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A solution to small cases of the honeymoon Oberwolfach problem

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ABSTRACT

The honeymoon Oberwolfach problem $\mathrm{HOP}(2m_1, 2m_2, \ldots, 2m_t)$ asks the following question. Given $n = m_1 + m_2 + \ldots + m_t$ newlywed couples at a conference and t round tables of sizes $2m_1, 2m_2, \ldots, 2m_t$, is it possible to arrange the $2n$ participants at these tables for $2n-2$ meals so that each participant sits next to their spouse at every meal, and sits next to every other participant exactly once? A solution to $\mathrm{HOP}(2m_1, 2m_2, \ldots, 2m_t)$ is a decomposition of $K_{2n} + (2n-3)I$, the complete graph K_{2n} with $2n-3$ additional copies of a fixed 1-factor I, into 2-factors, each consisting of disjoint I-alternating cycles of lengths $2m_1, 2m_2, \ldots, 2m_t$. The honeymoon Oberwolfach problem was introduced in a 2019 paper by Lepine and Šajna. The authors conjectured that $\mathrm{HOP}(2m_1, 2m_2, \ldots, 2m_t)$ has a solution whenever the obvious necessary conditions are satisfied, and proved the conjecture for several large cases, including the uniform cycle length case $m_1 = \ldots = m_t$, and the small cases with $n \le 9$. In the present paper, we extend the latter result to all cases with $n \le 20$ using a computer-assisted search.

Keywords: honeymoon Oberwolfach problem, 2-factorization, Semi-uniform 1-factorization, HOP-colouring-orientation

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1. Introduction

The well-known Oberwolfach problem, denoted $\mathrm{OP}(m_1, \ldots, m_t)$, asks whether $n = m_1 + \ldots + m_t$ participants can be seated at t tables of sizes $m_1, \ldots, m_t$ for several nights in a row so that each participant gets to sit next to every other participant exactly once. Thus, we

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are asking whether K_n, the complete graph on n vertices, admits a 2-factorization such that each 2-factor is a disjoint union of t cycles of lengths $m_1, \ldots, m_t$. The problem has been solved in many special cases — see [1, 2, 3, 4, 5, 6, 7, 8, 9, 12, 13, 14] — but is in general still open.

In the 2019 paper [11], Lepine and the second author introduced a new variant of the Oberwolfach problem, called the honeymoon Oberwolfach problem. This problem, denoted HOP$(m_1, \ldots, m_t)$, can be described as follows. We have $n = \frac{1}{2}(m_1 + \ldots + m_t)$ newlywed couples attending a conference and t tables of sizes $m_1, \ldots, m_t$ (where each $m_i \geq 3$). Is it possible to arrange the participants at these t round tables on $2n-2$ consecutive nights so that each couple sit together every night, and every participant sits next to every other participant exactly once?

In graph-theoretic terms we are asking whether $K_{2n}+(2n-3)I$, the multigraph obtained from the complete graph K_{2n} by adjoining $2n-3$ additional copies of a chosen 1-factor I, admits a decomposition into 2-factors, each a vertex-disjoint union of cycles of lengths $m_1, \ldots, m_t$, so that in each of these cycles, every other edge is a copy of an edge of I. A solution to HOP$(m_1, m_2, \ldots, m_t)$ is equivalent to a *semi-uniform 1-factorization of K_{2n} of type* $(m_1, m_2, \ldots, m_t)$; that is, a 1-factorization $\{F_1, F_2, \ldots, F_{2n-1}\}$ such that for all $i \bullet 1$, the 2-factor $F_1 \cup F_i$ consists of disjoint cycles of lengths $m_1, m_2, \ldots, m_t$.

If $m_1 = \ldots = m_t = m$ and $tm = 2n$, then the symbol HOP$(m_1, \ldots, m_t)$ is abbreviated as HOP$(2n; m)$. Note that if HOP$(m_1, \ldots, m_t)$ has a solution, then the m_i are all even and at least 4; these are the obvious necessary conditions.

In [11], the authors proposed the following conjecture.

Conjecture 1.1. [11] *The obvious necessary conditions for HOP$(m_1, \ldots, m_t)$ to have a solution are also sufficient.*

They also proved the conjecture in the following cases.

Theorem 1.2. [11] *Let m and n be positive integers, $2 \leq m \leq n$. Then HOP$(2n; 2m)$ has a solution if and only if $n \equiv 0 \pmod{m}$.*

Theorem 1.3. [11] *Let $2 \leq m_1 \leq \ldots \leq m_t$ be integers, and $n = m_1 + \ldots + m_t$. Then HOP$(2m_1, \ldots, 2m_t)$ has a solution in each of the following cases.*

(a) $m_i \equiv 0 \pmod 4$ *for all i.*
(b) *n is odd and OP$(m_1, \ldots, m_t)$ has a solution.*
(c) *n is odd and $t = 2$.*
(d) *n is odd, $n < 40$, and $m_1 \geq 3$.*
(e) $n \leq 9$.

We remark that case (d) of Theorem 1.3 is extended to $n \leq 100$ by the results of [13, 12]. In this paper, we extend case (e) of Theorem 1.3, thus proving the following result.

Theorem 1.4. *Let $m_1, \ldots, m_t$ be integers with $m_i \geq 2$ for all i, and $n = m_1 + \ldots + m_t$ such that $n \leq 20$. Then HOP$(2m_1, \ldots, 2m_t)$ has a solution.*

To prove Theorem 1.4, we use the approach described in [11], combined with a computer-assisted search. In Sections 2 and 3, we present the relevant terminology and tools from [11], and in Section 4, we give the framework of the proof of Theorem 1.4, referring to the longer version of the paper [10] for the long list of computational results supporting the proof.

2. Terminology

In this paper, graphs may contain parallel edges, but not loops. As usual, K_n and λK_n denote the complete graph and the λ-fold complete graph, respectively, on n vertices. For $m \geq 2$, the symbol C_m denotes the cycle of length m, or m-cycle.

Let G be a graph, and let $H_1, \ldots, H_t$ be subgraphs of G. The collection $\{H_1, \ldots, H_t\}$ is called a *decomposition* of G if $\{E(H_1), \ldots, E(H_t)\}$ is a partition of $E(G)$.

An *r-factor* in a graph G is an r-regular spanning subgraph of G, and an *r-factorization* of G is a decomposition of G into r-factors. A 2-factor of G consisting of disjoint cycles of lengths $m_1, \ldots, m_t$, respectively, is called a *$(C_{m_1}, \ldots, C_{m_t})$-factor* of G, and a decomposition into $(C_{m_1}, \ldots, C_{m_t})$-factors is called a *$(C_{m_1}, \ldots, C_{m_t})$-factorization* of G.

For a positive integer n and $S \subseteq \mathbb{Z}_n^*$ such that $S = -S$, we define a *circulant* $\mathrm{Circ}(n; S)$ as the graph with vertex set $\{x_i : i \in \mathbb{Z}_n\}$ and edge set $\{x_i x_{i+d} : i \in \mathbb{Z}_n, d \in S\}$. An edge of the form $x_i x_{i+d}$ is said to be of *difference* d. Note that an edge of difference d is also of difference $n-d$, so we may assume that each difference is in $\{1, 2, \ldots, \lfloor \frac{n}{2} \rfloor\}$.

In this paper, the complete graph K_n will be viewed as the join of the circulant $\mathrm{Circ}(n-1; \mathbb{Z}_{n-1}^*)$ and the complete graph K_1 with vertex x_∞. Thus, $V(K_n) = \{x_i : i \in \mathbb{Z}_{n-1}\} \cup \{x_\infty\}$ and $E(K_n) = \{x_i x_j : i, j \in \mathbb{Z}_{n-1}, i \bullet j\} \cup \{x_i x_\infty : i \in \mathbb{Z}_{n-1}\}$. If this is the case, then an edge of the form $x_i x_\infty$ will be called of *difference infinity*.

