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Group Codes over Algebra of Wreath Products

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ABSTRACT

In this paper, the hyperoctahedral group algebra $\mathscr{F}[\overrightarrow{S_n}]$ over a splitting field $\mathscr{F}$ of wreath product $\overrightarrow{S_n}$ with $char(\mathscr{F}) \nmid |\overrightarrow{S_n}|$, is considered and the unique idempotents corresponding to the four linear characters of the group $\overrightarrow{S_n}$ are explored. Also, by establishing the minimum weights and dimensions, all group codes generated by the linear idempotents in the aforementioned group algebra are completely characterized for every n. The nonlinear idempotents corresponding to nonlinear characters of $\overrightarrow{S_3}$ are also obtained and various group codes in $\mathscr{F}[\overrightarrow{S_3}]$ generated by linear and nonlinear idempotents are examined.

Keywords: Hyperoctahedral group, group algebra, character, idempotent, group code

1. Introduction

Coding theory originated long back in 1948 after Shannon [21] published a classic paper "A Mathematical Theory of Communication"and was later explored by numerous researchers in different directions (see [4, 10, 7, 6, 8]). For instance, algebraic coding theory is a fast growing and fascinating topic which is popular for its mathematical structure and practical applications. Out of all mathematical techniques used to construct codes in algebraic coding theory, group algebra codes are of great importance and play a vital role in the error correction and detection for reliable communications while transferring information through noisy channels. In 1967, Berman [5] proved that cyclic codes and Reed Muller codes are ideals or subspaces in a group algebra $\mathscr{F}[\mathscr{G}]$ of a field $\mathscr{F}$ and a finite group $\mathscr{G}$ under consideration. For a deeper study of these types of codes one can refer to [2, 5, 16]. However, there exist some group algebra codes which are constructed from zero divisors and units in group rings [11], which do not necessarily form ideals in the algebraic structure. An important study regarding Abelian and non-Abelian group codes in a group algebra of a group over

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any field has been described in [3, 15, 19, 20]. Further, the nature of group codes, whether Abelian or non-Abelian for $|\mathscr{G}| < 128$ with $|\mathscr{G}| \notin \{24, 48, 54, 60, 64, 72, 96, 108, 120\}$ is explored in [19]. The aforementioned work motivated us to construct group codes in a group algebra of hyperoctahedral groups $\overrightarrow{S_n}$. Furthermore, a study of group algebra codes over binary tetrahedral group can be found in [7].

Recall from [18] that a group algebra $\mathscr{F}[\mathscr{G}] = \{\sum_{g\in\mathscr{G}} a_g g : a_g \in \mathscr{F}\}$ is the free $\mathscr{F}$-module over $\mathscr{G}$ with the elements of group $\mathscr{G}$ as an $\mathscr{F}$-basis for $\mathscr{F}[\mathscr{G}]$, where $\mathscr{F}$ is a field and $|\mathscr{G}|$ is finite. The addition and multiplication of elements in $\mathscr{F}[\mathscr{G}]$ are defined naturally by extending the operations in both $\mathscr{F}$ and $\mathscr{G}$ as: $\sum_{g\in\mathscr{G}} a_g g + \sum_{g\in\mathscr{G}} b_g g = \sum_{g\in\mathscr{G}} (a_g + b_g)g$; $\Big(\sum_{g\in\mathscr{G}} a_g g\Big)\Big(\sum_{h\in\mathscr{G}} b_h h\Big) = \sum_{x\in\mathscr{G}} \Big(\sum_{gh=x} a_g b_h\Big)x$. Note that under these operations, $\mathscr{F}[\mathscr{G}]$ is an associative $\mathscr{F}$-algebra with identity $1 = 1_{\mathscr{F}} 1_{\mathscr{G}}$, where $1_{\mathscr{F}}$ and $1_{\mathscr{G}}$ are identity elements of $\mathscr{F}$ and $\mathscr{G}$ respectively. The scalar multiplication in $\mathscr{F}[\mathscr{G}]$ is defined by $\alpha\Big(\sum_{g\in\mathscr{G}} a_g g\Big) = \sum_{g\in\mathscr{G}} (\alpha a_g)g$, for $\alpha \in \mathscr{F}$. Also, it has been found that an element $g \in \mathscr{G}$ can be identified as an element $1.g$ of $\mathscr{F}[\mathscr{G}]$ i.e., $\mathscr{G} \subset \mathscr{F}[\mathscr{G}]$ and hence the elements of $\mathscr{G}$ constitute the coding basis for group codes when viewed as subspaces in $\mathscr{F}[\mathscr{G}]$.

Further, a group algebra code in $\mathscr{F}[\mathscr{G}]$ is defined as a one sided (left or right) ideal in $\mathscr{F}[\mathscr{G}]$, whereas if $\mathscr{F}[\mathscr{G}]$ is regarded as a vector space, then a code can be viewed as a subspace in $\mathscr{F}[\mathscr{G}]$. In addition, if $\mathscr{G}$ is cyclic or Abelian, then a code in a group algebra $\mathscr{F}[\mathscr{G}]$ is a cyclic code or an Abelian code. On the other hand, if a code is a vector subspace of $\mathscr{F}[\mathscr{G}]$, then it is called a linear code. More importantly, if $char(\mathscr{F}) \nmid |\mathscr{G}|$, then $\mathscr{F}[\mathscr{G}]$ is semisimple. Thus, $\mathscr{F}[\mathscr{G}]$ is decomposable into a direct sum $\mathscr{F}[\mathscr{G}] = \oplus_i \mathscr{F}\mathscr{G}_{e_k}$, where $\mathscr{F}\mathscr{G}_{e_k}$ is the minimal ideal generated by the idempotent e_k (see [12]). Suppose that $\mathscr{E} = \{e_i\}_{i=1}^{s}$ is the set of idempotents in a semisimple group algebra $\mathscr{F}[\mathscr{G}]$, then any ideal $\mathscr{I}$ in $\mathscr{F}[\mathscr{G}]$ is also expressible as a direct sum of minimal ideals in $\{\mathscr{F}\mathscr{G}_{e_j}\}_{e_j\in\mathscr{E}}$ which are generated by a single element $e_j \in \mathscr{E}$, i.e., $\mathscr{I} = \oplus_{x\in\mathscr{E}_1} \mathscr{F}\mathscr{G}_x$, for some subset $\mathscr{E}_1$ of $\mathscr{E}$. Let $\mathscr{I}$ be an ideal generated by a subset $\mathscr{E}' = \{e_i\}_{i=1}^{t}$ of the set of idempotents $\mathscr{E} \subseteq \mathscr{F}[\mathscr{G}]$. Then $\mathscr{I} = \{u \in \mathscr{F}\mathscr{G} : ue_j = 0 \text{ for all } e_j \in \mathscr{E} \setminus \mathscr{E}'\}$ and for simplicity $\mathscr{I}$ is denoted by $\mathscr{I}_\mu$, where $\mu = \mathscr{E} \setminus \mathscr{E}'$, the complement of $\mathscr{E}'$ in $\mathscr{E}$. Moreover, the length of a group code $\mathscr{I}_\mu$ in $\mathscr{F}[\mathscr{G}]$ is the order of the group $\mathscr{G}$ and is denoted by n. The weight $wt(u)$ of an element $u = \sum_{g\in\mathscr{G}} a_g g$ is the order of the set which consists of all non-zero coefficients in the presentation of u with elements of $\mathscr{G}$ as basis, i.e., $wt(u) = |\{a_g : a_g \neq 0\}|$. Let $\mathscr{I}$ be a group algebra code in $\mathscr{F}[\mathscr{G}]$ with its dimension as a subspace over $\mathscr{F}$ is k and the minimum distance d, where $d = d(\mathscr{I}) = min\{wt(u) : 0 \neq u \text{ and } ue_i = 0 \text{ for all } e_i \in \mu\}$. Then the group algebra code $\mathscr{I}$ is called (n, k, d) group code. Moreover, a linear (n, k, d) group code over $\mathscr{F}$ with $d = n - k + 1$ is called a maximum distance separable (MDS) group code.

Additionally, the algebraic structure of group algebra codes helps in detecting and correcting errors that may occur during reading or writing data. Further, algebraic properties in group algebra possess the capacity to design efficient decoding algorithms, reducing the time required for error correction. The use of group algebras lead to more efficient decoding processes compared to other coding schemes, particularly in terms of computational complexity. For instance, it follows from the above definition that producing a group code as an ideal in a group algebra involves the computations of product of elements with idempotents in the underlying group algebra. The complexity analysis of product of elements in group algebra has already been discussed in [1]. The results in [1] conclude that a computation of a product in group algebra apparently requires time $O(n^2)$ and the number of active multiplications can certainly be reduced to $O(n^{\frac{3}{2}})$, where n is the order of the group in group algebra.

In fact, the study of group codes in a group algebra $\mathscr{F}[\mathscr{G}]$ depends on the choice of the field $\mathscr{F}$

and the group $\mathscr{G}$. The hyperoctahedral group $\overrightarrow{S_n}$ is the wreath product of symmetric groups, i.e., $\overrightarrow{S_n} = \mathbb{Z}_2 \wr S_n$, the wreath product of a cyclic group of order 2 with the symmetric group over n symbols. This group can also be seen as the group of $n \times n$ signed permutation matrices. The group $\overrightarrow{S_n}$ of order $2^n n!$ has its abstract presentation as a group generated by $\sigma_1, \sigma_2, ..., \sigma_n$, satisfying the relations $\sigma_i^2 = 1$ $(i = 1, 2, ..., n)$, $(\sigma_i \sigma_{i+1})^3 = 1$ $(i = 1, 2, ..., n-2)$, $(\sigma_{n-1}\sigma_n)^4 = 1$ and $\sigma_j \sigma_k = \sigma_k \sigma_j$, $\forall j$ and $k \neq j \pm 1$.

Throughout this paper, $\overrightarrow{S_n}$ represents the hyperoctahedral group of dimension n and $\mathscr{F}$ is a splitting field of $\overrightarrow{S_n}$ whose order is relatively prime to $char(\mathscr{F})$. The main objective of this paper is to characterize completely the group codes generated by the linear idempotents in the group algebra $\mathscr{F}[\overrightarrow{S_n}]$ corresponding to the linear characters of $\overrightarrow{S_n}$, for every n. Also, various group codes of length 48 generated by linear and nonlinear idempotents in $\mathscr{F}[\overrightarrow{S_3}]$ are obtained in section 3.

2. Group codes generated by linear idempotents in $\mathscr{F}[\overrightarrow{S_n}]$

The group $\overrightarrow{S_n}$ has $p_b(n)$ conjugacy classes, where $p_b(n)$ represents the number of bi-partitions of a natural number n (see [17, 9]). Consequently, the group $\overrightarrow{S_n}$ has $p_b(n)$ characters, out of which four are linear characters (say) $\chi_1, \chi_2, \chi_3, \chi_4$, as discussed in the upcoming remark, while all the remaining characters are nonlinear. Recall that a linear character of a group $\mathscr{G}$ is a group homomorphism from $\mathscr{G}$ to the multiplicative group $\mathscr{F}^*$ of a field $\mathscr{F}$. In fact, the linear characters of a group are the only non zero characters which are homomorphisms from that group to the multiplicative group of a field and the set of all linear characters of a group forms an Abelian group under point wise multiplication which is called the character group of that group.

Remark 2.1. [17] Let $\sigma_1, \sigma_2, ..., \sigma_n$ be the generators of the hyperoctahedral group $\overrightarrow{S_n}$ of dimension $n \geq 2$. The character group of $\overrightarrow{S_n}$ is of order 4 i.e., $\overrightarrow{S_n}$ has four linear characters $\chi_1, \chi_2, \chi_3, \chi_4 : \overrightarrow{S_n} \longrightarrow \mathscr{F}^*$ defined by $\chi_1(\sigma_j) = 1$, $\forall\, j, 1 \leq j \leq n$; $\chi_2(\sigma_j) = -1$, $\forall\, j, 1 \leq j \leq n$;

$$\chi_3(\sigma_j) = \begin{cases} 1, & \forall\, j \leq n-1, \\ -1, & \text{for } j = n, \end{cases}$$

and

$$\chi_4(\sigma_j) = \begin{cases} -1, & \forall\, j \leq n-1, \\ 1, & \text{for } j = n. \end{cases}$$

As we have gathered all the necessary information required to obtain the linear idempotents corresponding to the linear characters in $\overrightarrow{S_n}$, so we list them in the following lemma by using Proposition 14.10 in [14], which states that: "If χ is a character of $\mathscr{F}[\mathscr{G}]$-module, then the unique idempotent corresponding to the character χ is given by $e = \frac{1}{|\mathscr{G}|} \sum_{g \in \mathscr{G}} \chi(g^{-1})g$". Moreover, an idempotent is termed as linear or nonlinear idempotent, if it corresponds to a linear or nonlinear character respectively.

Lemma 2.2. *The linear idempotents in $\mathscr{F}[\overrightarrow{S_n}]$ are given by* $e_1 = \frac{1}{2^n n!} \sum_{g \in \overrightarrow{S_n}} \chi_1(g^{-1})g = \frac{1}{2^n n!} \sum_{g \in \overrightarrow{S_n}} g$; $e_2 = \frac{1}{2^n n!} \sum_{g \in \overrightarrow{S_n}} \chi_2(g^{-1})g$; $e_3 = \frac{1}{2^n n!} \sum_{g \in \overrightarrow{S_n}} \chi_3(g^{-1})g$ *and* $e_4 = \frac{1}{2^n n!} \sum_{g \in \overrightarrow{S_n}} \chi_4(g^{-1})g$.

