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Article

Some Convex Polytopes Joining Paths and Their Metric Dimension

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Abstract: In recent years, there is a lot of interest in the topic of conveying the groups of planar graphs with an unvarying metric dimension. A few types of planar graphs have recently had their metric dimension determined, and an outstanding problem concerning these graphs was brought up that: Illustrate the types of planar graphs Υ that can be generated from a graph Φ through the addition of more edges to Φ, such that $dim(\Phi) = dim(\Upsilon)$ and $\mathbb{V}(\Phi) = \mathbb{V}(\Upsilon)$. While proceeding in a similar directives, we identify two families of radially identical planar graphs with unaltered metric dimension in this study: $F_{n,m}$ and $\gimel_{n,m}$. We do this by establishing that $dim(F_{n,m}) = dim(\gimel_{n,m})$ and $\mathbb{V}(F_{n,m}) = \mathbb{V}(\gimel_{n,m})$, respectively. We acquire another family of a radially symmetrical planar graph (i.e., $\daleth_{n,m}$) with a constant metric dimension. We show that all the vertices of these classes of the plane graphs can potentially be identified with just three well-chosen nodes.

Keywords: Planar graph, Convex polytope, Metric dimension

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1. Introduction

Let $\Phi = \Phi(\theta, \xi)$ be a fundamental, related (i.e., connected and simple) and undirected (i.e., they have no associated direction, or all of the edges are single-directional) graphical representation (or graph or network) with an edge set and a vertex set ξ and θ respectively. The *metric dimension* of the graph $\Phi = \Phi(\theta, \xi)$ is the smallest number (amount) of vertices (hubs or nodes) in a set that allows a vertex to be unambiguously identified by the list of shortest paths from that vertex to those vertices. From the definition, the ordered t-tuple (or t-vector) $\eta(v|\zeta) := (d_\Phi(v, v_1), d_\Phi(v, v_2), \ldots, d_\Phi(v, v_t))$ is the metric code (or metric representation) of a node v of Φ with respect to ζ in a graph Φ where $\zeta = \{v_1, v_2, v_3, \ldots, v_t\}$ of nodes v is an arranged (or ordered) subset. A set ζ that has been arranged is referred to as a resolving (locating) set of Φ if every pair of different vertices of Φ has a unambiguous metric representation. The cardinality (magnitude) of the subset ζ, or the location number (or metric dimension) of the graph Φ, denoted by $dim(\Phi)$ or $\beta(\Phi)$, is the minimum size of locating set on the graph Φ. This is known as the metric dimension or location number of Φ; mathematically $\beta(\Phi) = dim(\Phi) = minimum\{|\zeta| : \zeta \text{ is resolving set } (\text{or locating set})\}$. It is realized that the issue of registering this invariant is NP-hard. If a subset $\mathcal{T}$ of the arrangement of nodes $\theta(\Phi)$ is independent as well as resolving , then the set $\mathcal{T}$ is known as an *independent resolving set* for the graph Φ.

The p^{th} coordinate (or distance component) of the code $\eta(v|\zeta)$ for an ordered set of vertices of Φ is zero if and only if $v = v_\rho$. Consequently, it is ample to verify for any two distinct vertices $u, v \in \theta(\Phi) \setminus \zeta$ that $\eta(v|\zeta) \neq \eta(v|\zeta)$ in order to observe that the set ζ is a resolving set.

The notions of metric dimension and resolution/location trace back to the 1950s. L. M. Blumenthal [1] described them in terms of metric space. Unrelated to one another Melter and Harary [2] in 1976 and P. J. Slater [3] in 1975 introduced these concepts to graph networks. For trees, heptagonal circular ladders [4], circulant graphs [5], Harary, and other structures, the invariant metric dimension has been studied. Graph theory is a useful tool for studying the verification process in discrete science and has applications in many areas of the figure, social, and regular sciences. Applications of this invariant to problems of picture creation (or image processing) and design identification (or pattern identification) are discussed in [6]; scientific applications are presented in [7]; applications to combinatorial improvement (or optimisation) are obtained in [8]; and challenges of validation and system reveal (or network discovery) are reviewed in [9].

A family of connected graphs, or charts, let $\mho$ be formed. In particular, if $\beta(\Psi) = dim(\Psi)$ is free of the possibility of selecting of the graph Ψ in $\mho$ and is finite (or constrained), we assign the family $\mho$ to have stable (or constant) location number. Stated differently, $\mho$ is said to as a family with an unchanging location number if every graph in it has an indistinguishable location number [10]. Chartrand et al. showed in [7] that if a graph on n nodes is a path $\wp_n$, then it has location number one. Furthermore, for every positive integer n; $n \geq 3$, cycle C_n has location number two. Consequently, a collection of graphs with a constant location number is established by C_n $(n \geq 3)$ and $\wp_n$ $(n \geq 2)$. In addition, the families of graphs with unchanged location numbers also include the generalised Petersen graphs $P(n, 2)$ and the Harary graphs $H_{4,n}$ [5]. Recently, Sharma and Bhat studied the pane graphs and their metric dimension in [11] heptagonal circular ladder in [12, 13]. Also, G. Gnecco et. al in [14] studied the optimal data collection designs in machine learning; and N. Iqbal in [15] reviewed the fractional study of the non-linear burgers' equation.

Joining two graphs $\Psi_1 = \Psi_1(\theta_1, \xi_1)$ and $\Psi_2 = \Psi_2(\theta_2, \xi_2)$, denoted by $\Phi = \Psi_1 + \Psi_2$, meaning a graph $\Phi = \Phi(\theta, \xi)$ such that $\theta = \theta_1 \cup \theta_2$ and $\xi = \xi_1 \cup \xi_2 \cup \{\nu\varsigma : \nu \in \theta_1 \text{and } \varsigma \in \theta_2\}$. Next, we define a wheel W_κ as $W_\kappa = K_1 + C_\kappa$ for $\kappa \geq 3$, a fan $F_\kappa = K_1 + \wp_\kappa$ for $\kappa \geq 1$, and a Jahangir graph $J_{2\kappa}$ $(\kappa \geq 2)$ resulting from the wheel graph $W_{2\kappa}$ by deleting κ spokes of the wheel graph (also known as a gear graph). The locating number for the fan graph F_κ $(\kappa \geq 1)$, as determined by Caceres et al. in [16], is $\lfloor \frac{2\kappa+2}{5} \rfloor$ for $\kappa \notin \{1, 2, 3, 6\}$. In [17] Chartrand et al., they determined the location number of the wheel graph W_κ $(\kappa \geq 3)$, which is $\lfloor \frac{2\kappa+2}{5} \rfloor$ for $\kappa \notin \{3, 6\}$. Tomescu and Javaid [18] obtained the location number of the Jahangir graph $J_{2\kappa}$ $(\kappa \geq 4)$, which is $\lfloor \frac{2\kappa}{3} \rfloor$. It should be noted that the location numbers of the plane graphs in these three families—the Fan, Wheel, and Jahangir graphs—depend on the total number of vertices in the graphs. As a result, these families do not belong to the plane graphs having fixed location numbers, also known as constant metric dimensions.