Let I be a chosen 1-factor in the graph K_{2n}. An edge of K_{2n} is said to be an *I-edge* if it belongs to $E(I)$, and a *non-I-edge* otherwise. The symbol $K_{2n} + \lambda I$ denotes the graph K_{2n} with λ additional copies of each I-edge, for a total of $\lambda + 1$ copies of each I-edge. (Note that these additional copies of I-edges of K_{2n} are then also considered to be I-edges of $K_{2n} + \lambda I$.) A cycle C of $K_{2n} + \lambda I$, necessarily of even length, is said to be *I-alternating* if the I-edges and non-I-edges along C alternate. A 2-factor (or 2-factorization) of $K_{2n} + \lambda I$ is said to be *I-alternating* if each of its cycles is I-alternating.

Thus, a solution to the honeymoon Oberwolfach problem $\mathrm{HOP}(m_1, \ldots, m_t)$ is an I-alternating $(C_{m_1}, \ldots, C_{m_t})$-factorization of $K_{2n} + (2n-3)I$ for $2n = m_1 + m_2 + \ldots + m_t$.

3. The tools

As in [11], we use the symbol $4K_n^\bullet$ to denote the 4-fold complete graph with n vertices whose edges are coloured pink, blue, and black, and black edges are oriented so that each 4-set of parallel edges contains one pink edge, one blue edge, and two opposite black arcs.

Definition 3.1. [11] A 2-factorization $\mathcal{D}$ of $4K_n^\bullet$ is said to be *HOP* if each cycle of $\mathcal{D}$ satisfies the following condition:

(C) any two adjacent (that is, consecutive) edges satisfy one of the following:
- one is blue and the other pink; or
- both are black and directed in the same way;
- one is blue and the other black, directed towards the blue edge; or
- one is pink and the other black, directed away from the pink edge.

Theorem 3.2. [11] *Let $m_1, \ldots, m_t$ be integers greater than 1, and let $n = m_1 + \ldots + m_t$. Then HOP$(2m_1, \ldots, 2m_t)$ has a solution if and only if $4K_n^\bullet$ admits an HOP $(C_{m_1}, \ldots, C_{m_t})$-factorization.*

In the next proposition, the symbol $2K_n^{Circ}$ denotes the multigraph $2K_n$ whose edges are coloured pink and black so that each 2-set of parallel edges contains one pink edge and one black edge.

Proposition 3.3. [11] *Assume n is even, and let the vertex set of $2K_n^{Circ}$ be $\{x_i : i \in \mathbb{Z}_{n-1}\} \cup \{x_\infty\}$. Let ρ be the permutation $\rho = (x_\infty)(x_0\ x_1\ x_2\ \ldots\ x_{n-2})$, and let ρ_{Circ} denote the permutation on the edge set of $2K_n^{Circ}$ that is induced by ρ and that preserves the colour of the edges.*

Suppose $2K_n^{Circ}$ admits a $(C_{m_1}, \ldots, C_{m_t})$-factor F such that

(A1) *each cycle in F of length at least 3 contains an even number of pink edges, and*

(A2) *F contains exactly one edge from each of the orbits of $\langle \rho_{Circ} \rangle$.*

Then $4K_n^\bullet$ admits an HOP $(C_{m_1}, \ldots, C_{m_t})$-factorization.

Proposition 3.4. [11] *Assume n is even, and let the vertex set of $4K_n^\bullet$ be $\{x_i : i \in \mathbb{Z}_{n-1}\} \cup \{x_\infty\}$. Let ρ be the permutation $\rho = (x_\infty)(x_0\ x_1\ x_2\ \ldots\ x_{n-2})$, and let $\rho_\bullet$ denote the permutation on the edge set of $4K_n^\bullet$ that is induced by ρ and that preserves the colour (and orientation) of the edges.*

Suppose $4K_n^\bullet$ admits edge-disjoint $(C_{m_1}, \ldots, C_{m_t})$-factors F_1 and F_2 such that

(D1) *each cycle in F_1 and F_2 satisfies Condition (C) in Definition 3.1, and*

(D2) *F_1 and F_2 jointly contain exactly one edge from each of the orbits of $\langle \rho_\bullet \rangle$.*

Then $\mathcal{D} = \{\rho_\bullet^i(F_1), \rho_\bullet^i(F_2) : i \in \mathbb{Z}_{n-1}\}$ is an HOP $(C_{m_1}, \ldots, C_{m_t})$-factorization of $4K_n^\bullet$.

Proposition 3.5. [11] *Assume n is odd, and let the vertex set of $4K_n^\bullet$ be $\{x_i : i \in \mathbb{Z}_{n-1}\} \cup \{x_\infty\}$. Let ρ be the permutation $\rho = (x_\infty)(x_0\ x_1\ x_2\ \ldots\ x_{n-2})$, and let $\rho_\bullet$ denote the permutation on the edge set of $4K_n^\bullet$ that is induced by ρ and that preserves the colour (and orientation) of the edges.*

Suppose $4K_n^\bullet$ admits pairwise edge-disjoint $(C_{m_1}, \ldots, C_{m_t})$-factors F_1, F_2, and F_3 such that

(E1) *each cycle in F_1, F_2, and F_3 satisfies Condition (C) in Definition 3.1;*

(E2) *each orbit of $\langle \rho_\bullet \rangle$ has edges either in $F_1 \cup F_2$ or in F_3;*

(E3) *if $e \in E(F_1 \cup F_2)$, then $\rho_\bullet^{\frac{n-1}{2}}(e) \in E(F_1 \cup F_2)$; and*

(E4) *$F_1 \cup F_2$ contains a pink and a blue edge of difference $\frac{n-1}{2}$.*

Then $\mathcal{D} = \{\rho_\bullet^i(F_1), \rho_\bullet^i(F_2) : i = 0, 1, \ldots, \frac{n-3}{2}\} \cup \{\rho_\bullet^i(F_3) : i \in \mathbb{Z}_{n-1}\}$ *is an HOP* $(C_{m_1}, \ldots, C_{m_t})$*-factorization of* $4K_n^\bullet$.

The required 2-factors F from Proposition 3.3, F_1 and F_2 from Proposition 3.4, and F_1, F_2, and F_3 from Proposition 3.5 will are called the *starter 2-factors* (or *starters*) of the resulting 2-factorizations. Thus, we are referring to Propositions 3.3, 3.4, and 3.5 as the one-starter, two-starter, and three-starter approach, respectively.

4. Proof of Theorem 1.4

By the *type* of a $(C_{m_1}, \ldots, C_{m_t})$-factor we mean the multiset $[m_1, \ldots, m_t]$.
Proof. By Theorem 1.3(e), it suffices to consider $10 \le n \le 20$. For each value of n, we list all possible 2-factor types $[m_1, \ldots, m_t]$, and refer to the result that guarantees existence of a solution to HOP$(2m_1, \ldots, 2m_t)$. In cases where we are referring to Proposition 3.3, 3.4, or 3.5, the computational results (that is, appropriate starter 2-factors) are given in the appendices of the extended version of this paper, namely [10].

Case $n = 10$: see [10, Appendix B] for the computational results.

2-factor type $[m_1, \ldots, m_t]$	HOP$(2m_1, \ldots, 2m_t)$ has a solution by...
$[2, 2, 2, 2, 2]$	Theorem 1.2
$[4, 2, 2, 2]$	Proposition 3.3 (one starter)
$[3, 3, 2, 2]$	Proposition 3.4 (two starters)
$[6, 2, 2]$	Proposition 3.4 (two starters)
$[5, 3, 2]$	Proposition 3.3 (one starter)
$[4, 4, 2]$	Proposition 3.3 (one starter)
$[4, 3, 3]$	Proposition 3.4 (two starters)
$[8, 2]$	Proposition 3.3 (one starter)
$[7, 3]$	Proposition 3.4 (two starters)
$[6, 4]$	Proposition 3.4 (two starters)
$[5, 5]$	Theorem 1.2
$[10]$	Theorem 1.2

Case $n = 11$: see [10, Appendix C] for the computational results.