Further, we compute the product of an arbitrary element and the linear idempotents in $\mathscr{F}[\overrightarrow{S_n}]$ which will be used to construct various group codes in $\mathscr{F}[\overrightarrow{S_n}]$. For this, let $u = \sum_{g\in\overrightarrow{S_n}} \lambda_g g$ be an arbitrary element in $\mathscr{F}[\overrightarrow{S_n}]$ with $\lambda_g \in \mathscr{F}$. Then

$$\begin{aligned} ue_i &= \Big(\sum_{g\in\overrightarrow{S_n}} \lambda_g g\Big)\Big(\frac{1}{2^n n!}\sum_{h\in\overrightarrow{S_n}} \chi_i(h^{-1})h\Big) \\ &= \frac{1}{2^n n!}\sum_{x\in\overrightarrow{S_n}}\Big(\sum_{g,h\in\overrightarrow{S_n},gh=x} \lambda_g\chi_i(h^{-1})\Big)x \\ &= \frac{1}{2^n n!}\sum_{x\in\overrightarrow{S_n}}\Big(\sum_{g,h\in\overrightarrow{S_n},gh=x} \lambda_g\chi_i(h^{-1})\Big)\chi_i(xx^{-1})x, \end{aligned}$$

as for any character χ, $\chi(xx^{-1}) = \chi(e) = 1$, where e is the identity element of $\overrightarrow{S_n}$.

Further, since χ_i is a linear character and every linear character is a group homomorphism, we have $\chi_i(xx^{-1}) = \chi_i(x)\chi_i(x^{-1})$, therefore

$$ue_i = \frac{1}{2^n n!}\sum_{x\in\overrightarrow{S_n}}\Big(\sum_{g,h\in\overrightarrow{S_n},gh=x} \lambda_g\chi_i(h^{-1})\Big)\chi_i(x)\chi_i(x^{-1})x.$$

Note that $\chi_i(g) \in \mathscr{F}$, for all $g \in \overrightarrow{S_n}$ and $i = 1, 2, 3, 4$, so $\chi_i(x)$ can be distributed inside the summation, thus the above can be written as

$$ue_i = \frac{1}{2^n n!}\sum_{x\in\overrightarrow{S_n}}\Big(\sum_{g,h\in\overrightarrow{S_n},gh=x} \lambda_g\chi_i(h^{-1})\chi_i(x)\Big)\chi_i(x^{-1})x.$$

Again, χ_i is a homomorphism, therefore

$$\begin{aligned} ue_i &= \frac{1}{2^n n!}\sum_{x\in\overrightarrow{S_n}}\Big(\sum_{g,h\in\overrightarrow{S_n},gh=x} \lambda_g\chi_i(xh^{-1})\Big)\chi_i(x^{-1})x \\ &= \frac{1}{2^n n!}\sum_{x\in\overrightarrow{S_n}}\Big(\sum_{g\in\overrightarrow{S_n}} \lambda_g\chi_i(g)\Big)\chi_i(x^{-1})x \\ &= \Big(\sum_{g\in\overrightarrow{S_n}} \lambda_g\chi_i(g)\Big)\Big(\frac{1}{2^n n!}\sum_{x\in\overrightarrow{S_n}} \chi_i(x^{-1})x\Big) \\ &= \Big(\sum_{g\in\overrightarrow{S_n}} \lambda_g\chi_i(g)\Big)e_i. \end{aligned}$$

Thus, we summarize

$$ue_1 = \Big(\sum_{g\in\overrightarrow{S_n}} \lambda_g\chi_1(g)\Big)e_1 = \Big(\sum_{g\in\overrightarrow{S_n}} \lambda_g\Big)e_1, \tag{1}$$

$$ue_2 = \Big(\sum_{g\in\overrightarrow{S_n}} \lambda_g \chi_2(g)\Big)e_2, \tag{2}$$

$$ue_3 = \Big(\sum_{g\in\overrightarrow{S_n}} \lambda_g \chi_3(g)\Big)e_3, \tag{3}$$

$$ue_4 = \Big(\sum_{g\in\overrightarrow{S_n}} \lambda_g \chi_4(g)\Big)e_4. \tag{4}$$

Proposition 2.3. *The system of four equations* $\sum_{g\in\overrightarrow{S_n}} \lambda_g\chi_i(g) = 0$, $i = 1, 2, 3, 4$ *is linearly independent.*

Proof. Let $AX = 0$ be the matrix representation for the system of equations $\sum_{g\in\overrightarrow{S_n}} \lambda_g\chi_i(g) = 0$ where $i = 1, 2, 3, 4$. Clearly, the matrix A is a matrix of order $4 \times 2^n n!$ whose j^{th} column contains the coefficients of the j^{th} unknown λ_j in the given system of four equations for $i = 1, 2, 3, 4$ and X is a $2^n n! \times 1$ column matrix of $2^n n!$ unknowns λ_j, $1 \leq j \leq 2^n n!$. Note that the given system of equations is linearly independent if and only if the rank of the matrix A is 4. Equivalently, the matrix A has a non vanishing minor of order 4.

Next, we will construct a square sub-matrix of A of order 4 with non zero determinant. For this, first observe that each column of the matrix A corresponds to an element $g \in \overrightarrow{S_n}$ which is for sure the column vector $\begin{bmatrix}\chi_1(g)\\ \chi_2(g)\\ \chi_3(g)\\ \chi_4(g)\end{bmatrix}$, where $\chi_1, \chi_2, \chi_3, \chi_4$ are linear characters of $\overrightarrow{S_n}$. Now, consider a square sub matrix B of order 4 of the matrix A, whose columns are the columns of the matrix A corresponding to four distinct elements ${\sigma_1}^2, \sigma_1, \sigma_n$ and $\sigma_1\sigma_n \in \overrightarrow{S_n}$ is

$$B = \begin{bmatrix}\chi_1({\sigma_1}^2) & \chi_1(\sigma_1) & \chi_1(\sigma_n) & \chi_1(\sigma_1\sigma_n)\\ \chi_2({\sigma_1}^2) & \chi_2(\sigma_1) & \chi_2(\sigma_n) & \chi_2(\sigma_1\sigma_n)\\ \chi_3({\sigma_1}^2) & \chi_3(\sigma_1) & \chi_3(\sigma_n) & \chi_3(\sigma_1\sigma_n)\\ \chi_4({\sigma_1}^2) & \chi_4(\sigma_1) & \chi_4(\sigma_n) & \chi_4(\sigma_1\sigma_n)\end{bmatrix}.$$

Since χ_1, χ_2, χ_3 and χ_4 are linear characters of $\overrightarrow{S_n}$, therefore the entries of the above matrix reduces to

$$B = \begin{bmatrix}\chi_1(\sigma_1)^2 & \chi_1(\sigma_1) & \chi_1(\sigma_n) & \chi_1(\sigma_1)\chi_1(\sigma_n)\\ \chi_2(\sigma_1)^2 & \chi_2(\sigma_1) & \chi_2(\sigma_n) & \chi_2(\sigma_1)\chi_2(\sigma_n)\\ \chi_3(\sigma_1)^2 & \chi_3(\sigma_1) & \chi_3(\sigma_n) & \chi_3(\sigma_1)\chi_3(\sigma_n)\\ \chi_4(\sigma_1)^2 & \chi_4(\sigma_1) & \chi_4(\sigma_n) & \chi_4(\sigma_1)\chi_4(\sigma_n)\end{bmatrix}.$$

Further, in view of Remark 2.1, the above matrix takes the form

$$B = \begin{bmatrix}1 & 1 & 1 & 1\\ 1 & -1 & -1 & 1\\ 1 & 1 & -1 & -1\\ 1 & -1 & 1 & -1\end{bmatrix} \text{ and } |B| \neq 0.$$

□

From the above proposition, it follows that the four equations $\sum_{g\in\overrightarrow{S_n}} \lambda_g\chi_i(g) = 0$, (for $i = 1, 2, 3, 4$) are linearly independent. It is pertinent to notice that out of these four equations if we choose two or three distinct equations at a time, again form a linearly independent set, as a subset of a linearly independent set is linearly independent. The above observation infers that the dimension of an ideal $\mathscr{I}_\mu = \{u \in \mathscr{F}[\overrightarrow{S_n}] : ux = 0 \text{ for all } x \in \mu\}$ where $\mu \subseteq \{e_1, e_2, e_3, e_4\}$, as a vector subspace of $\mathscr{F}[\overrightarrow{S_n}]$ is $2^n n! - |\mu|$, because $ue_i = 0$ if and only if $\sum_{g\in\overrightarrow{S_n}} \lambda_g\chi_i(g) = 0$, for $i = 1, 2, 3, 4$ and every equation imposes an independent constraint.

The upcoming theorems provide us with various group codes in $\mathscr{F}[\overrightarrow{S_n}]$.

Theorem 2.4. *Let e_1, e_2, e_3, e_4 be linear idempotents in $\mathscr{F}[\overrightarrow{S_n}]$. Then $\mathscr{I}_{\{e_i\}}$ for $i = 1, 2, 3, 4$, is $(2^n n!\,, 2^n n! - 1, 2)$ MDS group code.*

Proof. Assume $u = \lambda g$, for $g \in \overrightarrow{S_n}$ and $0 \neq \lambda \in \mathscr{F}$ so that $wt(u) = 1$. By using equations (1) to (4), we have $ue_i = \pm\lambda\chi_i(g)e_i \neq 0$ for $i = 1, 2, 3, 4$. This implies that $u \notin \mathscr{I}_{\{e_i\}}$ and hence the minimum distance of $\mathscr{I}_{\{e_i\}}$ given by $d(\mathscr{I}_{\{e_i\}}) = min\{wt(u) : 0 \neq u \text{ and } ue_i = 0\}$ is ≥ 2. Now, for any choice of two distinct elements σ_j, σ_k from the subset $\{\sigma_1, \sigma_2, ..., \sigma_{n-1}\}$ of the set of generators of $\overrightarrow{S_n}$, there exists an element $u = \sigma_j - \sigma_k \in \mathscr{F}[\overrightarrow{S_n}]$ with weight 2 such that $ue_i = (\chi_i(\sigma_j) - \chi_i(\sigma_k))e_i$. Further, Remark 2.1 infers that $ue_i = 0$, as $\chi_i(\sigma_j) = \chi_i(\sigma_k)$ for all $j, k \in \{1, 2, ..., n-1\}$. This implies that $u \in \mathscr{I}_{\{e_i\}}$ for every i and hence $d(\mathscr{I}_{\{e_i\}}) = 2$. Moreover, as discussed above, the dimension of $\mathscr{I}_{\{e_i\}}$ as a vector space over $\mathscr{F}$ is $2^n n! - 1$. Hence $\mathscr{I}_{\{e_i\}}$ is a $(2^n n!\,, 2^n n! - 1, 2)$ MDS group code. □

Theorem 2.5. *Let e_1, e_2, e_3, e_4 be linear idempotents in $\mathscr{F}[\overrightarrow{S_n}]$ and i, j, k be three distinct elements of the set $\{1, 2, 3, 4\}$. Then*

(i) $\mathscr{I}_{\{e_i,e_j\}}$ *is* $(2^n n!\,, 2^n n! - 2, 2)$ *group code.*

(ii) $\mathscr{I}_{\{e_i,e_j,e_k\}}$ *is* $(2^n n!\,, 2^n n! - 3, 2)$ *group code.*

(iii) $\mathscr{I}_{\{e_1,e_2,e_3,e_4\}}$ *is* $(2^n n!\,, 2^n n! - 4, 2)$ *group code.*

Proof. Assume that $\mathscr{I}$ denotes the corresponding ideal in (i), (ii) and (iii). First, it is observed from equations (1) to (4) that for $i \in \{1, 2, 3, 4\}$, $ue_i = 0$ if and only if $\sum_{g\in\overrightarrow{S_n}} \lambda_g\chi_i(g) = 0$ is satisfied.

Now, in any of the parts (i), (ii) and (iii), if we choose an element $u = \lambda_g g$, for $g \in \overrightarrow{S_n}$, $0 \neq \lambda_g \in \mathscr{F}$ with $wt(u) = 1$, then it follows from equations (1) to (4) that $ue_i = 0$ for $i = 1, 2, 3, 4$, if and only if $\lambda_g = 0$. This implies that $u \notin \mathscr{I}$ and so $d(\mathscr{I}) \geq 2$. We claim that for any two distinct choices of generators σ_s, σ_t from a subset $\{\sigma_1, \sigma_2, ..., \sigma_{n-1}\}$ of the set of generators of $\overrightarrow{S_n}$, there exists an element $u = \sigma_s - \sigma_t \in \mathscr{F}[\overrightarrow{S_n}]$ such that $u \in \mathscr{I}$. In order to show this, it is enough to prove that $ue_i = 0$, for each $i = 1, 2, 3, 4$. Clearly, for $u = \sigma_s - \sigma_t$, we have $\lambda_{\sigma_s} = 1, \lambda_{\sigma_t} = -1$ and $\lambda_g = 0$, for all $g \in \overrightarrow{S_n} \setminus \{\sigma_s, \sigma_t\}$. Therefore, by using equations (1) to (4), we have $ue_i = (\chi_i(\sigma_s) - \chi_i(\sigma_t))e_i$. Again, by using Remark 2.1, we have $ue_i = 0$, as $\chi_i(\sigma_s) = \chi_i(\sigma_t)$ for all $s, t \in \{1, 2, ..., n-1\}$. Thus, $d(\mathscr{I}) = 2$. Moreover, the dimension of ideals $\mathscr{I}$ over $\mathscr{F}$ in the case (i), (ii) or (iii) is $2^n n! - 2$, $2^n n! - 3$ or $2^n n! - 4$ respectively and this finishes the proof. □

3. Group codes generated by linear idempotents in $\mathscr{F}[\overrightarrow{A_n}]$

The group $\overrightarrow{A_n}$ of order $2^{n-1}n!$ can be defined as the kernel of the homomorphism $\epsilon : \overrightarrow{S_n} \longrightarrow \{\pm 1\}$. Also, it can be viewed as the wreath product of a cyclic group of order 2 with the alternating group A_n i.e., $\overrightarrow{A_n} = \mathbb{Z}_2 \wr A_n$. For $n \geq 2$, $\overrightarrow{A_n}$ has $\frac{1}{2}p_b(n)$ or $\frac{1}{2}p_b(n) + \frac{3}{2}p(n)$ characters depending on whether n is even or odd respectively (see [9]). There are two linear characters of $\overrightarrow{A_n}$, namely, ${\chi_1}'$ the trivial character and ${\chi_2}'$, the restriction of the linear character χ_1 of $\overrightarrow{S_n}$ as already discussed in Remark 2.1.

