101 Anti-Pasch Steiner Triple Systems of Order 19

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Abstract. An algorithm to construct anti-Pasch Steiner triple systems is described and used to construct 101 such systems of order 19. It is also proved that no anti-Pasch STS(19) contains a non-trivial subsystem. Anti-Pasch STS(19)s with given automorphisms are identified.

1. Introduction

A Steiner triple system on v points, briefly STS(v), is said to be *anti-Pasch* (some authors use the term *quadrilateral free*) if it contains no collection of four blocks whose union has cardinality six. Such a collection must take the form $\{a, b, c\}$, $\{a, y, z\}$, $\{x, b, z\}$ and $\{x, y, c\}$ and is known as a *Pasch configuration or quadrilateral.* The spectrum of v for which there exists an anti-Pasch STS(v) is not completely determined though the case $v \equiv 3 \pmod 6$ has been settled by Brouwer [2]. When $v \equiv 1 \pmod 6$, known results are much more fragmentary and a good summary of these is given in the paper by Chee and Lim [3]. The unique STS(7) and both of the two non-isomorphic STS(13)s are not anti-Pasch but it is conjectured that these two values for v are the only exceptions i.e. there exists an anti-Pasch STS(v) for all $v \equiv 1 \pmod 6$, $v \geq 19$.

In this paper we consider the case where $v = 19$. The first anti-Pasch STS(19) was identified by Robinson [12] who proved that the so-called Netto systems STS(v) where $v \equiv 7 \pmod{12}$ and v is a prime power are anti-Pasch precisely in the case $v \equiv 19 \pmod{24}$. In contrast, Stinson and Ferch [14] generated over 2000000 non-isomorphic STS(19)s, none of which was anti-Pasch. However the Netto system is not an isolated example: section 2 describes a simple modification of the hill-climbing algorithm whereby 101 non-isomorphic anti-Pasch STS(19)s were constructed and undoubtedly there are many more. 101 was where we stopped. In section 3 we prove the result that no anti-Pasch STS(19) contains a non-trivial subsystem and in section 4 present various results concerning classes of STS(19)s with given automorphisms which we have examined for their quadrilateral content. Several anti-Pasch STS(19)s are identified.

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2. Systems constructed by hill-climbing

Hill-climbing algorithms for the construction of combinatorial designs are described in the paper of the same title by Stinson [13]. The basic method is to extend a partial Steiner triple system, which initially contains no blocks, according to the following algorithm. Firstly a point x which has not yet occured with all other points is chosen at random (Stinson calls such a point a live point). Then two further points y and z with which x has not occured are chosen also at random to form a block $B_0 = \{x, y, z\}$. Finally if the pair $\{y, z\}$ does not appear in the partial STS(v) then B_0 is added to the design (moving vertically up the hill) and otherwise if $\{y, z\} \subset B_1$, a block of the system, then B_1 is replaced by B_0 (moving horizontally round the hill).

Our modification of the algorithm is quite simply to test whether the addition of B_0 or replacement of B_1 by B_0 introduces a quadrilateral into the partial STS(v). If it does the block B_0 is rejected and another generated. Thus at every stage of the construction the partial system is anti-Pasch. Now of course this algorithm is not guaranteed to succeed: indeed if used to try to construct an anti-Pasch STS(13) for example it will surely fail. For this reason a count is set to zero after each new block has been added and the number of trial new blocks generated until the next new block is added is counted. However if this count reaches a certain number, say N, and we found a value of $N = 250000$ was an appropriate figure, the whole process is abandoned and the algorithm re-started from scratch. Empirical data showed that for STS(19)s about 1.5% of all attempts to construct anti-Pasch systems were successful and we used the program to construct 101 non-isomorphic anti-Pasch STS(19)s. These were obtained from only 103 successful runs. In addition the anti-Pasch STS(21) given in our paper with J.S. Phelan [8] was also constructed using this program.

Formally the algorithm is

```
begin
restart: b := c := 0;
B := φ;
while b < v(v − 1)/6 do
    begin choose a live point x;
    choose y, z which have not occured with x;
    B0 := {x, y, z};
    c := c + 1;
    if c > N then goto restart;
    if {y, z} occurs in a block of B then
    B1 := that block else B1 := φ;
    if B ∪ {B0}\{B1} is anti-Pasch then
        begin B := B ∪ {B0}\{B1};
        if B1 = φ then begin b := b + 1; c := 0 end
        end
    end
end
```

Listings of the systems so constructed are given in the Appendix using a compact notation similar to, though in detail different from, that used in the paper by Colbourn, Magliveras and Stinson [4]. The fact that the 101 systems listed are indeed pairwise non-isomorphic was proved by computing the train structure of each system. Any STS(v) can be used to define a function f from the set of all triples, not just those in the STS(v), to itself by the rule $f\{l,m,n\} = \{p,q,r\}$ if $\{l,m,r\}$, $\{l,q,n\}$ and $\{p,m,n\}$ are blocks of the system. A *directed graph* or *train* is thus formed which is an invariant of the STS(v). We found that of the 103 systems constructed, 101 had different train structures, the other 2 being isomorphic to systems already obtained.

