of increasing the frequency of response.

In the process of piano teaching, “strengthening exercises” cannot be clearly defined, because each person's technical mastery is different, and can only say that the difficulty coefficient is more difficult than the original. In sports training often used to strengthen training to the weak parts of the muscle training or technology consolidation and improvement. The main practice methods are: changing fingers, shifting position, shifting tone, expanding, reducing, increasing fingers, decreasing fingers, adding flowers, keeping tone, changing volume, changing rhythm, changing speed, changing special techniques and so on.

3. Monte Carlo-based piano finger training device design

3.1. Monte Carlo and reduced-density Monte Carlo methods

3.1.1. Monte Carlo method. According to the equations of kinematics, the set of spatial position points accessible to the actuators at the end of the training device is the reachable workspace of the device, the set of points $W(p)$:

$$W(p)=\begin{cases} p_X(\theta_i) \\ p_Y(\theta_i)\,\theta_i \subseteq R_i \\ p_Z(\theta_i) \end{cases} \tag{1}$$

where denotes R_i is the limiting range of θ_i joint variables.

Monte Carlo method is a non-deterministic numerical computation method based on probability statistics [15-16], which calculates the solutions in the probabilistic model by building a probabilistic model with a large number of randomly sampled model parameters, and then obtains an approximation of the solution of the problem by a large set of solutions. This method is an effective method for solving high-dimensional complex problems because its error is independent of the number of dimensions.

The Monte Carlo method solution steps are as follows:

1) Establish a mathematical model T of the positive kinematics of the device and determine the joint limits $[\theta_{i\min}\theta_{i\max}]$ of the device.

2) Generate N random variables between $[0,1]$ based on the random function $rand$ and calculate the random joint angles of each joint:

$$\theta_i=\theta_{i\min}+(\theta_{i\max}-\theta_{i\min})rand(N,1) \tag{2}$$

where i is the number of joints.

3) The random values of each joint are freely matched into N groups and substituted into the positive kinematic equation to obtain the position of the reference point of the actuator at the end of the device $P_{(X,Y,Z)}$.

4) Based on the training device base coordinate, plot the coordinates of each reference point to obtain the point cloud map of the training device workspace.

In order to study the shape of the device reachable workspace and its boundary more intuitively, the point cloud map of the workspace in the XOY, XOZ, and YOZ coordinate planes can be obtained by slicing the 3D workspace map.

3.1.2. Density reduction Monte Carlo method. Aiming at the deficiencies of the Monte Carlo method in solving the workspace of the device, the reduced-density Monte Carlo method is proposed, and the influence of the settings of each parameter in the method on the accuracy of the workspace is investigated.

1) Voxel grid method. According to the initial random point cloud P_{init} of the device workspace generated by the Monte Carlo method and the voxel grid method, the initial point cloud is partitioned

into a number of rectangular regions, each of which is called a voxel. The specific steps of the voxel grid method are as follows:

(1) Build the enclosing box containing the random point cloud. According to the position coordinates of the random point cloud P_{init} obtain the extrema $X_{\max}$, $Y_{\max}$, $Z_{\max}$, $X_{\min}$, $Y_{\min}$ and $Z_{\min}$ of P_{init} in each axial direction, and calculate the lengths of the point cloud in each axial direction $l=|X_{\max}-X_{\min}|$, $w=|Y_{\max}-Y_{\min}|$ and $h=|Z_{\max}-Z_{\min}|$. In order to avoid the initial random point cloud not covering the boundary of the workspace, resulting in the lack of the workspace, this paper introduces the expansion factor φ to expand the boundary range of the random point cloud, and determines the length of the expansion range of the random point cloud by the equation (3) d. Therefore, the selection of the expansion factor φ is related to the range of the initial random point cloud, and when the range of the initial random point cloud is small, φ a larger value should be taken, which is conducive to a more comprehensive coverage of the boundary of the real workspace. The opposite is also true. By expanding the range length d, the actual boundary value of the enclosing box is determined as shown in Eq. (4) and the enclosing box is built.

$$d=\frac{\varphi}{16}\left(l+w+2h-|l-w|-\big|2h-l-w+|l-w|\big|\right) \tag{3}$$

where $\varphi\in[0 \quad 1]$. When $\varphi=0$, the extension length d of the enclosing box is 0, when the enclosing box does not extend. When $\varphi=1$, the extension length d of the enclosing box is 1/4 times the minimum side length of the enclosing box.

$$\begin{gathered}X_{b\max}=X_{\max}+dX_{b\min}=X_{\min}-d\\ Y_{b\max}=Y_{\max}+dY_{b\min}=Y_{\min}-d\\ Z_{b\max}=Z_{\max}+dZ_{b\min}=Z_{\min}-d\end{gathered} \tag{4}$$

(2) Split the enclosing box into a number of voxels. According to the axial extreme value range of the enclosing box, the enclosing box is divided into n_X, n_Y and n_Z equal parts along the X, Y and Z axes respectively, and the edge lengths of the axial voxels are calculated to be l_b, w_b and h_b as shown in Eq. (5), and the boundary extreme value coordinates of each voxel are also derived, which lays the foundation for the division of the initial randomized point cloud in the later stage.