Finally, for $\overrightarrow{A_n}$, using the same arguments with the necessary variations as in case of $\overrightarrow{S_n}$, one can easily obtain the linear idempotents and the group codes generated by these idempotents in a group algebra $\mathscr{F}[\overrightarrow{A_n}]$ as discussed in the following results, where $\mathscr{F}$ is a splitting field of $\overrightarrow{A_n}$ with $char(\mathscr{F}) \nmid |\overrightarrow{A_n}|$.

Lemma 3.1. *The linear idempotents in $\mathscr{F}[\overrightarrow{A_n}]$ are given by* $f_1 = \frac{1}{2^{n-1}n!} \sum\limits_{g \in \overrightarrow{A_n}} {\chi_1}'(g^{-1}) g = \frac{1}{2^{n-1}n!} \sum\limits_{g \in \overrightarrow{A_n}} g$ *and* $f_2 = \frac{1}{2^{n-1}n!} \sum\limits_{g \in \overrightarrow{A_n}} {\chi_2}'(g^{-1}) g$.

Theorem 3.2. *Let f_1, f_2 be linear idempotents in $\mathscr{F}[\overrightarrow{A_n}]$. Then*
(i) *$\mathscr{I}_{\{f_i\}}$ for $i = 1, 2$, is $(2^{n-1}n!\,, 2^{n-1}n! - 1, 2)$ MDS group code.*
(ii) *$\mathscr{I}_{\{f_1, f_2\}}$ is $(2^{n-1}n!\,, 2^{n-1}n! - 2, 2)$ group code.*

4. Various group codes of length 48 in $\mathscr{F}[\overrightarrow{S_3}]$

The hyperoctahedral group of dimension 3 is $\overrightarrow{S_3}$ which has its abstract presentation as

$$\langle a, b, c : a^2 = b^2 = c^2 = 1, (ab)^3 = 1, (bc)^4 = 1, ab \neq ba, bc \neq cb, ac = ca \rangle.$$

Thus, the elements of $\overrightarrow{S_3}$ as words, using the letters a, b and c can be listed in a set as follows:

$\overrightarrow{S_3} = \{1, a, b, aba, cbc, acbac, bacbacb, ab, ba, acbc, cbac, abacbc, acbacb, bacbac, bcbacb, c, bcb, abcba, ac, bacb, abacba, cbacbc, abcbacbc, bacbacbc, bcbc, abcbac, abacbacb, bc, cb, abac, abcb, acba, bcba, abacbacbc, abcbc, bacbc, bcbac, cbacb, abacbac, abcbacb, abc, acb, bac, cba, abacb, bacba, acbacbc, bcbacbc\}$.

Moreover, $\overrightarrow{S_3}$ has ten conjugacy classes which are given by

$$\begin{aligned}
C_1 &= \{1\},\\
C_2 &= \{a, b, aba, cbc, acbac, bacbacb\},\\
C_3 &= \{ab, ba, acbc, cbac, abacbc, acbacb, bacbac, bcbacb\},\\
C_4 &= \{c, bcb, abcba\},\\
C_5 &= \{ac, bacb, abacba, cbacbc, abcbacbc, bacbacbc\},\\
C_6 &= \{bcbc, abcbac, abacbacb\}\\
, C_7 &= \{bc, cb, abac, abcb, acba, bcba\},\\
C_8 &= \{abacbacbc\},\\
C_9 &= \{abcbc, bacbc, bcbac, cbacb, abacbac, abcbacb\},\\
C_{10} &= \{abc, acb, bac, cba, abacb, bacba, acbacbc, bcbacbc\}.
\end{aligned}$$

Consequently, the group $\overrightarrow{S_3}$ has ten characters, precisely, four linear characters $\chi_1, \chi_2, \chi_3, \chi_4$ and six nonlinear characters $\chi_5, \chi_6, \chi_7, \chi_8, \chi_9, \chi_{10}$, out of which the characters χ_5, χ_6 are of degree 2 and $\chi_7, \chi_8, \chi_9, \chi_{10}$ are of degree 3 discussed in Table 1.

Table 1. [13] The character table of $\vec{S_3}$

Characters	Conjugacy Classes									
	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
χ_1	1	1	1	1	1	1	1	1	1	1
χ_2	1	-1	1	-1	1	1	1	-1	-1	-1
χ_3	1	1	1	-1	-1	1	-1	-1	1	-1
χ_4	1	-1	1	1	-1	1	-1	1	- 1	1
χ_5	2	0	-1	2	0	2	0	2	0	-1
χ_6	2	0	-1	-2	0	2	0	-2	0	1
χ_7	3	1	0	1	-1	-1	1	-3	-1	0
χ_8	3	-1	0	-1	-1	-1	1	3	1	0
χ_9	3	1	0	-1	1	-1	-1	3	-1	0
χ_{10}	3	-1	0	1	1	-1	-1	-3	1	0

Now onwards, for our own convenience, we denote the class sum of a conjugacy class C_i by $\overline{C_i}$ and the i^{th} element of the set $\overrightarrow{S_3}$ defined in the beginning of this section by g_i. At this point, we are in a position to obtain the idempotents corresponding to the characters of $\overrightarrow{S_3}$, so in view of Proposition 14.10 in [14], we list them in the following two lemmas.

Lemma 4.1. *The linear idempotents in* $\mathscr{F}[\overrightarrow{S_3}]$ *are given by*

$$\begin{aligned} e_1 &= \tfrac{1}{48}[1 + \overline{C_2} + \overline{C_3} + \overline{C_4} + \overline{C_5} + \overline{C_6} + \overline{C_7} + \overline{C_8} + \overline{C_9} + \overline{C_{10}}], \\ e_2 &= \tfrac{1}{48}[1 - \overline{C_2} + \overline{C_3} - \overline{C_4} + \overline{C_5} + \overline{C_6} + \overline{C_7} - \overline{C_8} - \overline{C_9} - \overline{C_{10}}], \\ e_3 &= \tfrac{1}{48}[1 + \overline{C_2} + \overline{C_3} - \overline{C_4} - \overline{C_5} + \overline{C_6} - \overline{C_7} - \overline{C_8} + \overline{C_9} - \overline{C_{10}}], \\ e_4 &= \tfrac{1}{48}[1 - \overline{C_2} + \overline{C_3} + \overline{C_4} - \overline{C_5} + \overline{C_6} - \overline{C_7} + \overline{C_8} - \overline{C_9} + \overline{C_{10}}] \end{aligned}$$

where $\overline{C_i} = \sum_{g \in C_i} g$.

Lemma 4.2. *The nonlinear idempotents in* $\mathscr{F}[\overrightarrow{S_3}]$ *are given by*

$$\begin{aligned} v_1 &= \tfrac{1}{48}[(2)1 - \overline{C_3} + (2)\overline{C_4} + (2)\overline{C_6} + (2)\overline{C_8} - \overline{C_{10}}], \\ v_2 &= \tfrac{1}{48}[(2)1 - \overline{C_3} - (2)\overline{C_4} + (2)\overline{C_6} - (2)\overline{C_8} + \overline{C_{10}}], \\ v_3 &= \tfrac{1}{48}[(3)1 + \overline{C_2} + \overline{C_4} - \overline{C_5} - \overline{C_6} + \overline{C_7} - (3)\overline{C_8} - \overline{C_9}], \\ v_4 &= \tfrac{1}{48}[(3)1 - \overline{C_2} - \overline{C_4} - \overline{C_5} - \overline{C_6} + \overline{C_7} + (3)\overline{C_8} + \overline{C_9}], \\ v_5 &= \tfrac{1}{48}[(3)1 + \overline{C_2} - \overline{C_4} + \overline{C_5} - \overline{C_6} - \overline{C_7} + (3)\overline{C_8} - \overline{C_9}], \\ v_6 &= \tfrac{1}{48}[(3)1 - \overline{C_2} + \overline{C_4} + \overline{C_5} - \overline{C_6} - \overline{C_7} - (3)\overline{C_8} + \overline{C_9}] \end{aligned}$$

where $\overline{C_i} = \sum_{g \in C_i} g$.

Further, we compute the product of an arbitrary element and the idempotents in $\mathscr{F}[\overrightarrow{S_3}]$ which will be used to construct various group codes in $\mathscr{F}[\overrightarrow{S_3}]$. For this, let $u = \sum_{i=1}^{48} \lambda_i g_i$ be an arbitrary element in $\mathscr{F}[\overrightarrow{S_3}]$ with $\lambda_i \in \mathscr{F}$ for $i = 1, 2, ..., 48$, then calculating by SageMath [22], we have

$$ue_1 = \Big(\sum_{i=1}^{48} \lambda_i\Big) e_1, \tag{5}$$

$$ue_2 = \Big(\sum_{i \in \mathscr{S}_2} \lambda_i - \sum_{i \notin \mathscr{S}_2} \lambda_i\Big) e_2, \tag{6}$$

$$ue_3 = \Big(\sum_{i \in \mathscr{S}_3} \lambda_i - \sum_{i \notin \mathscr{S}_3} \lambda_i\Big) e_3, \tag{7}$$

$$ue_4 = \Big(\sum_{i \in \mathscr{S}_4} \lambda_i - \sum_{i \notin \mathscr{S}_4} \lambda_i\Big) e_4, \tag{8}$$

where $\mathscr{S}_2 = \{1, 8, 9, ..., 15, 19, 20, ..., 33\}$, $\mathscr{S}_3 = \{1, 2, ..., 15, 25, 26, 27, 35, 36, ..., 40\}$ and $\mathscr{S}_4 = \{1, 8, 9, ..., 18, 25, 26, 27, 34, 41, 42, ..., 48\}$.

Furthermore, Using SageMath [22] the products corresponding to the nonlinear idempotents are $uv_1 = \frac{1}{48}[2\lambda_1 - \lambda_8 - \lambda_9 - \lambda_{10} - \lambda_{11} - \lambda_{12} - \lambda_{13} - \lambda_{14} - \lambda_{15} + 2\lambda_{16} + 2\lambda_{17} + 2\lambda_{18} + 2\lambda_{25} + 2\lambda_{26} + 2\lambda_{27} + 2\lambda_{34} - \lambda_{41} - \lambda_{42} - \lambda_{43} - \lambda_{44} - \lambda_{45} - \lambda_{46} - \lambda_{47} - \lambda_{48}](g_1 + g_{16} + g_{17} + g_{18} + g_{25} + g_{26} + g_{27} + g_{34}) + [2\lambda_2 - \lambda_3 - \lambda_4 - \lambda_5 - \lambda_6 + 2\lambda_7 + 2\lambda_{19} - \lambda_{20} - \lambda_{21} - \lambda_{22} - \lambda_{23} + 2\lambda_{24} - \lambda_{28} - \lambda_{29} - \lambda_{30} + 2\lambda_{31} - \lambda_{32} + 2\lambda_{33} + 2\lambda_{35} - \lambda_{36} + 2\lambda_{37} - \lambda_{38} - \lambda_{39} - \lambda_{40}](g_2 + g_7 + g_{19} + g_{24} + g_{31} + g_{33} + g_{35} + g_{37}) + [-\lambda_2 + 2\lambda_3 - \lambda_4 + 2\lambda_5 - \lambda_6 - \lambda_7 - \lambda_{19} - \lambda_{20} + 2\lambda_{21} - \lambda_{22} + 2\lambda_{23} - \lambda_{24} + 2\lambda_{28} + 2\lambda_{29} - \lambda_{30} - \lambda_{31} - \lambda_{32} - \lambda_{33} - \lambda_{35} - \lambda_{36} - \lambda_{37} - \lambda_{38} + 2\lambda_{39} + 2\lambda_{40}](g_3 + g_5 + g_{21} + g_{23} + g_{28} + g_{29} + g_{39} + g_{40}) + [-\lambda_2 - \lambda_3 + 2\lambda_4 - \lambda_5 + 2\lambda_6 - \lambda_7 - \lambda_{19} + 2\lambda_{20} - \lambda_{21} + 2\lambda_{22} - \lambda_{23} - \lambda_{24} - \lambda_{28} - \lambda_{29} + 2\lambda_{30} - \lambda_{31} + 2\lambda_{32} - \lambda_{33} - \lambda_{35} + 2\lambda_{36} - \lambda_{37} + 2\lambda_{38} - \lambda_{39} - \lambda_{40}](g_4 + g_6 + g_{20} + g_{22} + g_{30} + g_{32} + g_{36} + g_{38}) + [-\lambda_1 + 2\lambda_8 - \lambda_9 + 2\lambda_{10} - \lambda_{11} - \lambda_{12} - \lambda_{13} + 2\lambda_{14} + 2\lambda_{15} - \lambda_{16} - \lambda_{17} - \lambda_{18} - \lambda_{25} - \lambda_{26} - \lambda_{27} - \lambda_{34} + 2\lambda_{41} + 2\lambda_{42} - \lambda_{43} - \lambda_{44} - \lambda_{45} + 2\lambda_{46} - \lambda_{47} + 2\lambda_{48}](g_8 + g_{10} + g_{14} + g_{15} + g_{41} + g_{42} + g_{46} + g_{48}) + [-\lambda_1 - \lambda_8 + 2\lambda_9 - \lambda_{10} + 2\lambda_{11} + 2\lambda_{12} + 2\lambda_{13} - \lambda_{14} - \lambda_{15} - \lambda_{16} - \lambda_{17} - \lambda_{18} - \lambda_{25} - \lambda_{26} - \lambda_{27} - \lambda_{34} - \lambda_{41} - \lambda_{42} + 2\lambda_{43} + 2\lambda_{44} + 2\lambda_{45} - \lambda_{46} + 2\lambda_{47} - \lambda_{48}](g_9 + g_{11} + g_{12} + g_{13} + g_{43} + g_{44} + g_{45} + g_{47})$.

