Dudeney's Round Table Problem – a Survey

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Abstract We survey Dudeney's round table problem which asks for a set of Hamilton cycles in the complete graph that uniformly covers the 2-paths of the graph. The problem was proposed about one hundred years ago but it is still unsettled. We mention the history of the problem, known results, generalizations, related designs, and some open problems.

1 Dudeney's round table problem

Dudeney proposed a problem in his book [8] as follows:

> "Seat the same n persons at a round table on $(n-1)(n-2)/2$ occasions so that no person shall ever have the same two neighbours twice. This is, of course, equivalent to saying that every person must sit once, and once only, between every possible pair."

We call the problem *Dudeney's round table problem.* Denoting the n persons by $1, 2, \ldots, n$, solutions for small n are obtained as follows. When $n = 3$, we have $\{(1,2,3)\}$, when $n = 4$, we have $\{(1,2,3,4), (1,2,4,3), (1,3,2,4)\}$, when $n = 5$, we have $\{(1,2,3,4,5), (1,2,4,5,3), (1,2,5,3,4), (1,3,2,5,4), (1,4,2,3,5), (1,5,2,4,3)\}$ for example, where $(a, b, \ldots, c)$ represents a cycle in which c is followed by a.

In graph terminology, the problem asks for a set of Hamilton cycles in the complete graph K_n with the property that every path of length two (2-path) of K_n lies on exactly one of the cycles. (The length of a path is the number of edges in the path.) We call such a set of cycles in K_n a *Dudeney set* in K_n. The cardinality of a Dudeney set in K_n is $(n-1)(n-2)/2$.

Dudeney's round table problem is equivalent to the problem of decomposing the line graph $L(K_n)$ of the complete graph K_n into n-cycles with the property that each of these n-cycles contains exactly one edge from each of the n maximum cliques of $L(K_n)$ which arise out of the stars at the vertices of K_n.

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It has been conjectured that there exists a solution of Dudeney's round table problem for every n people ($n \geq 3$); however, it is still unsettled.

Conjecture 1.1 (Dudeney)[4] *There exists a solution of Dudeney's round table problem for every n people ($n \geq 3$).*

In this paper, we survey Dudeney's round table problem. We mention the history of the problem, known results, generalizations, related designs, and some open problems.

2 The history of Dudeney's round table problem

In 1899, Judson proposed the following problem in *American Mathematical Monthly* [15]:

> "Seven persons met at a summer resort, and agreed to remain as many days as there are ways of sitting at a round table, so that no one shall sit twice between the same two companions. They remained fifteen days. It is required to show in what way they may have been seated."

In 1900, Judson showed solutions for 6 and 8 people and he wrote that he had failed to get a solution for 7 people in *Amer. Math. Monthly* [16], and the editor of the journal offered one year's free subscription to the first person to provide a solution for 7 people.

In 1904, Safford showed a solution for 7 people and expressed his belief that it is the unique solution[5], and he also provided another solution for 6 people and he wrote that the solution appeared to be non-isomorphic to the one given by Judson [45]. Safford showed that there are at most two non-isomorphic solutions for 6 people (one is Judson's solution, another is Safford and Dickson's solution) by exhaustive search [46], and Dickson proved that the two solutions are, in fact, non-isomorphic [5].

In 1905, Dickson generalized the problem to seating n people and obtained solutions for $n = 6, 8, 10, 12$ using group theory [6].

Dudeney posed the problem for 6 people in the English newspaper *Daily Mail* on 13th (the problem) and 16th (the answer) Octorber, 1905. Dudeney posed the problem for 7 people in his book *The Canterbury Puzzles*

[4] Dudeney wrote "I discovered a subtle method for solving all cases" in his book [7] (p. 237), but he appears never to have published the method. No one knows whether he really discoverd it, but at least he must have believed that there are solutions for all cases, so we may call it Dudeney's conjecture.

[5] His belief is correct. We can see that the numbers of non-isomorphic solutions for 6, 7, and 8 people are 2, 1, and 1066610, respectively with the aid of a computer [39].

[7] (Problem 90), and he posed the problem for n people in his book *Amusements in Mathematics* [8] (Problem 273). Thus, the problem has become popular and received his name.