2-factor type $[m_1, \ldots, m_t]$	HOP$(2m_1, \ldots, 2m_t)$ has a solution by...
$[3, 2, 2, 2, 2]$	Proposition 3.5 (three starters)
$[5, 2, 2, 2]$	Proposition 3.5 (three starters)
$[4, 3, 2, 2]$	Proposition 3.5 (three starters)
$[3, 3, 3, 2]$	Proposition 3.5 (three starters)
$[7, 2, 2]$	Proposition 3.5 (three starters)
$[6, 3, 2]$	Proposition 3.5 (three starters)
$[5, 4, 2]$	Proposition 3.5 (three starters)
$[5, 3, 3]$	Theorem 1.2

2-factor type $[m_1, \ldots, m_t]$	HOP$(2m_1, \ldots, 2m_t)$ has a solution by...
$[4, 4, 3]$	Theorem 1.2
$[9, 2]$	Theorem 1.2
$[8, 3]$	Theorem 1.2
$[7, 4]$	Theorem 1.2
$[6, 5]$	Theorem 1.2
$[11]$	Theorem 1.2

Case $n = 12$: see [10, Appendix D] for the computational results.

2-factor type $[m_1, \ldots, m_t]$	HOP$(2m_1, \ldots, 2m_t)$ has a solution by...
$[2, 2, 2, 2, 2, 2]$	Theorem 1.2
$[4, 2, 2, 2, 2]$	Proposition 3.3 (one starter)
$[3, 3, 2, 2, 2]$	Proposition 3.4 (two starters)
$[6, 2, 2, 2]$	Proposition 3.4 (two starters)
$[5, 3, 2, 2]$	Proposition 3.3 (one starter)
$[4, 4, 2, 2]$	Proposition 3.3 (one starter)
$[4, 3, 3, 2]$	Proposition 3.4 (two starters)
$[3, 3, 3, 3]$	Theorem 1.2
$[8, 2, 2]$	Proposition 3.3 (one starter)
$[7, 3, 2]$	Proposition 3.4 (two starters)
$[6, 4, 2]$	Proposition 3.4 (two starters)
$[5, 5, 2]$	Proposition 3.4 (two starters)
$[6, 3, 3]$	Proposition 3.3 (one starter)
$[5, 4, 3]$	Proposition 3.3 (one starter)
$[4, 4, 4]$	Theorem 1.2
$[10, 2]$	Proposition 3.4 (two starters)
$[9, 3]$	Proposition 3.3 (one starter)
$[8, 4]$	Theorem 1.2
$[7, 5]$	Proposition 3.3 (one starter)
$[6, 6]$	Theorem 1.2
$[12]$	Theorem 1.2

Case $n = 13$: see [10, Appendix E] for the computational results.

2-factor type $[m_1, \ldots, m_t]$	HOP$(2m_1, \ldots, 2m_t)$ has a solution by...
$[3, 2, 2, 2, 2, 2]$	Proposition 3.5 (three starters)
$[5, 2, 2, 2, 2]$	Proposition 3.5 (three starters)
$[4, 3, 2, 2, 2]$	Proposition 3.5 (three starters)
$[3, 3, 3, 2, 2]$	Proposition 3.5 (three starters)

2-factor type $[m_1, \ldots, m_t]$	HOP$(2m_1, \ldots, 2m_t)$ has a solution by...
$[7, 2, 2, 2]$	Proposition 3.5 (three starters)
$[6, 3, 2, 2]$	Proposition 3.5 (three starters)
$[5, 4, 2, 2]$	Proposition 3.5 (three starters)
$[5, 3, 3, 2]$	Proposition 3.5 (three starters)
$[4, 4, 3, 2]$	Proposition 3.5 (three starters)
$[4, 3, 3, 3]$	Theorem 1.2
$[9, 2, 2]$	Proposition 3.5 (three starters)
$[8, 3, 2]$	Proposition 3.5 (three starters)
$[7, 4, 2]$	Proposition 3.5 (three starters)
$[6, 5, 2]$	Proposition 3.5 (three starters)
$[7, 3, 3]$	Theorem 1.2
$[6, 4, 3]$	Theorem 1.2
$[5, 5, 3]$	Theorem 1.2
$[5, 4, 4]$	Theorem 1.2
$[11, 2]$	Theorem 1.2
$[10, 3]$	Theorem 1.2
$[9, 4]$	Theorem 1.2
$[8, 5]$	Theorem 1.2
$[7, 6]$	Theorem 1.2
$[13]$	Theorem 1.2

Case $n = 14$: see [10, Appendix F] for the computational results.

2-factor type $[m_1, \ldots, m_t]$	HOP$(2m_1, \ldots, 2m_t)$ has a solution by...
$[2, 2, 2, 2, 2, 2, 2]$	Theorem 1.2
$[4, 2, 2, 2, 2, 2]$	Proposition 3.3 (one starter)
$[3, 3, 2, 2, 2, 2]$	Proposition 3.4 (two starters)
$[6, 2, 2, 2, 2]$	Proposition 3.4 (two starters)
$[5, 3, 2, 2, 2]$	Proposition 3.3 (one starter)
$[4, 4, 2, 2, 2]$	Proposition 3.3 (one starter)
$[4, 3, 3, 2, 2]$	Proposition 3.4 (two starters)
$[3, 3, 3, 3, 2]$	Proposition 3.3 (one starter)
$[8, 2, 2, 2]$	Proposition 3.3 (one starter)
$[7, 3, 2, 2]$	Proposition 3.4 (two starters)
$[6, 4, 2, 2]$	Proposition 3.4 (two starters)
$[5, 5, 2, 2]$	Proposition 3.4 (two starters)
$[6, 3, 3, 2]$	Proposition 3.3 (one starter)
$[5, 4, 3, 2]$	Proposition 3.3 (one starter)
$[4, 4, 4, 2]$	Proposition 3.3 (one starter)

2-factor type $[m_1, \ldots, m_t]$	HOP$(2m_1, \ldots, 2m_t)$ has a solution by...
$[5, 3, 3, 3]$	Proposition 3.4 (two starters)
$[4, 4, 3, 3]$	Proposition 3.4 (two starters)
$[10, 2, 2]$	Proposition 3.4 (two starters)
$[9, 3, 2]$	Proposition 3.3 (one starter)
$[8, 4, 2]$	Proposition 3.3 (one starter)
$[7, 5, 2]$	Proposition 3.3 (one starter)
$[6, 6, 2]$	Proposition 3.3 (one starter)
$[8, 3, 3]$	Proposition 3.4 (two starters)
$[7, 4, 3]$	Proposition 3.4 (two starters)
$[6, 5, 3]$	Proposition 3.4 (two starters)
$[6, 4, 4]$	Proposition 3.4 (two starters)
$[5, 5, 4]$	Proposition 3.4 (two starters)
$[12, 2]$	Proposition 3.3 (one starter)
$[11, 3]$	Proposition 3.4 (two starters)
$[10, 4]$	Proposition 3.4 (two starters)
$[9, 5]$	Proposition 3.4 (two starters)
$[8, 6]$	Proposition 3.4 (two starters)
$[7, 7]$	Theorem 1.2
$[14]$	Theorem 1.2

CASE $n = 15$: see [10, Appendix G] for the computational results.