Similarly, the products uv_2, uv_3, uv_4, uv_5 and uv_6 have been obtained in Appendix I.

Next, in support of Theorem 2.4, we present the following example for $n = 3$.

Example 4.3. Let e_1, e_2, e_3, e_4 be linear idempotents in $\mathscr{F}[\overrightarrow{S_3}]$. Then $\mathscr{I}_{\{e_i\}}$ for $i = 1, 2, 3, 4$, is $(48, 47, 2)$ MDS group code.

The result stated below can also be derived from Theorem 2.5. However, we are giving the proof of this result for better understanding, so that we can use equations (9) to (12) in the later part of this paper.

Theorem 4.4. *Let e_1, e_2, e_3, e_4 be linear idempotents in $\mathscr{F}[\overrightarrow{S_3}]$ and i, j, k be three distinct elements of the set $\{1, 2, 3, 4\}$. Then*
(i) $\mathscr{I}_{\{e_i, e_j\}}$ is $(48, 46, 2)$ group code.
(ii) $\mathscr{I}_{\{e_i, e_j, e_k\}}$ is $(48, 45, 2)$ group code.
(iii) $\mathscr{I}_{\{e_1, e_2, e_3, e_4\}}$ is $(48, 44, 2)$ group code.

Proof. Assume that $\mathscr{I}$ denotes the corresponding ideal in (i), (ii) and (iii). First, observe that it follows from equations (5) to (8) that $ue_i = 0$ for $i \in \{1, 2, 3, 4\}$ if and only if the i^{th} equation in the following equations is satisfied.

$$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12} + \lambda_{13} + \lambda_{14} + \lambda_{15} + \lambda_{16} + \lambda_{17} + \lambda_{18} + \lambda_{19} + \lambda_{20} + \lambda_{21} + \lambda_{22} + \lambda_{23} + \lambda_{24} + \lambda_{25} + \lambda_{26} + \lambda_{27} + \lambda_{28} + \lambda_{29} + \lambda_{30} + \lambda_{31} + \lambda_{32} + \lambda_{33} + \lambda_{34} + \lambda_{35} + \lambda_{36} + \lambda_{37} + \lambda_{38} + \lambda_{39} + \lambda_{40} + \lambda_{41} + \lambda_{42} + \lambda_{43} + \lambda_{44} + \lambda_{45} + \lambda_{46} + \lambda_{47} + \lambda_{48} = 0, \quad (9)$$

$$\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4 - \lambda_5 - \lambda_6 - \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12} + \lambda_{13} + \lambda_{14} + \lambda_{15} - \lambda_{16} - \lambda_{17} - \lambda_{18} + \lambda_{19} + \lambda_{20} + \lambda_{21} + \lambda_{22} + \lambda_{23} + \lambda_{24} + \lambda_{25} + \lambda_{26} + \lambda_{27} + \lambda_{28} + \lambda_{29} + \lambda_{30} + \lambda_{31} + \lambda_{32} + \lambda_{33} - \lambda_{34} - \lambda_{35} - \lambda_{36} - \lambda_{37} - \lambda_{38} - \lambda_{39} - \lambda_{40} - \lambda_{41} - \lambda_{42} - \lambda_{43} - \lambda_{44} - \lambda_{45} - \lambda_{46} - \lambda_{47} - \lambda_{48} = 0, \quad (10)$$

$$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12} + \lambda_{13} + \lambda_{14} + \lambda_{15} - \lambda_{16} - \lambda_{17} - \lambda_{18} - \lambda_{19} - \lambda_{20} - \lambda_{21} - \lambda_{22} - \lambda_{23} - \lambda_{24} + \lambda_{25} + \lambda_{26} + \lambda_{27} - \lambda_{28} - \lambda_{29} - \lambda_{30} - \lambda_{31} - \lambda_{32} - \lambda_{33} - \lambda_{34} + \lambda_{35} + \lambda_{36} + \lambda_{37} + \lambda_{38} + \lambda_{39} + \lambda_{40} - \lambda_{41} - \lambda_{42} - \lambda_{43} - \lambda_{44} - \lambda_{45} - \lambda_{46} - \lambda_{47} - \lambda_{48} = 0, \quad (11)$$

$$\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4 - \lambda_5 - \lambda_6 - \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12} + \lambda_{13} + \lambda_{14} + \lambda_{15} + \lambda_{16} + \lambda_{17} + \lambda_{18} - \lambda_{19} - \lambda_{20} - \lambda_{21} - \lambda_{22} - \lambda_{23} - \lambda_{24} + \lambda_{25} + \lambda_{26} + \lambda_{27} - \lambda_{28} - \lambda_{29} - \lambda_{30} - \lambda_{31} - \lambda_{32} - \lambda_{33} + \lambda_{34} - \lambda_{35} - \lambda_{36} - \lambda_{37} - \lambda_{38} - \lambda_{39} - \lambda_{40} + \lambda_{41} + \lambda_{42} + \lambda_{43} + \lambda_{44} + \lambda_{45} + \lambda_{46} + \lambda_{47} + \lambda_{48} = 0. \quad (12)$$

Now, in any of the parts (i), (ii) and (iii), if we choose an element $u = \lambda g$, for $g \in \overrightarrow{S_3}$, $0 \neq \lambda \in \mathscr{F}$ with $wt(u) = 1$, then it follows from equations (9) to (12) that $ue_i = 0$ for $i = 1, 2, 3, 4$, if and only if $\lambda = 0$. This implies that $u \notin \mathscr{I}$ and so $d(\mathscr{I}) \geq 2$. We claim that for any two distinct choices of elements s, t from the set $\{19, 20, ..., 24, 28, 29, ..., 33\}$, there exists an element $u = g_s - g_t \in \mathscr{F}[\overrightarrow{S_3}]$ such that $u \in \mathscr{I}$. For this, it is enough to show that $ue_i = 0$, for each $i = 1, 2, 3, 4$. Clearly, for $u = g_s - g_t$, we have $\lambda_{g_s} = 1, \lambda_{g_t} = -1$ and $\lambda_{g_k} = 0$, for all $k \in \{1, 2, ..., 48\} \setminus \{s, t\}$. Therefore, by using equations (9) to (12), we have $ue_i = (1-1)e_i = 0$, for $i = 1, 2, 3, 4$. Thus, $d(\mathscr{I}) = 2$. Moreover, the dimension of ideals $\mathscr{I}$ over $\mathscr{F}$ in the case (i), (ii) or (iii) is 46, 45 or 44 respectively. Hence the result. □

The upcoming theorems provide us with various group codes in $\mathscr{F}[\overrightarrow{S_3}]$, which are generated nonlinear idempotents or a combination of linear and nonlinear idempotents in $\mathscr{F}[\overrightarrow{S_3}]$.

Theorem 4.5. *Let v_1 and v_2 be nonlinear idempotents in $\mathscr{F}[\overrightarrow{S_3}]$ corresponding to nonlinear characters of degree 2. Then*
(i) $\mathscr{I}_{\{v_i\}}$ for $i = 1, 2$, is $(48, 44, 2)$ group code.
(ii) $\mathscr{I}_{\{v_1, v_2\}}$ is $(48, 40, 2)$ group code.

Proof. Let $u = \sum_{i=1}^{48} \lambda_i g_i \in \mathscr{F}[\overrightarrow{S_3}]$ and $\mu \subseteq \{v_1, v_2\}$. Then $\mathscr{I}_\mu = \{u \in \mathscr{F}[\overrightarrow{S_3}] : ux = 0 \text{ for all } x \in \mu\}$. Note that $uv_1 = 0$ if and only if the following equations are satisfied.

$$\lambda_1 + \lambda_8 - 2\lambda_9 + \lambda_{10} - 2\lambda_{11} - 2\lambda_{12} - 2\lambda_{13} + \lambda_{14} + \lambda_{15} + \lambda_{16} + \lambda_{17} + \lambda_{18} + \lambda_{25} + \lambda_{26} + \lambda_{27} + \lambda_{34} + \lambda_{41} + \lambda_{42} - 2\lambda_{43} - 2\lambda_{44} - 2\lambda_{45} + \lambda_{46} - 2\lambda_{47} + \lambda_{48} = 0, \quad (13)$$

$$\lambda_2 + \lambda_3 - 2\lambda_4 + \lambda_5 - 2\lambda_6 + \lambda_7 + \lambda_{19} - 2\lambda_{20} + \lambda_{21} - 2\lambda_{22} + \lambda_{23} + \lambda_{24} + \lambda_{28} + \lambda_{29} - 2\lambda_{30} + \lambda_{31} - 2\lambda_{32} + \lambda_{33} + \lambda_{35} - 2\lambda_{36} + \lambda_{37} - 2\lambda_{38} + \lambda_{39} + \lambda_{40} = 0, \quad (14)$$

$$3\lambda_3 - 3\lambda_4 + 3\lambda_5 - 3\lambda_6 - 3\lambda_{20} + 3\lambda_{21} - 3\lambda_{22} + 3\lambda_{23} + 3\lambda_{28} + 3\lambda_{29} - 3\lambda_{30} - 3\lambda_{32} - 3\lambda_{36} - 3\lambda_{38} + 3\lambda_{39} + 3\lambda_{40} = 0, \quad (15)$$

$$3\lambda_8 - 3\lambda_9 + 3\lambda_{10} - 3\lambda_{11} - 3\lambda_{12} - 3\lambda_{13} + 3\lambda_{14} + 3\lambda_{15} + 3\lambda_{41} + 3\lambda_{42} - 3\lambda_{43} - 3\lambda_{44} - 3\lambda_{45} + 3\lambda_{46} - 3\lambda_{47} + 3\lambda_{48} = 0, \quad (16)$$

whereas, $uv_2 = 0$ if and only if the following equations are satisfied.

$$\lambda_1 + \lambda_8 - 2\lambda_9 + \lambda_{10} - 2\lambda_{11} - 2\lambda_{12} - 2\lambda_{13} + \lambda_{14} + \lambda_{15} - \lambda_{16} - \lambda_{17} - \lambda_{18} + \lambda_{25} + \lambda_{26} + \lambda_{27} - \lambda_{34} - \lambda_{41} - \lambda_{42} + 2\lambda_{43} + 2\lambda_{44} + 2\lambda_{45} - \lambda_{46} + 2\lambda_{47} - \lambda_{48} = 0, \quad (17)$$

$$\lambda_2 + \lambda_3 - 2\lambda_4 + \lambda_5 - 2\lambda_6 + \lambda_7 - \lambda_{19} + 2\lambda_{20} - \lambda_{21} + 2\lambda_{22} - \lambda_{23} - \lambda_{24} - \lambda_{28} - \lambda_{29} + 2\lambda_{30} - \lambda_{31} + 2\lambda_{32} - \lambda_{33} + \lambda_{35} - 2\lambda_{36} + \lambda_{37} - 2\lambda_{38} + \lambda_{39} + \lambda_{40} = 0, \quad (18)$$

$$3\lambda_3 - 3\lambda_4 + 3\lambda_5 - 3\lambda_6 + 3\lambda_{20} - 3\lambda_{21} + 3\lambda_{22} - 3\lambda_{23} - 3\lambda_{28} - 3\lambda_{29} + 3\lambda_{30} + 3\lambda_{32} - 3\lambda_{36} - 3\lambda_{38} + 3\lambda_{39} + 3\lambda_{40} = 0, \quad (19)$$

$$3\lambda_8 - 3\lambda_9 + 3\lambda_{10} - 3\lambda_{11} - 3\lambda_{12} - 3\lambda_{13} + 3\lambda_{14} + 3\lambda_{15} - 3\lambda_{41} - 3\lambda_{42} + 3\lambda_{43} + 3\lambda_{44} + 3\lambda_{45} - 3\lambda_{46} + 3\lambda_{47} - 3\lambda_{48} = 0. \quad (20)$$

(i) Let $I_{\{v_i\}}$ be the set of elements of the form $u = \sum_{i=1}^{48} \lambda_i g_i \in \mathscr{F}[\overrightarrow{S_3}]$ such that the coefficients in the presentation of u satisfy equations (13) to (16), if $i = 1$, while equations (17) to (20), if $i = 2$. Now, as observed in the previous theorem, we can similarly prove that $\mathscr{I}_{\{v_i\}}$ does not contain any element with weight 1. Also, an element with the minimum weight in $\mathscr{I}_{\{v_i\}}$, for $i = 1, 2$, is $u = g_3 - g_5$ with weight 2, as the coefficients of λ_3 and λ_5 in equations (13) to (20) are equal. Thus, we conclude that $d(\mathscr{I}_{\{v_i\}}) = 2$. Moreover, the dimension of $\mathscr{I}_{\{v_i\}}$ is 44. Hence $\mathscr{I}_{\{v_i\}}$ is $(48, 44, 2)$ group code.

