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Constructing an Equilibrium Optimization Model of Teaching Strategies between English Teachers and Students Based on Game Theory

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ABSTRACT

Teachers and students will form a variety of dependent behaviors and interactions centered on teaching activities in the teaching process, thus, the teaching process can be regarded as a typical game process. This paper invokes game theory, takes teacher-student behavioral interaction as the research object, constructs a game model of teacher-student behavior in the process of English teaching, and proposes a teaching optimization strategy for English flipped classroom. At the same time, numerical simulation of the teacher-student game model is carried out to explore the dynamic game equilibrium under the cooperative behavior of teachers and students. The simulation results show that in the teacher-student game network, the strategy choices of teachers and students change over time, and different benefit-loss parameter μ, additional gain parameter β_0, and cost-saving parameter ψ have a greater impact on the replication of the strategy choice behaviors of the game parties. In addition, the increase of the parameters of the gain PT obtained by the instructor's conscientious instruction, the gain PS obtained by the student's conscientious learning, and the loss KS of the punishment that the student receives for not learning conscientiously are conducive to the promotion of the instructor and the student's strategy evolution towards cooperation (conscientious instruction, conscientious learning), while the increase of the instructional cost CT of the instructor's conscientious instruction and the learning cost CT paid by the student's conscientious learning are not conducive to the promotion of the two parties' cooperation,. And when the proportion of instructors and students initially choosing cooperation is larger, the likelihood of both parties evolving toward cooperation is greater. This paper provides theoretical support for the optimization of English teaching process.

Keywords: game theory, dynamic game equilibrium, English teaching, flipped classroom

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1. Introduction

With the rapid development of science and technology, the resources and ways of English learning have become rich and diverse, but the massive amount of information also brings some problems, such as information overload and fragmented learning [1]. In this context, the traditional English teaching methods have been difficult to meet the learning needs of students, and the channels for students to acquire knowledge are more diversified, and the requirements for students' practical application of English are also higher [2-4]. Therefore, it is especially important to optimize English teaching strategies to adapt to the characteristics of the information age.

For teachers who are users of informatization technology, the application of informatization technology can enrich the content of teaching and improve the "texture" of the teaching content, thus significantly improving the teaching effect [5-6]. Using information technology, teachers can fully mobilize the four factors of scenario, collaboration, conversation and meaning, so that the teaching form is richer, the teaching content is more in-depth, and the speed of knowledge dissemination is improved to a certain extent [7-9]. On the other hand, the application of informatization technology is also conducive to improving the transferable ability of teachers, keeping pace with the times and adapting to the development of informatization [10-11]. For the students who are the actors of information technology, the application of information technology changes the role of students from passive receivers of knowledge to active acquirers and processors of knowledge [12-13]. In the process of learning, students experience the process of actively searching for information, identifying good and bad, processing information, exchanging and discussing until the information is transformed into knowledge, which improves the ability of students to adapt to the rapid development of the information age, the acquisition of knowledge, and broadens and deepens the breadth and depth of knowledge content [14-17]. However, while information technology brings many advantages, there are also great risks. The application of informatization technology will distract students' attention and concentration on the knowledge itself to the main body of informatization tools, and many students will be distracted or even addicted to the informatization network in the process of accepting the application of informatization technology [18-21]. Therefore, starting from the contradiction between teaching and learning in informatized English teaching, a game matrix is constructed to analyze the application of informatized technology in English teaching, in order to explore the ways and means for both sides to achieve win-win situation and maximize benefits [22-24].

This paper explores the teacher-student game behavior in the process of English teaching based on game theory, realizes the game model construction of teacher-student interaction behavior, and proposes the optimization strategy of English flipped classroom teaching based on game evolution. In order to verify the effectiveness of the proposed teaching optimization strategies, simulations were conducted for teachers adopting the flipped classroom and traditional teaching strategies, as well as for students adopting conscientious or non-conscientious learning, respectively. The initial strategy ratio, the costs and benefits of teachers' serious instruction and students' serious learning, and the effect of students' penalty loss on the evolutionary path of the game are also explored.

2. Optimization of Teaching Strategies Based on the Behavioral Game between English Teachers and Students

2.1. Game Theory

Game theory is a theory that studies the decision-making when the behaviors of decision-making subjects have direct interactions and the equilibrium problem of such decisions [25]. Game theory is divided into cooperative and non-cooperative games, and the difference between the two lies mainly in whether the parties can reach a binding agreement when people's behaviors interact. If so, it is a cooperative game. On the contrary, it is a non-cooperative game. All the games in this paper refer to non-cooperative games [26].

2.1.1. Classical theory of noncooperative game theory：

1) Complete information static game: Nash equilibrium. A Nash equilibrium refers to the optimal combination of strategies for all participants, which is defined as a strategic formulation of game $G=\{S_l,\cdots,S_n;u_l,\cdots,u_n\}$ with n participant, and the combination of strategies $s^*=\left(s_1^*,\cdots,s_i^*,\cdots,s_n^*\right)$ is a Nash equilibrium if, for each i, s_i^* is the optimal strategy for the i th participant given that the other participants choose $s_{-i}^*=\left(s_I^*,\cdots,s_{-i}^*,s_{+i}^*,\cdots,s_n^*\right)$ [27]. Namely:

$$u_i\left(s_i^*,s_{-i}^*\right)\geq u_i\left(s_i,s_{-i}^*\right),\forall s_i\in S_i,\forall i \tag{1}$$

Or, in another formulation, s_i^* is a solution to the following maximization problem:

$$s_i^*\in\arg\max_{s_i\in S_i} u_i\left(s_1^*,\cdots,s_{i-1}^*,s_i,s_{i-1}^*,\cdots,s_n^*\right),i=1,2,\cdots,n \tag{2}$$

where: G denotes a game and G has n participants, and the strategy space of each participant's full set of optional strategies is denoted by $S_l,\cdots,S_n$, respectively. $s_i\in S_i$ denotes the strategy chosen by the i th participant. u_i is the payment (utility level) of the i th participant, and $u_i\left(s_1^*,\cdots,s_i^*,\cdots,s_n^*\right)$ is the optimal payment combination of the n th participant.

2) Complete information dynamic games: subgame refinement Nash equilibrium. A subgame refined Nash equilibrium is defined as a subgame refined Nash equilibrium if the combination of strategies of the extended formulation game $s^*=\left(s_1^*,\cdots,s_i^*,\cdots,s_n^*\right)$ is a Nash equilibrium of the original game and it gives a Nash equilibrium on each subgame. It can be seen that the strategies that make up the refined Nash equilibrium must be optimal in each subgame.

3) Incomplete information static game: Bayesian Nash equilibrium. The temporal order of the static Bayesian game is: natural selection of type vector $\theta=(\theta_1,\cdots,\theta_n)$, where $\theta\in\Theta_i$, participant i observes θ_i, but participant $j(\neq i)$ knows only $p_j\left(\theta_{-j}/\theta_j\right)$ and does not observe θ_i. n participants simultaneously choose action $a=(a_1,\cdots,a_n)$, where $a_i\in A_i(\theta_i)$. Participant i gets $u_i(a_i,\cdots,a_n;\theta_i)$.