Dudeney claims in *Amusements in Mathematics* to have recorded schedules for $3 \leq n \leq 25$ and $n = 33$. But he displays the solutions only for $3 \leq n \leq 12$ and he explains that good many mathematicians are still considering the case of $n = 13$ and he will not rob them of the pleasure of solving it by showing the answer. Dudeney further writes in *The Canterbury Puzzles* that Bergholt solved the problem when $n = p+1$ (p is a prime), Bewley found a general method for all even numbers, and the problem for the odd numbers are extremely difficult, and for a long time no progress could be made with their solution. And he writes "At last, however (though not without much difficulty), I discovered a subtle method for solving all cases." As stated in a footnote of §1, he appears never to have provided his method.

In 1971, Meally posed the problem of asking solutions for $n = 13$ and any number n in the *Journal of Recreational Mathematics* [37]. Huang and Rosa investigated the problem and gave a construction for $n = p+1$ (p is a prime) and found solutions for $n = 9, 11, 13, 15$ with the aid of a computer [13]. Nakamura independently gave solutions for $n = 13, 15, 16$ [38].

The problem is also treated in Rouse Ball and Coxeter's book [4] (p. 49).

3 Known results for Dudeney's round table problem

Dudeney's round table problem has been solved for the following n. We list them in chronological order.

1. $n = p + 1$ (p is a prime)[6]
2. $n = 2p$ (p is a prime) [2, 3, 42][7]
3. $n = p^e + 1$ (p is a prime, $e \geq 1$) [43]
4. $n = p + 2$ (p is an odd prime and 2 is a primitive root of $GF(p)$) [9]

[6] The easy case $n = p + 1$ (p is a prime) has been solved by many people. Published examples are [13, 42, 48].

[7] Anderson constructed a perfect 1-factorization of K_{2p} (p is a prime) in [2, 3]. We put his papers on the list since a perfect 1-factorization induces a Dudeney set immediately (see §5.2).

5. $n = p^e + 1$[8], $n = pq + 1$ (p, q are odd primes, $e \geq 1$); $n = p^e q^f + 1$ (p, q are distinct odd primes with $p \geq 5$, $q \geq 11$, and $e, f \geq 1$) [10][9]

6. n is even [18]

7. $n = p + 2$ (p is an odd prime, 2 is the square of a primitive root of $GF(p)$ and $p \equiv 3 \pmod 4$) [17] (Theorem 1.1)

8. $n = p + 2$ (p is an odd prime and -2 is a primitive root of $GF(p)$) [17] (Theorem 1.2)

9. $n = p + 2$ (p is an odd prime, 2 is the square of a primitive root of $GF(p)$, $p \equiv 1 \pmod 4$, 3 is not a quadratic residue modulo p) [24]

10. $n = p + 2$ (p is an odd prime, -2 is the square of a primitive root of $GF(p)$, and either

    (10-1) $p \equiv 1 \pmod 4$ and 3 is not a quadratic residue modulo p, or

    (10-2) $p \equiv 3 \pmod 4$ [24]

11. some sporadic cases ($n = 11$ [8]; 23, 45 [9]; 27, 29, 35, 37, 41, 47 [40]; 75, 91 [21]).

In conclusion it has been solved when n is even. In the case that n is odd, it has been solved when $n = 2^e + 1$ ($e \geq 1$), and $n = p + 2$ (p is an odd prime) with some conditions, and some sporadic cases. The smallest values for which the conjecture remains unsolved are $n = 51, 53$.

4 Dudeney designs

Dudeney's round table problem asks for a uniform covering of 2-paths with Hamilton cycles in K_n. It is natural to consider Hamilton paths, k-cycles, or k-paths instead of Hamilton cycles; and the complete bipartite graph, the complete digraph, or the complete bipartite digraph instead of K_n.

A generalization of a Dudeney set has been considered by Heinrich et al. [11]. Consider a graph G and a subgraph H of G. A $D(G, H, \lambda)$ *design* is a multiset $\mathcal{D}$ of subgraphs of G, each isomorphic to H, so that every 2-path of G lies in exactly λ subgraphs in $\mathcal{D}$. Analogously, when G is a directed graph (digraph) and H is a subgraph of G (a subgraph of a directed graph means a directed subgraph), a $D(G, H, \lambda)$ *design* is a multiset $\mathcal{D}$ of

[8] The methods for $n = p^e + 1$ of the cases 3 and 5 are different. Note that p is a prime including 2 in the case 3.

[9] The paper [10] was submitted earlier than the paper [18], but was published later.

subgraphs of G, each isomorphic to H, so that every directed 2-path lies in exactly λ subgraphs in $\mathcal{D}$. We call these designs *Dudeney designs*[10] [11].