3. Systems containing non-trivial subsystems

In this section we prove

Theorem. *No anti-Pasch STS(19) contains a non-trivial subsystem.*

Proof: The only possible non-trivial subsystems of an STS(19) are the unique STS(7) which contains quadrilaterals and the unique STS(9) which does not. So it is enough to show that any STS(19) containing an STS(9) also contains a quadrilateral. Let the STS(9) be on base set $U = \{1,2,3,\ldots,9\}$ and let $D = \{0',1',2',\ldots,9'\}$ be the further ten elements required for the STS(19). Then in addition to the 12 blocks of the STS(9), the STS(19) has 45 blocks of the form $\{a,b',c'\}$ where $a \in U$ and $b',c' \in D$. Further all pairs contained in a block with a fixed point $a \in U$ form a one-factor of the complete graph K_{10} on D and the set of all one-factors over all points of U form a one-factorization.

Now one possible form for a quadrilateral in the STS(19) are blocks $\{a,b',c'\}$, $\{a,y',z'\}$, $\{x,b',z'\}$ and $\{x,y',c'\}$ where $a,x \in U$ and $b',c',y',z' \in D$. This implies that the union of two of the one-factors of the one-factorization of K_{10} contains a four-cycle. Since the only two possibilities for such a union are either a four-cycle and a six-cycle or alternatively a ten-cycle i.e. Hamiltonian circuit, it follows that for the STS(19) to be anti-Pasch the union of every two one-factors must form a Hamiltonian circuit. Such a one-factorization is called perfect and it is known that up to isomorphism it is unique on K_{10} and is the one-factorization usually denoted by GA_{10} [10]. On the set D the one-factors are given by $\{\{a',(a+1)'\},\{(a+2)',(a+8)'\},\{(a+3)',(a+9)'\},\{(a+4)',(a+6)'\},\{(a+5)',(a+7)'\}\}$, $a = 0,2,4,6,8$: denote these one-factors as type A, and $\{\{0',b'\},\{2',(b+2)'\},\{4',(b+4)'\},\{6',(b+6)'\},\{8',(b+8)'\}\}$, $b = 3,5,7,9$: denote these one-factors as type B, all additions being in modulo 10 arithmetic.

The only other possible form for a quadrilateral in the STS(19) are blocks $\{a,b,c\}$, $\{a,y',z'\}$, $\{b,x',z'\}$ and $\{c,x',y'\}$ where $a,b,c \in U$ and $x',y',z' \in D$. Observing that $\{y',z'\}$, $\{x',z'\}$ and $\{x',y'\}$ form a triangle and must therefore occur in different one-factors, it follows that in order to construct an anti-Pasch

STS(19) containing an STS(9) it is necessary to assign the elements of U to the one-factors of the one-factorization in such a way that the three one-factors corresponding to any triple of the STS(9) do not contain a triangle as above. However, it can be easily verified by looking at the structure of GA_{10}, that if $\{l, m, n\}$ is a triple of the STS(9), and if two of the elements are assigned to one-factors of type B and the third is assigned to a one-factor of type A, then a triangle is formed (as described above). Hence, if two of the elements are assigned to a one-factor of type B then so must the third. This in turn would imply the existence of a Steiner triple system on 4 points (the number of one-factors of type B) which is impossible and so the theorem is proved.

The number of non-isomorphic STS(19)s containing a subsystem STS(9) have been enumerated by Stinson and Seah [15]: there are precisely 284457 of them, none of which is of course anti-Pasch. The above theorem does raise one interesting question. The least value of v for which an STS(u) can be embedded in an STS(v) is $v = 2u+1$ and the classic result of Doyen and Wilson [6] states that any STS(u) can be embedded in an STS(v) for all admissible $v \geq 2u + 1$. Clearly this result does not hold if it is demanded that the STS(v), and so also the STS(u), is anti-Pasch. In particular we ask for an example of an anti-Pasch STS(u) embedded in an anti-Pasch STS($2u + 1$). We have proved this is impossible when $u = 9$ and the next possible case to consider is $u = 15$.

4. Systems with given automorphisms

Recently Colbourn, Magliveras and Stinson [4] have enumerated all Steiner triple systems of order 19 with non-trivial automorphism group. There are 172248 of them. The method used is to consider all the basic automorphism types at least one of which any STS(19) having a non-trivial automorphism group must admit. If P is a permutation of degree 19, then P is said to have *basic automorphism type* $a_1^{n_1} \dots a_t^{n_t}$ if P has n_i cycles of length a_i for $1 \leq i \leq t$.

It is then found that the basic automorphism types are 19^1, $1^1 2^9$, $1^1 3^6$, $1^3 2^8$, $1^7 2^6$ and $1^7 3^4$. The paper also gives a table of basic automorphism types for all 172248 systems and explicit listings of all STS(19)s having an automorphism group of order greater than or equal to 9. Using this information and the fact that any system admitting an automorphism of type $1^7 2^6$ or $1^7 3^4$ must contain a subsystem STS(7) and hence quadrilaterals the only candidates for anti-Pasch STS(19)s having an automorphism group of order greater than or equal to 9 fall into four categories.