2-factor type $[m_1, \ldots, m_t]$	HOP$(2m_1, \ldots, 2m_t)$ has a solution by...
$[3, 2, 2, 2, 2, 2, 2]$	Proposition 3.5 (three starters)
$[5, 2, 2, 2, 2, 2]$	Proposition 3.5 (three starters)
$[4, 3, 2, 2, 2, 2]$	Proposition 3.5 (three starters)
$[3, 3, 3, 2, 2, 2]$	Proposition 3.5 (three starters)
$[7, 2, 2, 2, 2]$	Proposition 3.5 (three starters)
$[6, 3, 2, 2, 2]$	Proposition 3.5 (three starters)
$[5, 4, 2, 2, 2]$	Proposition 3.5 (three starters)
$[5, 3, 3, 2, 2]$	Proposition 3.5 (three starters)
$[4, 4, 3, 2, 2]$	Proposition 3.5 (three starters)
$[4, 3, 3, 3, 2]$	Proposition 3.5 (three starters)
$[3, 3, 3, 3, 3]$	Theorem 1.2
$[9, 2, 2, 2]$	Proposition 3.5 (three starters)
$[8, 3, 2, 2]$	Proposition 3.5 (three starters)
$[7, 4, 2, 2]$	Proposition 3.5 (three starters)
$[6, 5, 2, 2]$	Proposition 3.5 (three starters)
$[7, 3, 3, 2]$	Proposition 3.5 (three starters)
$[6, 4, 3, 2]$	Proposition 3.5 (three starters)

2-factor type $[m_1,\ldots,m_t]$	HOP$(2m_1,\ldots,2m_t)$ has a solution by...
$[5,5,3,2]$	Proposition 3.5 (three starters)
$[5,4,4,2]$	Proposition 3.5 (three starters)
$[6,3,3,3]$	Theorem 1.2
$[5,4,3,3]$	Theorem 1.2
$[4,4,4,3]$	Theorem 1.2
$[11,2,2]$	Proposition 3.5 (three starters)
$[10,3,2]$	Proposition 3.5 (three starters)
$[9,4,2]$	Proposition 3.5 (three starters)
$[8,5,2]$	Proposition 3.5 (three starters)
$[7,6,2]$	Proposition 3.5 (three starters)
$[9,3,3]$	Theorem 1.2
$[8,4,3]$	Theorem 1.2
$[7,5,3]$	Theorem 1.2
$[6,6,3]$	Theorem 1.2
$[7,4,4]$	Theorem 1.2
$[6,5,4]$	Theorem 1.2
$[5,5,5]$	Theorem 1.2
$[13,2]$	Theorem 1.2
$[12,3]$	Theorem 1.2
$[11,4]$	Theorem 1.2
$[10,5]$	Theorem 1.2
$[9,6]$	Theorem 1.2
$[8,7]$	Theorem 1.2
$[15]$	Theorem 1.2

CASE $n = 16$: see [10, Appendix H] for the computational results.

2-factor type $[m_1,\ldots,m_t]$	HOP$(2m_1,\ldots,2m_t)$ has a solution by...
$[2,2,2,2,2,2,2,2]$	Theorem 1.2
$[4,2,2,2,2,2,2]$	Proposition 3.3 (one starter)
$[3,3,2,2,2,2,2]$	Proposition 3.4 (two starters)
$[6,2,2,2,2,2]$	Proposition 3.4 (two starters)
$[5,3,2,2,2,2]$	Proposition 3.3 (one starter)
$[4,4,2,2,2,2]$	Proposition 3.3 (one starter)
$[4,3,3,2,2,2]$	Proposition 3.4 (two starters)
$[3,3,3,3,2,2]$	Proposition 3.3 (one starter)
$[8,2,2,2,2]$	Proposition 3.3 (one starter)
$[7,3,2,2,2]$	Proposition 3.4 (two starters)
$[6,4,2,2,2]$	Proposition 3.4 (two starters)
$[5,5,2,2,2]$	Proposition 3.4 (two starters)
$[6,3,3,2,2]$	Proposition 3.3 (one starter)

2-factor type $[m_1, \ldots, m_t]$	HOP$(2m_1, \ldots, 2m_t)$ has a solution by...
$[5, 4, 3, 2, 2]$	Proposition 3.3 (one starter)
$[4, 4, 4, 2, 2]$	Proposition 3.3 (one starter)
$[5, 3, 3, 3, 2]$	Proposition 3.4 (two starters)
$[4, 4, 3, 3, 2]$	Proposition 3.4 (two starters)
$[4, 3, 3, 3, 3]$	Proposition 3.3 (one starter)
$[10, 2, 2, 2]$	Proposition 3.4 (two starters)
$[9, 3, 2, 2]$	Proposition 3.3 (one starter)
$[8, 4, 2, 2]$	Proposition 3.3 (one starter)
$[7, 5, 2, 2]$	Proposition 3.3 (one starter)
$[6, 6, 2, 2]$	Proposition 3.3 (one starter)
$[8, 3, 3, 2]$	Proposition 3.4 (two starters)
$[7, 4, 3, 2]$	Proposition 3.4 (two starters)
$[6, 5, 3, 2]$	Proposition 3.4 (two starters)
$[6, 4, 4, 2]$	Proposition 3.4 (two starters)
$[5, 5, 4, 2]$	Proposition 3.4 (two starters)
$[7, 3, 3, 3]$	Proposition 3.3 (one starter)
$[6, 4, 3, 3]$	Proposition 3.3 (one starter)
$[5, 5, 3, 3]$	Proposition 3.3 (one starter)
$[5, 4, 4, 3]$	Proposition 3.3 (one starter)
$[4, 4, 4, 4]$	Theorem 1.2
$[12, 2, 2]$	Proposition 3.3 (one starter)
$[11, 3, 2]$	Proposition 3.4 (two starters)
$[10, 4, 2]$	Proposition 3.4 (two starters)
$[9, 5, 2]$	Proposition 3.4 (two starters)
$[8, 6, 2]$	Proposition 3.4 (two starters)
$[7, 7, 2]$	Proposition 3.4 (two starters)
$[10, 3, 3]$	Proposition 3.3 (one starter)
$[9, 4, 3]$	Proposition 3.3 (one starter)
$[8, 5, 3]$	Proposition 3.3 (one starter)
$[7, 6, 3]$	Proposition 3.3 (one starter)
$[8, 4, 4]$	Theorem 1.2
$[7, 5, 4]$	Proposition 3.3 (one starter)
$[6, 6, 4]$	Proposition 3.3 (one starter)
$[6, 5, 5]$	Proposition 3.3 (one starter)
$[14, 2]$	Proposition 3.4 (two starters)
$[13, 3]$	Proposition 3.3 (one starter)
$[12, 4]$	Theorem 1.2
$[11, 5]$	Proposition 3.3 (one starter)
$[10, 6]$	Proposition 3.3 (one starter)
$[9, 7]$	Proposition 3.3 (one starter)
$[8, 8]$	Theorem 1.2
$[16]$	Theorem 1.2

CASE $n = 17$: see [10, Appendix I] for the computational results.

2-factor type $[m_1, \dots, m_t]$	HOP$(2m_1, \dots, 2m_t)$ has a solution by...
$[3, 2, 2, 2, 2, 2, 2, 2]$	Proposition 3.5 (three starters)
$[5, 2, 2, 2, 2, 2, 2]$	Proposition 3.5 (three starters)
$[4, 3, 2, 2, 2, 2, 2]$	Proposition 3.5 (three starters)
$[3, 3, 3, 2, 2, 2, 2]$	Proposition 3.5 (three starters)
$[7, 2, 2, 2, 2, 2]$	Proposition 3.5 (three starters)
$[6, 3, 2, 2, 2, 2]$	Proposition 3.5 (three starters)
$[5, 4, 2, 2, 2, 2]$	Proposition 3.5 (three starters)
$[5, 3, 3, 2, 2, 2]$	Proposition 3.5 (three starters)
$[4, 4, 3, 2, 2, 2]$	Proposition 3.5 (three starters)
$[4, 3, 3, 3, 2, 2]$	Proposition 3.5 (three starters)
$[3, 3, 3, 3, 3, 2]$	Proposition 3.5 (three starters)
$[9, 2, 2, 2, 2]$	Proposition 3.5 (three starters)
$[8, 3, 2, 2, 2]$	Proposition 3.5 (three starters)
$[7, 4, 2, 2, 2]$	Proposition 3.5 (three starters)
$[6, 5, 2, 2, 2]$	Proposition 3.5 (three starters)
$[7, 3, 3, 2, 2]$	Proposition 3.5 (three starters)
$[6, 4, 3, 2, 2]$	Proposition 3.5 (three starters)
$[5, 5, 3, 2, 2]$	Proposition 3.5 (three starters)
$[5, 4, 4, 2, 2]$	Proposition 3.5 (three starters)
$[6, 3, 3, 3, 2]$	Proposition 3.5 (three starters)
$[5, 4, 3, 3, 2]$	Proposition 3.5 (three starters)
$[4, 4, 4, 3, 2]$	Proposition 3.5 (three starters)
$[5, 3, 3, 3, 3]$	Theorem 1.2
$[4, 4, 3, 3, 3]$	Theorem 1.2
$[11, 2, 2, 2]$	Proposition 3.5 (three starters)
$[10, 3, 2, 2]$	Proposition 3.5 (three starters)
$[9, 4, 2, 2]$	Proposition 3.5 (three starters)
$[8, 5, 2, 2]$	Proposition 3.5 (three starters)
$[7, 6, 2, 2]$	Proposition 3.5 (three starters)
$[9, 3, 3, 2]$	Proposition 3.5 (three starters)
$[8, 4, 3, 2]$	Proposition 3.5 (three starters)
$[7, 5, 3, 2]$	Proposition 3.5 (three starters)
$[6, 6, 3, 2]$	Proposition 3.5 (three starters)
$[7, 4, 4, 2]$	Proposition 3.5 (three starters)
$[6, 5, 4, 2]$	Proposition 3.5 (three starters)
$[5, 5, 5, 2]$	Proposition 3.5 (three starters)
$[8, 3, 3, 3]$	Theorem 1.2
$[7, 4, 3, 3]$	Theorem 1.2
$[6, 5, 3, 3]$	Theorem 1.2

