J. COMBIN. MATH. COMBIN. COMPUT. 131 (2026) 183–191

Journal of Combinatorial Mathematics
and Combinatorial Computing
www.combinatorialpress.com/jcmcc

Combinatorial Press

Constructing Hadamard 3-balanced incidence structures with small automorphism groups

Ivica Martinjak*

ABSTRACT

It is known that an automorphism group G acting on a symmetric, uniform, and balanced incidence structure $\mathcal{D}$ has the same number of orbits on the set of points and the set of lines of $\mathcal{D}$. Moreover, a group G induces a tactical decomposition of $\mathcal{D}$. These facts are often used to perform efficient constructions of various combinatorial designs and other incidence structures. In this paper, we use a variation of this approach to construct Hadamard 3-balanced incidence structures by employing an automorphism of order 3. We are also able to reach structures with a small automorphism group.

Keywords: Hadamard 3-design, incidence structure, symmetric design, tactical decomposition, automorphism group

2020 Mathematics Subject Classification: 05B05, 05B20.

1. Introduction

Combinatorial t-designs are a central topic in discrete mathematics due to their deep connections with group theory, finite geometries, coding theory, and other branches of combinatorics. While 2-designs are relatively well understood and classified for many parameter sets, 3-designs remain significantly more difficult to construct and analyze.

While within 2-designs every two points lie on λ lines, 3-designs are even more regular having every triple of points lying on the λ lines. Obviously, every t-design, $t \geq 2$, is also a 2-design. More precisely, every t-(v, k, λ) is a s-(v, k, λ_s) design, $0 \leq s \leq t$, where

$$\lambda_s = \lambda \frac{(v-s)(v-s-1)\cdots(v-t+1)}{(k-s)(k-s-1)\cdots(k-t+1)}. \tag{1}$$

* Corresponding author.

Received 02 Oct 2025; Revised 20 Jun 2026; Accepted 27 Jun 2026; Published Online 20 Jul 2026.

DOI: 10.61091/jcmcc131-10

As an example, note that a design with parameters 3-$(14, 4, 1)$ is also a 2-design with $\lambda = 6$. The four structures with these parameters and having large automorphism group, are found by N. Mendelsohn [12]. There are several infinite series of 3-designs [4, 10]. A common property of these structures is that they have many more blocks than points.. In particular, a class of 3-designs having parameters

$$3 - (4n, 2n, n-1), \quad n \geq 2, \tag{2}$$

is usually called *Hadamard 3-designs*. If a Hadamard matrix of order $4n$ exist, then there is also a 2-design with parameters

$$2 - (4n-1, 2n-1, n-1).$$

Conversely, every design with these parameters corresponds to a Hadamard matrix..

The smallest representative of the family (2) has parameters 3-$(8, 4, 1)$ and up to isomorphism there is only one such structure whose automorphism group has order 1344. An elementary construction of this structure by considering cube vertices is well known. To construct such structures for larger parameters, one usually uses methods based on group action [1, 3, 8]. In particular, one can use the method of tactical decomposition [7, 11]. An overview of classification algorithms for combinatorial designs can be found in [6].

In what follows we perform constructions of Hadamard 3-designs admitting an automorphism of order 3. More precisely, we use a variation of the standard tactical decomposition from [9], in order to obtain structures with small automorphism groups.

Previous computational classifications of block designs frequently relied on relatively large automorphism groups in order to reduce the search space. In contrast, we assume only the existence of an automorphism of order 3. Although this leads to significantly larger computational tasks, it allows the construction of designs with very small automorphism groups, including rigid designs with trivial automorphism group.

2. Preliminaries

An *incidence structure* is a triple $\mathcal{I} = (\mathcal{P}, \mathcal{B}, I)$ where

$$\mathcal{P} = \{p_1, \ldots, p_v\}$$

is a set of *points*,

$$\mathcal{B} = \{B_1, \ldots, B_b\}$$

is a set of *lines* or *blocks* and $I \subseteq \mathcal{P} \times \mathcal{B}$ is an *incidence relation*. If every point $p \in \mathcal{P}$ is incident with the same number r of lines, the structure $\mathcal{I}$ is called *regular* with the degree of regularity equal to r, denoted $d(p) = r$. Similarly, the structure is *uniform*, with the degree of uniformity equal to k if $d(L) = k, \ \forall L \in \mathcal{L}$. Finally, the structure is *balanced* if every t points is on the same number λ of lines. These three properties are not independent: uniformity and balanced property imply the regularity. The *order* n of a regular incidence structure is defined as the difference $n := r - \lambda$. Uniform and balanced incidence structure is uniquely determined by the 4-tuple of parameters $t-(v,k,\lambda)$ and it is called *t-design* or *balanced incomplete block design*. When lines $B_i, i=1,\dots,b$ are consisted of points, $B_i=\{p_{i_1},\dots,p_{i_k}\}$, we write $B_i=\{i_1 i_2\cdots i_k\}$, in short.