Now, a characteristic of a connected graph with respect to metric dimension equal to two. $\Psi = \Psi(\theta, \xi)$ was demonstrated by Khuller et al. in [19] and is

Theorem 1. [19] *Let $\mathcal{A} \subseteq \theta(\Psi)$ be the set that forms a basis of the connected graph $\Psi = \Psi(\theta, \xi)$ with cardinal number two i.e., $|\mathcal{A}| = dim(\Psi) = \beta(\Psi) = 2$, and say $\mathcal{A} = \{\nu, \xi\}$. Then, the below listed three points are satisfied:*

1. *Between the pair of vertices ν and ξ, there exists a unique as well as shortest path $\wp$.*
2. *The vertices (or nodes) ν and ξ have degrees (or valencies) that are never more than* 3.
3. *The valency of any other vertex (or node) on $\wp$ can never be more than* 5.

All vertex indices in this article are assumed to be modulo n. The location number of the plane graph $F_{n,m}$ is obtained in the subsequent section, and we show that $\beta(F_{n,m}) = 3$ for positive integers m, n, where $m \geq 1$ and $n \geq 6$.

2. The Plane Graph $F_{n,m}$

The plane graph $F_{n,m}$ comprises of $n(m + 5)$ number of nodes and $n(m + 8)$ number of edges. It has $3n$ 5-sided faces, and an n-sided face (see Figure 1). By $\xi(F_{n,m})$ and $\theta(F_{n,m})$, We represent the plane graph's $F_{n,m}$ vertex and edge ordering separately. Consequently, we have

$$\theta(F_{n,m}) = \{\rho_\tau, q_\tau, \ell_\tau, s_\tau, \tau_\tau, u_{\tau l} : 1 \leqslant \tau \leqslant n \text{ and } 1 \leqslant l \leqslant m\},$$

and

$$\begin{aligned}\xi(F_{n,m}) = \; & \{\rho_\tau q_\tau, q_\tau \ell_\tau, \ell_\tau s_\tau, s_\tau \tau_\tau, \tau_\tau u_{\tau 1}, u_{\tau 1} u_{\tau 2}, u_{\tau 2} u_{\tau 3}, \ldots, u_{\tau m-1} u_{\tau m} : 1 \\ & \leqslant \tau \leqslant n\} \cup \{\rho_\tau \rho_{\tau+1}, \ell_\tau q_{\tau+1}, u_{\tau 1} \tau_{\tau+1}, s_\tau s_{\tau+1} : 1 \leqslant \tau \leqslant n\}.\end{aligned}$$

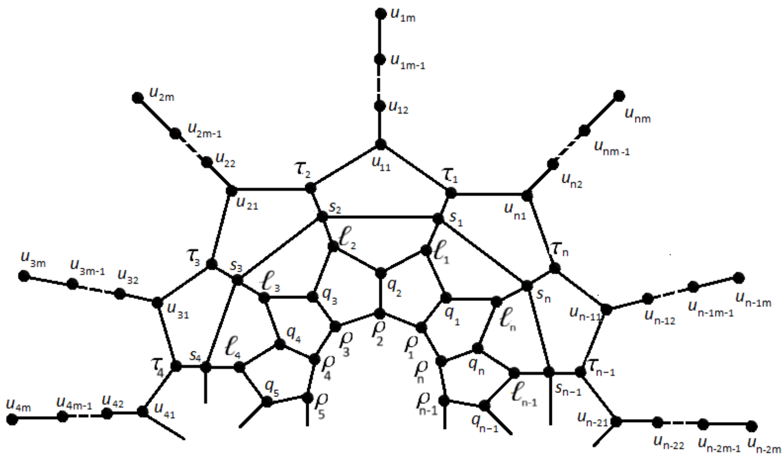

Figure 1. The Plane Graph $F_{n,m}$

For our motive, let us call the cycle brought forth by the arrangement of nodes $\{\rho_\tau : 1 \leqslant \tau \leqslant n\}$ as the ρ-cycle, the cycle brought forth by the arrangement of nodes $\{q_\tau : 1 \leqslant \tau \leqslant n\} \cup \{\ell_\tau : 1 \leqslant \tau \leqslant n\}$ as the $q\ell$-cycle, the cycle that the node layout creates $\{s_\tau : 1 \leqslant \tau \leqslant n\}$ as the s-cycle, the cycle that the node layout creates $\{\tau_\tau : 1 \leqslant \tau \leqslant n\} \cup \{u_{\tau 1} : 1 \leqslant \tau \leqslant n\}$ as the τu-cycle, and the rest of the vertices of the graphs $F_{n,m}$ as the outer vertices. In the accompanying theorem, we discover that the location number of the plane graph, $F_{n,m}$ is 3 i.e., just 3 vertices properly chosen are adequate to determine all the vertices of the plane graph $F_{n,m}$. Note that the choice of appropriate basis node is the crux of the problem.

Theorem 2. *For any two positive integers $m \geqslant 1$ and $n \geqslant 6$, let $F_{n,m}$ be the planar graph on $n(m+5)$ vertices as discussed earlier. Then, we have $dim(F_{n,m}) = 3$ i.e., the location number is* 3.

Proof. In order to illustrate this, we excitedly examine the two ensuing possibilities that depend on the positive integer n, namely, the even and odd values of the positive whole number n.

Case(slowromancapi@) When the integer n is even.

In this case, we can write $n = 2v$, $v \geq 3$, $v \in \mathcal{N}$. Let $\Re = \{\rho_2, \rho_{v+1}, \rho_n\} \subset \theta(F_{n,m})$, we show that $\Re$ is a locating set for $F_{n,m}$ in this case. For this, we give the metric codes for every node of $\theta(F_{n,m})$ concerning the set $\Re$.

Presently, the metric codes for the vertices of ρ-cycle $\{\rho_\tau : 1 \leqslant \tau \leqslant n\}$ are

$$\eta(\rho_\tau|\Re) = \begin{cases} (1, v, 1), & \tau = 1; \\ (0, v-1, 2), & \tau = 2; \\ (\tau-1, v-\tau+1, \tau), & 5 \le \tau \le v; \\ (v-1, 0, v-1), & \tau = v+1; \\ (2v-\tau+2, \tau-v-1, 2v-\tau), & v+2 \le \tau \le 2v-\tau : \\ (2, v-1, 0), & \tau = 2v. \end{cases}$$

The vertices of $q\ell$-cycle $\{q_\tau : 1 \leqslant \tau \leqslant n\} \cup \{\ell_\tau : 1 \leqslant \tau \leqslant n\}$ have metric codes:

$$\eta(q_\tau|\Re) = \eta(\rho_\tau|\Re) + (1, 1, 1),$$

and

$$\eta(\ell_\tau|\Re) = \begin{cases} (2, v+1, 3), & \tau = 1; \\ (\tau, v-\tau+2, \tau+2), & 2 \le \tau \le v-1; \\ (v, 2, v+1), & \tau = v; \\ (v+1, 2, v), & \tau = v+1; \\ (2v-\tau+3, \tau-v+1, 2v-\tau+1), & v+2 \le \tau \le 2v-1; \\ (2v-\tau+3, \tau-v+1, 2), & \tau = 2v. \end{cases}$$