2-factor type $[m_1, \dots, m_t]$	HOP$(2m_1, \dots, 2m_t)$ has a solution by...
$[6, 4, 4, 3]$	Theorem 1.2
$[5, 5, 4, 3]$	Theorem 1.2
$[5, 4, 4, 4]$	Theorem 1.2
$[13, 2, 2]$	Proposition 3.5 (three starters)
$[12, 3, 2]$	Proposition 3.5 (three starters)
$[11, 4, 2]$	Proposition 3.5 (three starters)
$[10, 5, 2]$	Proposition 3.5 (three starters)
$[9, 6, 2]$	Proposition 3.5 (three starters)
$[8, 7, 2]$	Proposition 3.5 (three starters)
$[11, 3, 3]$	Theorem 1.2
$[10, 4, 3]$	Theorem 1.2
$[9, 5, 3]$	Theorem 1.2
$[8, 6, 3]$	Theorem 1.2
$[7, 7, 3]$	Theorem 1.2
$[9, 4, 4]$	Theorem 1.2
$[8, 5, 4]$	Theorem 1.2
$[7, 6, 4]$	Theorem 1.2
$[7, 5, 5]$	Theorem 1.2
$[6, 6, 5]$	Theorem 1.2
$[15, 2]$	Theorem 1.2
$[14, 3]$	Theorem 1.2
$[13, 4]$	Theorem 1.2
$[12, 5]$	Theorem 1.2
$[11, 6]$	Theorem 1.2
$[10, 7]$	Theorem 1.2
$[9, 8]$	Theorem 1.2
$[17]$	Theorem 1.2

CASE $n = 18$: see [10, Appendix J] for the computational results.

2-factor type $[m_1, \dots, m_t]$	HOP$(2m_1, \dots, 2m_t)$ has a solution by...
$[2, 2, 2, 2, 2, 2, 2, 2, 2]$	Theorem 1.2
$[4, 2, 2, 2, 2, 2, 2, 2]$	Proposition 3.3 (one starter)
$[3, 3, 2, 2, 2, 2, 2, 2]$	Proposition 3.4 (two starters)
$[6, 2, 2, 2, 2, 2, 2]$	Proposition 3.4 (two starters)
$[5, 3, 2, 2, 2, 2, 2]$	Proposition 3.3 (one starter)
$[4, 4, 2, 2, 2, 2, 2]$	Proposition 3.3 (one starter)
$[4, 3, 3, 2, 2, 2, 2]$	Proposition 3.4 (two starters)
$[3, 3, 3, 3, 2, 2, 2]$	Proposition 3.3 (one starter)
$[8, 2, 2, 2, 2, 2]$	Proposition 3.3 (one starter)
$[7, 3, 2, 2, 2, 2]$	Proposition 3.4 (two starters)

2-factor type $[m_1, \ldots, m_t]$	HOP$(2m_1, \ldots, 2m_t)$ has a solution by...
$[6, 4, 2, 2, 2, 2]$	Proposition 3.4 (two starters)
$[5, 5, 2, 2, 2, 2]$	Proposition 3.4 (two starters)
$[6, 3, 3, 2, 2, 2]$	Proposition 3.3 (one starter)
$[5, 4, 3, 2, 2, 2]$	Proposition 3.3 (one starter)
$[4, 4, 4, 2, 2, 2]$	Proposition 3.3 (one starter)
$[5, 3, 3, 3, 2, 2]$	Proposition 3.4 (two starters)
$[4, 4, 3, 3, 2, 2]$	Proposition 3.4 (two starters)
$[4, 3, 3, 3, 3, 2]$	Proposition 3.3 (one starter)
$[3, 3, 3, 3, 3, 3]$	Theorem 1.2
$[10, 2, 2, 2, 2]$	Proposition 3.4 (two starters)
$[9, 3, 2, 2, 2]$	Proposition 3.3 (one starter)
$[8, 4, 2, 2, 2]$	Proposition 3.3 (one starter)
$[7, 5, 2, 2, 2]$	Proposition 3.3 (one starter)
$[6, 6, 2, 2, 2]$	Proposition 3.3 (one starter)
$[8, 3, 3, 2, 2]$	Proposition 3.4 (two starters)
$[7, 4, 3, 2, 2]$	Proposition 3.4 (two starters)
$[6, 5, 3, 2, 2]$	Proposition 3.4 (two starters)
$[6, 4, 4, 2, 2]$	Proposition 3.4 (two starters)
$[5, 5, 4, 2, 2]$	Proposition 3.4 (two starters)
$[7, 3, 3, 3, 2]$	Proposition 3.3 (one starter)
$[6, 4, 3, 3, 2]$	Proposition 3.3 (one starter)
$[5, 5, 3, 3, 2]$	Proposition 3.3 (one starter)
$[5, 4, 4, 3, 2]$	Proposition 3.3 (one starter)
$[4, 4, 4, 4, 2]$	Proposition 3.3 (one starter)
$[6, 3, 3, 3, 3]$	Proposition 3.4 (two starters)
$[5, 4, 3, 3, 3]$	Proposition 3.4 (two starters)
$[4, 4, 4, 3, 3]$	Proposition 3.4 (two starters)
$[12, 2, 2, 2]$	Proposition 3.3 (one starter)
$[11, 3, 2, 2]$	Proposition 3.4 (two starters)
$[10, 4, 2, 2]$	Proposition 3.4 (two starters)
$[9, 5, 2, 2]$	Proposition 3.4 (two starters)
$[8, 6, 2, 2]$	Proposition 3.4 (two starters)
$[7, 7, 2, 2]$	Proposition 3.4 (two starters)
$[10, 3, 3, 2]$	Proposition 3.3 (one starter)
$[9, 4, 3, 2]$	Proposition 3.3 (one starter)
$[8, 5, 3, 2]$	Proposition 3.3 (one starter)
$[7, 6, 3, 2]$	Proposition 3.3 (one starter)