$$\begin{gathered}l_b=\left(X_{b\max}-X_{b\min}\right)/n_x\\ w_b=\left(Y_{b\max}-Y_{b\min}\right)/n_y\\ h_b=\left(Z_{b\max}-Z_{b\min}\right)/n_z\end{gathered} \tag{5}$$

(3) Mark voxel index n. In order to better identify the position of each voxel, this paper proposes the marking rules for voxel index n, where $n\in[123\cdots]$. Voxel labeling rule: start labeling along the X-axis positive direction first, starting from 1 until the last voxel n_X. Next, move one voxel in the positive direction along the Y-axis and continue labeling sequentially along the X-axis until the last voxel $n_X n_Y$ of the entire bottom layer is marked. Then move one voxel in the positive direction along the Z-axis and continue the first two steps with the sequential labeling index. And so on until the last voxel $n_X n_Y n_Z$ is labeled.

2) Uniformly encrypt the density of the point cloud inside the workspace. First, a voxel with an initial random number of point clouds within the voxel below an accuracy threshold N_ε is searched for in accordance with the voxel index n and the number of point clouds within the voxel is recorded N_n. The index value n and the number of point clouds N_n of the voxel are saved in the database Database_1. At the same time, all point cloud coordinates within the voxel n and their corresponding angular values $\theta_1\theta_2\cdots\theta_i$ for each joint of the device are saved in the database Database_2.

Next, the number of encrypted point clouds required for voxel n is determined to be $N_{need}=N_\varepsilon-N_n$ for the voxel index n retained in database Database_1 and the number of point clouds contained within that voxel N_n. In order to accurately encrypt the number of point clouds within a voxel and reduce the waste of random point clouds, the joint angles corresponding to all point clouds for voxel n in Database_2 are extracted and the extremes of each joint within voxel n are explored sequentially in the order of the joints. The N_{need} added joint angle values for the device joints i are obtained from the number of encrypted point clouds N_{need} required for voxel n and the poles $\theta_{i\min}$ and $\theta_{i\max}$ for each joint:

$$\theta_{i\_new} = \theta_{i\min} + \left(\theta_{i\max} - \theta_{i\min}\right) rand\left(N_{need}, 1\right) \tag{6}$$

where θ_{i_new} denotes the added joint angle of the i nd joint.

According to the principle of free matching, the randomly generated joint angles in Eq. (6) are matched into N_{need} groups, and the N_{need} groups of joint angle values are utilized to calculate the positive kinematic solution of the device.

Finally, determine whether the coordinates of the newly added point cloud are included in voxel n, if so, save them in DataDatabase_3 according to index n, and merge the values of each joint angle corresponding to the point cloud into Database_2 according to index n. If they are not included, repeat the previous operation. In order to avoid the waste of resources caused by the fact that the voxels at the boundary of the workspace never reach the required number of points, set the number of repeated encryptions to 2 and then stop the encryption operation.

3) Encrypt the density of random point cloud at the workspace boundary. The voxels that still have less than N_ε point cloud in the encrypted voxel are considered as boundary voxels in the workspace. Based on the number of point clouds contained in the boundary voxel n in the databases Database_1 and Database_3, the number of encrypted point clouds required for the boundary voxel n is determined N_{nneed}. If the number of point clouds within the voxel n is 1, the value of each joint of the voxel n in Database_2 is extracted and used as an extended joint θ_i. Otherwise, the extreme values $\theta_{i\min}$ and $\theta_{i\max}$ of each joint of the voxel in Database_2 are explored and used as the maximum and minimum values of the extended joint. Extreme values $\theta_{i\_new\min}$ and $\theta_{i\_new\max}$ of each joint after extension are obtained by means of the Extended Joint Angle J_e parameter:

$$\begin{cases} \theta_{i\_new\min} = \theta_i - J_e \\ \theta_{i\_new\max} = \theta_i + J_e \end{cases} \quad if\ N_n = 1$$
$$\begin{cases} \theta_{i\_new\min} = \theta_{i\min} - J_e \\ \theta_{i\_new\max} = \theta_{i\max} + J_e \end{cases} \quad if\ N_n \neq 1 \tag{7}$$

Substituting the expanded extremes $\theta_{i\_new\min}$ and $\theta_{i\_new\max}$ of each joint into Eq. (6), we obtain N_{nneed} groups of new joint angles of the device.

The uneven distribution of random point clouds in the workspace and the blurring of the workspace boundary can be solved by the reduced-density Monte Carlo method, which ultimately achieves the purpose of reducing the waste of the newly added point clouds while guaranteeing the accuracy of the workspace, and thus improves the efficiency of solving the workspace.

3.2. Mechanical design of the finger trainer

3.2.1. Analysis of the physiological structure of the hand. The human hand is a complex structure composed of ligaments, bones, tendons, muscles, as well as soft tissues and skin. During finger movement, the relative angles between the joints determine the final posture of the hand, therefore, studying the form of joint movement will determine the movements required for finger training and lay the foundation for mechanical design.

