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A Data-Driven Computational Analysis of Piano Playing Technique Evolution

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ABSTRACT

As the father of musical instruments, the piano is commonly used in solo, repertoire, accompaniment and other performance processes. In the process of piano playing, the quality of its sound is closely related to the playing skills. The article analyzes the structural composition of the piano as well as the physical mechanism of sound generation, and summarizes the characteristics of the four elements of piano music, namely pitch, intensity, timbre and duration, on the mathematical basis of the twelve equal temperament laws and the vibration equations of the strings. Subsequently, we analyze the time and frequency domain characteristics of the piano's musical technique evolution, and calculate the main physical parameters that can affect the piano timbre. Finally, based on the theoretical study and characterization, the corresponding result evaluation experiments were conducted. It is concluded that by analyzing the root-mean-square and mean values of the vibration time-domain signals of piano soundboards excited at different points, it can be seen that, for different structures of piano soundboards, there are excitation points that can maximize their vibration signals. At the same time, the time-domain characteristic index crag factor is analyzed, and it is found that there is no obvious pattern in the crag factor value of the vibration signal of the soundboard with different point excitations.

Keywords: time-domain characteristics; piano sound; string vibration equation; playing technique

1. Introduction

In the context of the network era, the rapid development of information technology, the music industry teaching needs to keep pace with the times, increase the theoretical and practical ability of information technology, the traditional theory of piano teaching and new information technology organic combination [1-3]. The application of digital forms of piano teaching, which not only provides unprecedented convenience and support for piano playing skills education and teaching, brings revolutionary significance to “teaching” and “learning”, but also brings challenges [4-5].

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Piano performance teaching is highly specialized and requires a large amount of time to learn. The traditional teaching method is one-on-one teaching through teachers, and this form leads to a large number of students whose economic level cannot be afforded, and the current number of teachers cannot realize one-on-one teaching for a large number of students [6-9]. Therefore, students can use wearable sensors and other intelligent devices to collect a large amount of behavioral data generated in the state of piano playing, and obtain feedback on the evaluation of the performance effect with the help of deep learning, artificial intelligence, and other information technologies in order to carry out independent learning [10-13]. This data-driven learning of piano playing skills, on the one hand, reduces the time consumed by teachers, so that teachers do not need to supervise students every moment, and only need to analyze the students' practice data through the network to obtain comprehensive access to students' learning and training [14-17]. On the other hand, it allows teachers to customize personalized teaching plans for different students' performance data, so that students can obtain more targeted teaching methods [18-21]. Therefore, the study of data-driven technology can optimize and integrate educational resources, guide teachers to pay attention to the cultivation of practical and comprehensive high-quality piano talents in piano teaching, give full play to the important value of networked piano teaching in colleges and universities, and improve the teachers' awareness of discipline construction, so as to provide reference for the reform of higher piano education curriculum [22-25].

The article firstly starts from the acoustic theory of musical instruments, and on the basis of the theoretical foundation of the physical structure of musical instruments and the mechanism of sound generation, as well as the mathematical foundation of analyzing the twelve equal temperament laws and vibrational equations, the concepts as well as the characteristics of the four elements of the piano musical sound, namely, the pitch, the intensity, the timbre and the length of the tone, are respectively analyzed in depth. The string vibration decay envelope is then analyzed in the time domain. Then the main objective parameters affecting the piano timbre are identified from the time and frequency domains respectively, and the spectral analysis of the piano sound signal is analyzed in all aspects. Then, transient excitations were applied to the soundboards of pianos of different structures at different points to make them vibrate freely, and the effects of percussion at different points on the time-domain and frequency-domain characteristics of the vibration signals were analyzed. Finally, the method proposed in this paper is used to carry out experiments on the sound attenuation process of the soundboard vibration and to analyze the vibration characteristics of the piano soundboard.

2. Fundamentals of music theory

2.1. Physical structure of the piano and the mechanism of sound production

2.1.1. Structural components of the piano. Piano is a sophisticated and complex structure of the instrument, the development so far, there is a relatively perfect structure, it is mainly by the shell, the sound source, the mechanical part of the three major components, and each part of the interconnected together to play a role in the piano.

2.1.2. Piano Sound Mechanisms. Piano, its basic sound source is the strings, the player through the keyboard to drive the hammer to stimulate the strings to vibrate, so as to produce sound waves. The resonance box uses the principle of acoustic resonance to respond to the sound waves from the vibration of the strings, increasing the amplitude of the sound waves, and at the same time, through the filtering of the material, spreading the sound waves out dramatically.

2.2. Pitch of piano notes

Pitch, i.e., the height of a tone, refers to the objective judgment of the auditory system on the pitch of a sound. A musical signal consists of a number of vibrations at different frequencies, the lowest of which is usually referred to as the pitch of the musical note. In musical instruments, the pitch of all notes is usually defined by a meter. To determine the pitch of a musical instrument, a base note is determined first, and the other notes are calculated by multiplication based on the frequency of this note according to the meter used [26]. The piano is one of the typical twelve equal temperament instruments, in the twelve equal temperament relationship, the same degree, octave intervals are very pure, the fourth and fifth intervals are also very harmonious, which makes it easy for the instrument to transpose, which is exactly what the other temperament systems can not do.

2.3. Intensity of piano notes

The sound intensity of music is an objective property of the auditory system to determine the strength of the sound, also known as loudness, volume. The intensity of sound is generally measured objectively in terms of sound pressure level, which is measured in decibels (db). The sound of a piano is a "musical sound". When striking the strings of a piano to make it sound, even if the speed, strength and manner of touching the keys are the same, the intensity of the sound depends on the strength and tension of the strings themselves, and is even affected by the structure of the soundboard and the quality of the mallets in the resonance system of the piano.

2.4. The timbre of piano musical notes

2.4.1. Equation of vibration of a piano string. The vibration of the strings comes from the hammer strikes. The movement of the drive system, driven by the keyboard playing, causes the mallets to strike the strings and vibrate them, thus making the sound of the piano. In general, the mallets strike the strings with a small contact surface and for a short period of time. The string length can be set to l and the string is struck with a cosine-shaped mallet of width $2d$ [27]. The position of the strike point is $x = x_0$, and the piano string vibrates with an initial velocity of v_0, an acceleration of a, and a vibration time of t. Because of the unique shape of the mallet, the velocity is maximum at the moment of strike x_0, and the velocities at the endpoints of the strike region, $x = x_0 - d$ and $x = x_0 + d$, are zero, and thereafter the string begins to vibrate freely, and the vibration equation can be solved by the following system of equations:

$$\begin{cases} \dfrac{\partial^2 u}{\partial t^2} = a^2 \dfrac{\partial^2 u}{\partial x^2} (0 < x < l, t > 0) \\ u|_{x=0} = u|_{x=l} = 0 \\ u|_{x=0} = \varphi(x) = 0 \\ \left.\dfrac{\partial u}{\partial t}\right|_{t=0} = \psi(x) = \begin{cases} 0, & (0 < x < x_0 - d, x_0 + d < x < l) \\ v_0 \cos \dfrac{x - x_0}{2d} \pi, & (x_0 - d < x < x_0 + d) \end{cases} \end{cases} \tag{1}$$

Based on the eigenfunction $\sin(n\pi x / l)$ known from the boundary conditions, the string vibration equation can be formulated as:

$$u(x,t) = \sum_{n=1}^{\infty} (C_n \cos \frac{n\pi a}{l} t + D_n \sin \frac{n\pi a}{l} t) \sin \frac{n\pi x}{l} \tag{2}$$

Substituting the above equation into the initial conditions yields:

$$\varphi(x) = \sum_{n=1}^{\infty} C_n \sin \frac{n\pi}{l} x = 0 \, (0 < x < l) \tag{3}$$

$$\psi(x)=\sum_{n=1}^{\infty}D_n\cdot\frac{n\pi a}{l}\cdot\sin\frac{n\pi}{l}x=\begin{cases}0, & (0<x<x_0-d, x_0+d<x<l)\\ v_0\cos\frac{x-x_0}{2d}\pi, & (x_0-d<x<x_0+d)\end{cases} \tag{4}$$