2-factor type $[m_1, \ldots, m_t]$	HOP$(2m_1, \ldots, 2m_t)$ has a solution by...
$[8, 4, 4, 2]$	Proposition 3.3 (one starter)
$[7, 5, 4, 2]$	Proposition 3.3 (one starter)
$[6, 6, 4, 2]$	Proposition 3.3 (one starter)
$[6, 5, 5, 2]$	Proposition 3.3 (one starter)
$[9, 3, 3, 3]$	Proposition 3.4 (two starters)
$[8, 4, 3, 3]$	Proposition 3.4 (two starters)
$[7, 5, 3, 3]$	Proposition 3.4 (two starters)
$[6, 6, 3, 3]$	Proposition 3.4 (two starters)
$[7, 4, 4, 3]$	Proposition 3.4 (two starters)
$[6, 5, 4, 3]$	Proposition 3.4 (two starters)
$[5, 5, 5, 3]$	Proposition 3.4 (two starters)
$[6, 4, 4, 4]$	Proposition 3.4 (two starters)
$[5, 5, 4, 4]$	Proposition 3.4 (two starters)
$[14, 2, 2]$	Proposition 3.4 (two starters)
$[13, 3, 2]$	Proposition 3.3 (one starter)
$[12, 4, 2]$	Proposition 3.3 (one starter)
$[11, 5, 2]$	Proposition 3.3 (one starter)
$[10, 6, 2]$	Proposition 3.3 (one starter)
$[9, 7, 2]$	Proposition 3.3 (one starter)
$[8, 8, 2]$	Proposition 3.3 (one starter)
$[12, 3, 3]$	Proposition 3.4 (two starters)
$[11, 4, 3]$	Proposition 3.4 (two starters)
$[10, 5, 3]$	Proposition 3.4 (two starters)
$[9, 6, 3]$	Proposition 3.4 (two starters)
$[8, 7, 3]$	Proposition 3.4 (two starters)
$[10, 4, 4]$	Proposition 3.4 (two starters)
$[9, 5, 4]$	Proposition 3.4 (two starters)
$[8, 6, 4]$	Proposition 3.4 (two starters)
$[7, 7, 4]$	Proposition 3.4 (two starters)
$[8, 5, 5]$	Proposition 3.4 (two starters)
$[7, 6, 5]$	Proposition 3.4 (two starters)
$[6, 6, 6]$	Theorem 1.2
$[16, 2]$	Proposition 3.3 (one starter)
$[15, 3]$	Proposition 3.4 (two starters)
$[14, 4]$	Proposition 3.4 (two starters)
$[13, 5]$	Proposition 3.4 (two starters)
$[12, 6]$	Proposition 3.4 (two starters)
$[11, 7]$	Proposition 3.4 (two starters)
$[10, 8]$	Proposition 3.4 (two starters)
$[9, 9]$	Theorem 1.2
$[18]$	Theorem 1.2

CASE $n = 19$: see [10, Appendix K] for the computational results.

2-factor type $[m_1, \ldots, m_t]$	HOP$(2m_1, \ldots, 2m_t)$ has a solution by...
$[3, 2, 2, 2, 2, 2, 2, 2, 2]$	Proposition 3.5 (three starters)
$[5, 2, 2, 2, 2, 2, 2, 2]$	Proposition 3.5 (three starters)
$[4, 3, 2, 2, 2, 2, 2, 2]$	Proposition 3.5 (three starters)
$[3, 3, 3, 2, 2, 2, 2, 2]$	Proposition 3.5 (three starters)
$[7, 2, 2, 2, 2, 2, 2]$	Proposition 3.5 (three starters)
$[6, 3, 2, 2, 2, 2, 2]$	Proposition 3.5 (three starters)
$[5, 4, 2, 2, 2, 2, 2]$	Proposition 3.5 (three starters)
$[5, 3, 3, 2, 2, 2, 2]$	Proposition 3.5 (three starters)
$[4, 4, 3, 2, 2, 2, 2]$	Proposition 3.5 (three starters)
$[4, 3, 3, 3, 2, 2, 2]$	Proposition 3.5 (three starters)
$[3, 3, 3, 3, 3, 2, 2]$	Proposition 3.5 (three starters)
$[9, 2, 2, 2, 2, 2]$	Proposition 3.5 (three starters)
$[8, 3, 2, 2, 2, 2]$	Proposition 3.5 (three starters)
$[7, 4, 2, 2, 2, 2]$	Proposition 3.5 (three starters)
$[6, 5, 2, 2, 2, 2]$	Proposition 3.5 (three starters)
$[7, 3, 3, 2, 2, 2]$	Proposition 3.5 (three starters)
$[6, 4, 3, 2, 2, 2]$	Proposition 3.5 (three starters)
$[5, 5, 3, 2, 2, 2]$	Proposition 3.5 (three starters)
$[5, 4, 4, 2, 2, 2]$	Proposition 3.5 (three starters)
$[6, 3, 3, 3, 2, 2]$	Proposition 3.5 (three starters)
$[5, 4, 3, 3, 2, 2]$	Proposition 3.5 (three starters)
$[4, 4, 4, 3, 2, 2]$	Proposition 3.5 (three starters)
$[5, 3, 3, 3, 3, 2]$	Proposition 3.5 (three starters)
$[4, 4, 3, 3, 3, 2]$	Proposition 3.5 (three starters)
$[4, 3, 3, 3, 3, 3]$	Theorem 1.2
$[11, 2, 2, 2, 2]$	Proposition 3.5 (three starters)
$[10, 3, 2, 2, 2]$	Proposition 3.5 (three starters)
$[9, 4, 2, 2, 2]$	Proposition 3.5 (three starters)
$[8, 5, 2, 2, 2]$	Proposition 3.5 (three starters)
$[7, 6, 2, 2, 2]$	Proposition 3.5 (three starters)
$[9, 3, 3, 2, 2]$	Proposition 3.5 (three starters)
$[8, 4, 3, 2, 2]$	Proposition 3.5 (three starters)
$[7, 5, 3, 2, 2]$	Proposition 3.5 (three starters)
$[6, 6, 3, 2, 2]$	Proposition 3.5 (three starters)
$[7, 4, 4, 2, 2]$	Proposition 3.5 (three starters)
$[6, 5, 4, 2, 2]$	Proposition 3.5 (three starters)
$[5, 5, 5, 2, 2]$	Proposition 3.5 (three starters)
$[8, 3, 3, 3, 2]$	Proposition 3.5 (three starters)
$[7, 4, 3, 3, 2]$	Proposition 3.5 (three starters)

2-factor type $[m_1, \dots, m_t]$	HOP$(2m_1, \dots, 2m_t)$ has a solution by...
$[6, 5, 3, 3, 2]$	Proposition 3.5 (three starters)
$[6, 4, 4, 3, 2]$	Proposition 3.5 (three starters)
$[5, 5, 4, 3, 2]$	Proposition 3.5 (three starters)
$[5, 4, 4, 4, 2]$	Proposition 3.5 (three starters)
$[7, 3, 3, 3, 3]$	Theorem 1.2
$[6, 4, 3, 3, 3]$	Theorem 1.2
$[5, 5, 3, 3, 3]$	Theorem 1.2
$[5, 4, 4, 3, 3]$	Theorem 1.2
$[4, 4, 4, 4, 3]$	Theorem 1.2
$[13, 2, 2, 2]$	Proposition 3.5 (three starters)
$[12, 3, 2, 2]$	Proposition 3.5 (three starters)
$[11, 4, 2, 2]$	Proposition 3.5 (three starters)
$[10, 5, 2, 2]$	Proposition 3.5 (three starters)
$[9, 6, 2, 2]$	Proposition 3.5 (three starters)
$[8, 7, 2, 2]$	Proposition 3.5 (three starters)
$[11, 3, 3, 2]$	Proposition 3.5 (three starters)
$[10, 4, 3, 2]$	Proposition 3.5 (three starters)
$[9, 5, 3, 2]$	Proposition 3.5 (three starters)
$[8, 6, 3, 2]$	Proposition 3.5 (three starters)
$[7, 7, 3, 2]$	Proposition 3.5 (three starters)
$[9, 4, 4, 2]$	Proposition 3.5 (three starters)
$[8, 5, 4, 2]$	Proposition 3.5 (three starters)
$[7, 6, 4, 2]$	Proposition 3.5 (three starters)
$[7, 5, 5, 2]$	Proposition 3.5 (three starters)
$[6, 6, 5, 2]$	Proposition 3.5 (three starters)
$[10, 3, 3, 3]$	Theorem 1.2
$[9, 4, 3, 3]$	Theorem 1.2
$[8, 5, 3, 3]$	Theorem 1.2
$[7, 6, 3, 3]$	Theorem 1.2
$[8, 4, 4, 3]$	Theorem 1.2
$[7, 5, 4, 3]$	Theorem 1.2
$[6, 6, 4, 3]$	Theorem 1.2
$[6, 5, 5, 3]$	Theorem 1.2
$[7, 4, 4, 4]$	Theorem 1.2
$[6, 5, 4, 4]$	Theorem 1.2
$[5, 5, 5, 4]$	Theorem 1.2
$[15, 2, 2]$	Proposition 3.5 (three starters)
$[14, 3, 2]$	Proposition 3.5 (three starters)
$[13, 4, 2]$	Proposition 3.5 (three starters)
$[12, 5, 2]$	Proposition 3.5 (three starters)
$[11, 6, 2]$	Proposition 3.5 (three starters)
$[10, 7, 2]$	Proposition 3.5 (three starters)
$[9, 8, 2]$	Proposition 3.5 (three starters)