The joints of the fingers can perform up to 2 types of movements, namely bending movements and oscillating movements [17]. The bending motion is similar to the fist-clenching motion and the oscillating motion is something that can only be done by the joints that are directly connected to the palm of the hand. Each joint of the finger has a name; the joint connected to the palm is called the metacarpophalangeal joint MP, the middle joint is called the proximal interphalangeal joint PIP, and the most distal joint is called the distal interphalangeal joint DIP. Each of the four fingers except the thumb has 4 degrees of freedom, i.e., MP has 2 degrees of freedom and PIP and DIP have 1 degree of freedom each. The thumb has 2 degrees of freedom for MP and 1 degree of freedom for PIP. Thus the whole hand has 19 degrees of freedom and the finger joints are shown in Fig. 1.

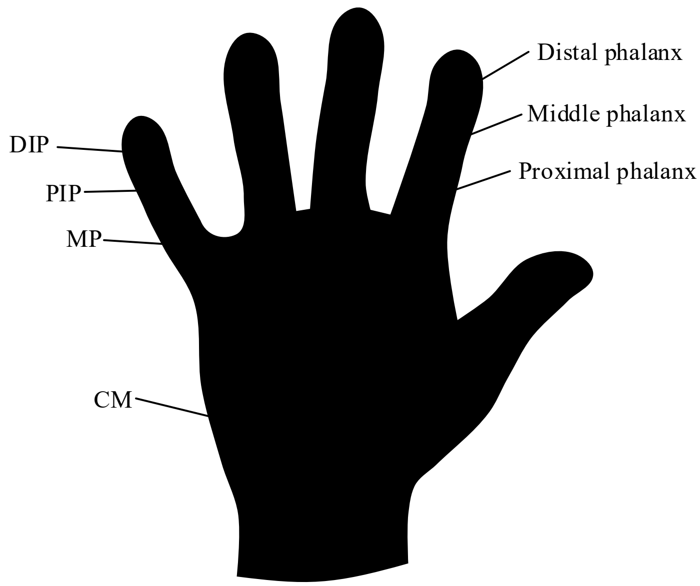


Fig. 1. Finger joint

3.2.2. Design of mechanical structures. The mechanical structure design of the finger trainer is divided into two parts, which are thumb training mechanism and four-finger training mechanism.

The thumb training is mainly composed of motor, motor base, slide rail, pulley, nylon line, thumb holder, etc. No.1 motor is installed on the base of the whole machine, and a wheel disk is installed on its reducer output shaft. Meanwhile, a pulley is installed at the end of the slide rail, and they form a belt drive system through the nylon thread. The nylon line and the thumb thumb holder where the bracket is solidly connected, so that when the motor rotation can drive the thumb bracket in the slide back and forth movement, can realize the thumb horizontal direction training, that is, swing movement. Thumb training in the vertical direction and the horizontal direction of the training mechanism is similar to the No. 2 motor where the bracket is solidly connected to the thumb frame, it can slide back and forth with the thumb frame on the horizontal slide. On the left side of the thumb holder is a specially machined slide, and a small slider is mounted inside the thumb sleeve, which slides up and down on the slide. Motor reducer output shaft is also installed on a wheel, which is mounted on the top of the bracket pulley and nylon rope composed of a vertical direction of the belt drive mechanism, when the motor forward and reverse, the nylon rope drives the finger sleeve in the slide rail to move, you can realize the thumb in the vertical direction of the movement.

Four fingers training mechanism where the bracket and the bracket where the No. 3 motor is solidly connected to the support plate, the support plate through a conical sleeve and the No. 3 motor reducer output shaft to form an interference fit, so that when the No. 3 motor positive and negative rotation, can drive the support plate to do a small range of circular motion, which realizes the four-finger bending movement.

3.3. Finger training exercise modality planning

3.3.1. Acceleration control. When the system is powered up and running, the motor is controlled so that it operates at a constant acceleration and is able to speed up to reach the target position, assuming that the acceleration a remains constant during acceleration, and the velocity V_n during each control cycle:

$$V_n = aT_n \tag{8}$$

Repeat the above equation for each cycle until $V_n = V_{\max}$, $V_{\max}$ is the maximum motor speed value.

3.3.2. Deceleration control. In each deceleration calculation, it is necessary to judge the difference S_n between the current position of the motor and the target position to determine whether the motor should start decelerating or not, and the formula for calculating the distance of the deceleration zone is S_d:

$$S_d = \frac{V_{\max}^2}{2a} \tag{9}$$

If $S_n \le S_d$, it means that the motor is going to enter the deceleration zone, then the speed at the next moment is:

$$V_{n+1} = V_n - aT_n \tag{10}$$

From that moment onwards, this velocity decelerates to 0 with constant deceleration and the motor reaches the target position.