Thus, the four parameters, t,v,k and λ, uniquely determine a t-design $\mathcal{D}$. We let $\mathcal{D}_n$ denote a class of t-designs $\mathcal{D}$ of order n. There are several notable series of t-designs [10]. For example, a wide class of t-designs are finite geometries, with finite projective planes and biplanes as the best known representatives.

Recall that an isomorphism on a combinatorial design is a map preserving the structure. We let $\mathcal{D}=(\mathcal{P},\mathcal{B},I)$ and $\mathcal{D}'=(\mathcal{P}',\mathcal{B}',I')$ be two t-designs. A bijection $\phi:\mathcal{P}\cup\mathcal{B}\to\mathcal{P}'\cup\mathcal{B}'$ is an isomorphism if

i) ϕ maps points onto points and blocks onto blocks, and

ii) $(p,B)\in I \Leftrightarrow (\phi(p),\phi(B))\in I',\ \ \forall p\in\mathcal{P},\ \ \forall B\in\mathcal{B}$.

Two t-designs $\mathcal{D}$ and $\mathcal{D}'$ are *isomorphic*, $\mathcal{D}\approx\mathcal{D}'$, if there exists an isomorphism of $\mathcal{D}$ on $\mathcal{D}'$. An isomorphism of t-design $\mathcal{D}$ is an *automorphism* of $\mathcal{D}$. It is known that a set of all automorphisms of $\mathcal{D}$ form a group, which is called a *a full automorphisms group* and it is denoted by $Aut(\mathcal{D})$. An *automorphism group order* $|Aut(\mathcal{D})|$ is the cardinality of a full automorphism group of $\mathcal{D}$.

Naturally, an incidence structure is represented by its incidence matrix, whose entries define the incidence relation I. We let $\mathcal{D}=(\mathcal{P},\mathcal{B},I)$ be a t-design where $\mathcal{P}=\{p_1,\dots,p_v\}$ and $\mathcal{B}=\{B_1,\dots,B_b\}$ are set of points and set of blocks, respectively. An incidence matrix of $\mathcal{D}$ is a $v\times b$ binary matrix $M=[m_{ij}]$ defined by the rule

$$m_{ij}=\begin{cases}1, & \text{if } (p_i,B_j)\in I\\ 0, & \text{if } (p_i,B_j)\notin I.\end{cases}$$

Throughout this paper we represent considered Hadamard designs by its incidence matrices.

Sometimes, a design $\mathcal{D}$ is represented by five parameters t,v,b,r and k, denoted

$$t-(v,b,r,k,\lambda).$$

A remarkable property of an uniform and balanced incidence structure is relationship among these parameters,

$$v\cdot r=b\cdot k. \tag{3}$$

To see this, we count the elements of set

$$S=\{(p,B)\ \mid\ p\in\mathcal{P},B\in\mathcal{B},p\in B\},$$

in two different ways. Firstly, each $p\in\mathcal{P}$ is incident with r blocks. Thus, $|S|=v\cdot r$. On the other hand, each of b blocks contain k points.

3. Partial classifications and constructions of Hadamard 3-designs

In addition to the mentioned approaches on constructions and classification of t-designs, recall that the method of tactical decomposition were introduced and applied in papers

[2, 5]. In this work we use a variation of the method that is precisely described in [9]. Namely, in the second step of the construction procedure we allow combination not only among cyclic matrices but both cyclic and anticyclic matrices. We denote the basic type of construction by "Cyc" and the second type by "ACyc". It follows from the definition of an incidence matrix M of a t-(v, b, r, k, λ) design that it satisfies the following properties:

i) every column contains exactly k "1"s,

ii) every row contains exactly r "1"s, and

iii) every t-tuple of rows intersect in exactly λ "1"s.

In order to carry out the final step of our construction procedure, we develop an algorithm which checks these properties to determine whether a "matrix candidate" is a valid incidence matrix.

In all computations, every admissible tactical decomposition matrix was expanded by a custom C program generating all incidence matrix candidates compatible with the prescribed decomposition. Each candidate was tested against the defining conditions of a Hadamard 3-design. To eliminate isomorphic copies, every obtained design was transformed into its incidence graph and processed using the nauty package of McKay [13]. Canonical labeling was employed to partition the generated designs into isomorphism classes, and only one representative of each class was retained.

For the parameter sets 3-$(12, 6, 2)$, 3-$(16, 8, 3)$, 3-$(20, 10, 4)$, and 3-$(28, 14, 6)$, all incidence matrices arising from the considered tactical decomposition matrices were exhaustively generated and tested. Consequently, the reported numbers of non-isomorphic designs are exact within the class of designs admitting an automorphism of order 3.

