On the Enumeration of $2 - (7, 3, \lambda)$ block designs

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Abstract

We give the numbers of nonisomorphic $2 - (7, 3, \lambda)$ block designs for $\lambda = 6, 7, 8, 9$. We discuss the method of generation and present statistics concerning automorphism groups and multiple blocks. The 418 $2 - (7, 3, 6)$ block designs together with the order of their automorphism groups are listed.

1. Introduction

A $2-(v,k,\lambda)$ *design* is a pair $\mathcal{D} = (V, \mathcal{B})$ where V is a v-set of elements and $\mathcal{B}$ is a set of k-subsets of V called blocks such that every 2-subset of V is contained in exactly λ blocks. The number of blocks b in $\mathcal{B}$ is then determined by:

$$b = \frac{v(v-1)}{k(k-1)}\lambda.$$

We generated the sets of all nonisomorphic $2-(7,3,\lambda)$ designs for $\lambda = 5, 6, 7, 8, 9$. Two designs $(V_1, \mathcal{B}_1)$, $(V_2, \mathcal{B}_2)$ are called *isomorphic* if there is a bijection $\phi : V_1 \longmapsto V_2$ such that for every block $B \in \mathcal{B}_1$ holds $\phi(B) \in \mathcal{B}_2$.

In the second section we refer to the method used to produce the designs and in the third section we present statistics concerning these sets of designs.

The generation was done on an IBM 3081D and the program was written in VSPascal 2.0. For the analysis we used an 80386/25 computer with Turbo Pascal 6.0.

2. Method of Generation

For the enumeration of nonisomorphic designs we used an implementation of an algorithm of Ivanov described in [1].

This algorithm constructs the incidence matrix of a block design by a backtrack row by row, where the *incidence matrix* $A = (a_{ij})$ of a design $(V, \mathcal{B})$ is a $(v \times b)$ matrix such that $a_{ij} = 1$ if the ith point is in the jth block and $a_{ij} = 0$ otherwise.

In addition an orderly argument is used to prune the search tree. Let $A = (a_{ij})$ be an incidence matrix of a $2-(v,k,\lambda)$ block design. Let $r_1, r_2, \ldots, r_v$ denote the rows of A and denote with $N(A)$ the binary number $r_1 r_2 \cdots r_v$. If A and B are two incidence matrices then we say B is *greater* than A ($A < B$) if and only if $N(A) < N(B)$.

Now an incidence matrix C is called *canonical* if for all row permutations Φ and all column permutations Ψ the following holds: $P_\Phi C P_\Psi \leq C$, where P_Φ and P_Ψ are the permutation matrices realizing the permutations Φ and Ψ. That means C is the greatest matrix in this isomorphism class where C lies. Only those branches of the search tree where canonical matrices can be found are considered.

When an incidence matrix A is completed during the backtrack one has to check whether A is canonical or not. For a detailed description of the algorithm see [1].

For $\lambda = 6$ we found one design more than Ivanov [1]. We checked our set of designs with a pairwise isomorphism test. In addition, we applied the trivial check of canonicity of an incidence matrix A by applying all $v!$ row permutations to A, then sorting the columns lexicographically decreasing and

checking whether the matrix appearing this way became greater or not for each permutation. In all cases we could confirm our results.

As far as we know the numbers of nonisomorphic $2 - (7, 3, \lambda)$ designs for $\lambda = 7, 8, 9$ are new [2].

3. Results and analysis

The numbers of nonisomorphic $2-(7, 3, \lambda)$ designs we found are given in Table 1. In addition Table 1 displays the numbers of distinct frequency vectors FV for each class of designs.

λ	#Designs	FV
6	418	160
7	1508	392
8	5413	923
9	17785	2117

Table 1: Numbers of nonisomorphic block designs and frequency vectors

The frequency vector of a block design is a λ-dimensional vector v, such that $v(i)$ contains the number of i-fold blocks ($i = 1 \ldots \lambda$).

Table 2 shows the number of designs with automorphism group of order N.

4. Acknowledgement

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N	λ				
	5	6	7	8	9
1	30	208	1085	4463	16043
2	25	89	214	558	1123
3	19	45	103	191	353
4	5	19	37	79	113
6	8	22	30	49	69
8	4	8	10	24,	26
12	3	7	8	17	22
16	1	2	2	4	3
21	3	3	5	5	7
24	7	8	10	15	17
36	0	0	0	0	1
42	0	2	1	2	2
48	0	1	1	2	1
60	1	0	0	0	0
72	0	1	0	0	1
144	0	1	0	1	1
168	1	2	2	3	3
5040	1	0	0	0	0

Table 2: Orders of Automorphism Groups

References

[1] A.V.Ivanov
Constructive Enumeration of Incidence Systems
Annals of Discrete Math. 26 (1985) 227-246.

[2] R.Mathon, A.Rosa
Tables of Parameters of BIBDs with $r \leq 41$ including Existence, Enumeration and Resolvability Results
Ann. Discr. Math. 26, 275 - 307.

List of the 418 $2-(7,3,6)$ block designs

Let $T_1, \ldots, T_{35}$ be the tripels of the elements $\{1, 2, \ldots, 7\}$ listed in increasing lexicographic order. In this sequence each tripel is assigned a symbol of the following sequence:

$$123456789abcdefghijklmnopqrstuvwxyz$$

For each design the following data is given:
After the first "#" the number of the design, after the second "#" the order of the automorphismgroup, after the third "#" the frequency vector and after the fourth "#" the design as list of tripels coded as described above.

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#  2# 24#  4  0  0  0  4  3 #111111aaaaaaffffffllllllmnoooooorssssssttttttu
#  3# 24#  0  4  0  4  0  3 #111111aaaaaaffffffllllmmnnoooorssssssttttuu
#  4# 48#  0  0  8  0  0  3 #111111aaaaaaffffffllllmmmnnnnooorrrsssstttuuuu
#  5#  8#  8  0  0  2  4  1 #111111aaaaabeffffffklllllmnooooprssssssttttttu
#  6#  4#  4  4  2  2  2  1 #111111aaaaabeffffffklllmannnoooprrrsssssttttttuu
#  7#  4#  4  2  6  0  2  1 #111111aaaaabeffffffklllmmmnnnnoooprrrsssssttttuu
#  8#  4#  6  2  0  4  2  1 #111111aaaaabeffffffklmmmmennnnnoprrrrssttttuuuu
#  9# 12#  6  0  0  0  6  1 #111111aaaaabeffffffkmmmmmmannnnnnprrrrrstuuuuuu
# 10#  8#  0 10  0  4  0  1 #111111aaaabbeeffffffkkllmmannnnooopprssssssttttuu
# 11#  4#  2  4  6  2  0  1 #111111aaaabbeeffffffkkllmmannnnoppprrrssssttttuuu
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# 21# 12#  6  6  6  0  0  1 #111111aaabbbcdeeeeffffkkllmmmannnnooppqrrrrsssttttuuv
# 22#144#  0 18  0  0  0  1 #111111aabbccddeeeffffkkllmmannnnooopppqqrrssttuuvv
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