A $D(G, H, \lambda)$ design is *resolvable* or *vertex-resolvable* if the subgraphs in the design can be partitioned into classes so that every vertex of G appears exactly once in each class. Each such class is called a *parallel class* of the design. A $D(G, H, \lambda)$ design is *edge-resolvable* if the subgraphs in the design can be partitioned into classes so that every edge of G appears exactly once in each class. *Arc-resolvable* are defined similarly.

The following notation will be used. K_n is the complete graph on n vertices, $K_{n,n}$ is the complete bipartite graph on partite sets with n vertices each, C_k is a cycle on k vertices, and P_k is a path on k vertices. K_n^* is the complete (loop-free) digraph on n vertices and $K_{n,n}^*$ is the complete bipartite digraph on partite sets with n vertices each. K_n^* and $K_{n,n}^*$ are digraphs which are obtained from K_n and $K_{n,n}$, respectively, by substituting two oppositely directed edges (arcs) for each edge. $\vec{C}_k$ is a directed cycle on k vertices and $\vec{P}_k$ is a directed path on k vertices.

The problems of constructing $D(G, H, \lambda)$ designs in which H is a cycle, a path, a directed cycle, or a directed path have been solved for the following cases. We say that a $D(G, H, \lambda)$ design is *solved* if we have the necessary and sufficient condition of n for the existence of the design.

1. $D(K_n, P_3, 1)$ designs (trivial)[12], resolvable $D(K_n, P_3, 1)$ designs [11] (Th. 2.9 (ii)), and edge-resolvable $D(K_n, P_3, 1)$ designs [11] (Th. 2.11, 2.13 (i))

2. $D(K_n, C_3, 1)$ designs (trivial)[13], resolvable $D(K_n, C_3, 1)$ designs [11] (Th. 2.9 (i)), and edge-resolvable $D(K_n, C_3, 1)$ designs [11] (Th. 2.12, 2.13 (ii))

3. $D(K_n, P_4, \lambda)$ designs[14] [11] (Th. 2.20, Cor. 2.23), [35] (Th. 2.3)

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[10] For more general definition, see [11].

[11] Applications of Dudeney designs are, for example, experimental designs balanced for pairs of residual (carry-over) effects of treatments [49, 50]. k-Cycle or k-circuit decompositions of K_n are similar to Dudeney designs (A *k-cycle [k-circuit] decomposition* of K_n is a set of k-cycles [k-circuits] in K_n so that every edge occurs on exactly one of the cycles [circuits]), and their applications are in [14, 34, 44] for example. Since these designs are balanced with respect to neighbors, they are generally called *neighbor designs*. A Dudeney design is a kind of neighbor design.

[12] A $D(K_n, P_3, 1)$ design is trivially constructed by taking all 2-paths in K_n. As a $D(K_n, P_3, 1)$ design exists for all n, a $D(K_n, P_3, \lambda)$ design trivially exists for all λ and n by taking λ $D(K_n, P_3, 1)$ designs. We don't put on the list such a trivial λ-fold design. The same applies hereinafter.

[13] A $D(K_n, C_3, 1)$ design is trivially constructed by taking all triangles in K_n.

[14] From [11] (Th. 2.20, Cor. 2.23), we see that there is a $D(K_n, P_4, \lambda)$ design if and only if (i) λ is even, or (ii) λ is odd and $n \equiv 0, 1, 2 \pmod 4$.

4. $D(K_n, C_4, \lambda)$ designs [12] and resolvable $D(K_n, C_4, 1)$ designs[15] [26]

5. $D(K_n, P_5, 1)$ designs [28, 36]

6. $D(K_n, P_6, 1)$ designs [31, 32]

7. $D(K_n, C_6, 1)$ designs [29]

8. $D(K_n, P_7, 1)$ designs [1]

9. $D(K_n^*, \overrightarrow{P}_3, 1)$ designs (trivial), resolvable $D(K_n^*, \overrightarrow{P}_3, 1)$ designs[16], and arc-resolvable $D(K_n^*, \overrightarrow{P}_3, 1)$ designs [11] (Th. 2.14)

10. $D(K_n^*, \overrightarrow{P}_4, 1)$ designs [11] (Th. 2.21, 2.22)

11. $D(K_{n,n}, P_4, 1)$ designs, resolvable $D(K_{n,n}, P_4, 1)$ designs, and edge-resolvable $D(K_{n,n}, P_4, 1)$ designs [11] (Th. 3.3)

12. $D(K_{n,n}, C_4, \lambda)$ designs [20], resolvable $D(K_{n,n}, C_4, 1)$ designs [11] (Th. 3.1 (ii)), and edge-resolvable $D(K_{n,n}, C_4, 1)$ designs [11] (Th. 3.1 (i))

13. $D(K_{n,n}^*, \overrightarrow{C}_4, \lambda)$ designs, resolvable $D(K_{n,n}^*, \overrightarrow{C}_4, 1)$ designs, and arc-resolvable $D(K_{n,n}^*, \overrightarrow{C}_4, 1)$ designs [20]

14. $D(K_{n,n}^*, \overrightarrow{C}_{2n}, 1)$ designs [19].