For the parameter set 3-$(24, 12, 5)$, the number of possible completions became prohibitively large and the search was not exhaustive. Therefore, the results reported in Section 3.2 represent only a partial classification and provide lower bounds on the number of non-isomorphic designs obtained.

The second class of designs $\mathcal{D}$ in the series (2) has parameters

$$3 - (12, 22, 11, 6, 2).$$

According to basic properties of the 3-designs, these designs $\mathcal{D}_9$ have $b = 22$ lines and every point is incident with $r = 11$ lines. In the first step of the construction procedure, we assume that an automorphism $g \in Aut(\mathcal{D}_9)$ of order 3 acts on $\mathcal{D}_9$. We obtain two candidates for tactical decomposition matrices, T_1 and T_2, one with four fixed blocks and no one fixed point and another one having three fixed points and four fixed blocks, respectively. When attempting to expand these matrices into incidence matrices of a design, only T_2,

$$T_2 = \left[\begin{array}{cccc|cccccc}
1 & 1 & 0 & 0 & 3 & 3 & 3 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 3 & 0 & 0 & 3 & 3 & 0 \\
1 & 1 & 0 & 0 & 0 & 3 & 0 & 3 & 0 & 3 \\
\hline
1 & 0 & 1 & 0 & 1 & 1 & 2 & 1 & 2 & 2 \\
0 & 1 & 0 & 1 & 1 & 1 & 2 & 1 & 2 & 2 \\
0 & 0 & 1 & 1 & 2 & 2 & 1 & 2 & 1 & 1
\end{array}\right],$$

produces valid designs, whereas in the first case no admissible combination of cyclic matrices leads to a valid incidence matrix. Matrix T_2 yields two isomorphic incidence matrices. Related automorphism group order is $|Aut(\mathcal{D}_9)|= 7920$.

To reach designs with smaller automorphism groups, we carry out constructions in which both cyclic and anticyclic matrices of order 3 are permitted. However, for these parameters of designs this approach results only with already constructed design. The facts presented above prove Theorem 3.1.

Theorem 3.1. *There is unique design $\mathcal{D}_6$ with parameters 3-$(12,6,2)$ admitting an action of an automorphism of order 3, having the order of the full automorphism group $|Aut(\mathcal{D}_6)|= 7920$.*

In the same manner we were able to partially classify Hadamard 3-design with parameters 3-$(16,8,3)$, 3-$(20,10,4)$ and 3-$(28,14,6)$. More precisely, we construct all structure of these parameters that admits an automorphism of order 3. We describe it in the following subsection 3.1.

3.1. Classifications assuming an action of automorphism of order 3

Hadamard designs $\mathcal{D}_{12}$ of order 12 are designs with parameters

$$3-(16,30,15,8,3).$$

These designs have $v = 16$ points and $b = 30$ blocks where every point is incident with $r = 15$ blocks. Every $t = 3$ rows of incidence matrix intersect in $\lambda = 3$ points within these designs. These regularities of the structure of designs $\mathcal{D}_{12}$ suggest huge number of the inner symmetries, as obtained results confirm. Our method of construction results with 5 non-isomorphic 3-designs $\mathcal{D}_{12}$. We constructed duals with the automorphism group orders of 2688, and three designs having $|Aut(\mathcal{D}_{12})|$ equal to 1536, 9216 and 322560. The incidence matrix of the most symmetric design is depicted in Figure 1 (with "+" instead of 1s and "-" instead of 0s).

These designs $\mathcal{D}_{12}$ are developed from total of 5 tactical decomposition matrices. In two tactical decomposition matrices, automorphism $g \in Aut(\mathcal{D}_{12})$ of order 3 fixes one point whereas in the rest of three matrices g fixes 4 points and 6 blocks. Neither within these designs our second type of construction result with any additional design. These facts are summarized in the following Theorem 3.2.

Theorem 3.2. *There are exactly 5 non-isomorphic designs $\mathcal{D}_{12}$ with parameters 3-$(16,8,3)$ admitting an action of the automorphism of order 3. The orders of the full automorphism group of these designs are 1536, 2688, 9216 and 322560. There are dual designs having the automorphism group order 2688.*

The forth class of designs $\mathcal{D}$ in the series (2) is the class of designs $\mathcal{D}_{15}$ with parameters

$$3-(20,38,19,10,4).$$

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Fig. 1. Incidence matrix of a design $\mathcal{D}_{12}$ with parameters 3-$(16, 8, 3)$, having the automorphism group order $|Aut(\mathcal{D}_{12})|= 322520$

These designs are consisted from $b = 38$ blocks, where every point is incident with $r = 19$ blocks. An incidence matrix of such design is a 20×38 matrix having every three rows intersecting in $\lambda = 4$ points. An automorphism of order 3 acts on these designs only forming two fixed points and two fixed blocks. There is a tactical decomposition matrix and it gives three non-isomorphic structures. The automorphism group orders of these designs are 96, 144 and 171.