(I) 4 systems having a basic automorphism type 19^1. These are the cyclic systems first enumerated by Bays [1].

(II) 1 system with automorphism group of order 18 and basic automorphism type $1^1 2^9$. This can be considered with all such systems having automorphism type $1^1 2^9$ of which there are 183 further systems with automorphism group of order either 2 or 6. These are the reverse STS(19)s enumerated by Denniston [5].

(III) 7 systems having an automorphism group of order 12.

(IV) 19 systems having an automorphism group of order 9 and basic automorphism type $1^1 3^6$ of which 9 are 9 of the 10 2-rotational systems found by Phelps and Rosa [11]. (The 10th system is also cyclic).

These are considered in turn.

(I) Listings of the 4 cyclic STS(19)s are given in the survey paper by Mathon, Phelps and Rosa [9]. We follow their notation and list the base triples from Z_{19} which generate the system under the action of $i \mapsto i+1 \pmod{19}$.

A1 $\{0,1,4\}, \{0,2,9\}, \{0,5,11\}$, group order = 19. This system contains 19 quadrilaterals generated under the same mapping by $\{0,1,4\}, \{1,2,5\}, \{2,4,11\}$ and $\{0,5,11\}$.

A2 $\{0,1,4\}, \{0,2,12\}, \{0,5,13\}$, group order = 57. This system is anti-Pasch.

A3 $\{0,1,8\}, \{0,2,5\}, \{0,4,10\}$, group order = 57. This system contains 38 quadrilaterals generated by $\{0,1,8\}$, $\{0,4,10\}$, $\{4,8,14\}$ and $\{1,10,14\}$ and $\{0,2,5\}, \{3,5,8\}, \{0,3,17\}$ and $\{2,8,17\}$.

A4 $\{0,1,8\}, \{0,2,5\}, \{0,4,13\}$, group order = 171. This is the Netto system mentioned in section 1 and is anti-Pasch.

(II) The basic automorphism type $1^1 2^9$ of these systems means that a convenient representation of them is on the set $\{0, \pm 1, \pm 2, \ldots, \pm 9\}$ with the permutation $(0)(+1\ -1)(+2\ -2)\ldots(+9\ -9)$ as their reversal. Then by ignoring blocks which contain the fixed point 0 and dropping the signs in the other blocks a 2-$(9,3,2)$ block design containing no repeated blocks (strictly two identical copies) is obtained. We call this the underlying configuration of the reverse STS(19). Hence all reverse STS(19)s can be constructed by considering all such non-isomorphic 2-$(9,3,2)$ block designs without repeated blocks on the set $\{1,2,\ldots,9\}$ and "re-signing" the elements. All such designs, there are precisely 13 of them, were first published by Gibbons [7] and Denniston uses this approach in his enumeration of reverse STS(19)s in his paper [5]. For economy of space and because we do not refer overmuch to these designs we do not list them but direct the reader to Denniston's paper [5] where they are also listed. We follow his labelling. That many of the reverse STS(19)s are not anti-Pasch can be deduced from the following simple result.

Theorem. *If a* 2-$(9,3,2)$ *block design contains a collection of four blocks of the form* $\{x,b,z\}, \{x,y,c\}, \{x,b,y\}$ *and* $\{x,c,z\}$ *then any reverse* STS(19) *having this* 2-$(9,3,2)$ *block design as its underlying configuration is not anti-Pasch.*

Proof: Denote by B the collection of blocks of the reverse STS(19). Then $\{x1, b1, z1\} \in B$ where $x1 = \pm x, b1 = \pm b$ and $z1 = \pm z$. Further $\{x2, b1, y1\} \in$

B where $x2 = -x1$ and $y1 = \pm y$, and in addition $\{x1, b2, y2\} \in B$ where $b2 = -b1$ and $y2 = -y1$. Similarly $\{x2, c1, z1\} \in B$ where $c1 = \pm c$, as well as $\{x1, c2, z2\} \in B$ where $c2 = -c1$ and $z2 = -z1$. Finally it must follow that $\{x1, y1, c1\} \in B$ since the blocks containing $\{x1, y2\}$ and $\{x1, c2\}$ are identified above. The blocks $\{x1, b1, z1\}$, $\{x2, b1, y1\}$, $\{x2, c1, z1\}$ and $\{x1, y1, c1\}$ form a quadrilateral and so the STS(19) is not anti-Pasch.