Bayesian Nash equilibrium: a purely strategic Bayesian Nash equilibrium of a n-player incomplete information static game $G=\{A_1,\cdots,A_n;\theta_1,\cdots,\theta_n;P;u_1,\cdots,u_n\}$ is a type-dependent strategy combination $\{a_i^*(\theta_i)\}_{i=1}^n$ in which each participant i maximizes his or her expected utility function v_i given his or her own type θ_i and the other participant's type-dependent strategies $a_{-i}^*(\theta_i)$. In other words, a strategy combination $a^*=\left(a_1^*(\theta_1),\cdots,a_n^*(\theta_n)\right)$ is a Bayesian Nash equilibrium if, for all i, $a_i\in A_i(\theta_i)$, there are:

$$a_i^*(\theta_i)\in\arg\max_{a_i}\sum p_i(\theta_{-i}/\theta_i)u_i\left(a_i,a_{-i}^*(\theta_{-i});\theta_i,\theta_{-i}\right) \tag{3}$$

A Bayesian Nash equilibrium is one in which, given the probability distribution of one's own type and the types of others, the expected utility of each participant is maximized, i.e., no one is motivated to choose another strategy.

4) Incomplete information dynamic games: refined Bayesian Nash equilibrium. The refined Bayesian equilibrium of the incomplete information dynamic game is a combination of the refined Nash equilibrium of the complete information dynamic game and the Bayesian equilibrium of the incomplete information static game.

Refined Bayesian Equilibrium Definition: let S_i be the strategy space of i, $s_i \in S_i$ be a particular type θ_i dependent strategy, $a_{-i}^h = \left(a_1^h, \cdots, a_{i-1}^h, a_{i+1}^h, \cdots, a_n^h\right)$ be the combination of actions of the other $n-1$ participants observed by the participant i on the h th information set, $\tilde{p}_i\left(\theta_{-i} / a_{-i}^h\right)$ be the posterior probability that the participant i believes that the other $n-1$ participants belong to the type θ_{-i} in the event of an observation of a_{-i}^h, and $u_i\left(s_i, s_{-i}, \theta_i\right)$ be the utility function of i. A refined Bayesian equilibrium is a combination of strategies $s^*(\theta) = \left(s_1^*\left(\theta_1\right), \cdots, s_n^*\left(\theta_n\right)\right)$ and a combination of posterior probabilities $\tilde{p} = \left(\tilde{p}_1, \cdots, \tilde{p}_n\right)$ that satisfies: for all participants i and at every information set h there is:

$$s_i^*\left(s_{-i}, \theta_i\right) \in \arg\max_{s_i} \sum_{\theta_{-i}} \tilde{p}_i\left(\theta_{-i} / a_{-i}^h\right) u_i\left(s_i, s_{-i}, \theta_i\right) \tag{4}$$

where $\tilde{p}_i\left(\theta_{-i} / a_{-i}^h\right)$ is obtained from the prior probability $p_i\left(\theta_{-i} / \theta_i\right)$, the observed a_{-i}^h and the optimal strategy s_{-i}^* using Bayes' rule.

2.1.2. Expressions of the game. In game theory, a game can be formulated in two ways: a strategic formulation and an extended formulation. Among them, the strategic formulation is more suitable for static games, while the extended formulation is more suitable for dynamic games.
1) Strategic formulation. The strategic formulation, also known as the standard formulation, means that all participants choose their strategies simultaneously, and the strategies chosen by all participants together determine the payoffs for each participant. The strategic formulation game is represented by $G = \left\{S_1, \ldots, S_n; u_1, \ldots, u_n\right\}$, which consists of three elements: the set of participants $i \in \Gamma, \Gamma = \left(1, 2, \ldots, n\right)$, the strategy space of each participant S_i, and the payment function of each participant u_i.
2) Extended formulation. The extended formulation of the game mainly "expands" the strategy space of the participants, i.e., gives a dynamic description of each strategy. The extended formulation consists of six elements: the set of participants, action order, action space, information set, payoff function and probability distribution of the natural N choices. The extended formulation of the n-player finite-strategy game can be represented by a game tree.

2.2. *Basic Assumptions of Teacher-Student Gaming Behavior in the English Teaching Process*

2.2.1. Participants. A participant refers to a decision-making subject in a game whose aim is to maximize the level of his or her payments (expected utility) by choosing actions (or strategies). In the teaching and learning process, the interaction between the participants is the core of the teaching and learning process, the participants are divided into educators and educated, then there are three permutations: the game between the educators, which revolves around power and interests. The game between educators is the key to effective learning. The game between the educator and the educated is the key to effective teaching and learning. In this paper, the two sides of the game mainly refer to English teachers and students.

2.2.2. Basic assumptions. The following premise assumptions are needed to analyze teacher-student behavior in the English teaching process using game theory:
1) Rational individual assumption. In order to study the behavioral game between teachers and students in this paper, it must first be assumed that English teachers and students in the teaching process are rational individuals, and what is required is the process rationality of both teachers and students, i.e., it is required that both sides of the game have the ability to obtain information, collect information, learn and summarize experience and correctly adjust their behavior. Under this

rationality assumption, teachers and students pursue the goal of gradual improvement.

2) Interactive behavior assumption. In this paper, for the purpose of modeling and game analysis, all teacher-student interaction behaviors are quantified as the degree of time and effort teachers devote to teaching and other teaching activities, and the degree of students' direct efforts towards learning, and measured by the proportion of time teachers and students devote to teaching and non-teaching activities.

3) Model Assumptions. All the teacher-student interaction behaviors addressed in this paper occur in the normative English teaching process. First of all, it is assumed that the English teaching process consists of typical students and typical English teachers, where "typical" refers to teacher-student subjects who are able to choose their behaviors rationally. Secondly, the game model of teacher-student interaction in this paper will consist of three components:

(1) The student achievement function, which is a quantifiable measure of the pedagogical benefits obtained by teachers and students, and assumes that the achievement is determined by the ratio of the time spent by teachers and students on educational and learning activities.

(2) The utility function for the "typical English teacher", which is also assumed to be determined by the proportion of time spent on achievement (gains from teaching) and gains from other activities performed by the English teacher, i.e., non-teaching activities such as leisure and recreation.

(3) The utility function for the "typical student", which is assumed to be determined by the proportion of time spent on grades (learning gains) and the student's gains from other non-learning activities, i.e., leisure and recreation.

2.3. Game Model of Teacher-Student Behavior in the English Teaching Process

2.3.1. Game relations between teachers and students. In this paper, the behavioral strategy space of English teachers (T) and students (S) is designated as (Conscientiousness, Non-Conscientiousness), abbreviated as (D, N), and the research hypotheses are as follows:

1) If both the teacher and the student are not serious, they gain V_T and V_S, where $V_T > 0$ and $V_S > 0$, respectively.

2) If only the teacher is serious and the student is not serious, the teacher's gain is $(1+\alpha_0)V_T - C_T$, where $\alpha_0 (\alpha_0 > 0)$ is the coefficient of increase in the teacher's gain when he is serious, C_T is the cost of the teacher's effort in choosing to be serious, and $C_T > 0$. Only the teacher is serious and the student is not serious but he shares in the gains from the teacher's seriousness, which constitutes the student's "free-riding" behavior. This constitutes "free-rider" behavior on the part of the student, whose gains are π_s and $\pi_s > V_s$.

3) When only the student is serious about learning and the teacher is not serious about instruction, the student's gain is $(1+\beta_0)V_S - C_S$, where $\beta_0 (\beta_0 > 0)$ is the coefficient of increase in the student's gain when he is serious, C_S is the cost of the student's effort in choosing to be serious about learning, and $C_S > 0$. When only the students are conscientious and the teacher does not share in the benefits of the students' conscientiousness, this constitutes "free-riding" behavior on the part of the teacher, whose benefits are π_T, $\pi_T > V_T$.

4) when the teacher and the students choose to be serious, their gains are $(1+\alpha_1)V_T - C_T$ and $(1+\beta_1)V_S - C_S$, where α_1 is the teacher and students are serious to the teacher's gains increased coefficient, and $\alpha_1 > \alpha_0 > 0$. β_1 is the teacher and students are serious to the student's gains increased coefficient, and $\beta_1 > \beta_0 > 0$.