The coefficients C_n and D_n can be obtained from Eq:

$$C_n=\frac{2}{l}\int_0^l\varphi(x)\sin\frac{n\pi x}{l}dx=0 \tag{5}$$

$$\begin{aligned}D_n&=\frac{2}{n\pi a}\int_{x_0-d}^{x_0+d}v_0\cos\frac{x-x_0}{2d}\pi\cdot\sin\frac{n\pi x}{l}dx\\&=\frac{8v_0d}{n\pi^2a}\cdot\frac{1}{1-\frac{4d^2n^2}{l^2}}\cdot\sin\frac{n\pi x_0}{l}\cdot\cos\frac{n\pi d}{l}\end{aligned} \tag{6}$$

The piano string vibration equation can be derived from the above equation as:

$$u(x,t)=\frac{8v_0d}{\pi^2a}\sum_{n=1}^{\infty}\frac{1}{n}\frac{1}{1-\frac{4d^2n^2}{l^2}}\sin\frac{n\pi x_0}{l}\cos\frac{n\pi d}{l}\sin\frac{n\pi at}{l}\sin\frac{n\pi x}{l} \tag{7}$$

2.4.2. Overtone characterization of piano string vibrations. Piano string vibration contains the fundamental frequency and many integer multiples of the fundamental frequency overtone components, and its vibration equation determines that the fundamental frequency is the lowest, and the vibration amplitude tends to be the largest, i.e. the highest energy. From the piano string vibration equation, it can be seen that the piano string vibration is simply superimposed on the fundamental and all harmonics, i.e., it is the superposition of all standing waves, and the amplitude of the n st harmonic is $\propto\frac{1}{n}\frac{1}{1-\frac{4d^2n^2}{l^2}}$, the overtone columns composed of sine and cosine functions decay proportionally with $1/n$, while the overtone columns of the string vibration of plucked string instruments decay proportionally with $1/n^2$. This is the difference between the percussive string vibration and other plucked string vibrations [28].

2.5. *Piano musical note duration*

Tone length, i.e. the length of the tone, is a very important component unit in music, which specifically refers to the time experienced by a certain musical note from the pronunciation to the end, which mainly depends on the length of the vibration duration of the strings, so the estimation of the vibration duration of the piano strings in the music simulation is essentially to obtain the tone length of the corresponding note. For a piece of music, there is a certain sense of rhythm, and there is a clear relativity between the notes, which ensures a specific relationship between the time values of the notes in the piece.

2.6. *String Attenuation Characterization*

The ideal string vibrates in a relatively simple, fundamental mode of vibration called simple harmonic vibration. It is a sinusoidal curve with no decay. The string is often subjected to some resistance during vibration, causing its amplitude to decrease over time. A single-tone time-domain waveform of a piano is shown in Figure 1. This is an important characteristic of the musical signal in the time domain, which reflects the dynamic change of the piano sound, and is also an important factor affecting the piano timbre. This characteristic can be described by the envelope of the time domain. The envelope of the time domain is a concrete reflection of the change of the amplitude of the musical sound over time during the performance process, and the schematic diagram of the ADSR envelope in the time domain is shown in Figure 2. Onset, indicates the time when the sound amplitude reaches its maximum. Load Decrease, which indicates the time when the amplitude of the sound waveform reaches its maximum and then decays. Hold, which indicates the amplitude maintained before the release mode for sustained sounds, and the intersection of the decay and release modes for decayed sounds; this is the only stage that differs from the other three in that it does not indicate the length of the hold. The release sound, to the final stage of the sound, indicates the length of time that the sound waveform gradually decays from the maintained amplitude to zero.

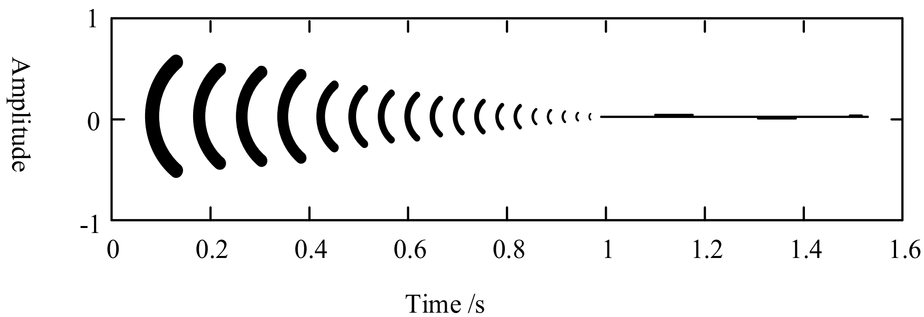


Fig. 1. Time domain waveform of piano single tone

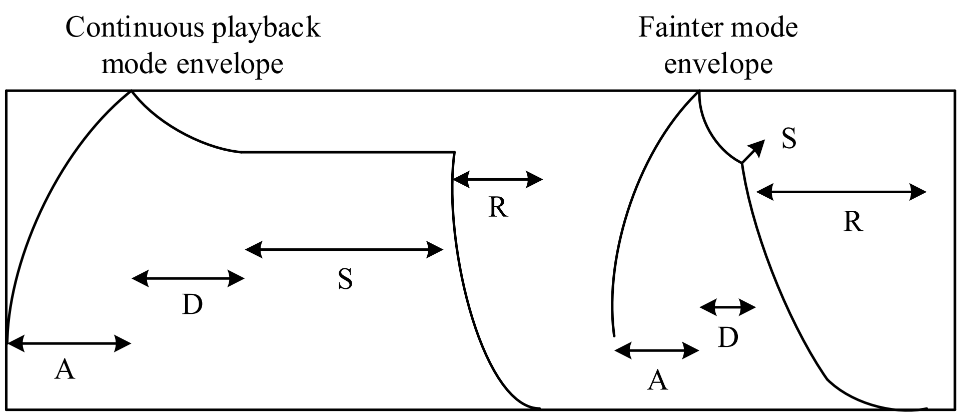


Fig. 2. Schematic diagram of ADSR envelope in the time domain

A simple short-time root-mean-square energy calculation method will be used to obtain the corresponding time-domain amplitude envelope for piano tones, which is tabulated as follows:

$$A_{RMS} = \sqrt{\frac{1}{N}\sum_{n=0}^{N-1} |x(n)|^2} \tag{8}$$

where $x(n)$ denotes a frame of time-domain signal and N is the frame length. After the signal is segmented into frames, the short-time rms energy of each frame can be calculated, and then the envelope of amplitude over time can be obtained.

A segmented envelope function matches well with the time domain amplitude envelope extracted in this paper. This envelope function is divided into three segments, the first segment is a straight line, the second segment is a parabola, and the third segment is an exponential decay. Its specific equation is:

$$f(x)=\begin{cases}25.17x & (0<x<=0.03)\\ -50\times(x-0.1)^2+1 & (0.03<x<=0.15)\\ 0.875\times e^{-6(x-0.5)} & (0.15<x<=1)\end{cases} \tag{9}$$

3. Techniques of piano performance music

Piano playing skills is the basic skills of piano playing, in the demonstration of a performance activity, the basic skills training is necessary, but also the first to be trained in the piano learning is also the same, in order to continue to improve their level of performance, a good way is to increase the amount of exercise every day, to increase the intensity of training and perseverance in training. The first thing a piano player needs to do in the early stages of training is to determine the correct way to play and touch the keys. Whether it is hand strength training, finger strength training, or training in the angle and strength of the keys, the training process is permeated with the training of playing skills. In the early and middle stages of playing, you will not be able to reach the advanced level of playing skills by playing more difficult pieces without practicing more practice pieces from the outside in.

3.1. Effects of fingers

Finger dexterity, independence and stability are all characteristics that affect performance. The requirement for a piano player is to have flexible fingers, which is the basic requirement to facilitate piano learning and training. If the fingers are not flexible enough, the player will not be able to perform the technique, and will not be able to express the emotion and the inner part of the piece.