The vertices of s-cycle $\{s_\tau : 1 \leqslant \tau \leqslant n\}$ have metric codes:

$$\eta(s_\tau|\Re) = \eta(\ell_\tau|\Re) + (1, 1, 1).$$

The metric codes for the vertices of τu-cycle $\{\tau_\tau : 1 \leqslant \tau \leqslant n\} \cup \{u_{\tau 1} : 1 \leqslant \tau \leqslant n\}$ are

$$\eta(\tau_\tau|\Re) = \eta(s_\tau|\Re) + (1, 1, 1),$$

and

$$\eta(u_{\tau 1}|\Re) = \begin{cases} (5, v+3, 6), & \tau = 1; \\ (\tau+3, v-\tau+4, \tau+5), & 2 \le \tau \le v-1; \\ (v+3, 5, v+3), & \tau = v; \\ (2v-\tau+5, \tau-v+4, 2v-\tau+3), & v+1 \le \tau \le 2v-2; \\ (2\varrho-\tau+5, \tau-\varrho+4, 5), & 2\varrho-1 \le \tau \le 2\varrho. \end{cases}$$

At last, the set of outer vertices $\{u_{\tau l} : 1 \leqslant \tau \leqslant n \text{ and } 2 \leqslant l \leqslant m\}$ have metric codes:

$$\eta(u_{\tau l}|\Re)) = \eta(u_{\tau 1}|\Re) + (l-1, l-1, l-1).$$

Since no two vertices have metric codes that are indistinguishable from one another, $\beta(F_{n,m}) \le 3$, it appears. Now, so as to finish the evidence for this case, we show that $\beta(F_{n,m}) \ge 3$ by working out that there does not exist a resolving set $\Re$ with the end goal that $|\Re| = 2$. Despite what might be expected, we guess that $\beta(F_{n,m}) = 2$. At that point by Theorem 1.1, we find that the valency of basis nodes can never exceed 3. But except the vertices of the set $\{s_\tau : 1 \leqslant \tau \leqslant n\}$, all other nodes of the radially symmetrical plane graph $F_{n,m}$ have a valency less than or equals to 3. At that point, we have the accompanying prospects to be talked about.

Resolving set	Contradiction
$\{\rho_1, \rho_g\}$, ρ_g $(2 \leq g \leq n)$	for $2 \leq g \leq \varrho$, we have $\eta(q_1\|\{\rho_1, \rho_g\}) = \eta(\rho_n\|\{\rho_1, \rho_g\})$, and when $g = \varrho + 1$, we have $\eta(\rho_n\|\{\rho_1, \rho_{\varrho+1}\}) = \eta(\rho_2\|\{\rho_1, \rho_{\varrho+1}\})$, a contradiction.
$\{q_1, q_g\}$, q_g $(2 \leq g \leq n)$	For $2 \leq g \leq \varrho - 2$, we have $\eta(q_{n-1}\|\{q_1, q_g\}) = \eta(\rho_{n-2}\|\{q_1, q_g\})$, when $\varrho - 1 \leq g \leq \varrho$, we have $\eta(q_2\|\{q_1, q_g\}) = \eta(s_1\|\{q_1, q_g\})$, and when $g = \varrho + 1$, we have $\eta(\rho_n\|\{q_1, q_{\varrho+1}\}) = \eta(\rho_2\|\{q_1, q_{\varrho+1}\})$, a contradiction.
$\{\ell_1, \ell_g\}$, ℓ_g $(2 \leq g \leq n)$	For $2 \leq g \leq \varrho$, we have $\eta(\tau_1\|\{\ell_1, \ell_g\}) = \eta(s_n\|\{\ell_1, \ell_g\})$, and when $g = \varrho + 1$, we have $\eta(s_n\|\{\ell_1, \ell_{\varrho+1}\}) = \eta(s_2\|\{\ell_1, \ell_{\varrho+1}\})$, a contradiction.

Resolving set	Contradiction
$\{\tau_1, \tau_g\}$, τ_g $(2 \leq g \leq n)$	For $2 \leq g \leq \varrho$, we have $\eta(\ell_1\|\{\tau_1, \tau_g\}) = \eta(s_n\|\{\tau_1, \tau_g\})$, and when $g = \varrho + 1$, we have $\eta(s_n\|\{\tau_1, \tau_{\varrho+1}\}) = \eta(s_2\|\{\tau_1, \tau_{\varrho+1}\})$, a contradiction.
$\{u_{1l}, u_{gl}\}$, u_{gl} $(1 \leq l \leq m, 2 \leq g \leq n)$	For $2 \leq g \leq \varrho - 1$, we have $\eta(\ell_1\|\{u_{1l}, u_{gl}\}) = \eta(s_n\|\{u_{1l}, u_{gl}\})$, when $g = \varrho$, we have $\eta(s_n\|\{u_{1l}, u_{\varrho l}\}) = \eta(\ell_2\|\{u_{1l}, u_{\varrho l}\})$, and when $g = \varrho + 1$, we have $\eta(s_1\|\{u_{1l}, u_{\varrho+1l}\}) = \eta(s_2\|\{u_{1l}, u_{\varrho+1l}\})$, a contradiction.
$\{\rho_1, q_g\}$, q_g $(1 \leq g \leq n)$	For $1 \leq g \leq \varrho - 2$, we have $\eta(q_{n-1}\|\{\rho_1, q_g\}) = \eta(\rho_{n-2}\|\{\rho_1, q_g\})$, when $\varrho - 1 \leq g \leq \varrho$, we have, $\eta(q_2\|\{\rho_1, q_g\}) = \eta(s_1\|\{\rho_1, q_g\})$, and when $g = \varrho + 1$, we have $\eta(\rho_n\|\{\rho_1, q_{\varrho+1}\}) = \eta(\rho_2\|\{\rho_1, q_{\varrho+1}\})$, a contradiction.
$\{\rho_1, \ell_g\}$, ℓ_g $(1 \leq g \leq n)$	For $1 \leq g \leq \varrho - 3$, we have $\eta(q_{n-1}\|\{\rho_1, \ell_g\}) = \eta(\rho_{n-2}\|\{\rho_1, r_g\})$, when $g = \varrho - 2$, we have, $\eta(q_2\|\{\rho_1, \ell_{\varrho-2}\}) = \eta(\ell_1\|\{\rho_1, \ell_{\varrho-2}\})$, and when $\varrho - 1 \leq g \leq \varrho + 1$, we have $\eta(s_{n-1}\|\{\rho_1, \ell_g\}) = \eta(q_{n-2}\|\{\rho_1, \ell_g\})$, a contradiction.
$\{\rho_1, \tau_g\}$, τ_g $(1 \leq g \leq n)$	For $g = 1$, we have $\eta(u_{11}\|\{\rho_1, \tau_1\}) = \eta(u_{n1}\|\{\rho_1, \tau_1\})$, and when $2 \leq g \leq \varrho + 1$, we have $\eta(\ell_1\|\{\rho_1, \tau_g\}) = \eta(q_2\|\{\rho_1, \tau_g\})$, a contradiction.
$\{\rho_1, u_{gl}\}$, u_{gl} $(1 \leq l \leq n, 1 \leq g \leq n)$	For $g = 1$, we have $\eta(\ell_2\|\{\rho_1, u_{1l}\}) = \eta(s_n\|\{\rho_1, u_{1l}\})$, when $2 \leq g \leq \varrho$, $\eta(\ell_1\|\{\rho_1, u_{gl}\}) = \eta(q_2\|\{\rho_1, u_{gl}\})$, and when $g = \varrho + 1$, we have $\eta(\ell_{n-1}\|\{\rho_1, u_{\varrho+1l}\}) = \eta(s_n\|\{\rho_1, u_{\varrho+1l}\})$, a contradiction.
$\{q_1, \ell_g\}$, ℓ_g $(1 \leq g \leq n)$	For $1 \leq g \leq \varrho - 1$, we have $\eta(s_{n-1}\|\{q_1, \ell_g\}) = \eta(\tau_n\|\{q_1, \ell_g\})$, and when $\varrho \leq g \leq \varrho + 1$, we have $\eta(s_1\|\{q_1, \ell_g\}) = \eta(\rho_2\|\{q_1, \ell_g\})$, a contradiction.