Based on the assumptions above, the benefit matrix of the teacher-student game can be established as shown in Table 1. Not serious teaching

Table 1. Benefit matrix of English teachers and students

English teachers	Students	
	Study carefully (D)	Not study seriously (N)
Serious guidance (D)	$(1+a_1)V_T-C_T,(1+\beta_1)V_s-C_s$	$(1+a_0)V_T-C_T,\pi_S$
Not providing serious guidance (N)	$\pi_r,(1+\beta_0)V_s-C_s$	V_T,V_s

2.3.2. Analysis of the evolutionary trend of the teacher-student game：

1) The equilibrium point of the evolutionary process. Assume that the probability of an English teacher adopting a serious instruction strategy is $x(0\le x\le 1)$, and the probability of adopting a non-serious instruction strategy is $1-x$. Meanwhile, assume that the probability of a student adopting a serious learning strategy is $y(0\le y\le 1)$, and the probability of adopting a non-serious learning strategy is $1-y$.

For the teacher, the expected benefits of choosing serious instruction and non-serious instruction strategies are respectively:

$$U_{1d}=y\left[(1+\alpha_1)V_T-C_T\right]+(1-y)\left[(1+\alpha_0)V_T-C_T\right] \tag{5}$$

$$U_{1n}=y\pi_T+(1-y)V_T \tag{6}$$

The average expected return is:

$$U_1=xU_{1d}+(1-x)U_{1n} \tag{7}$$

For students, the expected gain U_{2d}, U_{2n} and average gain U_2 for choosing serious and non-serious learning strategies are respectively:

$$U_{2d}=x\left[(1+\beta_1)V_S-C_S\right]+(1-x)\left[(1+\beta_0)V_S-C_S\right] \tag{8}$$

$$U_{2n}=x\pi_S+(1-x)V_S \tag{9}$$

$$U_2=yU_{2d}+(1-y)U_{2n} \tag{10}$$

According to the replication dynamics formula of the evolutionary game, the replication dynamics equation for Teacher A can be obtained as:

$$\begin{aligned}X=\frac{dx}{dt}&=x\left(U_{1d}-U_1\right)\\&=x(1-x)\left\{\alpha_0V_T-C_T-\left[\pi_T-(\alpha_1-\alpha_0+1)V_T\right]y\right\}\end{aligned} \tag{11}$$

Similarly, the replication dynamic equation for Student B is:

$$\begin{aligned}Y=\frac{dy}{dt}&=y\left(U_{2d}-U_2\right)\\&=y(1-y)\left\{\beta_0V_s-C_s-\left[\pi_s-(\beta_1-\beta_0+1)V_s\right]x\right\}\end{aligned} \tag{12}$$

A two-dimensional dynamical system (I) is obtained from the differential equations Eqs. (11) and (12):

$$\begin{aligned}\frac{dx}{dt}&=x\left(U_{1d}-U_1\right)\\&=x(1-x)\left\{\alpha_0V_T-C_T-\left[\pi_T-(\alpha_1-\alpha_0+1)V_T\right]y\right\}\\\frac{dy}{dt}&=y\left(U_{2d}-U_2\right)\\&=y(1-y)\left\{\beta_0V_S-C_S-\left[\pi_s-(\beta_1-\beta_0+1)V_S\right]x\right\}\end{aligned} \tag{13}$$

For this system (I), let $\frac{dx}{dt}=0$, $\frac{dy}{dt}=0$, respectively, it is clear that $(0,0)$, $(0,1)$, $(1,0)$, $(1,1)$ are the equilibrium points of the system. When $\frac{C_T}{V_T}<\alpha_0<\alpha_1<\frac{\pi_T+C_T-V_T}{V_T}$, $\frac{C_S}{V_S}<\beta_0<\beta_1<\frac{\pi_s+C_s-V_s}{V_s}$, $0<\frac{\beta_0 V_s-C_s}{\pi_s-(\beta_1-\beta_0+1)V_s}<1$, $0<\frac{\alpha_0 V_T-C_T}{\pi_T-(\alpha_1-\alpha_0+1)V_T}<1$, so (x_D, y_D) is also an equilibrium point of the system (I), where:

$$x_D=\frac{\beta_0 V_S-C_S}{\pi_S-(\beta_1-\beta_0+1)V_S} \tag{14}$$

$$y_D=\frac{\alpha_0 V_T-C_T}{\pi_T-(\alpha_1-\alpha_0+1)V_T} \tag{15}$$

2) Stability analysis of the equilibrium point. According to the calculation of the system of differential equations constitutes the group dynamics of the dynamic system, the stability analysis of its equilibrium point can be obtained by analyzing the local stability of the Jacobian matrix of this system. The partial derivatives with respect to x and y are obtained sequentially for the system of differential equations Eq. (13), and the Jacobian matrix can be derived as:

$$J=\begin{bmatrix}\frac{\partial X}{\partial x} & \frac{\partial X}{\partial y}\\ \frac{\partial Y}{\partial x} & \frac{\partial Y}{\partial y}\end{bmatrix}=\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix} \tag{16}$$

Among them:

$$a_{11}=(1-2x)\left\{\alpha_0 V_T-C_T-\left[\pi_T-(\alpha_1-\alpha_0+1)V_T\right]y\right\} \tag{17}$$

$$a_{12}=-x(1-x)\left[\pi_T-(\alpha_1-\alpha_0+1)V_T\right] \tag{18}$$

$$a_{21}=-y(1-y)\left[\pi_S-(\beta_1-\beta_0+1)V_S\right] \tag{19}$$

$$a_{22}=(1-2y)\left\{\beta_0 V_S-C_S-\left[\pi_S-(\beta_1-\beta_0+1)V_S\right]x\right\} \tag{20}$$

If the following conditions are satisfied:

(1) $a_{11}+a_{22}<0$ (the trace condition, whose value is denoted as $tr\,J$).

(2) $\begin{vmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{vmatrix}=a_{11}a_{22}-a_{12}a_{21}>0$ (Jacobian determinant condition, whose value is denoted as $\det J$), then the equilibrium point of the replicated dynamic equations is (asymptotically) locally stable, and that equilibrium point is the evolutionarily stable strategy ESS.

Let $S_1=\frac{\pi_T+C_T-V_T}{V_T}$, $S_2=\frac{\pi_S+C_S-V_S}{V_S}$.

Based on the above judgment method, the following conclusions can be drawn:

(1) when $0<\alpha_0<\frac{C_T}{V_T}$, $\alpha_0<\alpha_1<S_1$ and $0<\beta_0<\frac{C_S}{V_S}$, $\beta_0<\beta_1<S_2$, that is to say, regardless of one party seriously or jointly serious, serious to them to bring the incremental gains are smaller, but they pay a higher cost of effort, at this time $(0,0)$ is the evolution of the system is stable, $(0,1)$, $(1,0)$ is the saddle point, $(1,1)$ is the evolution of the game of instability, at this time, the teacher and student are not serious is the evolution of stable strategy.

(2) when $0<\alpha_0<\dfrac{C_T}{V_T}$, $\alpha_0<\alpha_1<S_1$ and $\dfrac{C_S}{V_S}<\beta_0<\beta_1<S_2$, when the student seriously study the gain is greater than the cost of the effort he paid for this, but less than its hitchhiking gain, and the teacher seriously guide can bring its gain is less than the cost of the effort he paid for this, the teacher will not seriously guide, this time the student has no hitchhiking can only be serious about learning, $(0,1)$ is the system is stable point of evolution, i.e., the teacher is not serious about guiding the student to study seriously is the evolutionarily stable strategy.