The above theorem means that any reverse STS(19) whose underlying configuration is $D2, D3, D4, D7, D8, D9$ or $D13$ can not be anti-Pasch since from Denniston's listings

$$\{1,4,5\},\ \{1,5,6\},\ \{4,5,9\},\ \{5,6,9\} \in D2,$$
$$\{1,6,7\},\ \{1,7,8\},\ \{2,6,7\},\ \{2,7,8\} \in D3,$$
$$\{1,3,4\},\ \{1,4,5\},\ \{3,4,8\},\ \{4,5,8\} \in D4,$$
$$\{1,4,5\},\ \{1,5,6\},\ \{4,5,9\},\ \{5,6,9\} \in D7,$$
$$\{1,2,7\},\ \{1,5,7\},\ \{2,7,9\},\ \{5,7,9\} \in D8,$$
$$\{1,3,9\},\ \{1,7,9\},\ \{3,5,9\},\ \{5,7,9\} \in D9, \text{ and}$$
$$\{1,5,6\},\ \{1,5,8\},\ \{3,5,6\},\ \{3,5,8\} \in D13.$$

This eliminates precisely 100 of the 184 reverse STS(19)s and since, as is proved in Denniston's paper, no reverse STS(19) can have $D1$ as its underlying configuration only leaves $D5, D6, D10, D11$ and $D12$ for consideration. This latter proved to be somewhat tedious and no easy mathematical argument similar to the above was found which would eliminate large numbers of such systems from being anti-Pasch. Essentially the calculations involved finding representatives of all of the reverse STS(19)s having $D5, D6, D10, D11$ and $D12$ as their underlying configurations and then examining these for their quadrilateral content firstly by mathematical reasoning and finally for those systems which were stubborn to attack by this method, by computer analysis. We omit the very lengthy calculations and state the results.

$D5$ None of the 32 non-isomorphic reverse STS(19)s having $D5$ as their underlying configuration are anti-Pasch.

$D6$ None of the 32 non-isomorphic reverse STS(19)s having $D6$ as their underlying configuration are anti-Pasch.

$D10$ Two of the 8 non-isomorphic reverse STS(19)s having $D10$ as their underlying configuration are anti-Pasch. One of these ($D10\,A$) has automorphism group of order 2 and the other ($D10\,B$) has the cyclic group of order 6 as its automorphism. 24 blocks of each of the systems are given below in a simplified notation omitting set brackets and commas, another 24 blocks being the reversals of these and the remaining 9 blocks being $\{0, +i, -i\}$,

$i = 1, 2, \ldots, 9$.

($D10A$)	+1+2-4	+1-2+7	+1-3+6	+1+3-8
	+1+4-5	+1+5-9	+1-6+9	+1-7+8
	+2-3+5	+2+3+8	+2+4+9	+2-5+6
	+2-6+7	+2-8-9	+3-4+6	+3+4-9
	+3+5+7	+3-7+9	+4+5-7	-4-6+8
	+4+7+8	+5+6+8	+5-8+9	+6+7+9
($D10B$)	+1+2-4	+1-2+7	+1-3+6	+1+3-8
	+1+4-5	+1+5-9	+1-6+9	+1-7+8
	+2-3+5	+2+3+8	+2+4-9	+2-5+6
	+2-6+7	+2-8+9	+3+4+6	+3-4-9
	+3+5+7	+3-7+9	+4+5-7	-4+6+8
	+4+7+8	-5-6+8	+5+8+9	+6+7+9

$D11$ Two of the 8 non-isomorphic reverse STS(19)s having $D11$ as their underlying configuration are anti-Pasch. The automorphism group of both is cyclic of order 6 and both systems are listed below using the same conventions as in $D10$.

($D11A$)	+1+2-3	+1-2+6	+1+3-9	+1+4-7
	+1-4+8	+1+5-6	+1-5+7	+1-8+9
	+2+3-4	+2+4+7	+2-5+8	+2+5+9
	+2+6-8	+2-7-9	-3-4+5	+3+5+8
	+3+6-7	+3-6+9	+3+7-8	+4+5+6
	+4-6-9	+4+8+9	+5+7-9	-6-7-8
($D11B$)	+1+2-3	+1-2+6	+1+3-9	+1+4-7
	+1-4+8	+1+5-6	+1-5+7	+1-8+9
	+2+3+4	-2+4+7	+2+5-8	+2-5+9
	+2+6+8	+2+7-9	+3-4+5	-3+5+8
	+3+6-7	+3-6+9	+3+7+8	+4+5+6
	+4-6-9	+4+8+9	+5+7+9	-6-7+8

$D12$ Only 4 of the 184 non-isomorphic reverse STS(19)s have $D12$ as their underlying configuration and their automorphism groups have orders 18, 6, 6 and 2. We discover that both of those whose automorphism group is C_6 are anti-Pasch but the other two are not. In particular the system whose automorphism group is of order 18 contains precisely 18 quadrilaterals. Again we list both systems using the conventions as in $D10$.