Resolving set	Contradiction
$\{q_1, \tau_g\}$, τ_g $(1 \le g \le n)$	For $g = 1$, we have $\eta(u_{n1}\|\{q_1, \tau_1\}) = \eta(u_{11}\|\{q_1, \tau_1\})$, when $2 \le g \le \varrho$, we have $\eta(s_n\|\{q_1, \tau_g\}) = \eta(q_2\|\{q_1, \tau_g\})$, and when $g = \varrho + 1$, we have $\eta(s_1\|\{q_1, \tau_{\varrho+1}\}) = \eta(q_n\|\{q_1, \tau_{\varrho+1}\})$, a contradiction.
$\{q_1, u_{gl}\}$, u_{gl} $(1 \le l \le n,\ 1 \le g \le n)$	For $g = 1$, we have $\eta(\ell_2\|\{q_1, u_{1l}\}) = \eta(\tau_n\|\{q_1, u_{1l}\})$, when $2 \le g \le \varrho - 1$, we have $\eta(s_n\|\{q_1, u_{gl}\}) = \eta(q_2\|\{q_1, u_{gl}\})$, when $g = \varrho$, we have $\eta(s_1\|\{q_1, u_{\varrho l}\}) = \eta(s_n\|\{q_1, u_{\varrho l}\})$, and when $g = \varrho + 1$, we have $\eta(s_1\|\{q_1, u_{\varrho+11}\}) = \eta(q_n\|\{q_1, u_{\varrho+11}\})$, a contradiction.
$\{\ell_1, \tau_g\}$, τ_g $(1 \le g \le n)$	For $g = 1$, we have $\eta(u_{n1}\|\{q_1, \ell_1\}) = \eta(u_{11}\|\{q_1, \ell_1\})$, when $2 \le g \le \varrho$, we have $\eta(s_n\|\{q_1, \ell_g\}) = \eta(\tau_1\|\{q_1, \ell_g\})$, and when $g = \varrho + 1$, we have $\eta(s_2\|\{q_1, \ell_{\varrho+1}\}) = \eta(s_n\|\{q_1, \ell_{\varrho+1}\})$, a contradiction.
$\{\ell_1, u_{gl}\}$, u_l $(1 \le l \le n,\ 1 \le g \le n)$	For $g = 1$, we have $\eta(q_2\|\{\ell_1, u_{1l}\}) = \eta(q_1\|\{\ell_1, u_{1l}\})$, when $g = 2$, we have $\eta(q_2\|\{\ell_2, u_{2l}\}) = \eta(\tau_1\|\{\ell_2, u_{2l}\})$, when $3 \le g \le \varrho - 1$, we have $\eta(s_n\|\{\ell_1, u_{gl}\}) = \eta(\tau_1\|\{\ell_1, u_{gl}\})$, when $g = \varrho$, we have $\eta(\ell_2\|\{\ell_1, u_{\varrho l}\}) = \eta(s_n\|\{\ell_1, u_{\varrho l}\})$, and when $g = \varrho + 1$, we have $\eta(s_2\|\{\ell_1, u_{\varrho+11}\}) = \eta(\ell_n\|\{\ell_1, u_{\varrho+11}\})$, a contradiction.
$\{\tau_1, u_{gl}\}$, u_{gl} $(1 \le l \le n,\ 1 \le g \le n)$	For $1 \le g \le \varrho - 1$, we have $\eta(s_n\|\{\tau_1, u_{gl}\}) = \eta(\ell_1\|\{\tau_1, u_{gl}\})$, when $g = \varrho$, we have $\eta(\tau_2\|\{\tau_1, u_{\varrho l}\}) = \eta(s_n\|\{\tau_1, u_{\varrho l}\})$, and when $g = \varrho + 1$, we have $\eta(s_2\|\{\tau_1, u_{\varrho+11}\}) = \eta(\tau_n\|\{\tau_1, u_{\varrho+11}\})$, a contradiction.

In this manner, the above conversation explains that there is no resolving set comprising of two vertices for $\theta(F_{n,m})$ inferring that $\beta(F_{n,m}) = 3$ in this case.

Case(slowromancapii@) When the integer n is odd.

In this case, we can write $n = 2\varrho + 1$, $\varrho \ge 3$, $\varrho \in \mathcal{N}$. Let $\Re = \{\rho_2, \rho_{\varrho+1}, \rho_n\} \subset \theta(F_{n,m})$, we show that $\Re$ is a locating set for $F_{n,m}$ in this case. For this, we give the metric codes for every node of $\theta(F_{n,m})$ concerning the set $\Re$.

Presently, the metric codes for the vertices of p-cycle $\{\rho_\tau : 1 \leqslant \tau \leqslant n\}$ are

$$\eta(\rho_\tau|\Re) = \begin{cases} (1, \varrho, 1), & \tau = 1; \\ (0, \varrho - 1, 2), & \tau = 2; \\ (\tau - 1, \varrho - \tau + 1, \tau), & 5 \le \tau \le \varrho; \\ (\varrho - 1, 0, \varrho), & \tau = \varrho + 1; \\ (2\varrho - \tau + 3, \tau - \varrho - 1, 2\varrho - \tau + 1), & \varrho + 2 \le \tau \le 2\varrho : \\ (2, \varrho, 0), & \tau = 2\varrho + 1. \end{cases}$$

The metric codes for the vertices of $q\ell$-cycle $\{q_\tau : 1 \leqslant \tau \leqslant n\} \cup \{\ell_\tau : 1 \leqslant \tau \leqslant n\}$ are

$$\eta(q_\tau|\Re) = \eta(\rho_\tau|\Re) + (1, 1, 1),$$

and

$$\eta(\ell_\tau|\Re) = \begin{cases} (2, \varrho+1, 3), & \tau = 1; \\ (\tau, \varrho-\tau+2, \tau+2), & 2 \leq \tau \leq \varrho; \\ (\varrho+1, 2, \varrho+1), & \tau = \varrho+1; \\ (2\varrho-\tau+4, \tau-\varrho+1, 2\varrho-\tau+2), & \varrho+2 \leq \tau \leq 2\varrho; \\ (2\varrho-\tau+4, \tau-\varrho+1, 2), & \tau = 2\varrho+1. \end{cases}$$