From the above analysis, it is easy to see that the velocity trajectory of the motor has three cases as shown in Fig. 2. In Fig. 2(a), the motor runs through only two stages, uniform acceleration and uniform deceleration, and the motor does not reach the maximum velocity value. In Fig. 2(b), the motor goes through three stages, uniform acceleration, uniform velocity, and uniform deceleration during position control. The velocity value in the uniform velocity stage is the required velocity value. In Fig. 2(c), the motor also operates through only two stages, uniform acceleration and uniform deceleration, but the motor reaches the maximum speed value.

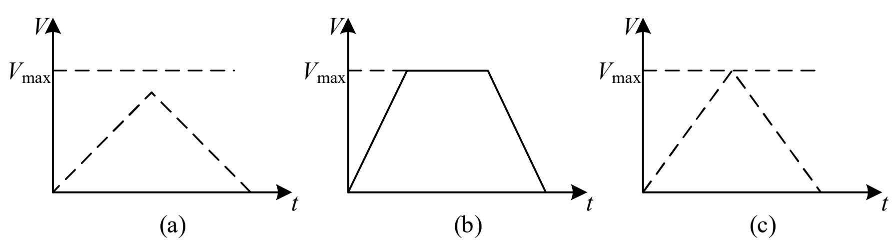


Fig. 2. Motor speed planning

By from the analysis of the several graphs above, the reason why the motor is in these situations is that the relationship between the remaining value of the motor's distance from the target position and the size of the deceleration zone is compared during each control cycle of the acceleration phase. If the remaining travel is less than or equal to the deceleration zone travel, the motor enters the deceleration zone, at which point the motor may not have reached the planned maximum speed. If the remaining travel is greater than the deceleration zone travel, the acceleration continues until it reaches the maximum speed, at which point it maintains a uniform rotation speed until it enters the deceleration zone.

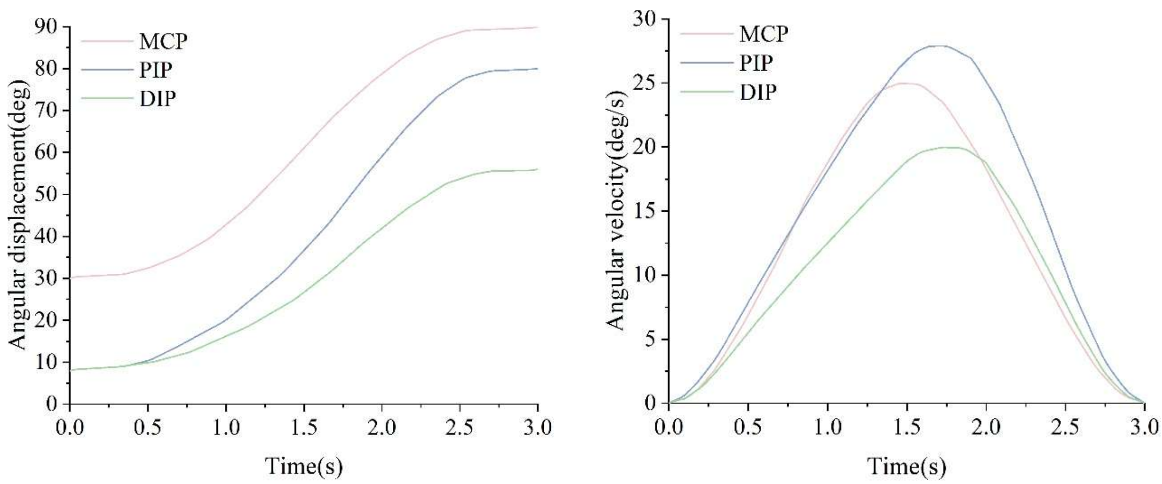
In order to ensure the synchronization of the two motors, in addition to the requirement that the two motors start at the same time, stop at the same time, but also according to the four finger sleeve end of the bending and swing movement stroke size, as well as the user on the training speed of the different requirements, acceleration and other indicators to adjust the planning.

4. Optimization analysis of piano training

4.1. Kinematic simulation analysis of finger training

Taking the index finger as an example, the finger training device designed in this paper was utilized for finger training simulation, and the change curves of the angular displacement and angular velocity of the MCP joint, PIP joint, and DIP joint of the index finger over time were recorded during the simulation as shown in Fig. 3, where (a) is the angular displacement and (b) is the angular velocity.

From Fig. 3(a), it can be seen that the maximum bending angles of each knuckle are ΔθMCP=60°, ΔθPIP=72°, and ΔθDIP=48°, which are basically consistent with the results of the analysis of the biological properties of the human hand. The angular velocity of each knuckle in Fig. 3(b) changes gently without sudden changes and shocks, demonstrating that the finger training device has good comfort.