(3) when $\dfrac{C_T}{V_T}<\alpha_0<\alpha_1<S_1$ and $0<\beta_0<\dfrac{C_S}{V_S}$, $\beta_0<\beta_1<S_2$, the teacher seriously guide the gain is greater than the cost of his efforts to pay for this, but less than its hitchhiking benefits, and students seriously study can bring its benefits less than the cost of its efforts, students will not seriously study, this time the teacher has no hitchhiker can ride, $(1,0)$ is the system's evolutionary stability, that is, the teacher is seriously guided by the student does not seriously study is an evolutionary stable strategy.

(4) when $\dfrac{C_T}{V_T}<\alpha_0<\alpha_1<S_1$ and $\dfrac{C_S}{V_S}<\beta_0<\beta_1<S_2$, the teacher is serious about guidance, students are serious about learning can bring more benefits than the cost they pay for this, but less than their “hitchhiking” and gain. At this time $(0,1)$, $(1,0)$ is the stability of the system, in this case there is always a teacher and students can hitchhike, i.e., one party is serious and the other party is not serious is the evolution of the system's stabilization strategy, the specific equilibrium to reach which is related to the initial state of the system.

(5) When $S_1<\alpha_0<\alpha_1$ and $S_2<\beta_0<\beta_1$, both sides of the seriousness of the input gain is greater than when they are not serious and the other party is serious “hitchhiking” gain. At this point, $(1,1)$ is the evolutionary stability point of the system, i.e., both teachers and students are serious as the evolutionary stability strategy of the system.

3) Evolutionary game of student education with punishment mechanism. From the above analysis, it can be seen that under different conditions, the game will evolve to different equilibrium points, and the resulting evolutionary stabilization strategies are also different, in some cases, teachers and students always have a party to hope for free-riding behavior, resulting in the outcome of the game is that both parties do not take seriously or only a single party to take seriously, especially in the case of free-riding behavior to obtain a large gain, so it is very necessary to adopt a certain mechanism to inhibit the free-riding behavior. Therefore, it is very necessary to adopt certain mechanisms to inhibit the free-riding behavior. In order to make the game can evolve to the ideal solution (serious, conscientious) strategy expected by the school, this paper introduces a punishment mechanism: in the process of the game between teachers and students, when one party is conscientious and the other party is not, the party that is not conscientious will be punished, assuming that the teacher's loss of punishment is K_T, and the student's loss of punishment is K_S. Under this circumstance, the benefit matrix of the game between the English teachers and the students is shown in Table 2.

Table 2. Benefit matrix for English teachers and students

English teachers	Students	
	Study carefully (D)	Not study seriously (N)
Serious guidance (D)	$(1+a_1)V_T-C_T,(1+\beta_1)V_s-C_s$	$(1+a_0)V_T-C_T,\pi_S-K_S$
Not providing serious guidance (N)	$\pi_T-K_T,(1+\beta_0)V_s-C_s$	V_T,V_s

From Table 2, the trace of Jacobian matrix J' when the equilibrium point is $(1,1)$ is:

$$\begin{aligned} trJ' = &-\left\{\alpha_0-V_T-\left[\pi_T-K_T-\left(1+\alpha_1-\alpha_0\right)V_T\right]\right\} \\ &-\left\{\beta_0 V_S-C_S-\left[\pi_S-K_S-\left(1+\beta_1-\beta_0\right)V_S\right]\right\} \end{aligned} \tag{21}$$

The rows and columns are:

$$\begin{aligned}\det J' = &\left\{\alpha_0 - V_T - \left[\pi_T - K_T - (1+\alpha_1-\alpha_0)V_T\right]\right\}\\ &\times\left\{\beta_0 V_S - C_S - \left[\pi_S - K_S - (1+\beta_1-\beta_0)V_s\right]\right\}\end{aligned} \tag{22}$$

Then the sufficient condition for $(1,1)$ to be the only evolutionarily stable strategy ESS is:

$$trJ'\langle 0, \det J'\rangle 0 \tag{23}$$

It follows that the sufficient condition for (seriously, seriously) to be the only ESS is:

$$K_T > \pi_T - (1+\alpha_1)V_T + C_T \tag{24}$$

$$K_S > \pi_S - (1+\beta_1)V_S + C_S \tag{25}$$

Among them, π_T and π_S are the "free-riding" benefits of teachers and students, $(1+\alpha_1)V_T$ and $(1+\beta_1)V_S$ represent the respective benefits brought by teachers and students, and C_T and C_S are the costs of teachers and students' serious efforts, so in order to make the stable strategy of the game choose a serious strategy for both teachers and students, it is necessary to form an unbalanced relationship between the punishment loss and the free-rider income when the teacher and student are not serious, the benefits brought by seriousness, and the cost of serious effort to form a certain imbalance relationship that satisfies equations (24) and (25).

2.4. Optimization Strategy of English Flipped Classroom Teaching Based on Game Evolution

From the results of the game model of teacher-student interaction behavior in the process of English teaching and the analysis of its evolution trend, it can be seen that: only when teachers teach seriously and students study seriously, English teaching can achieve better implementation results. Therefore, based on the perspective of teacher-student cooperation, this paper designs a balanced optimization strategy for English flipped classroom teaching. Since the flipped classroom game is characterized by both sequential and simultaneous actions, the following measures can be taken from the perspective of English teachers to contribute to the establishment of the implementation conditions.

2.4.1. Under the sequential approach to operations. Based on the characteristics of the teacher-student interaction mode under the sequential action approach, the key is how to increase the difference in the benefits of the English teachers who adopt the flipped teaching approach under the different learning styles of the students, so it is necessary to expand the outreach of the learning situation analysis, and adjust the flipping depth and dimension.

1) Expanding the Extension of Learning Situation Analysis. Traditional learning situation analysis focuses on teachers' analysis of students' existing knowledge and ability before class. However, the teaching activities of flipped teaching are based on students' active participation and independent development before, during and after class, so the analysis of learning needs to be data-driven and extended to inside and after class, based on the technical support of interactive systems, intelligent diagnostic systems for student learning, statistical systems and other aspects, to analyze the details of the interactions between the students and the online text, the courseware, the learning environment and the English teacher, in order to help English teachers focus on students' differences. English teachers to pay attention to students' differences.

2) Adjusting the depth and dimension of flipping. Flipped teaching should not be a “one-size-fits-all” behavior, and needs to be based on the analysis of the learning situation, providing individualized help and adjusting the depth and dimension of flipping according to the students' objective ability and subjective willingness to learn independently. The stronger the students' learning autonomy, the greater the depth of the flip, placing as much teaching focus and time as possible on the internalization of knowledge. The dimension of the flip can also be more, including the construction of knowledge

maps, students' ability development, etc. On the contrary, the mode of "half-flip" can be adopted in order to reduce the depth of the flip and the dimension of the flip.

2.4.2. Under the simultaneous mode of operation. Based on the characteristics of the teacher-student interaction mode under the simultaneous action mode, the key is how to increase the difference in benefits under the different teaching modes adopted by teachers for independent learning students, so it is necessary to reasonably determine the flipped content and process and improve the learning evaluation mechanism.

1) Determine the content and process of flipping according to the cognitive goal classification framework, different types of knowledge and skills in the cognitive process dimensions of memorization, comprehension, application, analysis, evaluation and creativity, should be different for students' cognition and English teachers' teaching design. Therefore, for independent learning students, English teachers should determine the process reengineering under flipped teaching according to different knowledge categories, and embody the lower dimensions of learning such as memorization and comprehension mainly before class, and the higher dimensions of learning such as application, analysis, evaluation and creation mainly in the middle of the class and after the class, in order to improve the efficiency of students' independent learning.