The metric codes for the vertices of s-cycle $\{s_\tau : 1 \leqslant \tau \leqslant n\}$ are

$$\eta(s_\tau|\Re) = \eta(\ell_\tau|\Re) + (1, 1, 1).$$

The metric codes for the vertices of τu-cycle $\{\tau_\tau : 1 \leqslant \tau \leqslant n\} \cup \{u_{\tau 1} : 1 \leqslant \tau \leqslant n\}$ are

$$\eta(\tau_\tau|\Re) = \eta(s_\tau|\Re) + (1, 1, 1),$$

and

$$\eta(u_{\tau 1}|\Re) = \begin{cases} (5, \varrho+3, 6), & \tau = 1; \\ (\tau+3, \varrho-\tau+4, \tau+5), & 2 \leq \tau \leq \varrho-1; \\ (\varrho+3, 5, \varrho+4), & \tau = \varrho; \\ (\varrho+4, 5, \varrho+3), & \tau = \varrho+1; \\ (2\varrho-\tau+6, \tau-\varrho+4, 2\varrho-\tau+4), & \varrho+2 \leq \tau \leq 2\varrho-1; \\ (2\varrho-\tau+6, \varrho+4, 5), & 2\varrho \leq \tau \leq 2\varrho+1. \end{cases}$$

At last, the metric codes for the set of outer vertices $\{u_{\tau l} : 1 \leqslant \tau \leqslant n \textit{ and } 2 \leqslant l \leqslant m\}$ are

$$\eta(u_{\tau l}|\Re)) = \eta(u_{\tau 1}|\Re) + (l-1, l-1, l-1).$$

Again we observe that no pair of vertices are having indistinguishable metric codes, suggesting $\beta(F_{n,m}) \leq 3$. As a result, we believe that $\beta(F_{n,m}) = 2$, as pointed out in Case(slowromancapi@), to be a parallel prospect, and logical contradiction can be deduced accordingly. Thus, $\beta(F_{n,m}) = 3$ also applies in this case, brought the proof to an a conclusion.

□

The desired result can alternatively be expressed as:

Theorem 3. *For two positive integers $m \geqslant 1$ and $n \geqslant 6$, let $F_{n,m}$ be the plane graph on $n(m+5)$ vertices as defined earlier. Then, its independent resolving number is* 3.

In the accompanying section, we acquire the location number of the plane graph $\beth_{n,m}$, and for positive integers m, n with $m \geq 1$ and $n \geq 6$ we demonstrate that $\beta(\beth_{n,m}) = 3$.

3. The Plane Graph $\beth_{n,m}$

The plane graph $\beth_{n,m}$ comprises of $n(m+5)$ number of nodes and $n(m+9)$ number of edges. This graph is obtained from the graph $F_{n,m}$ by placing the new edges between the nodes q_τ and $q_{\tau+1}$ $(1 \leqslant \tau \leqslant n)$. It has $2n$ 5-sided faces, n 4 and 3-sided faces, and an n-sided face (see Figure 2). By $\xi(\beth_{n,m})$ and $\theta(\beth_{n,m})$, we signify the arrangement of edges and vertices of the plane graph $\beth_{n,m}$ separately. Consequently, we have

$$\theta(\beth_{n,m}) = \{\rho_\tau, q_\tau, \ell_\tau, s_\tau, \tau_\tau, u_{\tau l} : 1 \leqslant \tau \leqslant n \textit{ and } 1 \leqslant l \leqslant m\},$$

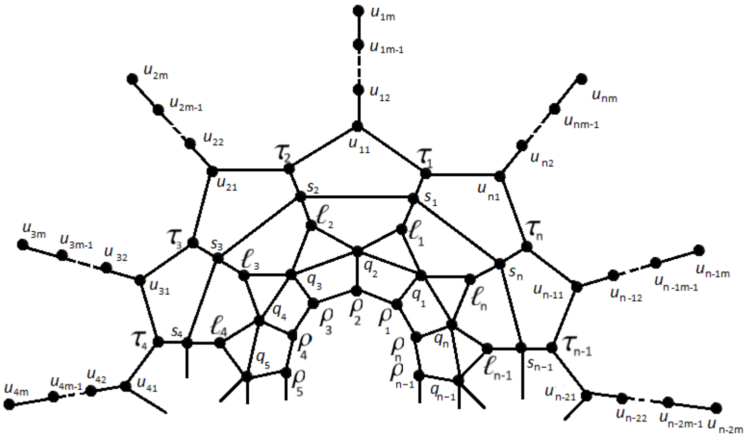

Figure 2. The Plane Graph $\gimel_{n,m}$

and

$$\begin{aligned}\xi(\gimel_{n,m}) \;=\; & \{\rho_\tau q_\tau, q_\tau \ell_\tau, \ell_\tau s_\tau, s_\tau \tau_\tau, \tau_\tau u_{\tau 1}, u_{\tau 1}u_{\tau 2}, u_{\tau 2}u_{\tau 3}, \ldots, u_{\tau m-1}u_{\tau m} : 1 \leqslant \\ & \tau \leqslant n\} \cup \{\rho_\tau \rho_{\tau+1}, q_\tau q_{\tau+1}, \ell_\tau q_{\tau+1}, u_{\tau 1}\tau_{\tau+1}, s_\tau s_{\tau+1} : 1 \leqslant \tau \leqslant n\}.\end{aligned}$$

For our motive, we call cycle brought forth by the arrangement of nodes $\{\rho_\tau : 1 \leqslant \tau \leqslant n\}$ as the ρ-cycle, the cycle brought forth by the arrangement of nodes $\{q_\tau : 1 \leqslant \tau \leqslant n\}$ as the q-cycle, the arrangement of nodes $\{\ell_\tau : 1 \leqslant \tau \leqslant n\}$ as the set of central nodes, the cycle brought forth by the arrangement of nodes $\{s_\tau : 1 \leqslant \tau \leqslant n\}$ as the s-cycle, the cycle brought forth by the arrangement of vertices $\{\tau_\tau : 1 \leqslant \tau \leqslant n\} \cup \{u_{\tau 1} : 1 \leqslant \tau \leqslant n\}$ as the τu-cycle, and the rest of the vertices of the graphs $\gimel_{n,m}$ as the outer vertices. In the accompanying theorem, we discover that the location number of the plane graph, $\gimel_{n,m}$ is 3 i.e., just 3 vertices properly chosen are adequate to determine all the vertices of the plane graph $\gimel_{n,m}$. Note that the choice of appropriate basis node is the crux of the problem.

Theorem 4. *For two positive integers $m \geqslant 1$ and $n \geqslant 6$, let $\gimel_{n,m}$ be the planar graph on $n(m+5)$ vertices as defined above. Then, we have $dim(\gimel_{n,m}) = 3$ i.e., it has location number 3.*

Proof. In order to illustrate this, we excitedly examine the two following cases that depend on the positive integer n, that is, the even and odd values of the positive integer n.

Case(slowromancapi@) When the integer n is even.