2-factor type $[m_1, \ldots, m_t]$	HOP$(2m_1, \ldots, 2m_t)$ has a solution by...
$[13, 3, 3]$	Theorem 1.2
$[12, 4, 3]$	Theorem 1.2
$[11, 5, 3]$	Theorem 1.2
$[10, 6, 3]$	Theorem 1.2
$[9, 7, 3]$	Theorem 1.2
$[8, 8, 3]$	Theorem 1.2
$[11, 4, 4]$	Theorem 1.2
$[10, 5, 4]$	Theorem 1.2
$[9, 6, 4]$	Theorem 1.2
$[8, 7, 4]$	Theorem 1.2
$[9, 5, 5]$	Theorem 1.2
$[8, 6, 5]$	Theorem 1.2
$[7, 7, 5]$	Theorem 1.2
$[7, 6, 6]$	Theorem 1.2
$[17, 2]$	Theorem 1.2
$[16, 3]$	Theorem 1.2
$[15, 4]$	Theorem 1.2
$[14, 5]$	Theorem 1.2
$[13, 6]$	Theorem 1.2
$[12, 7]$	Theorem 1.2
$[11, 8]$	Theorem 1.2
$[10, 9]$	Theorem 1.2
$[19]$	Theorem 1.2

CASE $n = 20$: see [10, Appendix L] for the computational results.

2-factor type $[m_1, \ldots, m_t]$	HOP$(2m_1, \ldots, 2m_t)$ has a solution by...
$[2, 2, 2, 2, 2, 2, 2, 2, 2, 2]$	Theorem 1.2
$[4, 2, 2, 2, 2, 2, 2, 2, 2]$	Proposition 3.3 (one starter)
$[3, 3, 2, 2, 2, 2, 2, 2, 2]$	Proposition 3.4 (two starters)
$[6, 2, 2, 2, 2, 2, 2, 2]$	Proposition 3.4 (two starters)
$[5, 3, 2, 2, 2, 2, 2, 2]$	Proposition 3.3 (one starter)
$[4, 4, 2, 2, 2, 2, 2, 2]$	Proposition 3.3 (one starter)
$[4, 3, 3, 2, 2, 2, 2, 2]$	Proposition 3.4 (two starters)
$[3, 3, 3, 3, 2, 2, 2, 2]$	Proposition 3.3 (one starter)
$[8, 2, 2, 2, 2, 2, 2]$	Proposition 3.3 (one starter)
$[7, 3, 2, 2, 2, 2, 2]$	Proposition 3.4 (two starters)
$[6, 4, 2, 2, 2, 2, 2]$	Proposition 3.4 (two starters)
$[5, 5, 2, 2, 2, 2, 2]$	Proposition 3.4 (two starters)
$[6, 3, 3, 2, 2, 2, 2]$	Proposition 3.3 (one starter)
$[5, 4, 3, 2, 2, 2, 2]$	Proposition 3.3 (one starter)

2-factor type $[m_1, \dots, m_t]$	HOP$(2m_1, \dots, 2m_t)$ has a solution by...
$[4, 4, 4, 2, 2, 2, 2]$	Proposition 3.3 (one starter)
$[5, 3, 3, 3, 2, 2, 2]$	Proposition 3.4 (two starters)
$[4, 4, 3, 3, 2, 2, 2]$	Proposition 3.4 (two starters)
$[4, 3, 3, 3, 3, 2, 2]$	Proposition 3.3 (one starter)
$[3, 3, 3, 3, 3, 3, 2]$	Proposition 3.4 (two starters)
$[10, 2, 2, 2, 2, 2]$	Proposition 3.4 (two starters)
$[9, 3, 2, 2, 2, 2]$	Proposition 3.3 (one starter)
$[8, 4, 2, 2, 2, 2]$	Proposition 3.3 (one starter)
$[7, 5, 2, 2, 2, 2]$	Proposition 3.3 (one starter)
$[6, 6, 2, 2, 2, 2]$	Proposition 3.3 (one starter)
$[8, 3, 3, 2, 2, 2]$	Proposition 3.4 (two starters)
$[7, 4, 3, 2, 2, 2]$	Proposition 3.4 (two starters)
$[6, 5, 3, 2, 2, 2]$	Proposition 3.4 (two starters)
$[6, 4, 4, 2, 2, 2]$	Proposition 3.4 (two starters)
$[5, 5, 4, 2, 2, 2]$	Proposition 3.4 (two starters)
$[7, 3, 3, 3, 2, 2]$	Proposition 3.3 (one starter)
$[6, 4, 3, 3, 2, 2]$	Proposition 3.3 (one starter)
$[5, 5, 3, 3, 2, 2]$	Proposition 3.3 (one starter)
$[5, 4, 4, 3, 2, 2]$	Proposition 3.3 (one starter)
$[4, 4, 4, 4, 2, 2]$	Proposition 3.3 (one starter)
$[6, 3, 3, 3, 3, 2]$	Proposition 3.4 (two starters)
$[5, 4, 3, 3, 3, 2]$	Proposition 3.4 (two starters)
$[4, 4, 4, 3, 3, 2]$	Proposition 3.4 (two starters)
$[5, 3, 3, 3, 3, 3]$	Proposition 3.3 (one starter)
$[4, 4, 3, 3, 3, 3]$	Proposition 3.3 (one starter)
$[12, 2, 2, 2, 2]$	Proposition 3.3 (one starter)
$[11, 3, 2, 2, 2]$	Proposition 3.4 (two starters)
$[10, 4, 2, 2, 2]$	Proposition 3.4 (two starters)
$[9, 5, 2, 2, 2]$	Proposition 3.4 (two starters)
$[8, 6, 2, 2, 2]$	Proposition 3.4 (two starters)
$[7, 7, 2, 2, 2]$	Proposition 3.4 (two starters)
$[10, 3, 3, 2, 2]$	Proposition 3.3 (one starter)
$[9, 4, 3, 2, 2]$	Proposition 3.3 (one starter)
$[8, 5, 3, 2, 2]$	Proposition 3.3 (one starter)
$[7, 6, 3, 2, 2]$	Proposition 3.3 (one starter)
$[8, 4, 4, 2, 2]$	Proposition 3.3 (one starter)
$[7, 5, 4, 2, 2]$	Proposition 3.3 (one starter)
$[6, 6, 4, 2, 2]$	Proposition 3.3 (one starter)
$[6, 5, 5, 2, 2]$	Proposition 3.3 (one starter)
$[9, 3, 3, 3, 2]$	Proposition 3.4 (two starters)
$[8, 4, 3, 3, 2]$	Proposition 3.4 (two starters)
$[7, 5, 3, 3, 2]$	Proposition 3.4 (two starters)
$[6, 6, 3, 3, 2]$	Proposition 3.4 (two starters)