In the process of piano playing, the flexibility, stability and independence of the fingers are very important, which directly affects the player's grasp of the keystrokes, as well as the effect of the player's performance, so it is necessary to enhance a number of training of the fingers. In terms of independence, it is required to train individual fingers, so that they are not affected by other fingers when playing, and the degree of flexibility of individual fingers in touching the keys will have a direct impact on the effect of the entire musical presentation. In terms of dexterity, if the fingers are not flexible, dissonance will occur, affecting the musical expression [29]. Performers need to enhance the flexibility of fingers and practice diligently to prepare for better music performance. In terms of stability, players are required to keep a good mindset when playing, so that the fingers can give full play to their functions.

3.2. Use of keystroke techniques

During the entire piano playing process, it is necessary to relate to the intrinsic nature of the piece being played, and then make the corresponding treatment to fully express the intrinsic emotion and the required sound quality of the piece. This requires effective control of the fingers' keystrokes. In piano playing, there are strict requirements for the strength of key touch, the height of key touch, the angle of key touch and the area of key touch. In terms of the strength of the keys, it should be decided according to the style of the piece, such as strong music requires strong wrist and finger strength, while soothing music requires not too much strength, but lightness. In terms of the height of the keys, the height of the keys in piano playing is categorized into the low finger and the high lifting finger. The low finger enhances the continuity between syllables and is suitable for harmonious and soft music, while the high finger is the opposite. In terms of the angle of touch, there are two main types of finger touch: vertical and horizontal. Different keystroke angles give different sound effects. For example, vertical touch is good for upbeat music, while horizontal touch is good for subdued music. In terms of the area of the keys touched, the different areas of the fingers touching the keys in piano playing will also show different effects. For example, a large contact area is good for deep music, while a small contact area is good for brighter music.

3.3. Personal qualities of the performer

In the process of piano playing, the mental state of the performer will be affected by the music's emotion and other factors, and may have a psychic interface with these factors. Therefore, it is necessary for the piano player to have a good psychological quality, not to be disturbed by the music, and to learn to relax and be confident in the process of playing, so as to present his/her best state. If the playing skills are high, but the psychological quality is not, the performance will not be satisfactory. Therefore, it is necessary to optimize your mental state in the course of training and performance.

3.4. Pedal utilization

The pedal is a means of piano playing technique, and the effect of playing can be directly affected by the pedal. If the function of the pedal is not truly explored, it will also affect the effect of the piano playing. There are two pedals in a piano, the soft pedal and the sustain pedal. If you don't step on the pedals enough, the continuity of the piece will be affected and the performance will not be as good as it could be. However, if the pedal is over-pedaled, the performance will also be affected. Therefore, it is necessary to use the pedals reasonably according to the specific piece of music. Beethoven's works, for example, have a lot of pedal markings, while Mozart's repertoire has very few. Therefore, in normal training, do not ignore the function of the pedal, in order to achieve the best effect of the piano performance, and to play the high quality of the pedal ability.

4. Calculation of the evolution of the piano sound signal

4.1. Frequency domain characterization of piano sound

Every sound is produced by the vibration of matter, so every sound can be represented by a sonogram. A sonogram refers to a graph of sound vibrations on a plane rectangular coordinate system with time as the horizontal axis and amplitude as the vertical axis. If a point is taken on the sonogram every certain time (s), then it can constitute a set of data, which is the data inside the commonly used sound file. The time interval δ is called the sampling period, and $\frac{1}{\delta}$ is called the sampling frequency. However, such a set of data does not reflect the characteristics of the sound, but only a discrete copy of the original sound. It must be further analyzed by digital signal processing methods to obtain the corresponding results.

Sound signals, in general, are more complex functions. It can be expressed as a Fourier function:

$$\begin{aligned} f(t) &= A_0 + \sum_{k=1}^{\infty} (A_k \cos k\omega_0 t + B_k \sin k\omega_0 t) \\ &= A_0 + \sum_{k=1}^{\infty} A'_k \cos(k\omega_0 t + \varphi_k) \end{aligned} \tag{10}$$

where A_0, A_k, B_0, A'_k, etc. are the Fourier coefficients. In this way, any complex periodic vibration can be decomposed into many simple harmonic vibrations whose frequencies are integer multiples of the original periodic vibration. The simple harmonic vibration of $k=1$ has the same frequency as the original periodic vibration and becomes the fundamental frequency vibration. The simple harmonic vibrations of $k=2,3,\cdots$ become the k th harmonic vibrations, which are usually also called the k th harmonics. This method of spectral analysis, which decomposes an arbitrary vibration into a number of simple harmonic vibrations, can be used to study the main parameters affecting sound quality.

From the Fourier function formula and the piano spectrum diagram can be derived to express a sound signal of several parameters:

1) Hum A_0: represents the frequency lower than the gene, often called “hum”, not every sound signal has such a hum.

$$A_0 = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\,dt \tag{11}$$

2) Fundamental A_1: In musical notes, almost all sound signals have harmonics, and the frequency of the fundamental is generally taken as the pitch of this musical note.

$$A_1 = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos\omega_0 t dt \tag{12}$$

3) Composition of the harmonic column A_k, B_k: They can indicate the composition of the harmonic column, the essence of the piano sound is string vibration, and its harmonics into a 1:2:3:4... integer ratio relationship. The peaks of the harmonics are found near the theoretical values. In the low frequency part the theoretical values are very close to the actual values, but the more you go to the high frequency, the more obvious the error is. The spectrum shows that the 12th and 14th harmonics are not present in the theoretical values, but in reality there are two peaks. This is due to the fact that the piano is a complex vibration system, and the coupling of various vibrations produces other sounds. Nevertheless, the distribution of each harmonic is not far from the string vibration theory.
4) Phase $\varphi_k : \varphi_k$ indicates the relationship of phase, responding to the fact that harmonics of different frequencies take off at different moments. The literature on speech acoustic analysis suggests that changes in phase generally do not affect human hearing. Similarly, changes in phase within a certain range have a relatively small effect on timbre.

4.2. Time domain characterization of piano sound

The process of decay of each harmonic of a single piano tone over time. According to the change of piano sound with time, it can be divided into the following parts:
1) Excitation: According to the recorded waveform of a^1 tone in 0~60ms, it can be seen that the waveform after 19ms shows a relatively regular periodic change, and before that is the excitation section. The excitation section is the wave generated when the string starts to vibrate after the hammer strikes the piano string. We believe that this is related to the strength of the hammer hitting the strings, i.e., the playing technique, and different methods of playing can produce different timbres.
2) Tapping: The synthesis of vibrations in the same direction and at different frequencies produces tapping. This is due to:

$$x_1 = A\cos\omega_1 t \tag{13}$$

$$x_2 = A\cos\omega_2 t \tag{14}$$

$$\begin{aligned} x &= x_1 + x_2 = A\cos\omega_1 t + A\cos\omega_2 t \\ &= 2A\cos\frac{(\omega_2-\omega_1)t}{2}\cos\frac{(\omega_2-\omega_1)t}{2} \end{aligned} \tag{15}$$

When the angular frequencies ω_1 and ω_2 of two vibrations are large, but the difference between them is small, we can approximate the combined vibration as a "quasi-simple harmonic vibration" with amplitude varying slowly in accordance with $2A\cos\frac{(\omega_2-\omega_1)t}{2}$, compared with frequency $\frac{(\omega_2+\omega_1)t}{2}$. This phenomenon, in which the amplitude of the combined vibration strengthens and weakens, is called beat. The high-frequency component of this chorus is modulated by the low frequencies, and its envelope has a period of half the period of $2Acos\frac{(\omega_2-\omega_1)t}{2}$, i.e., $T_b = \frac{1}{2}\frac{2\pi}{\frac{(\omega_2-\omega_1)}{2}} = \frac{2\pi}{(\omega_2-\omega_1)}$. However,

by programming the fluctuations in the amplitude of the beat to take the median value, and re-enveloping the envelope, it is found that the beat has little effect on the sound quality of the piano.
3) Decay: For many musical instruments, especially the piano, the sound has to decay after excitation. Let us find the function that describes the decay of a musical instrument. In vibration, if viscosity is considered, then the equation of motion for a horizontal spring oscillator can be expressed as $\frac{d^2x}{dt^2}+2\delta\frac{dx}{dt}+\omega_0^2x=0$ where δ characterizes the constant of the magnitude of the damping of the system. Bringing the solution of the form e^{λ} into equation $\frac{d^2x}{dt^2}+2\delta\frac{dx}{dt}+\omega_0^2x=0$ yields the characteristic equation $\lambda^2+2\delta\lambda+\omega_0^2=0$ with characteristic roots of $\lambda=-\delta\pm\sqrt{\delta^2-\omega^2}$. Depending on the degree of damping $\Lambda^*=\delta/\omega_0$, the differential equation has three different solutions representing the three forms of vibration:

(1) $\delta^2<\omega_0^2$, underdamped vibration:

$$x(t)=A_0e^{-\varphi t}\cos(\omega t+\varphi_0) \tag{16}$$

(2) $\delta^2>\omega_0^2$, overdamped vibration:

$$x(t)=c_1e^{-(\delta-\sqrt{\delta^2-\omega^2})t}+c_2e^{-(\delta-\sqrt{\delta^2-\omega^2})t} \tag{17}$$

(3) $\delta^2=\omega_0^2$, Critically damped vibration:

$$x(t)=(c_1+c_2)e^{-\delta t} \tag{18}$$

5. Experimentation and analysis

5.1. Time-domain signal characterization of piano soundboard vibration signals

5.1.1. Time domain characterization. Entering the corresponding program code in the MATLAB software editor, the root mean square (RMS) value and the average value of the time domain signal amplitude of the four structures of piano soundboards vibrating freely under different point excitations are calculated, and the RMS value of the time domain signal amplitude of the different structures of piano soundboards vibrating freely under different point excitations, xRMS and the average value of the time domain signal amplitude of the different structures of piano soundboards xMean and the average value of the time domain signal amplitude of the different structures of piano soundboards xRMS are shown in Figure 3 and Figure 4. From the figures, it can be observed that when the soundboard of the piano vibrates freely under different point excitations, the mean value of the time domain signal amplitude and the root mean square value are consistent with the time domain signal energy law of the vibration of the soundboard under different point excitations. For the soundboard T, the maximum vibration signal energy is obtained when the soundboard is excited at point [4,7], followed by point [2,9], and the minimum at point [5,6]. For soundboard L90, the maximum vibration signal energy is obtained when the excitation is located at the [3,8] point location, the root mean square (RMS) values of the signals at the [2,9] and [4,7] point locations are almost equal, and the [5,6] point location is the smallest. For soundboard L55, the maximum vibration signal energy is obtained when the excitation is located at the [3,8] point location, followed by the [5,6] point location, and the [4,7] point location is the smallest. For the soundboard L180, the maximum vibration signal energy is obtained when the excitation is located at the [5,6] point position, followed by the [4,7] point position, and the [2,9] point position is the smallest.

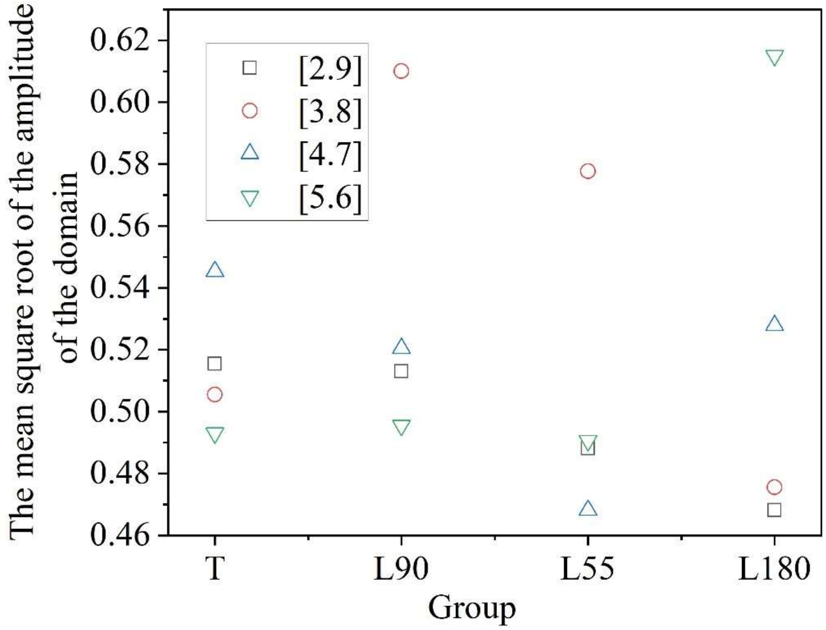


Fig. 3. x_{RMS} of piano soundboards with different structures

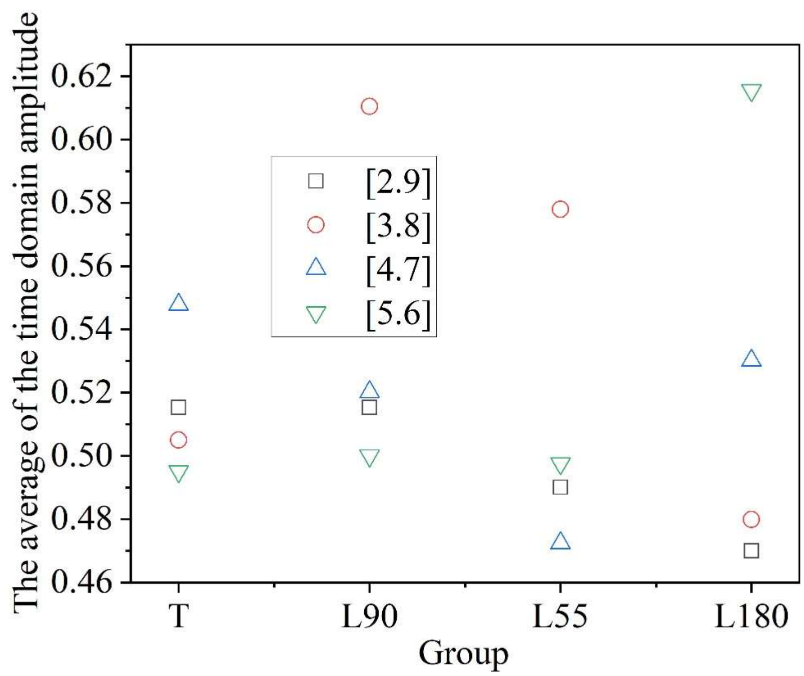


Fig. 4. x_{Mean} of piano soundboards with different structures

The above laws are related to the different unit splicing of the piano soundboard and the composite structure of the surface and core boards. For different structures of the soundboard, the relationship between the excitation point location lines and the structural texture of the soundboard is also different, and the relationship between the excitation point location lines and the structural texture of the soundboard is shown in Table 1.

Table 1. The angle between the excitation point connection lines and the texture of the soundboard

Point bit	Angle of point connection and sound board (or core plate) unit/°				Angle of point of point and vertical Angle of table plate/°			
	[2,9]	[3,8]	[4,7]	[5,6]	[2,9]	[3,8]	[4,7]	[5,6]

T	95	40	0	40	-			
L90	40	0	40	95	92	40	0	50
L55	95	40	0	40	92	40	0	50
L180	40	95	40	0	92	40	0	50

In the MATLAB software editor, the corresponding program code is input to calculate the crag factor of the vibration signals of different structures of piano sound boards under different excitation points, and the crag factor K of different structures of piano sound boards under different excitation points is shown in Fig. 5. Cliff factor is a parameter reflecting the degree of smoothness of the vibration signal waveform, and the increase of the cliff factor indicates that the shock component in the vibration signal increases and the signal waveform decays relatively fast. From the figure, it can be observed that the vibration signals of different structures of soundboards with different points of excitation have different crag factors. For the soundboard T, the [5,6] point excitation has the largest crag factor, followed by the [3,8] point, and the [4,7] point has the smallest crag factor. For soundboard L90, the [2,9] point location has the largest crag factor when excited, followed by the [3,8] point location, and the [5,6] point location is the smallest. For soundboard L55, the [5,6] point location has the largest crag factor when excited, the [3,8] and [2,9] point locations are very close, and the [4,7] point location is the smallest. For soundboard L180, the crag factor is largest for [3,8] point excitation, followed by [5,6] and smallest for [4,7]. There is no clear pattern in the effect of different point excitations on the crag factor value of the soundboard vibration signal.