In this case, we can write $n = 2\varrho$, $\varrho \geq 3$, $\varrho \in \mathcal{N}$. Let $\Re = \{\rho_2, \rho_{\varrho+1}, \rho_n\} \subset \theta(\gimel_{n,m})$, we show that $\Re$ is a locating set for $\gimel_{n,m}$ in this case. For this, we give the metric codes for every node of $\theta(\gimel_{n,m})$ concerning the set $\Re$. As of now, the p-cycle $\{\rho_\tau : 1 \leqslant \tau \leqslant n\}$ has the following metric codes for its vertices:

$$\eta(\rho_\tau|\Re) = \begin{cases} (1, \varrho, 1), & \tau = 1; \\ (0, \varrho-1, 2), & \tau = 2; \\ (\tau-1, \varrho-\tau+1, \tau), & 5 \leq \tau \leq \varrho; \\ (\varrho-1, 0, \varrho-1), & \tau = \varrho+1; \\ (2\varrho-\tau+2, \tau-\varrho-1, 2\varrho-\tau), & \varrho+2 \leq \tau \leq 2\varrho-\tau : \\ (2, \varrho-1, 0), & \tau = 2\varrho. \end{cases}$$

The metric codes for the vertices of q-cycle $\{q_\tau : 1 \leqslant \tau \leqslant n\}$ are

$$\eta(q_\tau|\Re) = \eta(\rho_\tau|\Re) + (1, 1, 1).$$

The metric codes for the set of the central vertices $\{\ell_\tau : 1 \leqslant \tau \leqslant n\}$ are

$$\eta(r_\tau|\Re) = \begin{cases} (2, \varrho+1, 3), & \tau = 1; \\ (\tau, \varrho-\tau+2, \tau+2), & 2 \le \tau \le \varrho - 1; \\ (\varrho, 2, \varrho+1), & \tau = \varrho; \\ (\varrho+1, 2, \varrho), & \tau = \varrho+1; \\ (2\varrho-\tau+3, \tau-\varrho+1, 2\varrho-\tau+1), & \varrho+2 \le \tau \le 2\varrho-1; \\ (2\varrho-\tau+3, \tau-\varrho+1, 2), & \tau = 2\varrho. \end{cases}$$

The metric codes for the vertices of s-cycle $\{s_\tau : 1 \leqslant \tau \leqslant n\}$ are

$$\eta(s_\tau|\Re) = \eta(\ell_\tau|\Re) + (1, 1, 1).$$

The metric codes for the vertices of τu-cycle $\{\tau_\tau : 1 \leqslant \tau \leqslant n\} \cup \{u_{\tau 1} : 1 \leqslant \tau \leqslant n\}$ are

$$\eta(\tau_\tau|\Re) = \eta(s_\tau|\Re) + (1, 1, 1),$$

and

$$\eta(u_{\tau 1}|\Re) = \begin{cases} (5, \varrho+3, 6), & \tau = 1; \\ (\tau+3, \varrho-\tau+4, \tau+5), & 2 \le \tau \le \varrho - 1; \\ (\varrho+3, 5, \varrho+3), & \tau = \varrho; \\ (2\varrho-\tau+5, \tau-\varrho+4, 2\varrho-\tau+3), & \varrho+1 \le \tau \le 2\varrho-2; \\ (2\varrho-\tau+5, \tau-\varrho+4, 5), & 2\varrho-1 \le \tau \le 2\varrho. \end{cases}$$

At last, the metric codes for the set of outer vertices $\{u_{\tau l} : 1 \leqslant \tau \leqslant n \textit{ and } 2 \leqslant l \leqslant m\}$ are

$$\eta(u_{\tau l}|\Re)) = \eta(u_{\tau 1}|\Re) + (l-1, l-1, l-1).$$

Since no two vertices have metric codes that are indistinguishable from one another, $\beta(\gimel_{n,m}) \le 3$, it emerges. Now, so as to finish the evidence for this case, we show that $\beta(\gimel_{n,m}) \ge 3$ by working out that there does not exist a resolving set $\Re$ with the end goal that $|\Re| = 2$. Despite what might be expected, we guess that $\beta(\gimel_{n,m}) = 2$. Now, on applying the same procedure as in Theorem 2.1, we obtain the contradictions in the similar fashion.

In this manner, the above conversation explains that there is no resolving set comprising of two vertices for $\theta(\gimel_{n,m})$ inferring that $\beta(\gimel_{n,m}) = 3$ in this case.

Case(slowromancapii@) When the integer n is odd.

In this case, we can write $n = 2\varrho + 1$, $\varrho \ge 3$, $\varrho \in \mathcal{N}$. Let $\Re = \{\rho_2, \rho_{\varrho+1}, \rho_n\} \subset \theta(\gimel_{n,m})$, we show that $\Re$ is a locating set for $\gimel_{n,m}$ in this case. For this, we give the metric codes for every node of $\theta(\gimel_{n,m})$ concerning the set $\Re$. Presently, the vertices of p-cycle $\{\rho_\tau : 1 \leqslant \tau \leqslant n\}$ have metric codes:

$$\eta(\rho_\tau|\Re) = \begin{cases} (1, \varrho, 1), & \tau = 1; \\ (0, \varrho-1, 2), & \tau = 2; \\ (\tau-1, \varrho-\tau+1, \tau), & 5 \le \tau \le \varrho; \\ (\varrho-1, 0, \varrho), & \tau = \varrho+1; \\ (2\varrho-\tau+3, \tau-\varrho-1, 2\varrho-\tau+1), & \varrho+2 \le \tau \le 2\varrho : \\ (2, \varrho, 0), & \tau = 2\varrho+1. \end{cases}$$

The vertices of q-cycle $\{q_\tau : 1 \leqslant \tau \leqslant n\}$ have metric codes:

$$\eta(q_\tau|\Re) = \eta(\rho_\tau|\Re) + (1, 1, 1).$$

The metric codes for the set of central vertices ℓ-cycle $\{\ell_\tau : 1 \leqslant \tau \leqslant n\}$ are

$$\eta(\ell_\tau|\Re) = \begin{cases} (2, \varrho+1, 3), & \tau = 1; \\ (\tau, \varrho-\tau+2, \tau+2), & 2 \leq \tau \leq \varrho; \\ (\varrho+1, 2, \varrho+1), & \tau = \varrho+1; \\ (2\varrho-\tau+4, \tau-\varrho+1, 2\varrho-\tau+2), & \varrho+2 \leq \tau \leq 2\varrho; \\ (2\varrho-\tau+4, \tau-\varrho+1, 2), & \tau = 2\varrho+1. \end{cases}$$

The vertices of s-cycle $\{s_\tau : 1 \leqslant \tau \leqslant n\}$ metric codes:

$$\eta(s_\tau|\Re) = \eta(\ell_\tau|\Re) + (1, 1, 1).$$

The vertices of τu-cycle $\{\tau_\tau : 1 \leqslant \tau \leqslant n\} \cup \{u_{\tau 1} : 1 \leqslant \tau \leqslant n\}$ have metric codes:

$$\eta(\tau_\tau|\Re) = \eta(s_\tau|\Re) + (1, 1, 1),$$

and

$$\eta(u_{\tau 1}|\Re) = \begin{cases} (5, \varrho+3, 6), & \tau = 1; \\ (\tau+3, \varrho-\tau+4, \tau+5), & 2 \leq \tau \leq \varrho-1; \\ (\varrho+3, 5, \varrho+4), & \tau = \varrho; \\ (\varrho+4, 5, \varrho+3), & \tau = \varrho+1; \\ (2\varrho-\tau+6, \tau-\varrho+4, 2\varrho-\tau+4), & \varrho+2 \leq \tau \leq 2\varrho-1; \\ (2\varrho-\tau+6, \varrho+4, 5), & 2\varrho \leq \tau \leq 2\varrho+1. \end{cases}$$