2-factor type $[m_1, \ldots, m_t]$	HOP$(2m_1, \ldots, 2m_t)$ has a solution by...
$[7,4,4,3,2]$	Proposition 3.4 (two starters)
$[6,5,4,3,2]$	Proposition 3.4 (two starters)
$[5,5,5,3,2]$	Proposition 3.4 (two starters)
$[6,4,4,4,2]$	Proposition 3.4 (two starters)
$[5,5,4,4,2]$	Proposition 3.4 (two starters)
$[8,3,3,3,3]$	Proposition 3.3 (one starter)
$[7,4,3,3,3]$	Proposition 3.3 (one starter)
$[6,5,3,3,3]$	Proposition 3.3 (one starter)
$[6,4,4,3,3]$	Proposition 3.3 (one starter)
$[5,5,4,3,3]$	Proposition 3.3 (one starter)
$[5,4,4,4,3]$	Proposition 3.3 (one starter)
$[4,4,4,4,4]$	Theorem 1.2
$[14,2,2,2]$	Proposition 3.4 (two starters)
$[13,3,2,2]$	Proposition 3.3 (one starter)
$[12,4,2,2]$	Proposition 3.3 (one starter)
$[11,5,2,2]$	Proposition 3.3 (one starter)
$[10,6,2,2]$	Proposition 3.3 (one starter)
$[9,7,2,2]$	Proposition 3.3 (one starter)
$[8,8,2,2]$	Proposition 3.3 (one starter)
$[12,3,3,2]$	Proposition 3.4 (two starters)
$[11,4,3,2]$	Proposition 3.4 (two starters)
$[10,5,3,2]$	Proposition 3.4 (two starters)
$[9,6,3,2]$	Proposition 3.4 (two starters)
$[8,7,3,2]$	Proposition 3.4 (two starters)
$[10,4,4,2]$	Proposition 3.4 (two starters)
$[9,5,4,2]$	Proposition 3.4 (two starters)
$[8,6,4,2]$	Proposition 3.4 (two starters)
$[7,7,4,2]$	Proposition 3.4 (two starters)
$[8,5,5,2]$	Proposition 3.4 (two starters)
$[7,6,5,2]$	Proposition 3.4 (two starters)
$[6,6,6,2]$	Proposition 3.4 (two starters)
$[11,3,3,3]$	Proposition 3.3 (one starter)
$[10,4,3,3]$	Proposition 3.3 (one starter)
$[9,5,3,3]$	Proposition 3.3 (one starter)
$[8,6,3,3]$	Proposition 3.3 (one starter)
$[7,7,3,3]$	Proposition 3.3 (one starter)
$[9,4,4,3]$	Proposition 3.3 (one starter)
$[8,5,4,3]$	Proposition 3.3 (one starter)
$[7,6,4,3]$	Proposition 3.3 (one starter)
$[7,5,5,3]$	Proposition 3.3 (one starter)
$[6,6,5,3]$	Proposition 3.3 (one starter)
$[8,4,4,4]$	Theorem 1.2

2-factor type $[m_1, \dots, m_t]$	HOP$(2m_1, \dots, 2m_t)$ has a solution by...
$[7, 5, 4, 4]$	Proposition 3.3 (one starter)
$[6, 6, 4, 4]$	Proposition 3.3 (one starter)
$[6, 5, 5, 4]$	Proposition 3.3 (one starter)
$[5, 5, 5, 5]$	Theorem 1.2
$[16, 2, 2]$	Proposition 3.3 (one starter)
$[15, 3, 2]$	Proposition 3.4 (two starters)
$[14, 4, 2]$	Proposition 3.4 (two starters)
$[13, 5, 2]$	Proposition 3.4 (two starters)
$[12, 6, 2]$	Proposition 3.4 (two starters)
$[11, 7, 2]$	Proposition 3.4 (two starters)
$[10, 8, 2]$	Proposition 3.4 (two starters)
$[9, 9, 2]$	Proposition 3.4 (two starters)
$[14, 3, 3]$	Proposition 3.3 (one starter)
$[13, 4, 3]$	Proposition 3.3 (one starter)
$[12, 5, 3]$	Proposition 3.3 (one starter)
$[11, 6, 3]$	Proposition 3.3 (one starter)
$[10, 7, 3]$	Proposition 3.3 (one starter)
$[9, 8, 3]$	Proposition 3.3 (one starter)
$[12, 4, 4]$	Theorem 1.2
$[11, 5, 4]$	Proposition 3.3 (one starter)
$[10, 6, 4]$	Proposition 3.3 (one starter)
$[9, 7, 4]$	Proposition 3.3 (one starter)
$[8, 8, 4]$	Theorem 1.2
$[10, 5, 5]$	Proposition 3.3 (one starter)
$[9, 6, 5]$	Proposition 3.3 (one starter)
$[8, 7, 5]$	Proposition 3.3 (one starter)
$[8, 6, 6]$	Proposition 3.3 (one starter)
$[7, 7, 6]$	Proposition 3.3 (one starter)
$[18, 2]$	Proposition 3.4 (two starters)
$[17, 3]$	Proposition 3.3 (one starter)
$[16, 4]$	Theorem 1.2
$[15, 5]$	Proposition 3.3 (one starter)
$[14, 6]$	Proposition 3.3 (one starter)
$[13, 7]$	Proposition 3.3 (one starter)
$[12, 8]$	Theorem 1.2
$[11, 9]$	Proposition 3.3 (one starter)
$[10, 10]$	Theorem 1.2
$[20]$	Theorem 1.2

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References

[1] P. Adams and D. Bryant. Two-factorisations of complete graphs of orders fifteen and seventeen. *Australasian Journal of Combinatorics*, 35:113–118, 2006.

[2] B. Alspach and R. Häggkvist. Some observations on the Oberwolfach problem. *Journal of Graph Theory*, 9(1):177–187, 1985. https://doi.org/10.1002/jgt.3190090114.

[3] B. Alspach, P. J. Schellenberg, D. R. Stinson, and D. Wagner. The Oberwolfach problem and factors of uniform odd length cycles. *Journal of Combinatorial Theory. Series A*, 52(1):20–43, 1989. https://doi.org/10.1016/0097-3165(89)90059-9.

[4] D. Bryant and P. Danziger. On bipartite 2-factorizations of $K_n - I$ and the Oberwolfach problem. *Journal of Graph Theory*, 68(1):22–37, 2011. https://doi.org/10.1002/jgt.20538.

[5] A. Deza, F. Franek, W. Hua, M. Meszka, and A. Rosa. Solutions to the Oberwolfach problem for orders 18 to 40. *Journal of Combinatorial Mathematics and Combinatorial Computing*, 74:95–102, 2010.

[6] F. Franek, J. Holub, and A. Rosa. Two-factorizations of small complete graphs. II. The case of 13 vertices. *Journal of Combinatorial Mathematics and Combinatorial Computing*, 51:89–94, 2004.

[7] F. Franek and A. Rosa. Two-factorizations of small complete graphs. In volume 86, number 2, pages 435–442. 2000. https://doi.org/10.1016/S0378-3758(99)00123-8. Special issue in honor of Professor Ralph Stanton.

[8] S. Glock, F. Joos, J. Kim, D. Kühn, and D. Osthus. Resolution of the Oberwolfach problem. *Journal of the European Mathematical Society (JEMS)*, 23(8):2511–2547, 2021. https://doi.org/10.4171/jems/1060.

[9] D. G. Hoffman and P. J. Schellenberg. The existence of C_k-factorizations of $K_{2n} - F$. *Discrete Mathematics*, 97(1-3):243–250, 1991. https://doi.org/10.1016/0012-365X(91)90440-D.

[10] M. R. Jerade and M. Šajna. The honeymoon oberwolfach problem: small cases. *arXiv*, (2407.00204):1–122, 2024. arxiv.org/abs/2407.00204.

[11] D. Lepine and M. Šajna. On the honeymoon Oberwolfach problem. *Journal of Combinatorial Designs*, 27(7):420–447, 2019. https://doi.org/10.1002/jcd.21656.

[12] M. Meszka. Solutions to the Oberwolfach problem for orders up to 100. *Australasian Journal of Combinatorics*, 89:243–248, 2024.

[13] F. Salassa, G. Dragotto, T. Traetta, M. Buratti, and F. Della Croce. Merging combinatorial design and optimization: the Oberwolfach problem. *Australasian Journal of Combinatorics*, 79:141–166, 2021.

[14] T. Traetta. A complete solution to the two-table Oberwolfach problems. *Journal of Combinatorial Theory. Series A*, 120(5):984–997, 2013. https://doi.org/10.1016/j.jcta.2013.01.003.

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