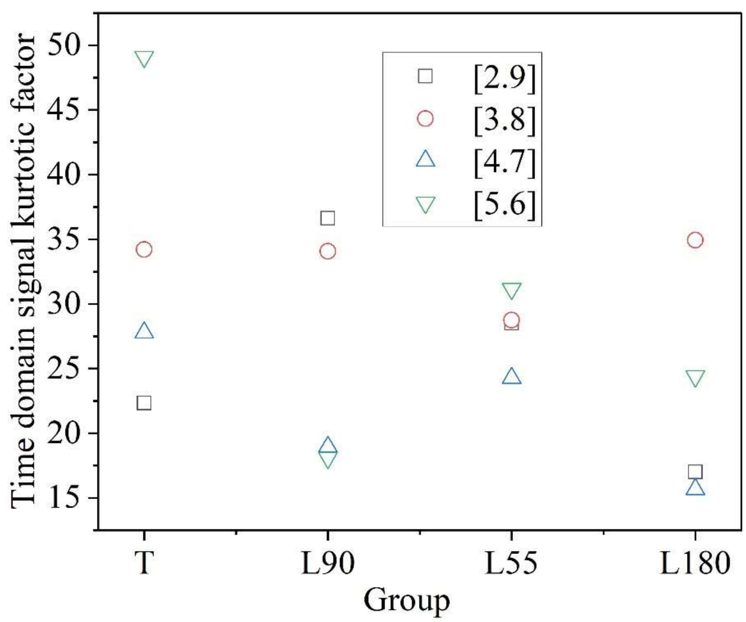
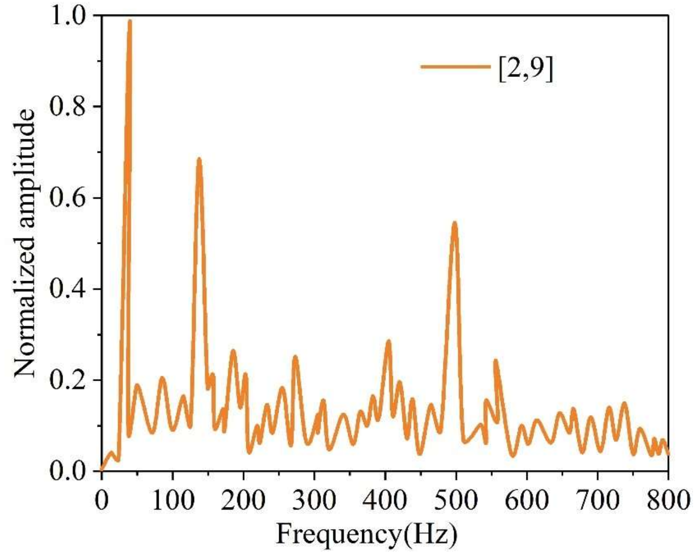
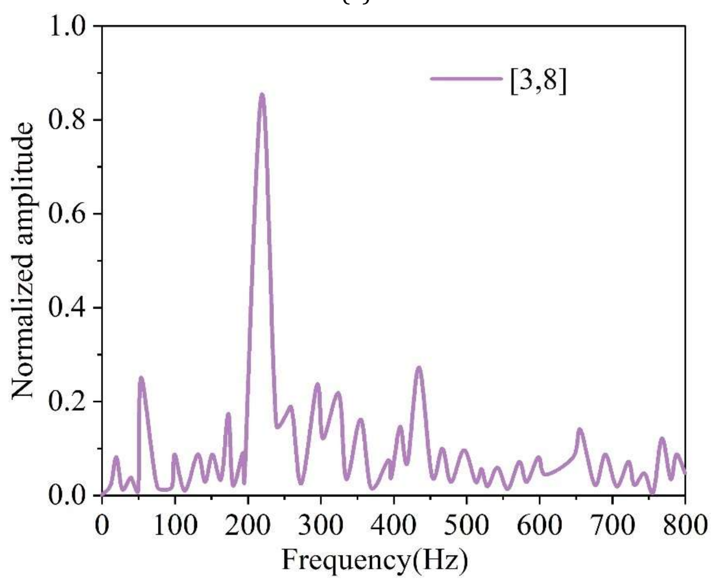


Fig. 5. *K* of piano soundboards with different structures at different excitation points

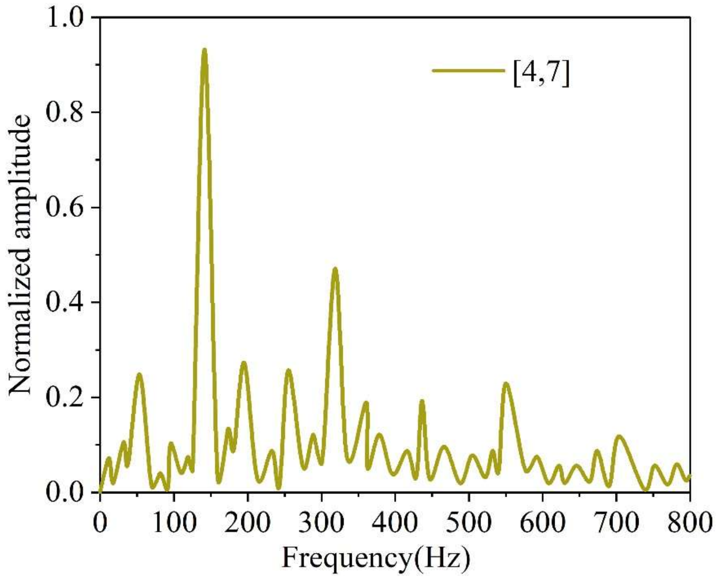
5.1.2. Frequency domain signal analysis. The resonant frequencies of each order remain basically the same when tapped at different point locations, but the amplitudes corresponding to the frequencies have large differences. The corresponding effects of different point excitations on the soundboard T vibration are shown in Fig. 6. The vibration spectrum of the European spruce beam-shaped specimen is shown in Fig. 7. It can be observed from the figure that there is a large difference between the amplitude of the frequency domain graph of the piano soundboard vibration and the frequency domain waveform of the beam-shaped solid wood specimen. The frequency domain diagram of the vibration of the pure solid wood beam-shaped specimen is clear, and the frequencies of each order are approximately multiplicative. In the frequency domain diagram of piano soundboard vibration, the amplitude of each order frequency changes with the change of excitation point position, and the amplitude increases with the order and has no obvious decreasing or other trends. Regardless of the point position of excitation, there is the phenomenon that the amplitude corresponding to individual orders is so small that it is difficult to recognize the frequency. It is also observed that certain frequencies can be made relatively high in amplitude by tapping at different point positions, such as 21 Hz, 485 Hz, 102 Hz, 377 Hz, 554 Hz, 169 Hz, 242 Hz for point excitation in [2,9], 221 Hz, 245 Hz, 323 Hz, 171 Hz, 446 Hz, 297 Hz for point excitation in [3,8], [136Hz, 193Hz, 320Hz, 149Hz, 445Hz, 369Hz under [4,7] point excitation, and [5,6] 579Hz, 98Hz, 137Hz, 54Hz, 370Hz, 171Hz under [5,6] point excitation are very obvious in the frequency domain plot and can be accurately recognized when reading the frequency.