At last, the metric codes for the set of outer vertices $\{u_{\tau l} : 1 \leqslant \tau \leqslant n \text{ and } 2 \leqslant l \leqslant m\}$ are

$$\eta(u_{\tau l}|\Re)) = \eta(u_{\tau 1}|\Re) + (l-1, l-1, l-1).$$

Again we see that no two vertices are having indistinguishable metric codes, suggesting that $\beta(\gimel_{n,m}) \leq 3$. Now, on expecting that $\beta(\gimel_{n,m}) = 2$, we consider that to be are parallel prospects as talked about in Case(slowromancapi@) and logical inconsistency can be inferred correspondingly. Consequently, $\beta(\gimel_{n,m}) = 3$ for this situation too, which concludes the theorem. □

This result can also be written as:

Theorem 5. *For two positive integers $m \geqslant 1$ and $n \geqslant 6$, let $\gimel_{n,m}$ be the planar graph on $n(m+5)$ vertices as defined above. Then, its independent resolving number is* 3.

In the accompanying section, we acquire the location number of the plane graph $\daleth_{n,m}$, and for positive integers m, n with $m \geq 1$ and $n \geq 6$ we demonstrate that $\beta(\daleth_{n,m}) = 3$.

4. The Plane Graph $\daleth_{n,m}$

The plane graph $\daleth_{n,m}$ comprises of $n(m+4)$ number of nodes and $n(m+9)$ number of edges. It has n 5-sided faces, n 4-sided faces, $2n$ 3-sided faces, and an n-sided face (see Figure 3). By $\xi(\daleth_{n,m})$ and $\theta(\daleth_{n,m})$, we signify the arrangement of edges and vertices of the plane graph $\daleth_{n,m}$ separately. Consequently, we have

$$\theta(\urcorner_{n,m}) = \{\rho_\tau, q_\tau, \ell_\tau, s_\tau, \tau_{\tau l} : 1 \leqslant \tau \leqslant n \ and \ 1 \leqslant l \leqslant m\},$$

and

$$\begin{aligned}\xi(\urcorner_{n,m}) &= \{\rho_\tau q_\tau, q_\tau \ell_\tau, \ell_\tau s_\tau, s_\tau \tau_{\tau 1}, \tau_{\tau 1}\tau_{\tau 2}, \tau_{\tau 2}\tau_{\tau 3}, \ldots, \tau_{\tau m-1}\tau_{\tau m} : 1 \leqslant \tau \leqslant n\} \\ &\cup \{\rho_\tau \rho_{\tau+1}, q_\tau \rho_{\tau+1}, q_\tau q_{\tau+1}, \ell_\tau \ell_{\tau+1}, \tau_{\tau 1} s_{\tau+1} : 1 \leqslant \tau \leqslant n\}.\end{aligned}$$

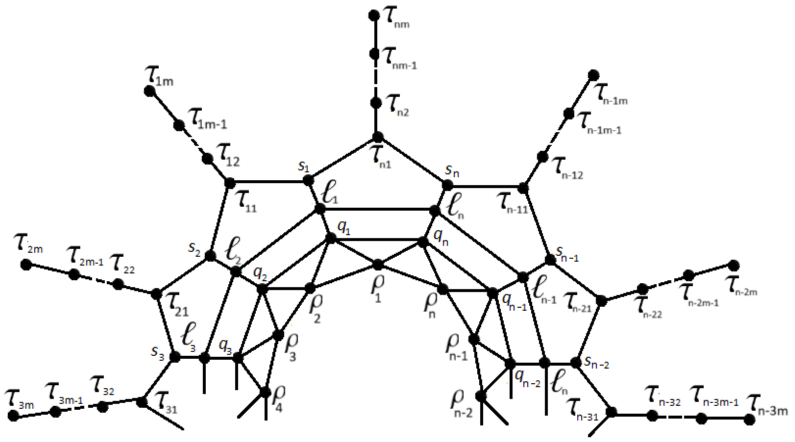

Figure 3. The Plane Graph $\urcorner_{n,m}$

For our motive, we call cycle brought forth by the arrangement of nodes $\{\rho_\tau : 1 \leqslant \tau \leqslant n\}$ as the ρ-cycle, the cycle brought forth by the arrangement of nodes $\{q_\tau : 1 \leqslant \tau \leqslant n\}$ as the q-cycle, the cycle brought forth by the arrangement of nodes $\{\ell_\tau : 1 \leqslant \tau \leqslant n\}$ as the ℓ-cycle, the cycle brought forth by the arrangement of vertices $\{s_\tau : 1 \leqslant \tau \leqslant n\} \cup \{\tau_{\tau 1} : 1 \leqslant \tau \leqslant n\}$ as the $s\tau$-cycle, and the rest of the vertices of the graph $\urcorner_{n,m}$ as the outer vertices. In the accompanying theorem, we discover that the location number of the plane graph, $\urcorner_{n,m}$ is 3 i.e., just 3 vertices properly chosen are adequate to determine all the vertices of the plane graph $\urcorner_{n,m}$. Note that the choice of appropriate basis node is the crux of the problem.

Theorem 6. *For two positive integers $m \geqslant 1$ and $n \geqslant 6$, let $\urcorner_{n,m}$ be the planar graph on $n(m+4)$ vertices as defined above. Then, we have $dim(\urcorner_{n,m}) = 3$ i.e., it has location number* 3.

Proof. To demonstrate this, we eagerly consider the resulting two cases relying on the positive integer n i.e., when the positive whole number n is even and when it is odd.
Case(slowromancapi@) When the integer n is even.

In this case, we can write $n = 2\varrho$, $\varrho \geq 3$, $\varrho \in \mathcal{N}$. Let $\Re = \{\rho_2, \rho_{\varrho+1}, \rho_n\} \subset \theta(\urcorner_{n,m})$, we show that $\Re$ is a locating set for $\urcorner_{n,m}$ in this case. For this, we give the metric codes for every node of $\theta(\urcorner_{n,m}) \setminus \Re$ concerning the set $\Re$. Presently, the metric codes for the vertices of ρ-cycle $\{\rho_\tau : 1 \leqslant t \leqslant n\}$ are

$$\eta(\rho_\tau|\Re) = \begin{cases} (1, \varrho, 1), & \tau = 1; \\ (\tau - 1, \varrho - \tau + 1, \tau), & 3 \leq \tau \leq \varrho; \\ (2\varrho - \tau + 2, \tau - \varrho - 1, 2\varrho - \tau), & \varrho + 2 \leq \tau \leq 2\varrho - 1. \end{cases}$$

The metric codes for the vertices of q-cycle $\{q_\tau : 1 \leqslant \tau \leqslant n\}$ are

$$\eta(q_\tau|\Re) = \begin{cases} (1,\varrho,2), & \tau = 1; \\ (\tau-1,\varrho-\tau+1,\tau-3), & 2 \leq \tau \leq \varrho-1; \\ (\varrho-1,1,\varrho), & \tau = \varrho; \\ (\varrho,1,\varrho-1), & \tau = \varrho+1; \\ (2\varrho-\tau+2,\tau-\varrho,2\varrho-\tau), & \varrho+2 \leq \tau \leq 2\varrho-1; \\ (2\varrho-\tau+2,\tau-\varrho,1), & \tau = 2\varrho. \end{cases}$$