When performing the "ACyc" type of construction, no one additional structure appears. These facts prove Theorem 3.3.

Theorem 3.3. *There are exactly* 3 *non-isomorphic designs* $\mathcal{D}_{15}$ *with parameters* 3-$(20, 10, 4)$ *admitting an action of the automorphism of order* 3. *Automorphism groups of these designs are of orders* $96, 144$ *and* 171.

Hadamard 3-design of order 21 is an uniform and balanced incidence structure with parameters

$$3 - (28, 54, 27, 14, 6).$$

Thus, an incidence matrix of this structure has size 28×54 where every three rows meet at 6 points. The automorphism g of order 3 act on this design on three different ways, fixing:

i) 1 point and no one block,

ii) 4 points and 6 blocks,

iii) 7 points and 12 blocks.

For these cases we get 5, 3 and 1 matrix of tactical decomposition, respectively. All these matrices were in scope for our algorithm only in case of "Cyc" type of construction (while in "ACyc" case we face too large number of possibilities to check). Obtained results are stated in the following Theorem 3.4. Frequencies of appearing automorphism group order are presented in Table 1. Parameter *niso* refers to the number of non-isomorphic structures among constructed incidence matrices.

Table 1. Structures with parameters 3-$(28, 14, 6)$ admitting an automorphism of order 3

$\|Aut(D)\|$	*niso*
3	152
6	14
9	2
12	1
18	3
27	1
39	1
156	1
1053	1

Theorem 3.4. *There are exactly* 176 *non-isomorphic designs* $\mathcal{D}_{21}$ *with parameters* 3-$(28, 14, 6)$ *admitting an action of the automorphism of order* 3. *Automorphism groups of these designs are of orders* $3, 6, 9, 12, 18, 27, 39156$ *and* 1053.

Now we summarize achieved classification results in Table 2.

Table 2. The number of Hadamard 3-designs admitting an automorphism of order 3

n	$t-(v,k,\lambda)$	*niso*
2	3-(8,4,1)	1
3	3-(12,6,2)	1
4	3-(16,8,3)	5
5	3-(20,10,4)	3
6	3-(24,12,5)	≥ 24
7	3-(28,14,6)	176

It is noteworthy that for the parameter set 3-(28,14,6), the overwhelming majority of the obtained designs have very small automorphism groups. In particular, 152 out of 176 non-isomorphic designs have automorphism group of order 3. This suggests that highly symmetric examples are relatively rare within the class of designs admitting an automorphism of order 3.

3.2. Structures with small automorphism groups

Hadamard designs $\mathcal{D}_{18}$ with parameters

$$3-(24, 46, 23, 12, 5),$$

have $b = 46$ blocks, where every of $v = 24$ points is incident with $r = 23$ blocks. Every three rows of incidence matrix intersect in $\lambda = 5$ points.

The automorphism g of order 3 act on this design on four different ways, fixing:

i) 4 blocks and no one points,

ii) 10 blocks and no one points,

iii) 3 points and 10 blocks, and

iv) 6 points and 10 blocks.

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Fig. 2. Incidence matrix of a 3-$(24, 12, 5)$ design, with no non-trivial automorphisms

In case when the automorphism g fixes 4 blocks and no one point, we obtain two tactical decomposition matrices while in every other case there are one tactical decomposition matrix. In the second step of the construction procedure we face difficulty with these designs. Namely, the number of combination to check was too large to be done in practical time. So, here we done only partial classification. We found structures with the automorphism group orders:

$$6, 12, 18, 24, 36, 48, 144, 240, 288, 1440, 15840.$$

Finally, the type "ACyc" of construction is fruitful within Hadamard 3-designs $\mathcal{D}_{18}$ of order 18. In addition to already listed orders, here we obtain designs having $|Aut(\mathcal{D}_{18})|$ equal to:

$$1, 2, 4, 8, 16, 32.$$

More precisely, we constructed 3 designs with trivial automorphism group, 11 designs with the order of automorphism group equal to 4, 5 structure is of orders 8 and 16 and 4 structures have the order of automorphism group equal to 32. Figure 2 shows the incidence matrix of a structure having trivial automorphism group (with "+" instead of 1s and "-" instead of 0s). Thus, there are at least 62 designs $\mathcal{D}_{18}$ with parameters 3-$(24, 12, 5)$ and at least 24 out of them admit an action of the automorphism $g \in Aut(\mathcal{D}_{18})$ of order 3. There are at least 3 designs with these parameters having the trivial automorphism group.

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Ivica Martinjak
Faculty of Electrical Engineering and Applied Computing
University of Dubrovnik, Dubrovnik, Croatia
E-mail imartinjak@unidu.hr