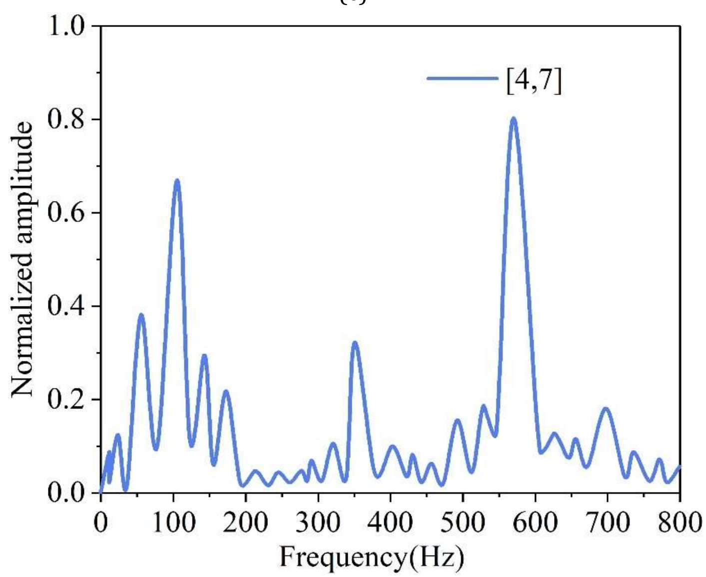
(a)



(b)



(c)



(d)

Fig. 6. The influence of different point excitation on the vibration of sound board T

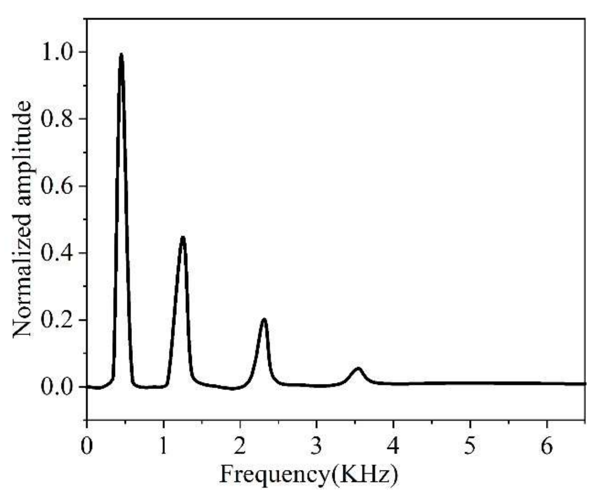
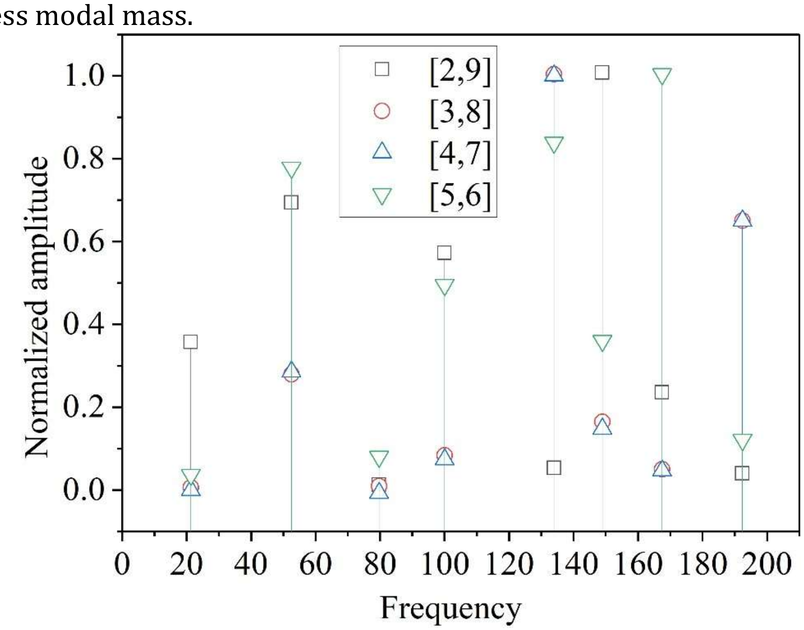
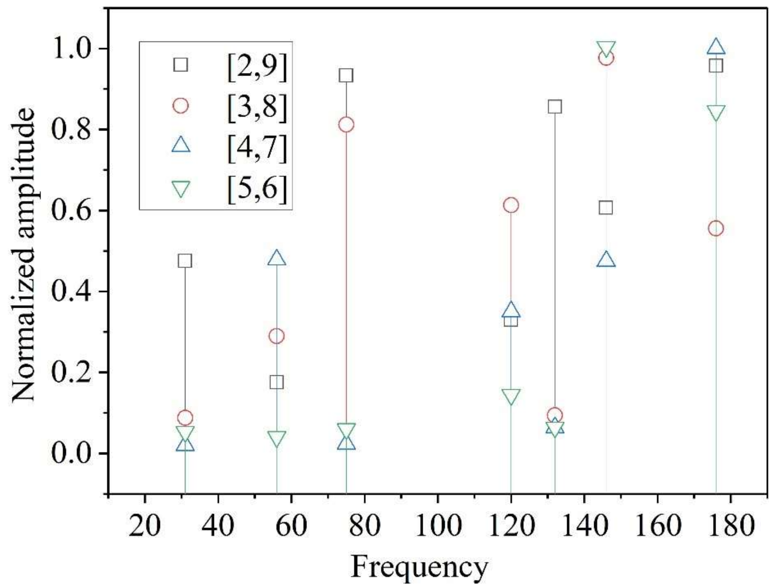


Fig. 7. European spruce beam specimen vibration spectrum

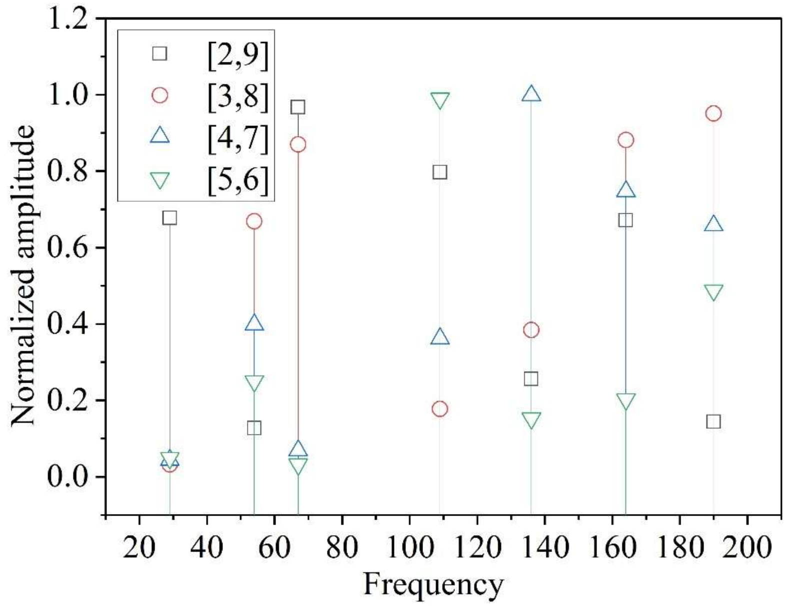
In order to investigate the effect of different point excitation on the vibration frequency domain signal of the piano sound board, identify the frequency and record the frequency corresponding to the amplitude in the signal spectrogram, the vibration frequency and amplitude of the piano sound board of different structures under different point excitation are shown in Fig. 8, (a) ~ (d) are solid wood sound board composite sound, vertical splicing core board solid wood composite sound, oblique splicing core board solid wood composite sound and horizontal splicing core board solid wood composite sound. The effect of different point excitations on the vibration frequency amplitude of each soundboard can be observed from Fig. From Fig. (a), it can be seen that the point excitation at [2,9] can make the vibration amplitude of the 1st, 2nd, 4th, and 6th order of the soundboard T significant. Excitation at [3,8] point position can make its 2nd, 7th order amplitude more obvious. Excitation at point position [4,7] can make its 5th and 8th order amplitudes more pronounced. The excitation at [5,6] point position can make its 2nd, 4th, 5th and 7th order amplitudes more obvious. From Fig. (b), it can be seen that the excitation at the point position [2,9] can make the 1st, 3rd, 5th, and 8th order amplitudes of the soundboard L90 more pronounced. Excitation at the [3,8] point position can make its 3rd, 4th, and 6th order amplitudes more pronounced. Excitation at the [4,7] point location can make its 2nd and 8th order amplitudes more pronounced. Excitation at [5,6] point location can make its 7th and 8th order amplitude more obvious. From Fig. (c), it can be seen that the excitation at [2,9] can make the 1st, 3rd, and 4th order amplitudes of the soundboard L55 more obvious. Excitation at the [3,8] point position can make its 2nd, 3rd, 7th, and 8th order amplitudes more pronounced. Excitation at the [4,7] point location can make its 6th and 7th order amplitudes more pronounced. Excitation at [5,6] point position can make its 4th and 8th order amplitude more obvious. From Fig. (d), it can be seen that the excitation at [2,9] can highlight the 1st, 3rd, and 5th order amplitudes of the soundboard L180 more obviously. Excitation at [3,8] can make its 2nd, 4th, and 5th order amplitudes more pronounced. Excitation at the [4,7] point location can make its 3rd, 4th, 6th, and 7th order amplitudes more pronounced. Excitation at the [5,6] point location can make its 4th and 7th order amplitudes more pronounced. It should be noted that obvious here is relative and refers to the excitation point position corresponding to the higher amplitude at that order frequency under 4 different point excitations. At the same time, it can be found that there are vibration frequencies that cannot be highlighted by all 4 excitation points, such as the 3rd order vibration of the soundboard T. The amplitudes corresponding to the frequencies measured at all 4 excitation points are smaller. This situation may be due to the fact that this order of frequency has less energy, which appears from the modal aspect, i.e., less modal mass.