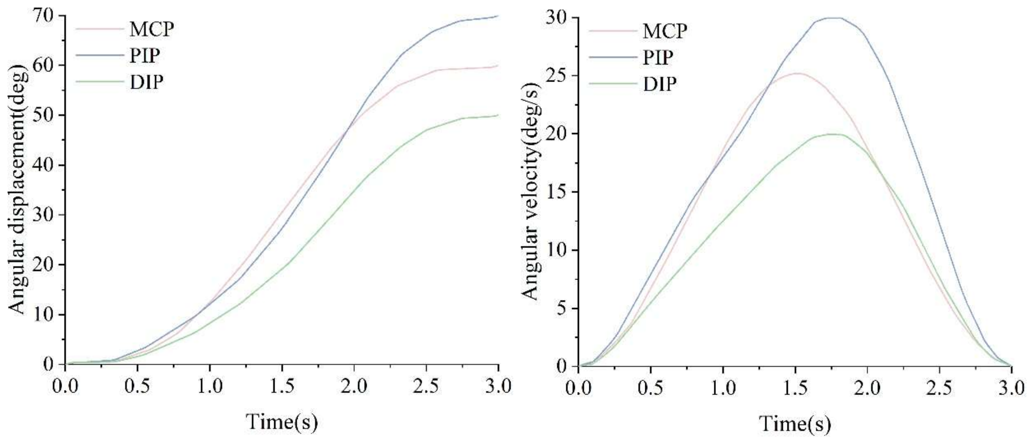


(a) Relation of joint position and time (b) Relation of angular velocity and time

Fig. 3. The relation between joint position, angular velocity and time of training device

In order to verify the correctness of the training device, the kinematic model of the index finger training mechanism was established by MATLAB, the motor drive function was exported as the input of the kinematic model, the sampling frequency was set to 25Hz, and the simulation time was 3 seconds. The rotation angle and angular velocity curves of each joint of the index finger are shown in Fig. 4.

The maximum angle range of each joint can be analyzed from the figure: ΔθMCP=60°, ΔθPIP=70°, ΔθDIP=50°, and the maximum angular velocities of the three joints are: ωMCP=25 deg/s, ωPIP=30 deg/s, andωDIP=25 deg/s. By comparing and analyzing with Figures 4 and 3, it is found that the errors between the kinematic model computation results and the simulation results are as follows: the error of the kinematic model calculation and the simulation results of the device are: the error of the motor drive function as the input of the kinematic model and the error of the device simulation. The errors between the kinematic model calculation results and the device simulation results are 0%, 2.8%, and 4.2% for the maximum rotation angles of MCP joint, PIP joint, and DIP joint, and 0%, 7.1%, and 0% for the maximum angular velocities of MCP joint, PIP joint, and DIP joint, which indicate that the kinematic model calculation results and the device simulation results are basically consistent, and it proves that the kinematic model of the finger training mechanism is correct, and can be be used to describe the motion changes of the finger training mechanism.



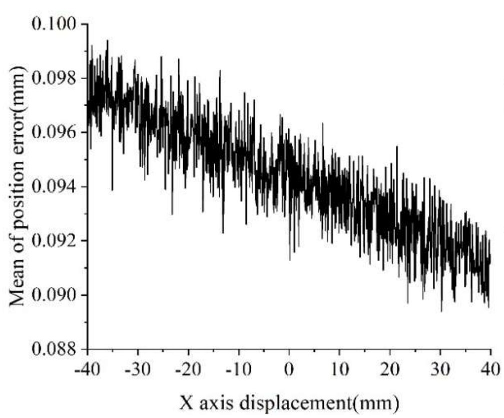
(a) Relation of joint position and time (b) Relation of angular velocity and time

Fig. 4. The relation between joint position, angular velocity and time

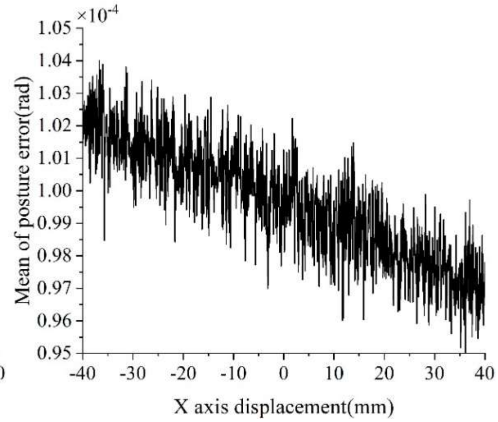
4.2. Error sensitivity analysis

4.2.1. Error sensitivity analysis of positional parameters to the training device. First, a number of discrete points were carefully selected within the workspace of the training device, which were uniformly distributed with the aim of fully reflecting the performance of the mechanism in different positions and attitudes. Subsequently, for each discrete point, a set of random errors was substituted to simulate the uncertainties in actual operation. By calculating the position and attitude errors of these discrete points under different error conditions, it is possible to get a clear picture of the specific performance of the parallel mechanism in terms of errors under different operating conditions. This comprehensive analysis method not only helps to understand the error characteristics of the parallel mechanism more deeply, but also provides valuable theoretical basis and data support for the subsequent error compensation and mechanism optimization. The average position error distribution of each mechanism in the training device along each axis of X, Y, and Z is shown in Fig. 5, with (a), (c), and (e) being the mean values of position errors in the X, Y, and Z axes, respectively, and (b), (d), and (f) being the mean values of attitude errors in the X, Y, and Z axes, respectively.

