Construction of some sequences with zero autocorrelation function

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Abstract. It is known that if there are base sequences of lengths $m+1$, $m+1$, m, m and y is a Yang number then there are T-sequences of lengths $y(2m+1)$. Base sequences of lengths $m+1$, $m+1$, m, m for $m = 26, 27, 28$ and some new decompositions into squares are constructed. T-sequences of lengths $61(2m+1)$ for some new decompositions into squares are also presented.

1. Introduction

Four sequences A,B,C,D of lengths $m+1, m+1, m, m$ with entries ± 1 and zero nonperiodic autocorrelation sum are called *base sequences*. If A, B, C, D are base sequences with lengths $m+1$, m pairs, then the sequences $(\frac{1}{2}(A+B)), (\frac{1}{2}(A-B)), (\frac{1}{2}(C+D)), (\frac{1}{2}(C-D))$ are called *suitable sequences* of lengths $m+1$, $m+1, m, m$ respectively.

Four sequences $X = (x_1, \ldots, x_n), Y = (y_1, \ldots, y_n), Z = (z_1, \ldots, z_n), W = (w_1, \ldots, w_n)$ of length n with entries $0, \pm 1$ are called *T-sequences* if:

(i) $|x_i| + |y_i| + |z_i| + |w_i| = 1, i = 1, \ldots, n$

(ii) they have zero nonperiodic autocorrelation sum.

Base sequences are crucial to Yang's [6,7,8] constructions for finding longer T-sequences of odd length. The most prolific method for constructing Hadamard matrices uses T-matrices or T-sequences. The essential defference between T-matrices and T-sequences is that the former have zero periodic autocorrelation function and the latter have zero non periodic autocorrelation function. For more details the reader is referred to Geramita and Seberry [1], and Seberry and Yamada [5].

Yang [6,7] found that if there are base sequences of lengths $m+1$, $m+1$, m, m then there are T-sequences of length $y(2m+1)$ for $y = 3, 7, 13$ and $2g+1$, where $g = 2^a 10^b 26^c$, $a, b, c \geq 0$ (*Golay numbers*). These are instances of what are termed *Yang numbers*. This method of multiplication of sequences was extended by Yang [8,9].

A quadruple $(E, F; G, H)$ of $(0, \pm 1)$-sequences is said to be a set of *near normal sequences* for length $n = 4m+1$ (abbreviated as NN(n)) if the following conditions are satisfied.

(i) $E = (X/0, 1), F = (Y/0)$, where X and Y are ± 1 sequences of length m, $0 = 0_{m-1}$ the sequence of zeros of length $m-1$, and $X/0 = (x_1, 0, x_2, 0, \ldots, x_{m-1}, 0, x_m)$

(ii) G and H are quasi-symmetric supplementary $(0, \pm 1)$-sequences of length $2m$, i.e. $G + H$ is a ± 1 sequence of length $2m$ and zeros appear symmetrically in G and H.

(iii) The sequences E, F, G, H have zero nonperiodic autocorrelation sum.

Yang [9] constructed near normal sequences for length 61 (NN(61)) where,

$$E = (-----+--+++-+-+/0_{14},+),$$
$$F = (++++--+-++++--+/0_{14}),$$
$$G = (--00-++0+0-++0+-0+++0+0--+00-+),$$
$$H = (00+-000+0+000-00+000-0+000++00).$$

Koukouvinos and Seberry [4] reformulated some results of Yang [8] who gave a method to multiply by y to get four T-sequences of lengths $y(2m + 1)$, where y is a Yang number and $m + 1, m + 1, m, m$ are the lengths of base sequences.

2. The New Results

Theorem 1. *Let a, b, c, d be the sums of the elements of suitable sequences with lengths $m + 1, m + 1, m, m$ so that $2m + 1 = a^2 + b^2 + c^2 + d^2$. Then using Yang's method to multiply by 61 we get four T-sequences of lengths $61(2m + 1)$ corresponding to one of four decompositions.*

$$61(2m+1) = (4a-4b-5c+2d)^2+(4a+4b-2c-5d)^2+(5a+2b+4c+4d)^2 +(-2a+5b-4c+4d)^2$$
$$61(2m+1) = (4b-4a-5c+2d)^2+(4b+4a-2c-5d)^2+(5b+2a+4c+4d)^2 +(-2b+5a-4c+4d)^2$$
$$61(2m+1) = (4a-4b-5d+2c)^2+(4a+4b-2d-5c)^2+(5a+2b+4d+4c)^2 +(-2a+5b-4d+4c)^2$$
$$61(2m+1) = (4b-4a-5d+2c)^2+(4b+4a-2d-5c)^2+(5b+2a+4d+4c)^2 +(-2b+5a-4d+4c)^2$$

Proof. If we apply Theorem 3 of [4], using the near normal sequences for length 61 constructed by Yang [9], we obtain the desired result by a straightforward verification.

If Williamson type matrices of order w exist and T-sequences of length n exist, then Hadamard matrices of order $4nw$ can be constructed (see [1,5]). The base sequences constructed in Table 1 for some new decompositions into squares can be used in Theorem 1 to obtain orthogonal designs OD$(4t; t, t, t, t)$ for $t = 61(2m+1)$ and $m = 26, 27, 28$. Williamson type matrices of order w are then used with these orthogonal designs to form Hadamard matrices of order $4tw$. This method sometimes leads to Hadamard matrices with maximum known excess (see [2,3,4]) and this is the motivation for looking for new decompositions into squares.

A Hadamard matrix of order $n = 4.61^2$ with maximal excess $\sigma(n) = 8.61^3$ constructed in [3].

Table 1
Base sequences BS(n) : $(A, B; C, D)$ with lengths $m + 1$, m pairs, $n = 2m + 1$

Length	Sums of squares	Sequences
$n = 2m+1$	$2n = a^2+b^2+c^2+d^2$	
53	$5^2+3^2+6^2+6^2$	A = (+−+++++−−+++−+−−+−−+−+−+++−)
		B = (−+−++++++−−−++++−++−−+−+−−−)
		C = (++−−−+−++−+−+−+++++++−++−−)
		D = (+−−++++++−−++−−−+++−++−++−)
53	$5^2+3^2+6^2+6^2$	A = (−+++−+−−−+−+−−+++−++++−++−+)
		B = (−+−++++−+++++−−+−−−+−+++−−−)
		C = (+−++−++++−−+−−++++−+−+++−−)
		D = (+++++−−++++−++−−−+−−+−+++−)
55	$8^2+6^2+3^2+1^2$	A = (+++++−+−−+−+−+++−−+++++−−++−)
		B = (+−−+−+++−++−−+++++++−+−+−−−+)
		C = (++++−+++−+−+−−+−−+−−+++−−−+)
		D = (−+−−−++−+−−+−++−−−+++++++−−)
57	$5^2+5^2+8^2+0^2$	A = (++++−+−−++−+−−−+−++++−+++−−+−)
		B = (−++−−+−−−+−++++−++++−−++++−+−)
		C = (+++++−++++−+−++−−+++−−−+−−++)
		D = (−+++−+−++++−−−−−+−++−++−−−+−)
57	$9^2+1^2+4^2+4^2$	A = (++−−+−−+++++++−++++−+−++−−−++)
		B = (+++−−−++−+−+−+++−+−−−−−++−++−)
		C = (+++++−−++++−++−−−+−−−+−++−+−)
		D = (+−−+++−+−−+−−+−+−+++−+++++−−)

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