($D12A$)	+1+2-3	+1-2+5	+1+3-4	+1+4-8
	+1-5+7	+1-6+8	+1+6-9	+1-7+9
	+2+3-6	+2+4-7	+2-4+9	+2+5-9
	+2+6+8	-2-7+8	+3+4+9	-3+5+7
	+3+5-8	+3+6+7	+3+8-9	+4-5+6
	+4+5+8	+4-6+7	+5+6+9	+7+8+9
($D12B$)	+1+2-3	+1-2+5	+1+3-4	+1+4-8
	+1-5+7	+1-6+8	+1+6-9	+1-7+9
	+2+3-6	+2+4+7	+2-4+9	+2+5-9
	+2+6+8	-2+7+8	+3+4+9	+3+5+7
	+3-5+8	+3+6-7	+3-8-9	+4-5+6
	+4+5+8	+4-6-7	+5+6+9	+7-8+9

(III) A computer analysis indicates that none of the 7 STS(19)s having an automorphism group of order 12 are anti-Pasch. For the record the number of quadrilaterals in each of the systems, in the order in which they are given in the supplement to the paper of Colbourn, Magliveras and Stinson [4] is 15, 7, 14, 22, 18, 18 and 23 respectively.
(IV) As in (III) we refer to the STS(19)s having an automorphism group of order 9 in the order in which they appear in the supplement to [4] and number them 1 to 19. For each system we give below the number of quadrilaterals and also identify which systems are 2-rotational using the listing of the latter by Mathon, Phelps and Rosa [9].

#system	#quadrilaterals	#system	#quadrilaterals
1	48	11	18
2	21	12($\sim B1$)	36
3	15	13	0 (anti-Pasch)
4	15	14($\sim B7$)	27
5	15	15	0 (anti-Pasch)
6	15	16($\sim B2$)	9
7($\sim B10$)	0 (anti-Pasch)	17($\sim B5$)	18
8	9	18($\sim B4$)	18
9($\sim B6$)	18	19($\sim B3$)	36
10($\sim B9$)	0 (anti-Pasch)		

For completeness, explicit listings of the four anti-Pasch systems follow.

#7($\sim B10$): Let the elements be $Z_9 \times \{1,2\}$ together with ∞. The system is generated under the action of the mappings $i_n \mapsto (i+1)_n \pmod 9$, $n = 1,2$

from the blocks

$$\{\infty,0_1,0_2\},\ \{0_1,3_1,6_1\},\ \{0_2,1_2,3_2\},$$
$$\{7_1,0_2,4_2\},\ \{1_1,8_1,0_2\},\ \{4_1,5_1,0_2\} \text{ and } \{2_1,6_1,0_2\}.$$

#10($\sim B9$): Using the same notation as above base blocks are

$$\{\infty,0_1,0_2\},\ \{0_1,3_1,6_1\},\ \{0_2,1_2,3_2\},$$
$$\{6_1,0_2,4_2\},\ \{1_1,8_1,0_2\},\ \{4_1,5_1,0_2\} \text{ and } \{3_1,7_1,0_2\}.$$

#13: Let the elements be {a,b,c,...,q,r,s}. The system comprises the following blocks again omitting set brackets and commas.

abd	ace	bcf	cdg	beh	dei	afi	dfj	efk	agh	bgk	egl
fgm	chj	dhn	fho	bip	ciq	gij	hir	ajs	bjr	ejp	ako
ckn	dkq	hks	ikm	jkl	alr	blq	clo	dlm	fls	hlp	iln
amn	bms	cmp	emr	hmq	jmo	bno	ens	fnp	gnr	jnq	dor
eoq	gop	ios	apq	dps	kpr	fqr	gqs	crs			

#15: Using the same notation as in #13 the blocks are

abd	ace	bcf	cdg	beh	dei	afi	dfh	efj	agj	bgk	egl
fgm	ahn	chl	gho	bio	cip	giq	hir	bjr	cjo	djp	hjk
ijs	akp	ckq	dkm	eks	fkn	ikl	alq	blm	dlr	fls	jln
amo	cmr	emp	hms	imn	jmq	bnp	cns	dnq	eno	gnr	dos
foq	kor	lop	fpr	gps	hpq	bqs	eqr	ars			

Finally to summarize the results of this section. We have exhaustively examined all non-isomorphic STS(19)s having an automorphism group whose order is greater than or equal to 9 using primarily the listings given by Colbourn, Magliveras and Stinson [4]. Precisely six are anti-Pasch. These comprise firstly the Netto system with automorphism group of order 171 and which is both cyclic and 2-rotational. One of the remaining three cyclic systems is also anti-Pasch and has automorphism group of order 57 as are two of the remaining nine 2-rotational systems with automorphism groups of order 9. Finally there are two further anti-Pasch systems with automorphism group of order 9.

Concerning reverse Steiner triple systems of order 19, the one with automorphism group of order 18 is not anti-Pasch, five of the fourteen having a cyclic group of order 6 are anti-Pasch, but only one of the remaining 169 which just have the reversal as their automorphism is anti-Pasch. Explicit listings of all these systems have been given above. We have checked that none of the twelve anti-Pasch STS(19)s described in this section are isomorphic to any of the systems listed in the Appendix. Hence we have in fact more than the 101 pairwise non-isomorphic anti-Pasch STS(19)s constructed by hill-climbing and to which the title of the paper refers. However as stated in section 1 there are undoubtedly very many more.