The metric codes for the vertices of ℓ-cycle $\{\ell_\tau : 1 \leqslant \tau \leqslant n\}$ are

$$\eta(\ell_\tau|\Re) = \eta(q_\tau|\Re) + (1,1,1).$$

The metric codes for the vertices of $s\tau$-cycle $\{s_\tau : 1 \leqslant \tau \leqslant n\} \cup \{\tau_{\tau 1} : 1 \leqslant \tau \leqslant n\}$ are

$$\eta(s_\tau|\Re) = \eta(\ell_\tau|\Re) + (1,1,1),$$

and

$$\eta(\tau_{\tau 1}|\Re) = \begin{cases} (4,\varrho+2,5), & \tau = 1; \\ (\tau+2,\varrho-\tau+3,\tau+4), & 2 \leq \tau \leq \varrho-1; \\ (\varrho+2,4,\varrho+2), & \tau = \varrho; \\ (2\varrho-\tau+4,\tau-\varrho+3,2\varrho-\tau+2), & \varrho+1 \leq \tau \leq 2\varrho-2; \\ (2\varrho-\tau+4,\tau-\varrho+3,4), & 2\varrho-1 \leq \tau \leq 2\varrho. \end{cases}$$

The metric codes for the set of vertices $\{\tau_{\tau l} : 1 \leqslant \tau \leqslant n \textit{ and } 2 \leqslant l \leqslant m\}$ are

$$\eta(\tau_{\tau l}|\Re)) = \eta(\tau_{\tau 1}|\Re) + (l-1,l-1,l-1).$$

We notice that no two vertices are having indistinguishable metric codes, suggesting that $\beta(\urcorner_{n,m}) \leq 3$. Now, so as to finish the evidence for this case, we show that $\beta(\urcorner_{n,m}) \geq 3$ by working out that there does not exist a resolving set $\Re$ with the end goal that $|\Re| = 2$. Despite what might be expected, we guess that $\beta(\urcorner_{n,m}) = 2$. Now, on applying the same procedure as in Theorem 2.1, we obtain the contradictions in the similar fashion.

In this manner, the above conversation explains that there is no resolving set comprising of two vertices for $\theta(\urcorner_{n,m})$ inferring that $\beta(\urcorner_{n,m}) = 3$ in this case.

Case(slowromancapii@) When the integer n is even.

In this case, we can write $n = 2\varrho + 1$, $\varrho \geq 3$, $\varrho \in \mathcal{N}$. We demonstrate that $\Re$ is a locating set for $\urcorner_{n,m}$ in this case, assuming $\Re = \{\rho_2, \rho_{\varrho+1}, \rho_n\} \subset \theta(\urcorner_{n,m})$. We provide the metric codes for each node in $\theta(\urcorner_{n,m}) \setminus \Re$ with respect to the set $\Re$ in order to do this. Presently, the metric codes for the vertices of p-cycle $\{\rho_\tau : 1 \leqslant \tau \leqslant n\}$ are

$$\eta(\rho_\tau|\Re) = \begin{cases} (1,\varrho,1), & \tau = 1; \\ (\tau-1,\varrho-\tau+1,\tau), & 3 \leq \tau \leq \varrho; \\ (2\varrho-\tau+3,\tau-\varrho-1,2\varrho-\tau+1), & \varrho+2 \leq \tau \leq 2\varrho. \end{cases}$$

The metric codes for the vertices of q-cycle $\{q_\tau : 1 \leqslant \tau \leqslant n\}$ are

$$\eta(q_\tau|\Re) = \begin{cases} (1,\varrho,2), & \tau = 1; \\ (\tau-1,\varrho-\tau+1,\tau-3), & 2 \leq \tau \leq \varrho; \\ (\varrho,1,\varrho), & \tau = \varrho+1; \\ (2\varrho-\tau+3,\tau-\varrho,2\varrho-\tau+1), & \varrho+2 \leq \tau \leq 2\varrho; \\ (2\varrho-\tau+3,\tau-\varrho,1), & \tau = 2\varrho+1. \end{cases}$$

The metric codes for the vertices of ℓ-cycle $\{\ell_\tau : 1 \leqslant \tau \leqslant n\}$ are

$$\eta(\ell_\tau|\Re) = \eta(q_\tau|\Re) + (1, 1, 1).$$

The metric codes for the vertices of $s\tau$-cycle $\{s_\tau : 1 \leqslant \tau \leqslant n\} \cup \{\tau_{\tau 1} : 1 \leqslant \tau \leqslant n\}$ are

$$\eta(s_\tau|\Re) = \eta(\ell_\tau|\Re) + (1, 1, 1),$$

and

$$\eta(\tau_{\tau 1}|\Re) = \begin{cases} (4, \varrho + 2, 5), & \tau = 1; \\ (\tau + 2, \varrho - \tau + 3, \tau + 4), & 2 \leq \tau \leq \varrho - 1; \\ (\varrho + 2, 4, \varrho + 3), & \tau = \varrho; \\ (\varrho + 3, 4, \varrho + 2), & \tau = \varrho + 1; \\ (2\varrho - \tau + 5, \tau - \varrho + 3, 2\varrho - \tau + 3), & \varrho + 2 \leq \tau \leq 2\varrho - 1; \\ (2\varrho - \tau + 5, \varrho + 3, 4), & 2\varrho \leq \tau \leq 2\varrho + 1. \end{cases}$$

The metric codes for the set of vertices $\{\tau_{\tau l} : 1 \leqslant \tau \leqslant n \textit{ and } 2 \leqslant l \leqslant m\}$ are

$$\eta(\tau_{\tau l}|\Re)) = \eta(\tau_{\tau 1}|\Re) + (l - 1, l - 1, l - 1).$$

Once more, we discover the fact that there are no two vertices with identical metric codes, implying that $\beta(\urcorner_{n,m}) \leq 3$. As a result, we believe that $\beta(\urcorner_{n,m}) = 2$, as discussed in Case(slowromancapi@), to be a parallel prospect, and logical contradiction can be deduced accordingly. Thus, $\beta(\urcorner_{n,m}) = 3$ also holds in this case, providing the theorem to a conclusion. □

This result can also be written as:

Theorem 7. *For two positive integers $m \geqslant 1$ and $n \geqslant 6$, let $\urcorner_{n,m}$ be the planar graph on $n(m + 4)$ vertices as defined earlier. Then, the independent resolving number of $\urcorner_{n,m}$ is* 3.

5. Conclusion

This article include the study of the *metric dimension* of three classes of radially symmetrical planar graphs viz., $F_{n,m}$, $\beth_{n,m}$, and $\urcorner_{n,m}$, and we have proved that the *location number* of these three classes of planar graphs is finite and is independent of the number of vertices in these graphs, and just three vertices chosen properly are adequate to determine all the vertices of these classes of radially symmetrical planar graphs. We besides saw that the basis set $\Re$ is independent for these three radially even families of plane graphs. We also note that the metric dimension of the graphs obtained from these three graphs by placing new edges between the vertices of the paths (making the complete cycle) remain the same as the *metric dimension* of these three families of planar graphs.

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