2) Improve the learning evaluation mechanism. Unlike conferred professors who mainly focus on students' learning results, the learning evaluation mechanism under flipped professors needs to be based on students' performance in the whole process of learning. Through online data such as the number of student logins, the number of views of learning materials, the active level of discussion questions, the percentage of homework completion, etc., we can instantly quantify the students' pre-course preparation and in-class knowledge internalization, track, feedback and even predict the students' learning behaviors, and reflect them in the formative evaluation results to form the students' additional learning benefits and incentive benefits, so as to ensure that the students' learning engagement and participation will be improved.

2.4.3. Under the sequential and simultaneous operations approach. Flipped teaching not only brings challenges to classroom teaching, but more importantly, it puts higher demands on English teachers' teaching quality in terms of determining teaching objectives, designing teaching content, and handling teacher-student relationship. In the short term, this will undoubtedly greatly increase the additional teaching cost of a single classroom flip for a single English teacher. Therefore a key factor in enhancing English teachers' motivation is how to reduce this extra cost.

1) Realizing economies of scale within subject systems. In the case of multiple lectures of related courses or the same course, the process of knowledge transfer such as the recording of pre-course videos and the design of test assignments has a certain degree of repeatability, which can be scientifically planned on the basis of considering the internal coordination of the subject system and the characteristics of the specific course, so as to realize the economies of scale in terms of the cost of instructional design and the cost of incentive implementation.

2) Learning effect. In the long run, from the viewpoint of the subject curriculum system and from the viewpoint of the teacher team, there is a possibility that flipped teaching can be endogenized into the "Leaming by Doing" model with high technological spillover, and the efficiency of subsequent assignments will be improved to a certain extent. Therefore, based on the English subject, the teaching and research department will set up a professional English flipped classroom teaching team, giving full play to the team's learning advantages and technology spillover effect, constructing a steeper team learning and imitation curve, and stimulating the "Leaming by Doing" energy of English teachers.

3. Numerical simulation analysis of teacher-student game models

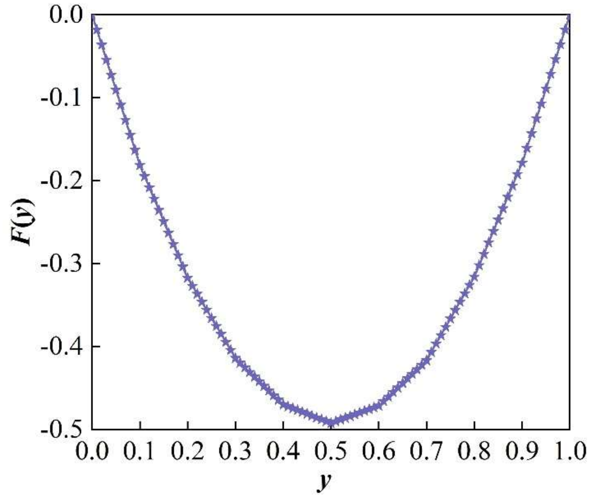
3.1. *Numerical Simulation for Teaching English in Flipped Classrooms*

In this paper, we use *MATLAB* software to simulate the proposed dynamic evolution game model of teacher-student behavior by setting the values of different parameters V_S, C_S, β_0, μ and ψ. Where ψ is the cost-saving parameter, which indicates ψC_T the cost saved when English teachers adopt the flipped classroom teaching model. μ is the gain-loss parameter, and the corresponding $\mu V_S = K_S$ represents the penalty loss of the gain when students do not study seriously. Other parameters are the same as the original model. The parameters of the model are set as follows: $V_S = V_T = 12$, $C_S = C_T = 5$, simulate the English teachers who adopt the flipped classroom, and the traditional teaching strategies, and the students who adopt serious learning or not serious learning, respectively.

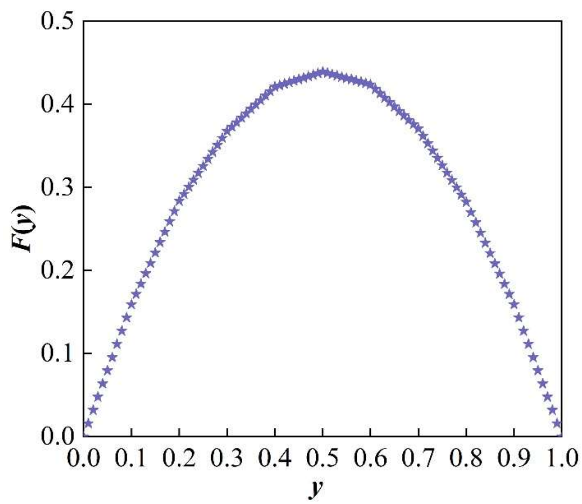
3.1.1. Changes in students' replication dynamic equations. The variation curves of the replication dynamic equations of students are shown in Fig. 1. Where the parameters of Fig. 1(a) are set as $\beta_0 = 0.4$, $q = 0.5$, $\mu = 0.7$ and satisfy inequalities $q > \frac{(C_S - \beta_0 I_S)}{(\mu V_S + C_S)}$, q for the proportion of teachers who adopt the strategy of flipped classroom versus traditional teaching model. The parameters of Fig. 1(b) are set as $\beta_0 = 0.2$, $q = 0.5$, $\mu = 0.2$ and satisfy inequality $q < \frac{(C_S - \beta_0 I_S)}{(\mu V_S + C_S)}$.

From Fig. 1(a), it can be seen that $F(y)$ is a parabola with an upward opening when condition $q > \frac{(C_S - \beta_0 I_S)}{(\mu V_S + C_S)}$ holds and when $0 < y < 1$, $F(y) < 0$, and $y = 0$, $y = 1$ are the two corresponding steady states of y. $y = 1$ is the evolutionary stabilization strategy, and all the students chose the serious learning strategy. Similarly, according to Fig. 1(b), when condition $q < \frac{(C_S - \beta_0 I_S)}{(\mu V_S + C_S)}$ holds, $F(y)$ is a downward opening parabola, and when $0 < y < 1$, $F(y) > 0$, and $y = 0$, $y = 1$ are the two corresponding steady states of y, and $y = 0$ is the evolutionary stable strategy. At this point, all the students choose the non-serious learning strategy.

Combining Fig. 1(a) and Fig. 1(b), it can be seen that the strategy choices of students in the teacher-student game network change over time, and different benefit-loss parameter μ, additional gain parameter β_0, and cost-saving parameter ψ have a greater impact on the replication of the strategy choice behaviors of the game parties. These parameters determine the gains of the game parties, and the theoretical results are consistent with the simulation results.