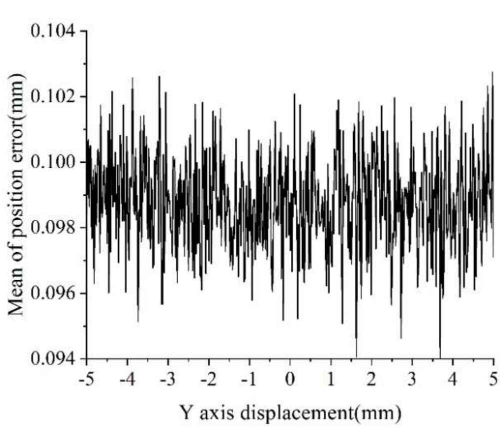
From Fig. 5, it can be seen that the attitude error of the mechanism position error in the X-axis and Z-direction from -40mm to 40mm moving attitude error have a tendency to gradually reduce, in the direction of the Y-axis attitude error is relatively consistent. In order to ensure the reliability of the subsequent research content, in the subsequent accuracy analysis process, this paper chooses (X, Y, Z) = (-40mm, -5mm, -40mm) as the reference point.



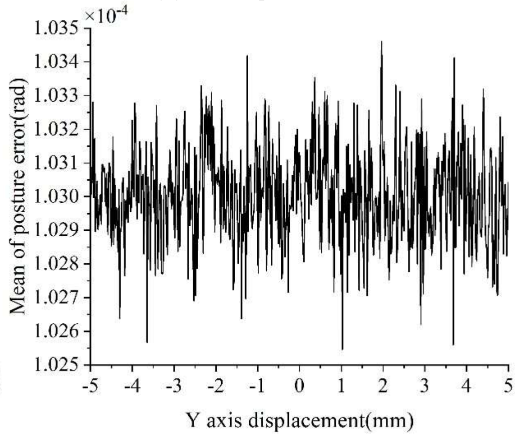
(a) X axis position error distribution



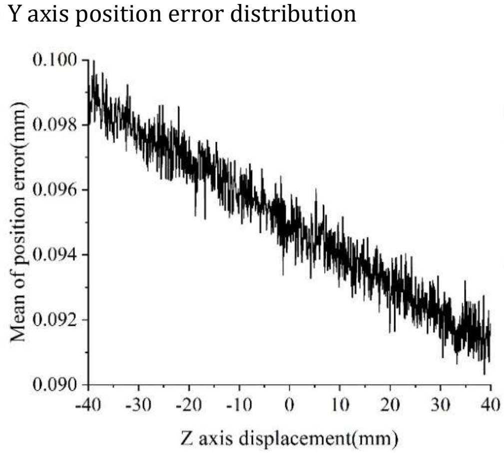
(b) X axis posture error distribution



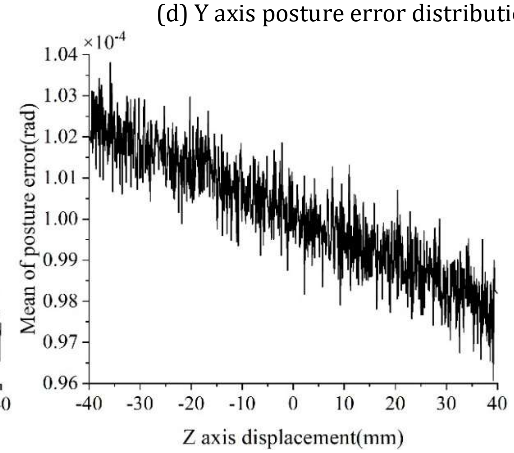
(c) Y axis position error distribution



(d) Y axis posture error distribution



(e) Z axis position error distribution



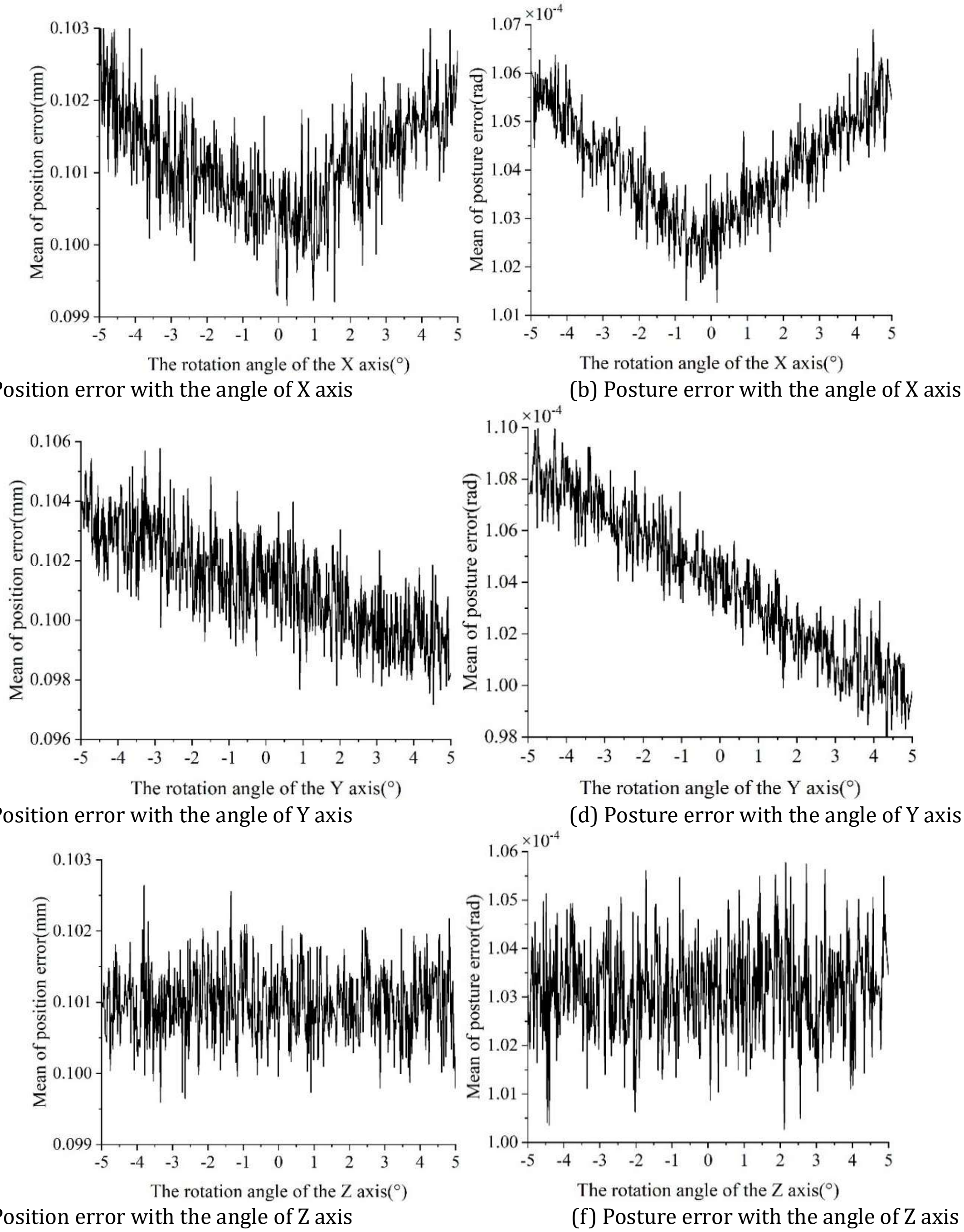
(f) Z axis posture error distribution

Fig. 5. Position error and posture error distribution

4.2.2. Error Sensitivity Analysis of Attitude Parameters on the Training Device. In practice, the factors affecting the working accuracy of the various mechanisms in the training device go far beyond the output position parameters of the moving platform. Attitude parameters such as the rotation angle of the moving platform around