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Appendix

Below are given listings of the 101 anti-Pasch STS(19)s constructed by the modified hill-climbing algorithm described in Section 2. Following the example of Colbourn, Magliveras and Stinson [4] we use a compact notation similar to, though in detail different from, the one used in that paper. In what follows the base set is $\{a, b, c, \ldots, r, s\}$ and the blocks are represented by a string of symbols $s_1, s_2, \ldots, s_{57}$. Using the usual lexicographical order, the symbol s_i is the largest element z_i in the ith triple $\{x_i, y_i, z_i\}$ where $x_i < y_i < z_i$. The remaining two elements implicitly have the property that there is no pair $x_i' < y_i'$ such that $\{x_i', y_i'\}$ does not appear in an earlier triple and either (i) $x_i' < x_i$ or (ii) $x_i' = x_i$ and $y_i' < y_i$.

No.	Blocks
1	JINGLQRSPQKNSIMORHRMPLNSQGOMRSJOMSPKPQRJLORSRSPKQSOQSPQNR
2	JMESILOPRDHRPSNMQKJSORPQNQPKLSRQLPSROKIOPRONNSQQNOMRRQSSS
3	IGSQPMONRLOFMKRSQNORSPMQIMQRLPLQNSRJILQSORSPNPKONSQSRQSPR
4	DFIMJSRQPILSRQPNOHONQMSRSOKMRQPGNKRQLQMPRSJPQROPLMSOSRNSS
5	LDPQINOMSJIQNHRPSRLKPNSQKHQRMSPMSILOJROSPNRSQROMSOQPQNSRR
6	JDQMORSLPFKLRPQNSRJISNPQNLIQPSRHKMOSSJQRPMPQLOSPRONQSROSR
7	SJIHQPLROKRJMNOQPENSMROQHMLOQSPOMQNSJORSQKRLPSRSPNORPQSRS
8	PGIHJONQSSORGNLMQRPKMNQOQLJSNPIONMSQRSPIRMPSPLRKSQRRSSQOR
9	DMQIPSNLRNRQJKSOPRFLJPOSJGNOQPSHKSOPPSRMOQNRSMRQRNMQPSQSR
10	HRNFMKQPSGSLQOPRNQOLSNMPPKRJLOQKRNSHMSRPJONSPMORSLRQSQSRQ
11	NLJOKHRSQGRFPMKOSEJKNQPSIQSPNOPMKQRSROQNSOMNSQLRSPPRMSRRQ
12	LKSGRNJQPMRKQNIOSNOGLSRQHOKQLPLMSQRSNMPRPQMSPQOORNSRPQSRS
13	GMKLPSJQRHRNQLPSOOPGQKRSQHJNPSIMKRSSOMRRSQOPMLOQRPNQNPSRS
14	HGPOJQRMSLJNOIPQSKFJRNPSRGISOQSMPKQLMNSRRNLPQQPSSONORQSRR
15	CKFPQMONSNOLJSPQRGMHNQLSRHRSPQOSKLPROJPQSNQMRLROPRSNRSPQS
16	IDMJLPOQSPSHNQORMSGMRKQOKPONQLRQIPORILSRKRNSQRSMNSPSPSPQR
17	EHOLJRSQPFMHQORNSRKSPQMOSNILPQPLQJNROKMRSNRQONPSLSMSPQRSR
18	RFOHMQKNSJQSPHMNORMLNKSQPMKLNSOILQRJRSLQSPRJOPSPRRPQSRQSO
19	NSILOHQPRQGPJMKRSMOGNJRPHLOQSRQNMKSRPSNORMQSQSLPSRNPOOSQR
20	HQGMRNOLSFKPORSQNPSNMJOROIQLRSLHQRNJKSQPKSPRPNOSMSMQRSPQR
21	NDMLJOKRSKEGMQRPSONSIMRQINJQSPRSRLPKQQPROKMQPRSLOSSQOSPNR
22	HOKFJRQPSPQRJISMOJHIRLNSGLMSPRLONSQSPORQNMPQQKOSNPOSQRRRS
23	HFLPMKNQSOIQKSPMRGHNMQSRFROSQPLRSNOPJLROSOJRQQMNSPSLRPRQS
24	RHLMQPKSOGQJNSPOMFPRQSNOSOINRPHKQORIKPRSLMSRPMQROSNRQPSQS
25	GLEMOJSRQEPHOKQNSKJIRPQSLMSRQOOJQPNSRQKSRNORSMLPNSSPRMRPQ
26	HIFRKOSNQEKJNLORSGPSNROQQOSRNPGMJNPSIQRSORQSMPPLQORPMQSSR
27	EOIPMSKQRQFNOSRMPGPKMRNSQPOSNRHOJNRSROSMQKLPSNLRQPQOROSQS
28	SRPHIKLNQQIKMLPRONPJOMKSMOSKQRGJOQSRNQSJMRQQNSPSROSPMRSPR
29	MHNIKJPRSRKSJPNOQJGOPNSQFLPRSQOPLNRISNRQQROSKMRSMQQORPSPS
30	LEMKRSOQPNGHOJMSRQMSKRPOIPLSROSPRLNQQJNROLMNPRQSQONQPSSSR
31	GJRNHQLPSHNPMSRQOFIQMRSPOLISPQGQLPNRSPRNSOKSROMOPQNQRS
32	RQNPKJIOSLPKJQOSNFIKNORSLOMKSRSRJOQHMNRQNSPLRSQRPMPRPQSQS
33	FGLINPRQSDMIONPRSFIMQSORHSMJNPRPRLSQOPKQMRRLSQSMQONQSOPRS
34	NDQSMJKORSHGPMRLQJHOPMQRPOJSRNQRSONMLNMQISPROPRQQRPSOSPSS
35	OHIRNSQLPIJHPLQRSEJKRNPSKRPSQOSMKONQQLMRJOPNMQSRQSNPSSROR