5 Related designs

In this section, we mention some designs related to Dudeney designs.

5.1 3-Designs

Let n, λ be positive integers and let K be a set of positive integers with $K \subseteq \{1, 2, \ldots, n\}$. A 3-$(n, K, \lambda)$ *design*, a 3-*design* in short, is a pair $(X, \mathcal{B})$ where X is an n-set of points and $\mathcal{B}$ is a multiset of subsets of X (blocks), with the property that every 3-subset of X is contained in exactly λ blocks and the size of each block is a member of the set K ([12] p. 52). It is easy to prove the following proposition.

Proposition 5.1 ([12] Lemma 1.3) *Let n, λ, μ be positive integers and let K be a set of positive integers with $K \subseteq \{1, 2, \ldots, n\}$. Let H be a graph.*

[15] There cannot exist edge-resolvable $D(K_n, C_4, 1)$ designs [11] (p. 106).

[16] It is known that there is a resolvable $D(K_n, P_3, 1)$ design for $n \geq 3$. Hence we obtain a resolvable $D(K_n^*, \overrightarrow{P}_3, 1)$ design for $n \geq 3$, by taking two directed 2-paths $\overrightarrow{(a, b, c)}$ and $\overrightarrow{(c, b, a)}$ for each 2-path (a, b, c) in the $D(K_n, P_3, 1)$ design.

If there exists a 3-(n, K, λ) design, and if for every $k \in K$ there exists a $D(K_k, H, \mu)$ design, then there exists a $D(K_n, H, \lambda\mu)$ design.

Heinrich et al. constructed $D(K_n, C_4, \lambda)$ designs for all admissible n and λ applying Prop. 5.1 [12].

5.2 Perfect 1-factorizations

Let $n \geq 4$ be even. A 1-factorization of K_n with the property that the union of any two of its 1-factors is a Hamilton cycle is called a *perfect 1-factorization.* It is easy to see that for a perfect 1-factorization $\mathcal{F}$ of K_n, $\{F \cup F' \mid F, F' \in \mathcal{F}\}$ is a Dudeney set of K_n, so we have the following proposition.

Proposition 5.2 *Let $n \geq 4$ be even. If there exists a perfect 1-factorization of K_n, then there exists a $D(K_n, C_n, 1)$ design.*

The problem of constructing a perfect 1-factorization of K_n is much more difficult than constructing a Dudeney set, and perfect 1-factorizations of K_n have been constructed only when $n = p+1$ and $n = 2p$ (p is a prime) and some sporadic cases [47, 51].

5.3 i-Perfect Hamilton decompositions

Let $n \geq 5$ be an odd integer and i be an integer with $2 \leq i \leq (n-1)/2$. A Hamilton decomposition $\mathcal{H}$ of K_n is called *i-perfect* if the set of chords at distance i in the Hamilton cycles in $\mathcal{H}$ is the edge set of K_n [22].

Proposition 5.3 ([22] Theorem A) *Let $n \geq 5$ be odd. If there exists a 2-perfect Hamilton decomposition of K_n, then there exists a Dudeney set of K_{n+1}.*

It is known that there exists a 2-perfect Hamilton decomposition of K_n for odd n with $5 \leq n \leq 29$ except $n = 9$ [25].

Problem 5.4 *Construct 2-perfect Hamilton decompositions of K_n for every odd $n \geq 11$.*

It is known that there exists a $D(K_n, C_n, 1)$ design when n is even [18], but the proof is long and complicated, so a simple proof is desirable. If Problem 5.4 is solved, it would give a simple proof[17].

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[17] Before solving the existence problem of $D(K_n, C_n, 1)$ designs for even n, Kobayashi et al. solved the existence problem of $D(K_n, C_n, 2)$ designs for even n [27]. The proof is truly simple. Such a simple proof is desirable also for $D(K_n, C_n, 1)$ design.