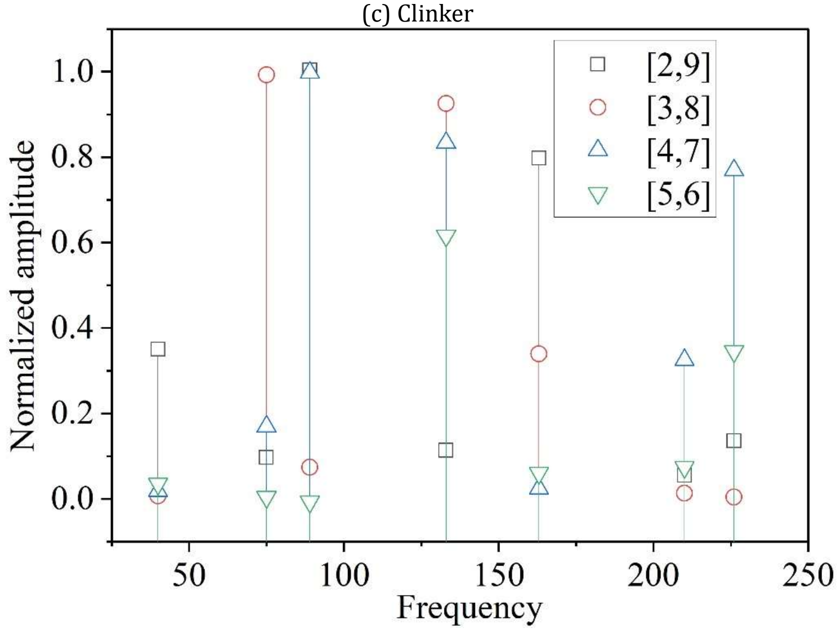
(a) Solid wood



(b) Vertical panel



(c) Clinker

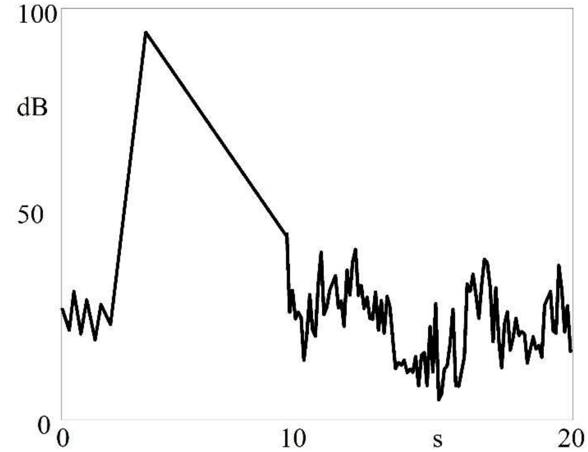


(d) Interwoven core wood composite sound board
Fig. 8. Vibration frequency and amplitude

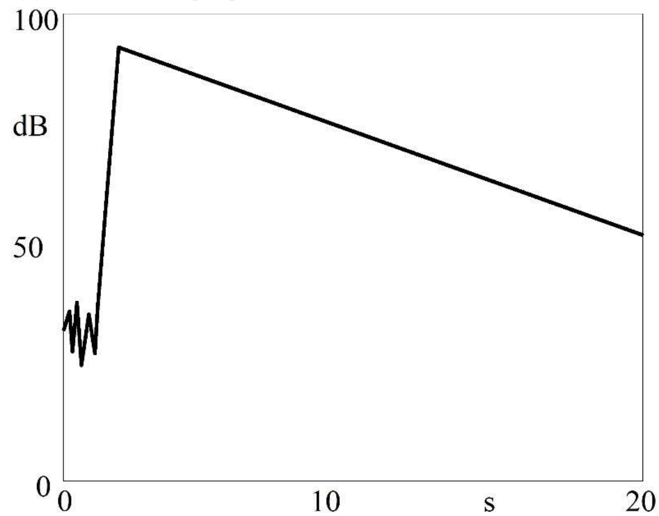
5.2. Vibration energy attenuation and vibration sound generation mechanism of soundboards

The sound board vibration sound wave attenuation process is shown in Figure 9, (a) ~ (b) are the sound perpendicular to the sound board direction and continuous articulation, (c) ~ (d) are the fundamental sound wave reaches the maximum decibel after the rapid attenuation, the overtone part of the slower attenuation. Figure (a) represents the moment when the key is played down, the sound emitted is perpendicular to the soundboard, which shows that the soundboard absorbs the sound waves emitted in the perpendicular direction more rapidly. Figure (b) represents the sustained articulation sound wave after the key is played down, the soundboard absorbs the overtones of the subsequent articulation more slowly, so when the key is played down, the sound emitted is a combination of the two, and the human ear receives the sound signal as a combination of the two vibration forms. There is a slight difference in the timing of the sound field formed by the vibration of the soundboard, the reason for which is determined by the relative phase relationship of the two components. The sound wave decay process as shown in Figure (c) Figure (d) indicates that the fundamental sound wave reaches the maximum decibel and then decays rapidly, while the overtone part decays more slowly, which is determined by the soundboard vibration energy decay characteristics, independent of how much force the player uses to play the keys. This is due to the attenuation of the vibration energy of the soundboard, which is independent of how much force the player uses to play the keys. Due to the differences in the sound regions, the attenuation of the sound wave is also related to the length of the strings. The bass strings are longer, with one or two layers of copper wire wrapped around the outer layer, and their sound attenuation is relatively slow. The sound attenuation time of the soundboard determines the fullness of the sound, when the sound attenuation of the soundboard is too fast, the human ear will feel the sound dry and tasteless, lack of infectious force and expressive power. When the sound board sound attenuation is too slow, the continuous sound will appear muddy phenomenon, so that it is difficult to distinguish the front of the sound is the end of the sound and will cause reverberation, therefore, the sound board needs to have a reasonable sound attenuation time, in the design process, the designer will be based on the positioning of the piano to determine the sound attenuation time of the sound board, in line with the performer and the hall's overall acoustic needs.

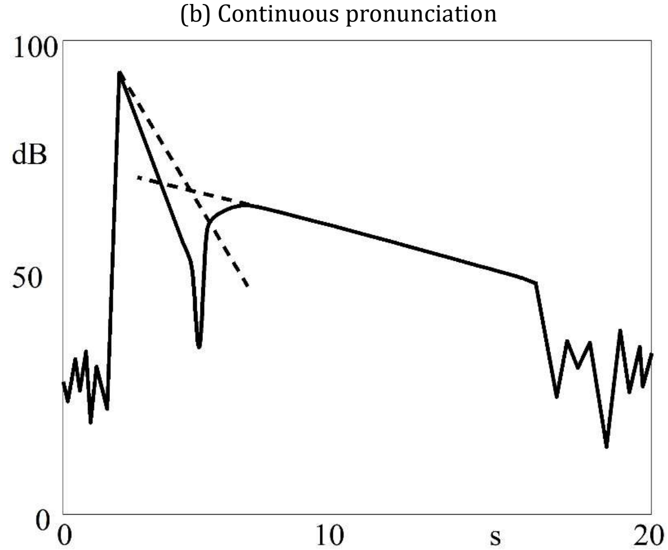
The soundboard is a continuous elastic vibration of the thin plate, the excitation will produce vibration, the soundboard has four main forms of vibration, different excitation will produce the corresponding form of vibration, the size of the excitation force to determine the amplitude of the size of the player to play down the keys mallet hit the strings, the strings will produce tension impulse, the impulse force through the string code to the soundboard, the soundboard will receive the impulse force and expand the impulse force excitation. At the same time, the soundboard is excited by the longitudinal vibration of the string. The height and width of this impulse, i.e. the amplitude of the soundboard, are determined by the player's key stroke and the amount of time spent on the key (the amount of time the damper is off the string). The width of the impulse determines the width of the frequency spectrum, and the excitation of the soundboard varies accordingly, which determines the style of each player. When the impulse force is all over, the vibration of the soundboard will decay rapidly until the soundboard stops vibrating, and this process is a steady-state forced vibration process, whose response frequency is proportional to the frequency of the excitation.