No. Blocks

36 DHILOQRSPGJORSNMQSRMNPOQKNJLOQRQNPSOPKRSSQLRMOQKPRNPSOSSR
37 DKJGQORPSGSPJKONROPHNRSQLQRSPMNOMIQRMSRNLJSPQPROQSOPQMSRS
38 EQKGJRSNPJQNIOLPSLKRHNOSOPMNSRHSQMPRLKSQLQPRMPRSPOQSRSQOR
39 JFINLOPSRQFMSKLORGPNJRSOHRONQSOQRLSHQSMPRKORPPQMSNSNPQRSQ
40 REIJNPSMQSNPHJQMORMOQKNPMLHKPSGSJONRORSQSPQRMOLRRPSQRQOSS
41 EOLRKJMPSHFJLOPQSPMSNKRQJHSQNRPRKNSQIQNLOOSQRMOSQPMPSSRRR
42 QOESRKNPMSFGNLOMRMKJNRPQJLQNRSLJMPQSQPKRQMSOPOSRSNQRQSSPR
43 DNSKQMRPOPNSMROLQOFHKQMSLNSJMRQRKPMQJPQROLOPSQONSRPNSRSRS
44 SOQFRLJPNJIRNOKMQEGRMQSPPLJNRSMINOPSSOLMRPQKSOQSNRQPSRQSR
45 EQPLKOMRSFLSRKQPOKNISPQRGQJSORMILOQSNKJRSQMRPMPNQRNSPSRSQ
46 OJMNGISQRKJQHLRNSQROMNSPFSOLRPOLJPSKNSRQPNRQKQSPQMPSNORSR
47 NHLKRIOQSFSMHLQRPEQKPMOSPROQNMONJPRSSLNMQKPOMSRQRSSOPQSRR
48 NHLIJKPSRIGOMQKSRELQMSRPPJNOQSSLKPRQNIRQPRSOMSRQPOSROQNPS
49 KMQIHNRSPSOLQPIMRPJNROLQNRMLSKSKROQOJLPSQLPNQMORQSRSRPQSS
50 OKLHNSMRQLMQJHKPSGNRIOPSPHQNRSSLJMRPOQMNMORKRSQPSRQSPSORQ
51 OILNRHKPSSMGLJRPQRLJQPNOQNOSKPHMSORIMSQRSMPNLRQOQSNQRPSSR
52 GHFQNSPORROMSJLNQNIOLQMSKISRQPNSPLRPMJLRQKORKROQSRNPQSSPS
53 HGRNPSQMOMJLQORNSFIPKNRSQLSKNPMHSORJNOSRMPQSQOLRLPPRRSQQS
54 EMRIHQSOPKNJMSLRQILQRNSPHGPMSQMQPONSPRNSSLPOQROMKORNQRSRS
55 LOGQHKSNRMNGPSRQOFKPQJRSIJRQOSOLMPSRQNSMIKOSQMRPSPOPRNQRS
56 EKGRSOQNPFIJRSONQMJOPSQRRQKPSOMKLNQSSJLOPNRPQQOMPNRSMRSSR
57 IGHROKQNSFMPJQORSROJLNSQIPNOLSNMLQSLISRQSOQRPORPMQSNRPSPR
58 EOFSRNQPMDPQOSRMNIQRKNSPKHQSORPOMNLQSNJLRSOPLMPSKROSQRQRS
59 NDLSHRMQPKRIGMSOQQLJIOSRGKOLNQSPJONSQMORQNPSRRSPPSOLRPRQS
60 JOGHKPSRQMPFRSQNOEQJRSPNOLMNSRKLMORSIMPRSOQNSKQPORLSQRPSQ
61 CJSHMLORQNGRSMKQPIJKRMOPSOQKLSRLNRPQJPNSPSNQKORQORPSMQSSR
62 SGFKILNRQEOMQJKNRSHJQMRPHKPMRQNJROQSOLPSRRNSPOQSPOSNQSPRS
63 DLSGPMOQREMHJLPQSHPIRNOSOSJQRNPRKJLNQOKNQOSQRNMSRRSSPQPRS
64 MDJQSPLROSOFKNPRQLOJQNPRPLRMJNSQKROSPINMSOMNSRQSOMQSPQSRR
65 DOPISRMNQFGQKNOSRSQILKPRMJROQLPKNROSMSROQPQONJMMSPPSSRQRS
66 NLSOMJQPRMRIKOPSQEJSRNOQPHJLNQRKNLQSQINSLPROMSQSRPRQSORPS
67 PMKFLNJSRGLORISMQIQLOPNSNQJSRPKJPRSHSMPOMSRQLQRRNQOQRSOSP
68 HQGMOKPNSLIRSQMOPRFJSOPNQKONMSKLNSPNPLQRRPSOMKQRSOQSRRQSR
69 GOPKSMJRQPFIJLOSRMGJQLSRJNLROSQOSRPONKMRRMSQPSPOQRNQPNQSS
70 DMHSRLKQPNOPHMQRSFKQRSPOIPKLQOSQLSNROLJNRNMSQOPSPRRMQPRSS
71 FLNKJIQRSMJOQKLPSKSGRNPQGMPROSJNQRPQOSPRLPRNSSOKSQROMQPSR
72 OMFKSJNRQFIGKQMPSLSJNRQPJPQSQRNPOQRHQLMSRMRNQLSRSPNPROSOS
73 KRMPJLOSQPRHLOQNSGJOSMQNLKJNSQGKRSQNRPSIMSPRLQROPRQSNPOSR
74 IQGOHNSMROFRSKLQPRJMNIPSPNLSMQLMQKSKNOSROJRQRPSRPSQPORQOS
75 DOLMNQPKSNOQJISPRSFQLKRPMJIKQRPKJSPRPOLRSSRONRMQRNQSMOQSS
76 CPGISLMORGLSJKRQPKMPSORQNIRLNOSHRNSQOKQNKQPMRNQROSPSPQSRS
77 MDHKLROPSRHGNOQSPQMHSNPOPJNKQSRLMSORQRPSJMPSLMNQSQLRRRSOQ
78 PIEONKQMSNLIRHSMQFQJMPSROSRMQPHLONRSSNKQPPRQJRQPLOSPRSOSR
79 PJHMKRSQOHSRGKNMQEROQNPSNKOLPRPQKNOSSLQOLPSNJQPRRMROQSRSS
80 JQNOILPRSIPLMQKOSHRNMOSPFOSQLRHQPMSRQLOSNSPRMNSKMPRQRSPQR
81 JKLOGPMRSOPRINSLQNGQJLRSQJKISRSKJMPLRNPRSOQPOQPORMNQSSQSR
82 CFONLQPRSLHIRKOPSQJGPMSRONSKOMRKIMQSSOPRQQLOPJNRPSRSRNQQS