(ii) Consider an ideal $\mathscr{I}_{\{v_1,v_2\}}$ of $\mathscr{F}[\overrightarrow{S_3}]$ which consists of the elements of the form $u = \sum_{i=1}^{48} \lambda_i g_i \in \mathscr{F}[\overrightarrow{S_3}]$ such that the coefficients in the presentation of u satisfy equations (13) to (20). Again, one can realize that $\mathscr{I}_{\{v_1,v_2\}}$ does not contain any element with weight 1 and an element with the minimum weight in $\mathscr{I}_{\{v_1,v_2\}}$ is $u = g_3 - g_5$ with its weight 2. Therefore, $d(\mathscr{I}_{\{v_1,v_2\}}) = 2$. Moreover, the dimension of $\mathscr{I}_{\{v_1,v_2\}}$ is 40. Hence $\mathscr{I}_{\{v_1,v_2\}}$ is $(48, 40, 2)$ group code. □

Theorem 4.6. *Let e_1, e_2, e_3, e_4 be linear idempotents and v_1, v_2 be nonlinear idempotents corresponding to nonlinear characters of degree 2 of $\mathscr{F}[\overrightarrow{S_3}]$. If X_i and Y_j are subsets of order i and j of the sets $\{e_1, e_2, e_3, e_4\}$ and $\{v_1, v_2\}$ respectively. Then*

(i) $\mathscr{I}_{X_1 \cup Y_1}$ is $(48, 43, 2)$ group code.
(ii) $\mathscr{I}_{X_1 \cup Y_2}$ is $(48, 39, 2)$ group code.
(iii) $\mathscr{I}_{X_2 \cup Y_1}$ is $(48, 42, 2)$ group code.
(iv) $\mathscr{I}_{X_2 \cup Y_2}$ is $(48, 38, 2)$ group code.
(v) $\mathscr{I}_{X_3 \cup Y_1}$ is $(48, 41, 2)$ group code.
(vi) $\mathscr{I}_{X_3 \cup Y_2}$ is $(48, 37, 2)$ group code.
(vii) $\mathscr{I}_{X_4 \cup Y_1}$ is $(48, 40, 2)$ group code.
$(viii)$ $\mathscr{I}_{X_4 \cup Y_2}$ is $(48, 36, 2)$ group code.

Proof. Assume that $\mathscr{I}_{X_i \cup Y_j}$ denotes the corresponding ideal in any possible case of (i), (ii), (iii) and (iv), where X_i and Y_j are some subsets of order i and j of the sets $\{e_1, e_2, e_3, e_4\}$ and $\{v_1, v_2\}$ respectively. We claim that $d(\mathscr{I}) = 2$. For this, consider an ideal $\mathscr{I}_{X_i \cup Y_j}$ of $\mathscr{F}[\overrightarrow{S_3}]$ containing the elements of the form $u = \sum_{i=1}^{48} \lambda_i g_i$ such that $ux = 0$, for all $x \in X_i \cup Y_j$. This infers that depending upon the subsets X_i and Y_j the coefficients in the presentation of u must satisfy suitable equations from equations (9) to (20), corresponding to $ux = 0$, for all $x \in X_i \cup Y_j$. Clearly, $\mathscr{I}_{X_i \cup Y_j}$ can not contain any element of weight 1 and so $d(\mathscr{I}_{X_i \cup Y_j}) \geq 2$. Further, an element with the smallest weight 2 in $\mathscr{I}_{X_i \cup Y_j}$ is $u = g_3 - g_5$, as the coefficient of λ_3 and λ_5 in all the equations (9) to (20) are equal. Thus, we conclude that $d(\mathscr{I}_{X_i \cup Y_j}) = 2$. Next, for the dimension of $\mathscr{I}_{X_i \cup Y_j}$, observe that all the equations (9) to (20) are linearly independent therefore, the dimension of $\mathscr{I}_{X_i \cup Y_j}$ is $48 - (i + 4j)$. This completes the proof. □

Theorem 4.7. *Let v_3, v_4, v_5 and v_6 be nonlinear idempotents in $\mathscr{F}[\overrightarrow{S_3}]$ corresponding to nonlinear characters of degree 3. Then*

(i) $\mathscr{I}_{\{v_i\}}$ for $i = 3, 4, 5, 6$, is $(48, 39, 2)$ group code.
(ii) $\mathscr{I}_{\{v_3,v_6\}}$ is $(48, 30, 2)$ group code.
(iii) $\mathscr{I}_{\{v_4,v_5\}}$ is $(48, 30, 2)$ group code.

Proof. Let $u = \sum_{i=1}^{48} \lambda_i g_i \in \mathscr{F}[\overrightarrow{S_3}]$ and $\mu \subseteq \{v_3, v_4, v_5, v_6\}$. Then $\mathscr{I}_\mu = \{u \in \mathscr{F}[\overrightarrow{S_3}] : ux = 0 \text{ for all } x \in \mu\}$. Note that $uv_3 = 0$ if and only if the following equations are satisfied.

$$\lambda_1 + 2\lambda_5 + 2\lambda_6 - 2\lambda_7 - \lambda_9 + 3\lambda_{11} + 3\lambda_{12} + 4\lambda_{14} + 3\lambda_{16} - \lambda_{17} - \lambda_{18} + 2\lambda_{19} - 2\lambda_{20} - 2\lambda_{21} + \lambda_{25} + \lambda_{26}$$

$$- 3\lambda_{27} + 6\lambda_{28} + 4\lambda_{30} - 2\lambda_{31} - 2\lambda_{32} - \lambda_{34} + 2\lambda_{36} + 2\lambda_{37} + 4\lambda_{39} + 4\lambda_{41} + 5\lambda_{43} - 3\lambda_{44} - 3\lambda_{45} + \lambda_{47} = 0, \quad (21)$$

$$\lambda_2 + 2\lambda_5 + 2\lambda_6 + \lambda_9 + 5\lambda_{11} + 3\lambda_{12} + 6\lambda_{14} + 4\lambda_{16} - 2\lambda_{17} - 2\lambda_{18} + 5\lambda_{19} - 2\lambda_{20} - 2\lambda_{21} - \lambda_{24} + 2\lambda_{25} + 2\lambda_{26} + 8\lambda_{28} + 6\lambda_{30} + \lambda_{33} - \lambda_{35} + 4\lambda_{36} + 5\lambda_{37} + 6\lambda_{39} + 6\lambda_{41} + 9\lambda_{43} - 3\lambda_{44} - \lambda_{47} = 0, \quad (22)$$

$$\lambda_3 + 3\lambda_5 + 2\lambda_6 + \lambda_9 + 2\lambda_{10} + 3\lambda_{11} + 3\lambda_{12} + 4\lambda_{14} + 6\lambda_{16} - 2\lambda_{17} + 6\lambda_{19} - 2\lambda_{20} - 3\lambda_{21} - \lambda_{23} + 4\lambda_{25} + 2\lambda_{26} + 7\lambda_{28} - 3\lambda_{29} + 6\lambda_{30} - 2\lambda_{33} + 2\lambda_{35} + 4\lambda_{36} + 4\lambda_{37} + 3\lambda_{39} + 6\lambda_{41} + 7\lambda_{43} - 3\lambda_{44} - 3\lambda_{45} - 2\lambda_{46} - \lambda_{47} = 0, \quad (23)$$

$$\lambda_4 + 2\lambda_5 + 3\lambda_6 + \lambda_9 + 2\lambda_{10} + 3\lambda_{11} + \lambda_{12} + 2\lambda_{14} + 6\lambda_{16} - 2\lambda_{18} + 6\lambda_{19} - 3\lambda_{20} - 2\lambda_{21} - \lambda_{22} + 2\lambda_{25} + 4\lambda_{26} + 4\lambda_{28} - 2\lambda_{29} + 5\lambda_{30} - \lambda_{32} - 2\lambda_{33} + 2\lambda_{35} + \lambda_{36} + 4\lambda_{37} + 2\lambda_{39} + 4\lambda_{41} - 2\lambda_{42} + 5\lambda_{43} - \lambda_{44} - 3\lambda_{45} - 2\lambda_{46} - \lambda_{47} = 0, \quad (24)$$

$$4\lambda_5 + 4\lambda_{11} + 4\lambda_{16} + 4\lambda_{19} + 4\lambda_{25} + 4\lambda_{28} + 4\lambda_{37} + 4\lambda_{43} = 0, \quad (25)$$

$$4\lambda_6 + 4\lambda_{11} + 4\lambda_{14} + 4\lambda_{16} + 4\lambda_{19} + 4\lambda_{26} + 4\lambda_{28} + 4\lambda_{30} + 4\lambda_{37} + 4\lambda_{39} + 4\lambda_{41} + 4\lambda_{43} = 0, \quad (26)$$

$$\lambda_8 - \lambda_9 + \lambda_{10} - \lambda_{11} + 3\lambda_{12} - \lambda_{13} + \lambda_{14} - 3\lambda_{15} + 2\lambda_{28} - 2\lambda_{29} + 2\lambda_{30} + 2\lambda_{31} - 2\lambda_{32} - 2\lambda_{33} + 2\lambda_{35} + 2\lambda_{36} - 2\lambda_{37} - 2\lambda_{38} + 2\lambda_{39} - 2\lambda_{40} + 3\lambda_{41} - \lambda_{42} + \lambda_{43} - 3\lambda_{44} + \lambda_{45} - \lambda_{46} + \lambda_{47} - \lambda_{48} = 0, \quad (27)$$

$$4\lambda_{10} + 4\lambda_{13} + 4\lambda_{29} + 4\lambda_{31} + 4\lambda_{35} + 4\lambda_{40} + 4\lambda_{42} + 4\lambda_{45} = 0, \quad (28)$$

$$4\lambda_{12} + 4\lambda_{14} + 4\lambda_{28} + 4\lambda_{30} + 4\lambda_{36} + 4\lambda_{39} + 4\lambda_{41} + 4\lambda_{43} = 0, \quad (29)$$

whereas, $uv_4 = 0$ if and only if the following equations are satisfied.

$$\lambda_1 + 2\lambda_5 + 2\lambda_6 - \lambda_9 + 3\lambda_{12} + 3\lambda_{13} + 5\lambda_{16} - 3\lambda_{17} - 3\lambda_{18} + 2\lambda_{20} + 2\lambda_{21} - 3\lambda_{25} - 3\lambda_{26} + 5\lambda_{27} - 2\lambda_{28} + 6\lambda_{31} - 2\lambda_{32} + \lambda_{34} - 2\lambda_{36} + 6\lambda_{37} - 2\lambda_{40} + 3\lambda_{43} + 3\lambda_{44} - \lambda_{47} = 0, \quad (30)$$

$$\lambda_2 + 2\lambda_5 + 2\lambda_6 - \lambda_9 + \lambda_{12} + 5\lambda_{13} - 2\lambda_{14} + 2\lambda_{15} + 4\lambda_{16} - 2\lambda_{17} - 2\lambda_{18} + 2\lambda_{20} + 2\lambda_{21}$$

$$+ \lambda_{24} - 2\lambda_{25} - 2\lambda_{26} + 4\lambda_{27} + 2\lambda_{29} - 2\lambda_{30} + 5\lambda_{31} - \lambda_{33} - \lambda_{35} + 5\lambda_{37} - 2\lambda_{38} + 2\lambda_{39} + 2\lambda_{41} - 2\lambda_{42} + 5\lambda_{43} + \lambda_{44} - \lambda_{47} = 0, \tag{31}$$

$$\lambda_3 + 3\lambda_5 + 2\lambda_6 - \lambda_9 + 2\lambda_{10} + \lambda_{12} + 7\lambda_{13} + 2\lambda_{15} + 6\lambda_{16} - 2\lambda_{17} + 2\lambda_{20} + 3\lambda_{21} + \lambda_{23} - 2\lambda_{26} + 6\lambda_{27} + 3\lambda_{29} - 2\lambda_{30} + 8\lambda_{31} - 2\lambda_{33} - 2\lambda_{35} + 8\lambda_{37} - 2\lambda_{38} + 3\lambda_{39} + 2\lambda_{41} + 7\lambda_{43} + \lambda_{44} + 2\lambda_{46} - \lambda_{47} = 0, \tag{32}$$

$$\lambda_4 + 2\lambda_5 + 3\lambda_6 - \lambda_9 + 2\lambda_{10} + 3\lambda_{12} + 5\lambda_{13} - 2\lambda_{14} + 6\lambda_{16} - 2\lambda_{18} + 3\lambda_{20} + 2\lambda_{21} + \lambda_{22} - 2\lambda_{25} + 6\lambda_{27} + 2\lambda_{29} - \lambda_{30} + 8\lambda_{31} - 3\lambda_{32} - 2\lambda_{33} - 2\lambda_{35} - 3\lambda_{36} + 8\lambda_{37} - \lambda_{38} + 2\lambda_{39} - 2\lambda_{42} + 5\lambda_{43} + 3\lambda_{44} + 2\lambda_{46} - \lambda_{47} = 0, \tag{33}$$

$$4\lambda_5 + 4\lambda_{13} + 4\lambda_{16} + 4\lambda_{21} + 4\lambda_{27} + 4\lambda_{31} + 4\lambda_{37} + 4\lambda_{43} = 0, \tag{34}$$

$$4\lambda_6 + 4\lambda_{13} + 4\lambda_{15} + 4\lambda_{16} + 4\lambda_{20} + 4\lambda_{27} + 4\lambda_{29} + 4\lambda_{31} + 4\lambda_{37} + 4\lambda_{39} + 4\lambda_{41} + 4\lambda_{43} = 0, \tag{35}$$

$$\lambda_8 - \lambda_9 + \lambda_{10} - \lambda_{11} + 3\lambda_{12} - \lambda_{13} + \lambda_{14} - 3\lambda_{15} + 2\lambda_{28} - 2\lambda_{29} + 2\lambda_{30} + 2\lambda_{31} - 2\lambda_{32} - 2\lambda_{33} - 2\lambda_{35} - 2\lambda_{36} + 2\lambda_{37} + 2\lambda_{38} - 2\lambda_{39} + 2\lambda_{40} - 3\lambda_{41} + \lambda_{42} - \lambda_{43} + 3\lambda_{44} - \lambda_{45} + \lambda_{46} - \lambda_{47} + \lambda_{48} = 0, \tag{36}$$

$$4\lambda_{10} + 4\lambda_{13} + 4\lambda_{29} + 4\lambda_{31} + 4\lambda_{37} + 4\lambda_{39} + 4\lambda_{43} + 4\lambda_{46} = 0, \tag{37}$$

$$4\lambda_{12} + 4\lambda_{14} + 4\lambda_{28} + 4\lambda_{30} + 4\lambda_{38} + 4\lambda_{40} + 4\lambda_{42} + 4\lambda_{44} = 0, \tag{38}$$

whereas, $uv_5 = 0$ if and only if the following equations are satisfied.