5.4 Kotzig's problem

Let $n \geq 5$ be odd. Kotzig defined a *perfect set of Hamilton decompositions of* K_n in 1979 [33]. It is a set of Hamilton decompositions of K_n $\{\mathcal{H}_1, \mathcal{H}_2, \ldots, \mathcal{H}_{n-2}\}$ such that each 2-path of K_n appears in one Hamilton cycle of $\mathcal{H}_1 \cup \mathcal{H}_2 \cup \cdots \cup \mathcal{H}_{n-2}$. This is an edge-resolvable Dudeney set of K_n in our terminology. He posed a problem.

Problem 5.5 (Kotzig) [11, 33] *What is the smallest odd number $n \geq 5$ for which there is a perfect set of Hamilton decompositions of K_n.*

There does not exist a perfect set of Hamilton decompositions of K_n when $n = 5, 7, 9$, and there exists a perfect set of Hamilton decompositions of K_{11} [41], so we see that the smallest number n of Problem 5.5 is $n = 11$.

6 Open problems

For Dudeney designs $D(G, H, \lambda)$, the most interesting cases would be that in which G is K_n, $K_{n,n}$, K_n^*, or $K_{n,n}^*$, and H is a Hamilton cycle or a Hamilton path[18]. In this section, we list open problems for those designs. The following partial results are known.

Theorem 6.1 [18, 23] *Let $n \geq 3$ be an integer.*
(1) *There exists a $D(K_n, C_n, 1)$ design when n is even.*
(2) *There exists a $D(K_n, C_n, 2)$ design when n is odd.*

Theorem 6.2 [30] *Let $n \geq 3$ be an integer.*
(1) *There exists a $D(K_n, P_n, 1)$ design when $n \equiv 0, 1, 3 \pmod 4$.*
(2) *There exists a $D(K_n, P_n, 2)$ design when $n \equiv 2 \pmod 4$.*

Theorem 6.3 [30] *Let $n \geq 2$ be an integer.*
(1) *There exists a $D(K_{n,n}, C_{2n}, 1)$ design when $n \equiv 0, 1, 3 \pmod 4$.*
(2) *There exists a $D(K_{n,n}, C_{2n}, 2)$ design when $n \equiv 2 \pmod 4$.*

We obtain the following corollaries immediately from the above theorems.

Corollary of Theorem 6.1 *Let $n \geq 3$ be an integer.*
(1) *There exists a $D(K_n^*, \overrightarrow{C}_n, 1)$ design when n is even.*
(2) *There exists a $D(K_n^*, \overrightarrow{C}_n, 2)$ design when n is odd.*

Corollary of Theorem 6.2 *Let $n \geq 3$ be an integer.*

[18] The existence problem of $D(K_n, C_n, 1)$ designs is Dudeney's round table problem, and that of $D(K_n, P_n, 1)$ designs is called Dudeney's counter table problem [30].

(1) *There exists a* $D(K_n^*, \overrightarrow{P}_n, 1)$ *design when* $n \equiv 0, 1, 3 \pmod 4$.
(2) *There exists a* $D(K_n^*, \overrightarrow{P}_n, 2)$ *design when* $n \equiv 2 \pmod 4$.

Note that the existence problem of a $D(K_{n,n}^*, \overrightarrow{C}_{2n}, 1)$ design is solved [19] as stated in §4. Thus, the remaining open problems are the following.

Problem 6.4 *Solve the existence problems of the following Dudeney designs.*

1. $D(K_n, C_n, 1)$ *designs for odd* n
2. $D(K_n, P_n, 1)$ *designs for* n *with* $n \equiv 2 \pmod 4$
3. $D(K_{n,n}, C_{2n}, 1)$ *designs for* n *with* $n \equiv 2 \pmod 4$
4. $D(K_{n,n}, P_{2n}, \lambda)$ *designs for* n *and* λ
5. $D(K_n^*, \overrightarrow{C}_n, 1)$ *designs for odd* n
6. $D(K_n^*, \overrightarrow{P}_n, 1)$ *designs for* n *with* $n \equiv 2 \pmod 4$
7. $D(K_{n,n}^*, \overrightarrow{P}_{2n}, \lambda)$ *designs for* n *and* λ.

If design 1 exists, then design 2 exists by deleting one vertex, design 5 exists trivially, and design 3 exists [30] (Prop. 5.1). And if design 2 exists, then design 6 exists trivially. That is, if design 1 exists, then designs 2, 3, 5, 6 exist. In this sense, the problem of constructing $D(K_n, C_n, 1)$ designs, i.e., Dudeney's round table problem has a fundamental position among these existence problems[19].

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