No. Blocks
83 ORMSNQPLKHFLIQSRPNPILOMSIRKSOQRMONQJMPSQNOSSRQPRQOMNRSPRS
84 QKNHGOLRSLPRMKNSOOFRIMPSQISMKROJSNPLRPQSPMNQSRQNPRQOQSOSR
85 PNEIQMLOSFQKRSMONIOMPQRSRLNKOSPIRSMQNQMLSJSPKOPROSRQPSQRR
86 MPOJNISLRNIHLPQRSGKSJQMRSMQLPROQPNRJIQRMNOSORPRMSOSNPQQSS
87 HPIRKMNSQMOSIJRQPJQHROLSKLPSQROHMPNSRPQNQOPQNMRLSRMSPROSS
88 KDMQJPLRSHLOGQMSRRLOKNPSKJHPSQRHISQPNROSSRNQMRNSQOSQPMPOR
89 GLQPSNKORKSJRINOQEOPSJNRMKLORPHRMLOSNQKPJLQSMQRRSQPSQPSSR
90 CMONHJSRQRNIOKQSPHJKSRMPQQLIPNSPMRSLQORSKNRNMQSQPOOSRPQRS
91 PJGQRLOSNLHNGJRSQPHKQNRSOQSNMRJSMLRSPNONLROPKMRQRPOSQQSPS
92 PDILRKSOQFMJHNLQSGIRNOSQNQORSKPOLSPRSPRNMPMQNQNSLOQPRRSRS
93 PHOKGMNRSIFHNORSQRJLSPNQQLNPKSOINRSNQPSRKMPRSQORLNRSQOPSQ
94 NDJGMQPSROLSRMKJQFHMRSQPKOJNPSRQIPORLNMQSSNRPPQSROONSMQRS
95 LGRIOKNSQIESNMPQRMNLOKRSNLPSQOKQSRPOJQPRIMPSLRMNORSRSQSPQ
96 PNFQILKOSOGSMRNQLPHSJLMRKQJOSRINRNPKNLQRPRQSOMSPRSOSQRSQP
97 ORHNMIPQSKLIQSMNRNJSOLPQGOKQRSHSRPQPLRNJMQRNRPSSOROMPOSQS
98 SFIOHRPNQMLGOKQPRSIOLNRQNKQROPPQLSRNJRMSPMLSMPSSNOQOSRSQR
99 SHQFORNPMIKQMHPROGROMLSQOIJPNSPNJMSLPSRQMRNSKQSRNQSQOSPRR
100 DPQNMJKSRNKOIMPQSMIJLKSRNSQLJORLORSPHQPRRSPSQPPORNMQRQOSS
101 PSLRIMQNOIMLHRKSQQMOPJRNSRKPJOQHKOPNPMSOLSRNMSPSQQSQSRRPR