each axis also have a significant impact on the accuracy. Therefore, when comprehensively analyzing the accuracy of the mechanism, the influence of these attitude parameters must also be considered to ensure the accuracy and completeness of the accuracy analysis.

In order to study the influence of attitude parameters on the output positional accuracy of the mechanism, the average positional error distribution of the mechanism in the workspace with the change of the angle of rotation of X, Y and Z axes is plotted as shown in Figure 6.

From the graphical information, it can be seen that as the X-axis turning angle gradually increases within the set range, the positioning error of the mechanism increases accordingly. And the output accuracy of the mechanism shows a steady increase in the process of rotating from -5° to 5° around the Y-axis. Comparatively speaking, the angular attitude around the Z-axis did not produce a significant change in the output accuracy of the mechanism.



(a) Position error with the angle of X axis (b) Posture error with the angle of X axis

(c) Position error with the angle of Y axis (d) Posture error with the angle of Y axis

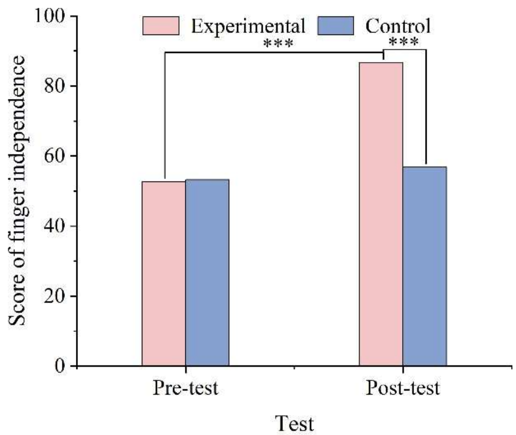
(e) Position error with the angle of Z axis (f) Posture error with the angle of Z axis