(a) Inequality $q > \dfrac{(C_S - \beta_0 I_S)}{(\mu V_S + C_S)}$ holds true

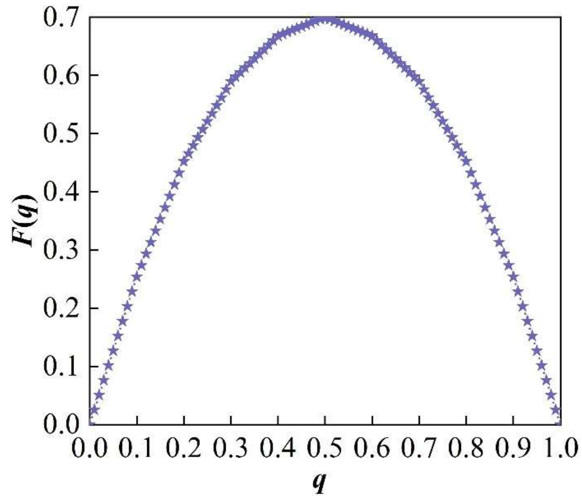


(b) Inequality $q < \dfrac{(C_S - \beta_0 I_S)}{(\mu V_S + C_S)}$ holds true

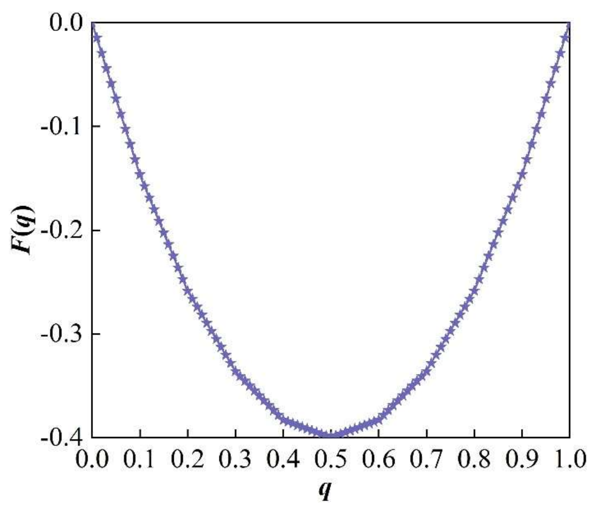
Fig. 1. Changes in the student's replication dynamic equation

3.1.2. Changes in the Replication Dynamics Equation for English Teachers. The variation curves of the replicated dynamic equations for English teachers are shown in Fig. 2. The corresponding parameters are chosen as follows: $\psi = 3.2, y = 0.5$ in Fig. 2(a), satisfying condition $y > \dfrac{1}{\psi}$. $\psi = 1.5, y = 0.5$ in Fig. 2(b), satisfying condition $y < \dfrac{1}{\psi}$.

According to Fig. 2(a), it can be seen that $F(q)$ is a parabola with downward opening when the inequality $y > \dfrac{1}{\psi}$ holds, and when $0 < q < 1$, $F(q) < 0$, while $q = 0$, $q = 1$ are two corresponding steady states, $q = 1$ is an evolutionary stable strategy, and all teachers choose the flipped classroom teaching strategy. Similarly, according to Fig. 2(b), when condition $y < \dfrac{1}{\psi}$ holds, $F(q)$ is a parabola with upward opening, and when $0 < q < 1$, $F(q) > 0$, and $q = 0$, $q = 1$ are two corresponding steady states, and $q = 0$ is an evolutionary stable strategy. At this point, all teachers choose the traditional teaching strategy.



(a) Inequality $y > \frac{1}{\psi}$ holds true



(b) Inequality $y < \frac{1}{\psi}$ holds true

Fig. 2. The teacher replicates dynamic equation changes

3.1.3. Temporal evolution of students' gaming strategies. In addition, the time evolution of students' game strategies is shown in Fig. 3. In which the parameters of students studying seriously are set as: $V_S = 12$, $C_S = 5$, $\beta_0 = 0.4$, $q = 0.5$, $\mu = 0.7$, satisfying condition $q > \frac{(C_S - \beta_0 I_S)}{(\mu V_S + C_S)}$, with the evolution of the game time, almost all students choose the strategy of studying seriously, and its proportion is stable at $y = 1$. The simulation results show that $y = 1$ is an evolutionary stable strategy. The parameters of students not studying seriously are set as: $V_S = 12$, $C_S = 5$, $\beta_0 = 0.2$, $q = 0.5$, $\mu = 0.2$, satisfying condition $q < \frac{(C_S - \beta_0 I_S)}{(\mu V_S + C_S)}$, with the evolution of game time, the proportion of students who finally choose the strategy of not studying seriously tends to be $y = 0$, and the simulation results show that $y = 0$ is an evolutionary stable strategy.

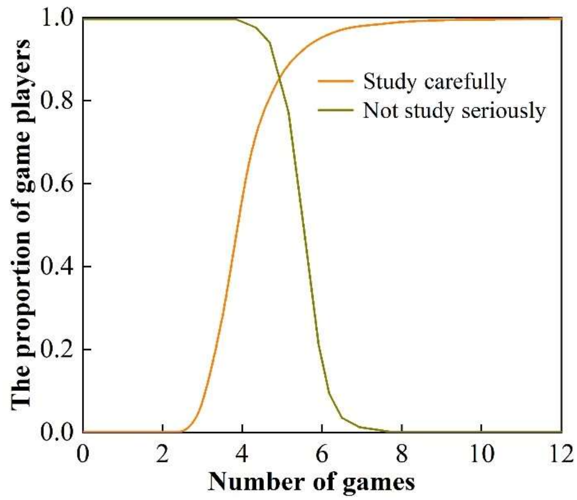


Fig. 3. Time evolution of students' game strategies

3.2. Numerical simulation of teaching influences on strategy evolution

In order to better investigate the influence of the influencing factors of English teachers and students on the strategy evolution in the process of English teaching, this paper, based on the relevant constraints and replicated dynamic equations, uses MATLAB to conduct numerical simulation analysis to simulate the evolution process under different values of influencing factors. Based on the basic assumptions of the model, the parameters are set to values $C_S = 4.8$, $C_T = 2.1$, $P_S = 9$, $P_T = 7$, $\pi_S = 4$, $\pi_T = 4$, $E_S = 6$, $K_s = 3$, where parameters P_S and P_T represent the gains of students and teachers when both teachers and students are serious, i.e., $P_S = (1 + \beta_1)V_S - C_S$ and $P_T = (1 + \alpha_1)V_T - C_T$, respectively, and parameter E_3 represents the gain of students when teachers are not serious about instruction and students are serious about learning, i.e., $E_S = (1 + \beta_0)V_S - C_S$.

3.2.1. The effect of initial strategy proportions on gaming systems. The initial values of setting (x, y) are $(0.2, 0.3)$, $(0.3, 0.4)$, $(0.5, 0.5)$, $(0.8, 0.3)$, $(0.8, 0.9)$, $(0.9, 0.1)$. The numerical simulation results for different initial strategy ratios of English teachers and students are shown in Fig. 4. In this simulation, the final evolution results of the system are (serious instruction, serious learning) and (no serious instruction, no serious learning). Regardless of the evolution results, when the difference between the initial values x and y of the two sides is larger, the slower the convergence speed is. This further suggests that the ratio of the initial strategies of both teachers and students is very important in the process of teaching English. When the ratio of both groups choosing cooperation at the beginning is low, the probability of both sides evolving to (serious instruction, serious learning) is low. When the ratio of one side choosing to cooperate is high and the ratio of the other side choosing to cooperate is low, both sides have the probability of evolving to (serious instruction, serious learning) or (no serious instruction, no serious learning), and the system is very unstable. However, when both parties choose to cooperate at a high percentage at the beginning, the system can easily maintain long-term stability.