$$\lambda_1 + 2\lambda_5 + 2\lambda_6 - 2\lambda_7 - \lambda_9 + 3\lambda_{11} + 3\lambda_{12} + 4\lambda_{14} - 3\lambda_{16} + \lambda_{17} + \lambda_{18} - 2\lambda_{19} + 2\lambda_{20} + 2\lambda_{21} + \lambda_{25} + \lambda_{26} - 3\lambda_{27} + 4\lambda_{29} + 2\lambda_{31} + 2\lambda_{32} + \lambda_{34} + 2\lambda_{36} + 2\lambda_{37} + 4\lambda_{39} + 4\lambda_{42} + 3\lambda_{44} + 3\lambda_{45} - \lambda_{47} = 0, \tag{39}$$

$$\lambda_2 + 2\lambda_5 + 2\lambda_6 + \lambda_9 + 5\lambda_{11} + 3\lambda_{12} + 6\lambda_{14} + 2\lambda_{17} + 2\lambda_{18} + 2\lambda_{20} + 2\lambda_{21} + \lambda_{24} + 2\lambda_{25} + 2\lambda_{26} + 6\lambda_{29} + 5\lambda_{31} + 4\lambda_{32} - \lambda_{33} - \lambda_{35} + 4\lambda_{36} + 5\lambda_{37} + 6\lambda_{39} + 6\lambda_{42} + 3\lambda_{44} + 5\lambda_{45} + \lambda_{47} = 0, \tag{40}$$

$$\lambda_3 + 3\lambda_5 + 2\lambda_6 + \lambda_9 + 2\lambda_{10} + 3\lambda_{11} + 3\lambda_{12} + 4\lambda_{14} + 2\lambda_{17} + 4\lambda_{18} + 2\lambda_{20} + 3\lambda_{21} + \lambda_{23} + 4\lambda_{25} + 2\lambda_{26}$$

$$+3\lambda_{29}+4\lambda_{31}+4\lambda_{32}+2\lambda_{33}+2\lambda_{35}+4\lambda_{36}+4\lambda_{37}+3\lambda_{39}+4\lambda_{42}+3\lambda_{44}+3\lambda_{45}+2\lambda_{46}+\lambda_{47}=0, \tag{41}$$

$$\lambda_4+2\lambda_5+3\lambda_6+\lambda_9+2\lambda_{10}+3\lambda_{11}+\lambda_{12}+2\lambda_{14}+4\lambda_{17}+2\lambda_{18}+3\lambda_{20}+2\lambda_{21}+\lambda_{22}+2\lambda_{25}+4\lambda_{26} +2\lambda_{29}+4\lambda_{31}+\lambda_{32}+2\lambda_{33}+2\lambda_{35}+\lambda_{36}+4\lambda_{37}+2\lambda_{39}+2\lambda_{42}+\lambda_{44}+3\lambda_{45}+2\lambda_{46}+\lambda_{47}=0, \tag{42}$$

$$4\lambda_5+4\lambda_{11}+4\lambda_{18}+4\lambda_{21}+4\lambda_{25}+4\lambda_{31}+4\lambda_{37}+4\lambda_{45}=0, \tag{43}$$

$$4\lambda_6+4\lambda_{11}+4\lambda_{14}+4\lambda_{17}+4\lambda_{20}+4\lambda_{26}+4\lambda_{29}+4\lambda_{31}+4\lambda_{37}+4\lambda_{39}+4\lambda_{42}+4\lambda_{45}=0, \tag{44}$$

$$\lambda_8-\lambda_9+\lambda_{10}-\lambda_{11}+3\lambda_{12}-\lambda_{13}+\lambda_{14}-3\lambda_{15}-2\lambda_{28}+2\lambda_{29}-2\lambda_{30}-2\lambda_{31}+2\lambda_{32}+2\lambda_{33}+2\lambda_{35} +2\lambda_{36}-2\lambda_{37}-2\lambda_{38}+2\lambda_{39}-2\lambda_{40}-3\lambda_{41}+\lambda_{42}-\lambda_{43}+3\lambda_{44}-\lambda_{45}+\lambda_{46}-\lambda_{47}+\lambda_{48}=0, \tag{45}$$

$$4\lambda_{10}+4\lambda_{13}+4\lambda_{28}+4\lambda_{33}+4\lambda_{35}+4\lambda_{40}+4\lambda_{43}+4\lambda_{46}=0, \tag{46}$$

$$4\lambda_{12}+4\lambda_{14}+4\lambda_{29}+4\lambda_{32}+4\lambda_{36}+4\lambda_{39}+4\lambda_{42}+4\lambda_{44}=0, \tag{47}$$

whereas, $uv_6=0$ if and only if the following equations are satisfied.

$$\lambda_1+2\lambda_5+2\lambda_6-\lambda_9+3\lambda_{12}+3\lambda_{13}+3\lambda_{17}+3\lambda_{18}+6\lambda_{19}-2\lambda_{20}-2\lambda_{21}-3\lambda_{25}-3\lambda_{26} +5\lambda_{27}+2\lambda_{28}+2\lambda_{32}-\lambda_{34}-2\lambda_{36}+6\lambda_{37}-2\lambda_{40}-3\lambda_{43}-3\lambda_{44}+5\lambda_{45}+\lambda_{47}=0, \tag{48}$$

$$\lambda_2+2\lambda_5+2\lambda_6-\lambda_9+\lambda_{12}+5\lambda_{13}-2\lambda_{14}+2\lambda_{15}+2\lambda_{17}+2\lambda_{18}+5\lambda_{19} -2\lambda_{20}-2\lambda_{21}-\lambda_{24}-2\lambda_{25}-2\lambda_{26}+4\lambda_{27}+4\lambda_{28}-2\lambda_{29}+2\lambda_{30}+\lambda_{33} -\lambda_{35}+5\lambda_{37}-2\lambda_{38}+2\lambda_{39}-2\lambda_{41}+2\lambda_{42}-\lambda_{44}+5\lambda_{45}+\lambda_{47}=0, \tag{49}$$

$$\lambda_3+3\lambda_5+2\lambda_6-\lambda_9+2\lambda_{10}+\lambda_{12}+7\lambda_{13}+2\lambda_{15}+2\lambda_{17}+4\lambda_{18}+6\lambda_{19} -2\lambda_{20}-3\lambda_{21}-\lambda_{23}-2\lambda_{26}+6\lambda_{27}+7\lambda_{28}-3\lambda_{29}+2\lambda_{30}+2\lambda_{33}-2\lambda_{35} +8\lambda_{37}-2\lambda_{38}+3\lambda_{39}-2\lambda_{41}+4\lambda_{42}-\lambda_{44}+7\lambda_{45}-2\lambda_{46}+\lambda_{47}=0, \tag{50}$$

$$\lambda_4+2\lambda_5+3\lambda_6-\lambda_9+2\lambda_{10}+3\lambda_{12}+5\lambda_{13}-2\lambda_{14}+4\lambda_{17}+2\lambda_{18}+6\lambda_{19}$$

$$- 3\lambda_{20} - 2\lambda_{21} - \lambda_{22} - 2\lambda_{25} + 6\lambda_{27} + 4\lambda_{28} - 2\lambda_{29} + \lambda_{30} + 3\lambda_{32} + 2\lambda_{33} - 2\lambda_{35} - 3\lambda_{36} + 8\lambda_{37} - \lambda_{38} + 2\lambda_{39} + 2\lambda_{42} - 3\lambda_{44} + 7\lambda_{45} - 2\lambda_{46} + \lambda_{47} = 0, \quad (51)$$

$$4\lambda_5 + 4\lambda_{13} + 4\lambda_{18} + 4\lambda_{19} + 4\lambda_{27} + 4\lambda_{28} + 4\lambda_{37} + 4\lambda_{45} = 0, \quad (52)$$

$$4\lambda_6 + 4\lambda_{13} + 4\lambda_{15} + 4\lambda_{17} + 4\lambda_{19} + 4\lambda_{27} + 4\lambda_{28} + 4\lambda_{30} + 4\lambda_{37} + 4\lambda_{39} + 4\lambda_{42} + 4\lambda_{45} = 0, \quad (53)$$

$$\lambda_8 - \lambda_9 + \lambda_{10} - \lambda_{11} + 3\lambda_{12} - \lambda_{13} + \lambda_{14} - 3\lambda_{15} - 2\lambda_{28} + 2\lambda_{29} - 2\lambda_{30} - 2\lambda_{31} + 2\lambda_{32} + 2\lambda_{33} - 2\lambda_{35} - 2\lambda_{36} + 2\lambda_{37} + 2\lambda_{38} - 2\lambda_{39} + 2\lambda_{40} + 3\lambda_{41} - \lambda_{42} + \lambda_{43} - 3\lambda_{44} + \lambda_{45} - \lambda_{46} + \lambda_{47} - \lambda_{48} = 0, \quad (54)$$

$$4\lambda_{10} + 4\lambda_{13} + 4\lambda_{28} + 4\lambda_{33} + 4\lambda_{37} + 4\lambda_{39} + 4\lambda_{42} + 4\lambda_{45} = 0, \quad (55)$$

$$4\lambda_{12} + 4\lambda_{14} + 4\lambda_{29} + 4\lambda_{32} + 4\lambda_{38} + 4\lambda_{40} + 4\lambda_{41} + 4\lambda_{43} = 0. \quad (56)$$

Suppose that $\mathscr{I}$ denotes the corresponding ideal in any possible case of (i), (ii), (iii) and (iv). We claim that $d(\mathscr{I}) = 2$. For this, consider the corresponding ideal $\mathscr{I}$ of $\mathscr{F}\overrightarrow{S_3}$ containing the elements of the form $u = \sum_{i=1}^{48} \lambda_i g_i$ satisfying $ux = 0$, for all $x \in \Lambda$, where Λ is a subset of the set $\{v_3, v_4, v_5, v_6\}$, which depends upon the case under consideration. This infers that the coefficients in the presentation of u must satisfy suitable equations from the set of equations (21) to (56), corresponding to $ux = 0$, for all $x \in \Lambda$. Clearly, $\mathscr{I}$ can not contain any element of weight 1 and so $d(\mathscr{I}) \geq 2$. Further, an element u with the smallest weight 2 in $\mathscr{I}$ is $u = g_1 + g_{34}$ if $\Lambda \subseteq \{v_3, v_6\}$, while $u = g_1 - g_{34}$ if $\Lambda \subseteq \{v_4, v_5\}$. Thus, we conclude that $d(\mathscr{I}) = 2$. Next, for the dimension of $\mathscr{I}$, observe that all the equations (21) to (56) are linearly independent, therefore the dimension of ideal $\mathscr{I}$ is $48 - 4|\Lambda|$. This completes the proof. □

We summarize some other results concerning group codes in $\mathscr{F}[\overrightarrow{S_3}]$ in the upcoming theorems. In fact these results can be proved by using similar techniques with necessary variation that we have used in our previous results of this section.

Theorem 4.8. *Let e_1, e_2, e_3, e_4 and v_1, v_2, v_3, v_4, v_5, v_6 be linear and nonlinear idempotents in $\mathscr{F}[\overrightarrow{S_3}]$. If X_i and Z_j are subsets of order i and j of the sets $\{e_1, e_4\}$ and $\{v_4, v_5\}$ respectively, then*
(i) $d(\mathscr{I}_{X_1 \cup Z_1}) = 2$ and $dim(\mathscr{I}_{X_1 \cup Z_1}) = 38$.
(ii) $d(\mathscr{I}_{X_1 \cup Z_2}) = 2$ and $dim(\mathscr{I}_{X_1 \cup Z_2}) = 29$.
(iii) $d(\mathscr{I}_{X_2 \cup Z_1}) = 2$ and $dim(\mathscr{I}_{X_2 \cup Z_1}) = 37$.
(iv) $d(\mathscr{I}_{X_2 \cup Z_2}) = 2$ and $dim(\mathscr{I}_{X_2 \cup Z_2}) = 28$.
(v) $d(\mathscr{I}_{\{v_2\} \cup Z_1}) = 2$ and $dim(\mathscr{I}_{\{v_2\} \cup Z_1}) = 35$.
(vi) $d(\mathscr{I}_{\{v_2\} \cup Z_2}) = 2$ and $dim(\mathscr{I}_{\{v_2\} \cup Z_2}) = 26$.

(vii) $d(\mathscr{I}_{X_1\cup\{v_1\}\cup Z_1}) = 2$ *and* $dim(\mathscr{I}_{X_1\cup\{v_1\}\cup Z_1}) = 34$.
(viii) $d(\mathscr{I}_{X_1\cup\{v_1\}\cup Z_2}) = 2$ *and* $dim(\mathscr{I}_{X_1\cup\{v_1\}\cup Z_2}) = 25$.
(ix) $d(\mathscr{I}_{X_2\cup\{v_1\}\cup Z_1}) = 2$ *and* $dim(\mathscr{I}_{X_2\cup\{v_1\}\cup Z_1}) = 33$.
(x) $d(\mathscr{I}_{X_2\cup\{v_1\}\cup Z_2}) = 2$ *and* $dim(\mathscr{I}_{X_2\cup\{v_1\}\cup Z_2}) = 24$.