Fig. 6. Position error and posture error distribution

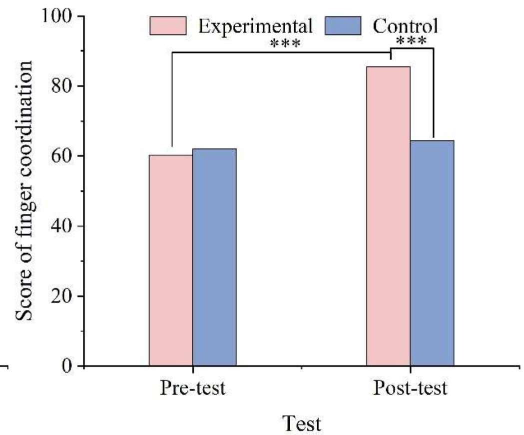
4.3. Analysis of the effect of piano training

The piano training device designed in this paper is put into practical application, and the piano students of S school are selected as the experimental objects to investigate the training effect of the Monte Carlo-based piano finger training device in this paper through the teaching experiment method. Finger independence, finger coordination, finger fluency and finger dexterity were tested. Fifty students were randomly selected and divided into the experimental group and the control group. The experimental group was trained with the training device designed in this paper, while the control group was trained with the conventional training method. The test results of the experimental and control groups are shown in Figure 7, where (a)~(d) are the test results of finger independence, finger coordination, finger fluency and finger dexterity, respectively.

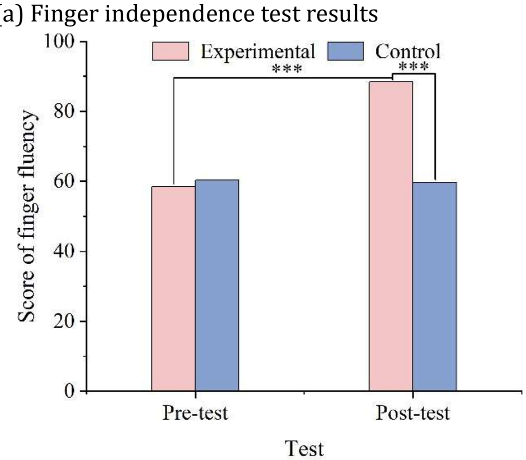
As can be seen in Figure 7, the scores of the experimental group on the four dimensions of finger independence, finger coordination, finger fluency and finger dexterity before the experiment were 52.63, 60.25, 58.46 and 60.47, respectively, while the scores of the control group were 53.24, 62.08, 60.31, and 59.84, respectively, none of which were significantly different from each other. After the experiment, the experimental group scored 86.74, 85.52, 88.41, and 84.58 on the four dimensions, while the control group scored 56.78, 64.38, 59.73, and 61.34. There remained no significant difference in the pre and post experimental scores of the control group. There are very significant differences in the pre- and post-test results of finger independence, finger coordination, finger fluency and finger dexterity in the experimental group. It can be seen that the Monte Carlo-based piano finger training device in this paper has a significant positive impact on improving students' piano finger playing skills.



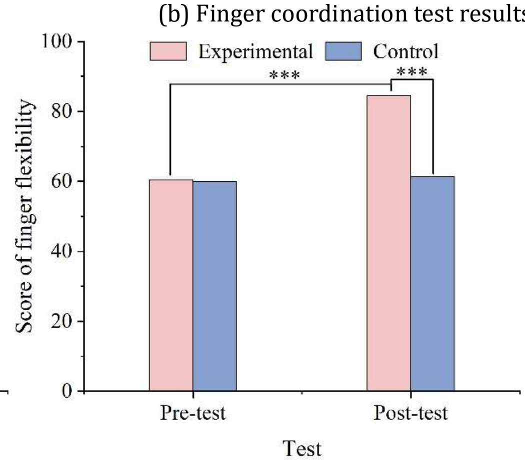
(a) Finger independence test results



(b) Finger coordination test results



(c) Finger fluency test results



(d) Finger flexibility test results

Fig. 7. Test results of experimental and control group

5. Conclusion

In this paper, starting from the most basic finger skills in piano playing, Monte Carlo method is introduced to optimize the traditional piano finger training and design the piano finger training device. The kinematic simulation of finger training is carried out to test the error sensitivity of the training device. Finally, the training effect of the piano finger training device proposed in this paper is investigated.

The Monte Carlo-based piano finger training device designed in this paper conforms to the biological characteristics of the human hand, has a good comfort level, and can be used to characterize the kinematic changes of finger training. When the position error of the mechanism moves from -40mm to 40mm, the position error gradually decreases in the X-axis and Z-axis, and the position error is smoother in the Y-axis direction. The postural error of the mechanism gradually increased with the increase of the X-axis rotation angle. The output accuracy of the mechanism increases steadily during the rotation from -5° to 5° around the Y-axis. The angular attitude around the Z-axis has no significant change on the output accuracy of the mechanism. There was no significant difference between the experimental and control groups in the four dimensions of finger independence, finger coordination, finger fluency and finger dexterity before the experiment. After the experiment, the experimental group scored much higher than the control group in all four dimensions, creating a significant difference between the two groups. The experimental group has a very significant difference between the pre- and post-test results, and the opposite is true for the control group. In this paper, the piano finger training device based on Monte Carlo optimization can effectively improve students' piano playing skills.

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