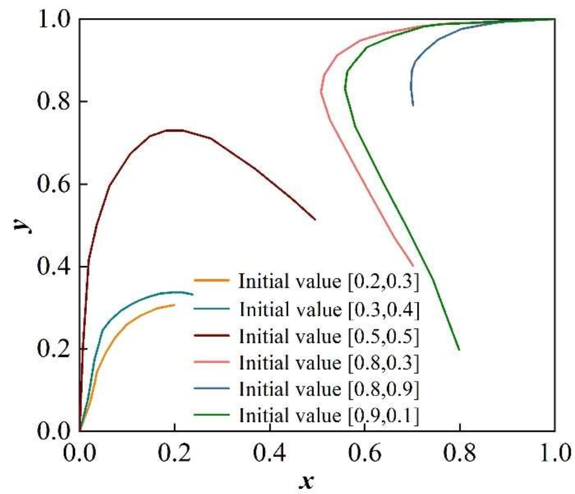
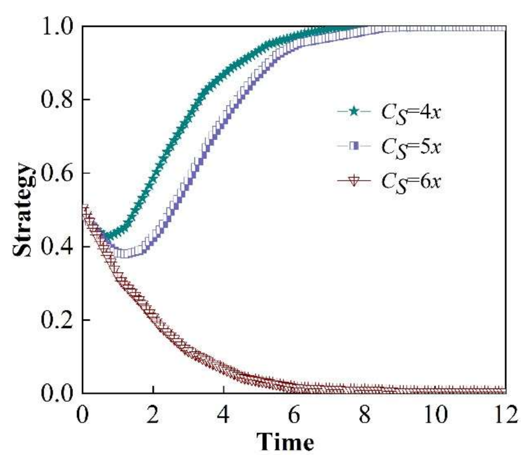


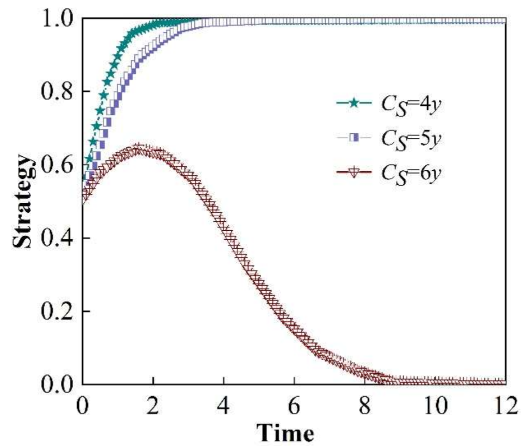
Fig. 4. Evolution results of the ratio of different initial strategies

3.2.2. Impact of CS and CT on evolutionary pathways. In order to examine the effect of the learning cost paid by the students' serious study on the teachers' and students' choice of strategies, the parameters other than the students' learning cost C_S are fixed so that the students' learning costs are 4, 5 and 6, respectively, and the data are analyzed by numerical simulation, and the effect of C_S on the evolutionary results is shown in Fig. 5. Where, Fig. (a) and Fig. (b) represent the effect of C_S on x and y, respectively.

Only at $C_S < 6$ will teachers and students finally choose the cooperative strategy (i.e., serious instruction and serious learning), and the lower the learning cost that students need to invest, the faster the two sides will evolve to a stable rate. In the process of English teaching, students will also face other courses, which makes the students in the learning cost of a certain upper limit, too high learning cost will bring pressure to the students, resulting in students' willingness to learn less seriously, which in turn prompts the teacher to choose not to seriously guide, accelerating the breakdown of cooperation between the two sides. Therefore, teachers can appropriately provide students with some help to reduce the cost of students' input in the process of English learning, and both sides are more likely to adopt cooperative strategies.



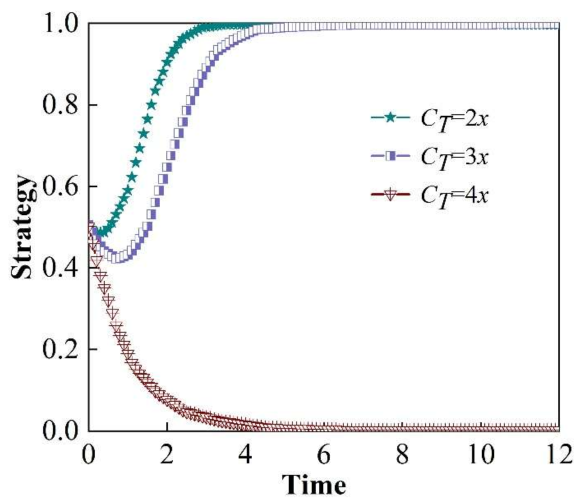
(a) The influence of CS on the outcome of x evolution



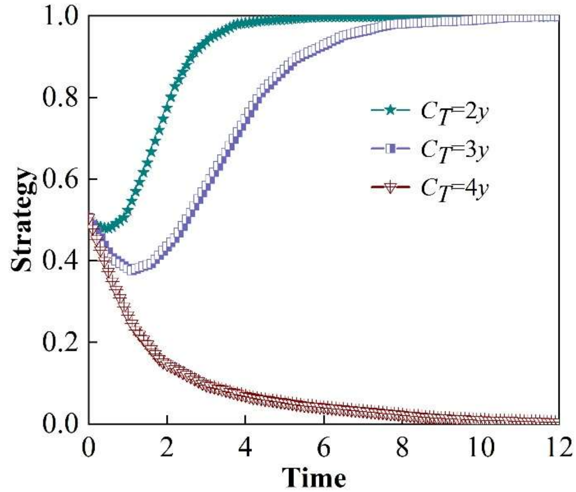
(b) The influence of CS on the outcome of y evolution

Fig. 5. The influence of CS on the evolution outcome

Similarly, fixing the parameters other than tutor's mentoring cost C_T so that the tutor's mentoring cost is 2, 3, and 4 respectively, numerical simulation is performed to analyze the data, and the effect of C_T on the evolution result is shown in Fig. 6, and Fig. (a) and Fig. (b) indicate the effect of C_T on x and y respectively. Only at $2\,C_T < 4$, the teacher and students evolve to the state of cooperative strategy (serious instruction, serious learning).



(a) The influence of CT on the outcome of x evolution

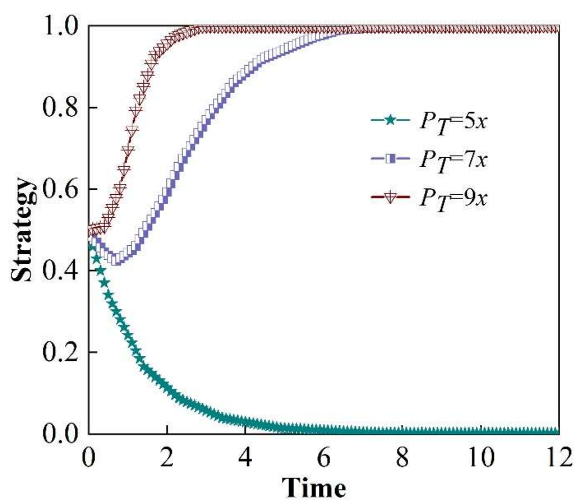


(b) The influence of CT on the outcome of y evolution

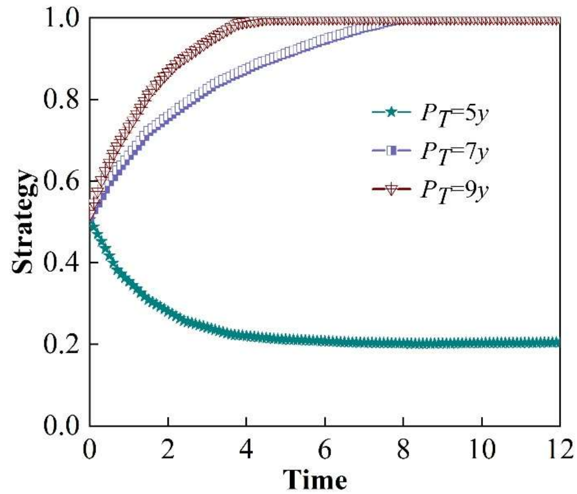
Fig. 6. The influence of CT on the evolution outcome

3.2.3. Effects of PS and PT on evolutionary pathways. In order to examine the effect of the gain P_T obtained by the English teacher's careful guidance on the teacher's and student's choice of strategy when the teacher and student's training goals are the same, parameters other than the gain P_T obtained by the teacher's careful guidance are fixed so that the tutor's gain is 5, 7, and 9, respectively, and the data are analyzed in a numerical simulation, and the effect P_T on the evolutionary results is shown in Fig. 7. Where, Fig. (a) and Fig. (b) represent the effect of P_T on the evolution results of x and y, respectively.