Theorem 4.9. *Let* e_1, e_2, e_3, e_4 *and* v_1, v_2, v_3, v_4, v_5, v_6 *be linear and nonlinear idempotents in* $\mathscr{F}[\overrightarrow{S_3}]$. *If* X_i *and* Z_j *are subsets of order* i *and* j *of the sets* $\{e_2, e_3\}$ *and* $\{v_3, v_6\}$ *respectively, then*
(i) $\mathscr{I}_{X_1\cup Z_1}$ *is* $(48, 38, 2)$ *group code.*
(ii) $\mathscr{I}_{X_1\cup Z_2}$ *is* $(48, 29, 2)$ *group code.*
(iii) $\mathscr{I}_{X_2\cup Z_1}$ *is* $(48, 37, 2)$ *group code.*
(iv) $\mathscr{I}_{X_2\cup Z_2}$ *is* $(48, 28, 2)$ *group code.*
(v) $\mathscr{I}_{\{v_2\}\cup Z_1}$ *is* $(48, 35, 2)$ *group code.*
(vi) $\mathscr{I}_{\{v_2\}\cup Z_2}$ *is* $(48, 26, 2)$ *group code.*
(vii) $\mathscr{I}_{X_1\cup\{v_2\}\cup Z_1}$ *is* $(48, 34, 2)$ *group code.*
(viii) $\mathscr{I}_{X_1\cup\{v_2\}\cup Z_2}$ *is* $(48, 25, 2)$ *group code.*
(ix) $\mathscr{I}_{X_2\cup\{v_2\}\cup Z_1}$ *is* $(48, 33, 2)$ *group code.*
(x) $\mathscr{I}_{X_2\cup\{v_2\}\cup Z_2}$ *is* $(48, 24, 2)$ *group code.*

5. Conclusions

This paper completely characterized the four families of group codes namely; $(2^n n!, 2^n n! - 1, 2)$ (MDS group code), $(2^n n!, 2^n n! - 2, 2)$, $(2^n n!, 2^n n! - 3, 2)$ and $(2^n n!, 2^n n! - 4, 2)$ group code which are generated by the linear idempotents corresponding to the linear characters of $\overrightarrow{S_n}$ in the hyperoctahedral group algebra $\mathscr{F}[\overrightarrow{S_n}]$ for every n and thereby established two families of group codes $(2^{n-1} n!, 2^{n-1} n! - 1, 2)$ (MDS group code) and $(2^{n-1} n!, 2^{n-1} n! - 2, 2)$ group code in $\mathscr{F}[\overrightarrow{A_n}]$ generated by linear idempotents corresponding to linear characters of $\overrightarrow{A_n}$, for every n. In addition, as the order and the number of nonlinear characters of the group $\overrightarrow{S_n}$ increases very rapidly with n, therefore the problem of complete characterization of group code generated by nonlinear idempotents for arbitrary n, in $\mathscr{F}[\overrightarrow{S_n}]$ is very lengthy to compute, in general. Moreover, we also attempted to explore various families of group codes of length 48 having minimum distance 2, generated by linear and nonlinear idempotents $\mathscr{F}[\overrightarrow{S_3}]$ of hyperoctahedral group of small order.

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Conflict of Interest The authors declare that there is no conflict of interest.

References

[1] M. D. Atkinson. The complexity of group algebra computations. *Theoretical Computer Science*, 5(2):205–209, 1977. https://doi.org/10.1016/0304-3975(77)90007-X.

[2] H. G. Aun and W. C. Keong. Group codes defined using extra-special p-group of order p3. *Bulletin of the Malaysian Mathematical Sciences Society*, 27(2):185–205, 2004.

[3] R. F. Bailey. Error-correcting codes from permutation groups. *Discrete Mathematics*, 309(13):4253–4265, 2009. https://doi.org/10.1016/j.disc.2008.12.027.

[4] A. Bayram and I. Siap. Cyclic and constacyclic codes over a non-chain ring. *Journal of Algebra Combinatorics Discrete Structures and Applications*, 1(1):1–12, 2014.

[5] S. Berman. Semisimple cyclic and abelian codes. ii. *Kibernetika*, 3(3):17–23, 1967. https://doi.org/10.1007/BF01119999.

[6] Y. Cao, Y. Cao, F.-W. Fu, and S. Wang. Left dihedral codes over galois rings gr (p2, m). *Discrete Mathematics*, 341(6):1816–1834, 2018. https://doi.org/10.1016/j.disc.2017.10.035.

[7] M. Dadhwal and Pankaj. Group codes over binary tetrahedral group. *Journal of Mathematical Cryptology*, 16(1):310–319, 2022. https://doi.org/10.1515/jmc-2022-0009.

[8] Y. Gao, Q. Yue, and Y. Wu. Lcd codes and self-orthogonal codes in generalized dihedral group algebras. *Designs, Codes and Cryptography*, 88(11):2275–2287, 2020. https://doi.org/10.1007/s10623-020-00778-z.

[9] D. Ghosh and S. Spallone. Determinants of representations of coxeter groups. *Journal of Algebraic Combinatorics*, 49(3):229–265, 2019. https://doi.org/10.1007/s10801-018-0842-2.

[10] K. Haymaker and J. O'pella. Locally recoverable codes from planar graphs. *Journal of Algebra Combinatorics Discrete Structures and Applications*, 7(1):35–53, 2020.

[11] P. Hurley and T. Hurley. Codes from zero-divisors and units in group rings. *International Journal of Information and Coding Theory*, 1(1):57–87, 2009. https://doi.org/10.1504/IJICOT.2009.024047.

[12] I. M. Isaacs. *Algebra: A Graduate Course*, volume 100. American Mathematical Soc., California: Books/Cole Publishin: Pacific Grove, 1992.

[13] A. Islami. *Symmetric Functions as Characters of Hyperoctahedral Group*. Ph D dissertation. York University, Toronto, 2020.

[14] G. D. James and M. W. Liebeck. *Representations and Characters of Groups*. Cambridge university press, 1993.

[15] S. Jitman and S. Ling. Self-dual and complementary dual abelian codes over galois rings. *Journal of Algebra Combinatorics Discrete Structures and Applications*, 6(2):75–94, 2019.

[16] F. MacWilliams. Codes and ideals in group algebras. *Combinatorial Mathematics and Its Applications*, 317:317–328, 1969.

[17] C. Musli. *Representations of Finite Groups*. Hindustan book agency, 1993.

[18] D. S. Passman. *The Algebraic Structure of Group Rings*. New York: Wiley, 1977.

[19] C. G. Pillado, S. González, V. T. Markov, C. Martínez, and A. A. Nechaev. When are all group codes of a noncommutative group abelian (a computational approach)? *Journal of Mathematical Sciences*, 186(4):578–585, 2012. https://doi.org/10.1007/s10958-012-1006-x.

[20] C. G. Pillado, S. González, C. Martínez, V. Markov, and A. Nechaev. Group codes over non-abelian groups. *Journal of Algebra and its Applications*, 12(07):1350037, 2013. https://doi.org/10.1142/S0219498813500370.

[21] C. E. Shannon. A mathematical theory of communication. *The Bell System Technical Journal*, 27(3):379–423, 1948. https://doi.org/10.1002/j.1538-7305.1948.tb01338.x.

[22] W. Stein et al. Sage mathematics software, (accessed on 04 May 2023). http://www.sagemath.org.

Appendix I- Products uv_2, uv_3, uv_4, uv_5 & uv_6

$$
\begin{aligned}
uv_2 = {} & \tfrac{1}{48}[2\lambda_1-\lambda_8-\lambda_9-\lambda_{10}-\lambda_{11}-\lambda_{12}-\lambda_{13}-\lambda_{14}-\lambda_{15}-2\lambda_{16}-2\lambda_{17}-2\lambda_{18}+2\lambda_{25}+2\lambda_{26}+2\lambda_{27}-2\lambda_{34}+\\
& \lambda_{41}+\lambda_{42}+\lambda_{43}+\lambda_{44}+\lambda_{45}+\lambda_{46}+\lambda_{47}+\lambda_{48}](g_1+g_{25}+g_{26}+g_{27}-g_{16}-g_{17}-g_{18}-g_{34})+[2\lambda_2-\lambda_3-\lambda_4-\lambda_5-\lambda_6+\\
& 2\lambda_7-2\lambda_{19}+\lambda_{20}+\lambda_{21}+\lambda_{22}+\lambda_{23}-2\lambda_{24}+\lambda_{28}+\lambda_{29}+\lambda_{30}-2\lambda_{31}+\lambda_{32}-2\lambda_{33}+2\lambda_{35}-\lambda_{36}+2\lambda_{37}-\lambda_{38}-\lambda_{39}-\\
& \lambda_{40}](g_2+g_7+g_{35}+g_{37}-g_{19}-g_{24}-g_{31}-g_{33})+[-\lambda_2+2\lambda_3-\lambda_4+2\lambda_5-\lambda_6-\lambda_7+\lambda_{19}+\lambda_{20}-2\lambda_{21}+\lambda_{22}-2\lambda_{23}+\\
& \lambda_{24}-2\lambda_{28}-2\lambda_{29}+\lambda_{30}+\lambda_{31}+\lambda_{32}+\lambda_{33}-\lambda_{35}-\lambda_{36}-\lambda_{37}-\lambda_{38}+2\lambda_{39}+2\lambda_{40}](g_3+g_5+g_{39}+g_{40}-g_{21}-g_{23}-g_{28}-\\
& g_{29})+[-\lambda_2-\lambda_3+2\lambda_4-\lambda_5+2\lambda_6-\lambda_7+\lambda_{19}-2\lambda_{20}+\lambda_{21}-2\lambda_{22}+\lambda_{23}+\lambda_{24}+\lambda_{28}+\lambda_{29}-2\lambda_{30}+\lambda_{31}-2\lambda_{32}+\lambda_{33}-\\
& \lambda_{35}+2\lambda_{36}-\lambda_{37}+2\lambda_{38}-\lambda_{39}-\lambda_{40}](g_4+g_6+g_{36}+g_{38}-g_{20}-g_{22}-g_{30}-g_{32})+[-\lambda_1+2\lambda_8-\lambda_9+2\lambda_{10}-\lambda_{11}-\lambda_{12}-\\
& \lambda_{13}+2\lambda_{14}+2\lambda_{15}+\lambda_{16}+\lambda_{17}+\lambda_{18}-\lambda_{25}-\lambda_{26}-\lambda_{27}+\lambda_{34}-2\lambda_{41}-2\lambda_{42}+\lambda_{43}+\lambda_{44}+\lambda_{45}-2\lambda_{46}+\lambda_{47}-2\lambda_{48}](g_8+\\
& g_{10}+g_{14}+g_{15}-g_{41}-g_{42}-g_{46}-g_{48})+[-\lambda_1-\lambda_8+2\lambda_9-\lambda_{10}+2\lambda_{11}+2\lambda_{12}+2\lambda_{13}-\lambda_{14}-\lambda_{15}+\lambda_{16}+\lambda_{17}+\lambda_{18}-\\
& \lambda_{25}-\lambda_{26}-\lambda_{27}+\lambda_{34}+\lambda_{41}+\lambda_{42}-2\lambda_{43}-2\lambda_{44}-2\lambda_{45}+\lambda_{46}-2\lambda_{47}+\lambda_{48}](g_9+g_{11}+g_{12}+g_{13}-g_{43}-g_{44}-g_{45}-g_{47}).
\end{aligned}
$$