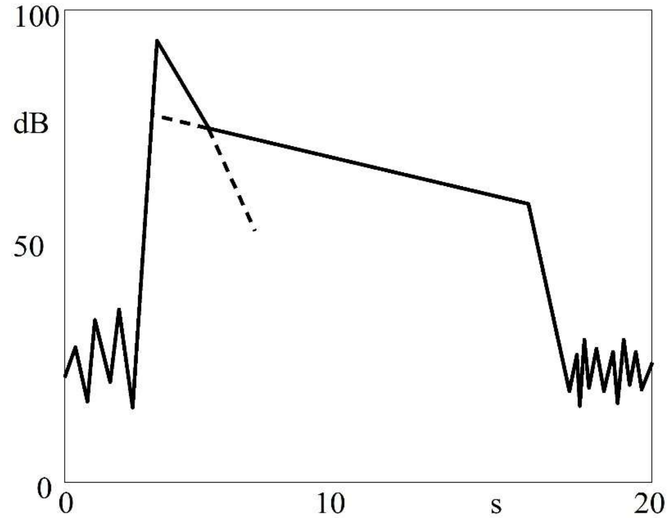
(a) The sound is perpendicular to the sound board direction



(b) Continuous pronunciation



(c) The overtone attenuation is slower



(d) The overtone attenuation is slower

Fig. 9. Sound attenuation of sound plate vibration

5.3. String vibration

5.3.1. Transverse vibration of strings. The transverse vibration of a string is a vibration in which the direction of vibration displacement is perpendicular to the length of the string. Transverse vibration is the dominant type of string vibration, and the working process of string instruments is mainly accomplished with the help of this vibration, and its basic acoustic characteristics are mainly dependent on the characteristics of the strings in transverse vibration, so this type of vibration is the basis on which all string instruments work, and it is a type of vibration that is hoped to be existed and should be strengthened. The main vibration pattern of the first four orders of transverse vibration of the string is shown in Fig. 10, and the first order vibration is the main vibration in practice.

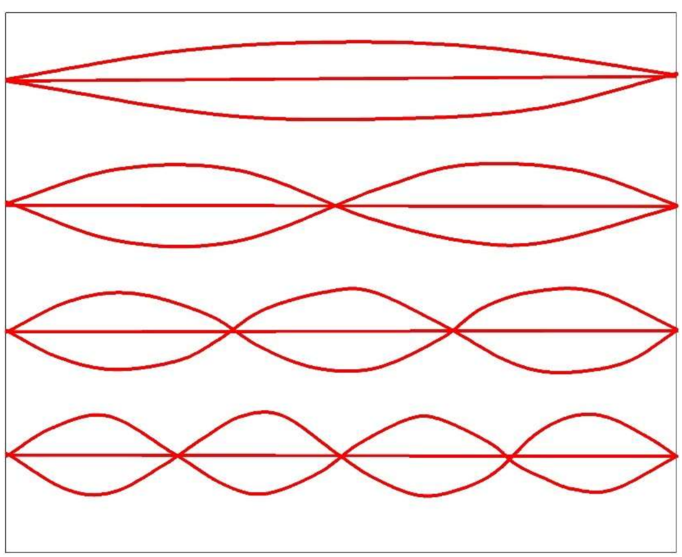


Fig. 10. The first four steps of the string vibration

5.3.2. Longitudinal vibration of strings. Longitudinal vibration of a string is the displacement vibration of the string cross-section in the axial direction. This vibration creates a longitudinal wave of stress within the string, and the resulting force will be transmitted to the soundboard via the yard bridge, with an effect on soundboard vibration. Longitudinal vibration is undesired and is an incidental result of the means of excitation which cannot be avoided.

1) Longitudinal vibration due to expansion and contraction. The string may be regarded as a spring, to which a large preload has been applied during operation. In the transverse vibration due to its own inertia and elastic force will be elongated and shortened, each 1 cross-section in its equilibrium position near the longitudinal vibration. This vibration is completed twice in 1 cycle of transverse vibration, so the frequency of this longitudinal vibration is equal to 2 times the fundamental frequency of transverse vibration of the string. This longitudinal vibration is an induced vibration, which is generated by the production of transverse vibration and exists due to the existence of transverse vibration until the end of articulation. Since the frequency of this longitudinal vibration can maintain a strict harmonic relationship with the fundamental frequency of the string, its presence will contribute to the improvement of the piano's tone, so it is a popular vibration.

2) Friction-induced longitudinal vibration. When the mallet strikes the string, friction occurs with the string. When the mallet at the point of striking the string to produce lateral displacement of the string, the string will elongate, in the contact area between the mallet and the string, the two will appear relative sliding, so the string is subjected to dry friction excitation, the direction parallel to the axis of the string. This friction can also cause longitudinal displacement of the string mass, resulting in longitudinal vibration. However, the mechanism of longitudinal vibration caused by friction and the formation of longitudinal vibration is different from the longitudinal vibration caused by expansion and contraction, expansion and contraction of the string mass to produce longitudinal displacement is due to traction, in this case the frequency of vibration and the size of the displacement of the point depends on the characteristics of the transverse vibration, the vibration is completely subordinate. Friction, on the other hand, produces a displacement of the string mass by virtue of "adhesion". When the hammer touches the string, there is a relative sliding trend between the two, keeping relatively stationary, at this time, the friction angle increases with the increase of transverse displacement, when the friction angle exceeds the maximum friction angle, it suddenly slips, until it remains relatively stationary again in the new position, and then enters the next "adhesion ———slippage" motion cycle. The resulting change in friction over time is sawtooth shaped, with a slower "sticking" phase and a steeper slipping phase.

When the mallet leaves the string, the string enters the free vibration state and vibrates in a damped decay cycle. The longitudinal vibration caused by friction is independent of the transverse vibration, and its vibration frequency and the size of the displacement of the point have nothing to do with the characteristics of the transverse vibration of the string, but only depends on the friction characteristics of the string and the mallet. Since the vibration state and frequency of this kind of longitudinal vibration have great randomness, the chance that its frequency can form a harmonic relationship with the fundamental frequency of the string is very small. In other words, in most cases it is a dissonant tone of the fundamental, so it is an undesirable vibration.

6. Conclusion

This study firstly elaborates the theoretical basis of the physical structure of the piano and the mechanism of sound generation, and investigates the piano attenuation characteristics on the basis of the analysis of the pitch, intensity, timbre and duration of the piano musical notes, and then calculates the evolution of the piano playing skills in the time and frequency domains, respectively, and finally carries out experimental exploration.

By observing the mean and root-mean-square values of the vibration time-domain signals of the piano soundboards excited at different points, the excitation points at which the individual soundboards obtain the maximum energy can be obtained. It is found that the maximum energy is obtained when the soundboard T is excited at the point position [4,7]. The soundboards L90 and L55 are excited at the [3,8] point position to obtain the maximum energy. The maximum energy is obtained when the soundboard L180 is excited at the [5,6] point. There is no obvious pattern in the values of the crag factor of the vibration signals of the soundboard for different point excitations. Observing the frequency domain characteristics of the vibration signal, it can be seen that the frequency of each order is basically the same when struck at different points, but the amplitude corresponding to the frequency has a large gap, and the resonance frequency of the desired order can be highlighted by excitation at different points.

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