When the value of P_T is taken as 7 and 9, the teacher and students will evolve to the state of cooperative (serious instruction, serious learning) strategy. In context, some teachers will initially choose to instruct their students seriously, and after a period of time, they find that the benefits gained from instructing their students are lower than the benefits gained from other work, so they choose to change their strategy and choose not to instruct seriously, and the students, due to the lack of guidance from the teacher, gradually choose to change their strategy as well, leading to the breakdown of cooperation between the two parties.



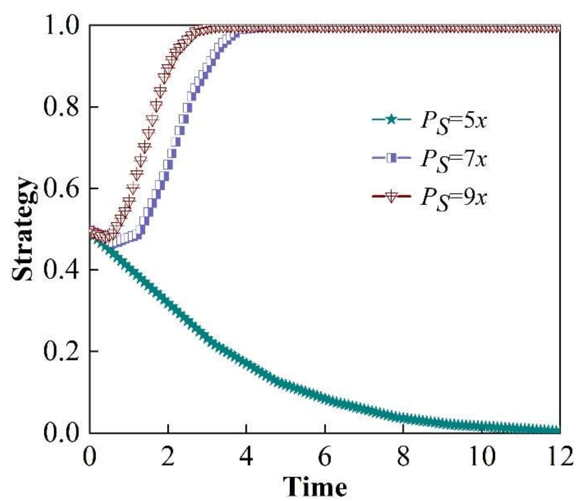
(a) The influence of PT on the outcome of x evolution



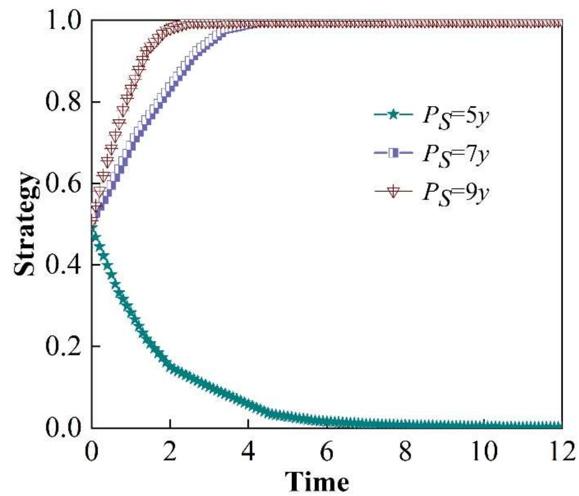
(b) The influence of PT on the outcome of y evolution

Fig. 7. The influence of PT on the evolution outcome

Similarly, fixing the parameters other than the gain P_S obtained by the student studying seriously, so that the student's gain is 5, 7, and 9, the data are analyzed by numerical simulation, and the effect of P_S on the evolution result is shown in Fig. 8, and Fig. (a) and Fig. (b) represent the effect of P_S on x and y, respectively. The tutor and student evolve to the state of cooperative strategy (serious guidance, serious learning) only when the value of P_S is 7 and 9.



(a) The influence of PS on the outcome of x evolution

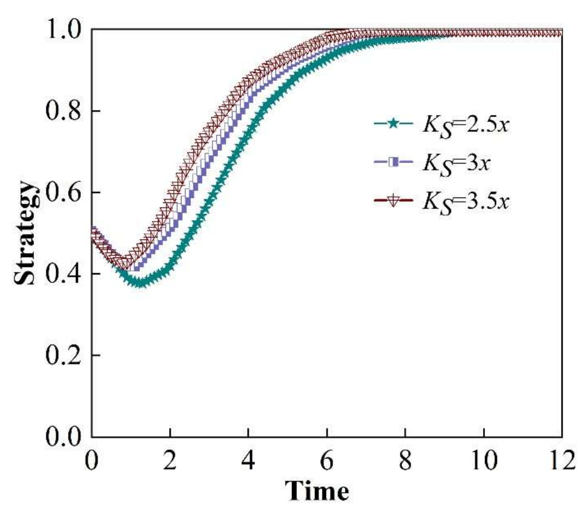


(b) The influence of PS on the outcome of y evolution

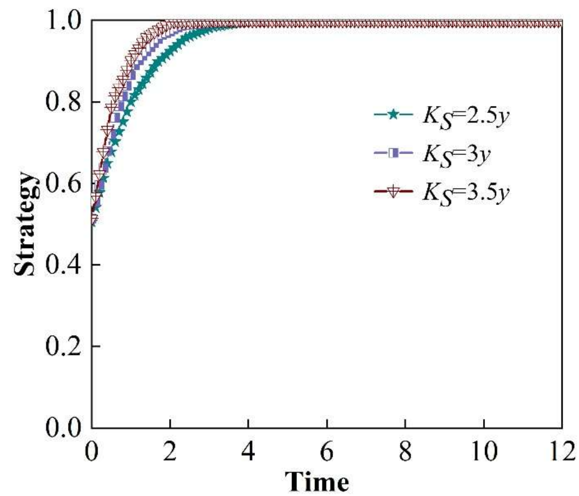
Fig. 8. The influence of PS on the evolution outcome

3.2.4. Impact of KS on evolutionary pathways. In order to examine the effect of the loss K_s suffered from not studying seriously on the choice of strategies by English teachers and students, the parameters other than the loss K_s suffered by students from punishment are fixed, so that K_s is 2.5, 3, 3.5 respectively, and numerical simulation analysis is carried out, and the effect of K_s on the evolutionary results is shown in Fig. 9, and Fig. (a) and Fig. (b) indicate the effect of K_s on x and y, respectively.

With the increase of punishment for students' negative learning, teachers and students are more inclined to choose the strategy of serious instruction and serious learning, and the evolution rate is accelerated. Therefore, the punishment for students who do not study seriously can be appropriately increased to promote cooperation between teachers and students in the English teaching process.



(a) The influence of KS on the outcome of x evolution



(b) The influence of KS on the outcome of y evolution

Fig. 9. The influence of KS on the evolution outcome

4. Conclusion

In this paper, game theory is used as an analytical tool to construct a game model of the interactive behavior of English teachers and students, and a numerical simulation of the optimization strategy of English flipped classroom teaching based on game evolution is proposed.

In the teacher-student game network, teachers' and students' strategy choices change over time, in which different benefit-loss parameter μ, additional gain parameter β_0, and cost-saving parameter ψ have a greater impact on the replication of teachers' and students' strategy choice behaviors. In order to improve classroom efficiency, English teachers should be advocated to adopt the teaching mode of flipped classroom and the strategy of serious teaching, while students adopt the strategy of serious and active learning, which in turn improves the overall interests of teachers and students, and makes teachers and students enjoy teaching and abandon the traditional passive teaching mode.

In the evolutionary game between teachers and students, the instructional cost of teachers instructing carefully C_T, the learning cost paid by students studying carefully C_S, the gain obtained by teachers instructing carefully P_T, the gain obtained by students studying carefully P_S, the personal gain obtained by teachers not instructing carefully π_T, the personal gain obtained by students not studying carefully π_S, the loss of punishment suffered by students not studying carefully K_s All of these parameters affect the development of the evolutionary process. Among them, the increase of parameters P_T, P_S, and K_s is favorable to promote the evolution of instructors and students toward the strategy of cooperation (serious instruction, serious learning), while the increase of parameters C_T and C_S is not favorable to promote the cooperation between the two parties. In addition, the larger the proportion of instructors and students initially choosing to cooperate, the greater the likelihood that both parties will evolve toward cooperation.

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