$$
\begin{aligned}
uv_3 = {} & \tfrac{1}{48}[3\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5+\lambda_6+\lambda_7+\lambda_{16}+\lambda_{17}+\lambda_{18}-\lambda_{19}-\lambda_{20}-\lambda_{21}-\lambda_{22}-\lambda_{23}-\lambda_{24}-\lambda_{25}-\lambda_{26}-\lambda_{27}+\\
& \lambda_{28}+\lambda_{29}+\lambda_{30}+\lambda_{31}+\lambda_{32}+\lambda_{33}-3\lambda_{34}-\lambda_{35}-\lambda_{36}-\lambda_{37}-\lambda_{38}-\lambda_{39}-\lambda_{40}]g_1+[\lambda_1+3\lambda_2-\lambda_7+\lambda_8+\lambda_9+\lambda_{10}+\\
& \lambda_{11}-\lambda_{12}-\lambda_{13}-\lambda_{14}-\lambda_{15}-\lambda_{16}+\lambda_{17}+\lambda_{18}+\lambda_{19}-3\lambda_{24}-\lambda_{25}-\lambda_{26}+\lambda_{27}+\lambda_{31}+\lambda_{33}-\lambda_{34}-\lambda_{35}-\lambda_{37}+\lambda_{41}+\\
& \lambda_{42}+\lambda_{43}+\lambda_{44}-\lambda_{45}-\lambda_{46}-\lambda_{47}-\lambda_{48}]g_2+[\lambda_1+3\lambda_3-\lambda_5+\lambda_8+\lambda_9-\lambda_{10}-\lambda_{11}-\lambda_{12}+\lambda_{13}+\lambda_{14}-\lambda_{15}+\lambda_{16}+\lambda_{17}-\\
& \lambda_{18}+\lambda_{21}-3\lambda_{23}+\lambda_{25}-\lambda_{26}-\lambda_{27}+\lambda_{28}+\lambda_{29}-\lambda_{34}-\lambda_{39}-\lambda_{40}+\lambda_{41}-\lambda_{42}-\lambda_{43}+\lambda_{44}+\lambda_{45}+\lambda_{46}-\lambda_{47}-\lambda_{48}]g_3+\\
& [\lambda_1+3\lambda_4-\lambda_6+\lambda_8+\lambda_9-\lambda_{10}-\lambda_{11}+\lambda_{12}-\lambda_{13}-\lambda_{14}+\lambda_{15}+\lambda_{16}-\lambda_{17}+\lambda_{18}+\lambda_{20}-3\lambda_{22}-\lambda_{25}+\lambda_{26}-\lambda_{27}+\lambda_{30}+\\
& \lambda_{32}-\lambda_{34}-\lambda_{36}-\lambda_{38}-\lambda_{41}+\lambda_{42}+\lambda_{43}-\lambda_{44}+\lambda_{45}+\lambda_{46}-\lambda_{47}-\lambda_{48}]g_4+[\lambda_1-\lambda_3+3\lambda_5-\lambda_8-\lambda_9+\lambda_{10}+\lambda_{11}+\\
& \lambda_{12}-\lambda_{13}-\lambda_{14}+\lambda_{15}+\lambda_{16}+\lambda_{17}-\lambda_{18}-3\lambda_{21}+\lambda_{23}+\lambda_{25}-\lambda_{26}-\lambda_{27}+\lambda_{28}+\lambda_{29}-\lambda_{34}-\lambda_{39}-\lambda_{40}-\lambda_{41}+\lambda_{42}+\\
& \lambda_{43}-\lambda_{44}-\lambda_{45}-\lambda_{46}+\lambda_{47}+\lambda_{48}]g_5+[\lambda_1-\lambda_4+3\lambda_6-\lambda_8-\lambda_9+\lambda_{10}+\lambda_{11}-\lambda_{12}+\lambda_{13}+\lambda_{14}-\lambda_{15}+\lambda_{16}-\lambda_{17}+\lambda_{18}-\\
& 3\lambda_{20}+\lambda_{22}-\lambda_{25}+\lambda_{26}-\lambda_{27}+\lambda_{30}+\lambda_{32}-\lambda_{34}-\lambda_{36}-\lambda_{38}+\lambda_{41}-\lambda_{42}-\lambda_{43}+\lambda_{44}-\lambda_{45}-\lambda_{46}+\lambda_{47}+\lambda_{48}]g_6+[\lambda_1-\\
& \lambda_2+3\lambda_7-\lambda_8-\lambda_9-\lambda_{10}-\lambda_{11}+\lambda_{12}+\lambda_{13}+\lambda_{14}+\lambda_{15}-\lambda_{16}+\lambda_{17}+\lambda_{18}-3\lambda_{19}+\lambda_{24}-\lambda_{25}-\lambda_{26}+\lambda_{27}+\lambda_{31}+\lambda_{33}-\\
& \lambda_{34}-\lambda_{35}-\lambda_{37}-\lambda_{41}-\lambda_{42}-\lambda_{43}-\lambda_{44}+\lambda_{45}+\lambda_{46}+\lambda_{47}+\lambda_{48}]g_7+[\lambda_2+\lambda_3+\lambda_4-\lambda_5-\lambda_6-\lambda_7+3\lambda_8-\lambda_{10}-\lambda_{14}-\\
& \lambda_{15}+\lambda_{19}+\lambda_{20}+\lambda_{21}-\lambda_{22}-\lambda_{23}-\lambda_{24}+\lambda_{28}-\lambda_{29}-\lambda_{30}+\lambda_{31}+\lambda_{32}-\lambda_{33}+\lambda_{35}-\lambda_{36}-\lambda_{37}+\lambda_{38}+\lambda_{39}-\lambda_{40}+\lambda_{41}+\\
& \lambda_{42}+\lambda_{46}-3\lambda_{48}]g_8+[\lambda_2+\lambda_3+\lambda_4-\lambda_5-\lambda_6-\lambda_7+3\lambda_9-\lambda_{11}-\lambda_{12}-\lambda_{13}+\lambda_{19}+\lambda_{20}+\lambda_{21}-\lambda_{22}-\lambda_{23}-\lambda_{24}-\lambda_{28}+\\
& \lambda_{29}+\lambda_{30}-\lambda_{31}-\lambda_{32}+\lambda_{33}-\lambda_{35}+\lambda_{36}+\lambda_{37}-\lambda_{38}-\lambda_{39}+\lambda_{40}+\lambda_{43}+\lambda_{44}+\lambda_{45}-3\lambda_{47}]g_9+[\lambda_2-\lambda_3-\lambda_4+\lambda_5+\lambda_6-\\
& \lambda_7-\lambda_8+3\lambda_{10}-\lambda_{14}-\lambda_{15}+\lambda_{19}-\lambda_{20}-\lambda_{21}+\lambda_{22}+\lambda_{23}-\lambda_{24}-\lambda_{28}+\lambda_{29}+\lambda_{30}+\lambda_{31}-\lambda_{32}-\lambda_{33}+\lambda_{35}+\lambda_{36}-\lambda_{37}-\\
& \lambda_{38}-\lambda_{39}+\lambda_{40}+\lambda_{41}+\lambda_{42}-3\lambda_{46}+\lambda_{48}]g_{10}+[\lambda_2-\lambda_3-\lambda_4+\lambda_5+\lambda_6-\lambda_7-\lambda_9+3\lambda_{11}-\lambda_{12}-\lambda_{13}+\lambda_{19}-\lambda_{20}-\lambda_{21}+\\
& \lambda_{22}+\lambda_{23}-\lambda_{24}+\lambda_{28}-\lambda_{29}-\lambda_{30}-\lambda_{31}+\lambda_{32}+\lambda_{33}-\lambda_{35}-\lambda_{36}+\lambda_{37}+\lambda_{38}+\lambda_{39}-\lambda_{40}+\lambda_{43}+\lambda_{44}-3\lambda_{45}+\lambda_{47}]g_{11}+\\
& [-\lambda_2-\lambda_3+\lambda_4+\lambda_5-\lambda_6+\lambda_7-\lambda_9-\lambda_{11}+3\lambda_{12}-\lambda_{13}-\lambda_{19}+\lambda_{20}-\lambda_{21}-\lambda_{22}+\lambda_{23}+\lambda_{24}+\lambda_{28}-\lambda_{29}+\lambda_{30}+\lambda_{31}-\\
& \lambda_{32}-\lambda_{33}+\lambda_{35}+\lambda_{36}-\lambda_{37}-\lambda_{38}+\lambda_{39}-\lambda_{40}+\lambda_{43}-3\lambda_{44}+\lambda_{45}+\lambda_{47}]g_{12}+[-\lambda_2+\lambda_3-\lambda_4-\lambda_5+\lambda_6+\lambda_7-\lambda_9-\\
& \lambda_{11}-\lambda_{12}+3\lambda_{13}-\lambda_{19}-\lambda_{20}+\lambda_{21}+\lambda_{22}-\lambda_{23}+\lambda_{24}-\lambda_{28}+\lambda_{29}-\lambda_{30}+\lambda_{31}+\lambda_{32}-\lambda_{33}+\lambda_{35}-\lambda_{36}-\lambda_{37}+\lambda_{38}-\\
& \lambda_{39}+\lambda_{40}-3\lambda_{43}+\lambda_{44}+\lambda_{45}+\lambda_{47}]g_{13}+[-\lambda_2+\lambda_3-\lambda_4-\lambda_5+\lambda_6+\lambda_7-\lambda_8-\lambda_{10}+3\lambda_{14}-\lambda_{15}-\lambda_{19}-\lambda_{20}+\lambda_{21}+\\
& \lambda_{22}-\lambda_{23}+\lambda_{24}+\lambda_{28}-\lambda_{29}+\lambda_{30}-\lambda_{31}-\lambda_{32}+\lambda_{33}-\lambda_{35}+\lambda_{36}+\lambda_{37}-\lambda_{38}+\lambda_{39}-\lambda_{40}+\lambda_{41}-3\lambda_{42}+\lambda_{46}+\lambda_{48}]g_{14}+\\
& [-\lambda_2-\lambda_3+\lambda_4+\lambda_5-\lambda_6+\lambda_7-\lambda_8-\lambda_{10}-\lambda_{14}+3\lambda_{15}-\lambda_{19}+\lambda_{20}-\lambda_{21}-\lambda_{22}+\lambda_{23}+\lambda_{24}-\lambda_{28}+\lambda_{29}-\lambda_{30}-\lambda_{31}+\\
& \lambda_{32}+\lambda_{33}-\lambda_{35}-\lambda_{36}+\lambda_{37}+\lambda_{38}-\lambda_{39}+\lambda_{40}-3\lambda_{41}+\lambda_{42}+\lambda_{46}+\lambda_{48}]g_{15}+[\lambda_1-\lambda_2+\lambda_3+\lambda_4+\lambda_5+\lambda_6-\lambda_7+3\lambda_{16}-\\
& \lambda_{17}-\lambda_{18}+\lambda_{19}-\lambda_{20}-\lambda_{21}-\lambda_{22}-\lambda_{23}+\lambda_{24}+\lambda_{25}+\lambda_{26}-3\lambda_{27}+\lambda_{28}+\lambda_{29}+\lambda_{30}-\lambda_{31}+\lambda_{32}-\lambda_{33}-\lambda_{34}+\lambda_{35}-\\
& \lambda_{36}+\lambda_{37}-\lambda_{38}-\lambda_{39}-\lambda_{40}]g_{16}+[\lambda_1+\lambda_2+\lambda_3-\lambda_4+\lambda_5-\lambda_6+\lambda_7-\lambda_{16}+3\lambda_{17}-\lambda_{18}-\lambda_{19}+\lambda_{20}-\lambda_{21}+\lambda_{22}-\lambda_{23}-\\
& \lambda_{24}+\lambda_{25}-3\lambda_{26}+\lambda_{27}+\lambda_{28}+\lambda_{29}-\lambda_{30}+\lambda_{31}-\lambda_{32}+\lambda_{33}-\lambda_{34}-\lambda_{35}+\lambda_{36}-\lambda_{37}+\lambda_{38}-\lambda_{39}-\lambda_{40}]g_{17}+[\lambda_1+\lambda_2-\\
& \lambda_3+\lambda_4-\lambda_5+\lambda_6+\lambda_7-\lambda_{16}-\lambda_{17}+3\lambda_{18}-\lambda_{19}-\lambda_{20}+\lambda_{21}-\lambda_{22}+\lambda_{23}-\lambda_{24}-3\lambda_{25}+\lambda_{26}+\lambda_{27}-\lambda_{28}-\lambda_{29}+\lambda_{30}+\\
& \lambda_{31}+\lambda_{32}+\lambda_{33}-\lambda_{34}-\lambda_{35}-\lambda_{36}-\lambda_{37}-\lambda_{38}+\lambda_{39}+\lambda_{40}]g_{18}+[-\lambda_1+\lambda_2-3\lambda_7+\lambda_8+\lambda_9+\lambda_{10}+\lambda_{11}-\lambda_{12}-\lambda_{13}-\\
& \lambda_{14}-\lambda_{15}+\lambda_{16}-\lambda_{17}-\lambda_{18}+3\lambda_{19}-\lambda_{24}+\lambda_{25}+\lambda_{26}-\lambda_{27}-\lambda_{31}-\lambda_{33}+\lambda_{34}+\lambda_{35}+\lambda_{37}+\lambda_{41}+\lambda_{42}+\lambda_{43}+\lambda_{44}-\\
& \lambda_{45}-\lambda_{46}-\lambda_{47}-\lambda_{48}]g_{19}+[-\lambda_1+\lambda_4-3\lambda_6+\lambda_8+\lambda_9-\lambda_{10}-\lambda_{11}+\lambda_{12}-\lambda_{13}-\lambda_{14}+\lambda_{15}-\lambda_{16}+\lambda_{17}-\lambda_{18}+3\lambda_{20}-
\end{aligned}
$$

$\lambda_{22}+\lambda_{25}-\lambda_{26}+\lambda_{27}-\lambda_{30}-\lambda_{32}+\lambda_{34}+\lambda_{36}+\lambda_{38}-\lambda_{41}+\lambda_{42}+\lambda_{43}-\lambda_{44}+\lambda_{45}+\lambda_{46}-\lambda_{47}-\lambda_{48}]g_{20}+[-\lambda_{1}+\lambda_{3}-$
$3\lambda_{5}+\lambda_{8}+\lambda_{9}-\lambda_{10}-\lambda_{11}-\lambda_{12}+\lambda_{13}+\lambda_{14}-\lambda_{15}-\lambda_{16}-\lambda_{17}+\lambda_{18}+3\lambda_{21}-\lambda_{23}-\lambda_{25}+\lambda_{26}+\lambda_{27}-\lambda_{28}-\lambda_{29}+\lambda_{34}+$
$\lambda_{39}+\lambda_{40}+\lambda_{41}-\lambda_{42}-\lambda_{43}+\lambda_{44}+\lambda_{45}+\lambda_{46}-\lambda_{47}-\lambda_{48}]g_{21}+[-\lambda_{1}-3\lambda_{4}+\lambda_{6}-\lambda_{8}-\lambda_{9}+\lambda_{10}+\lambda_{11}-\lambda_{12}+\lambda_{13}+$
$\lambda_{14}-\lambda_{15}-\lambda_{16}+\lambda_{17}-\lambda_{18}-\lambda_{20}+3\lambda_{22}+\lambda_{25}-\lambda_{26}+\lambda_{27}-\lambda_{30}-\lambda_{32}+\lambda_{34}+\lambda_{36}+\lambda_{38}+\lambda_{41}-\lambda_{42}-\lambda_{43}+\lambda_{44}-$
$\lambda_{45}-\lambda_{46}+\lambda_{47}+\lambda_{48}]g_{22}+[-\lambda_{1}-3\lambda_{3}+\lambda_{5}-\lambda_{8}-\lambda_{9}+\lambda_{10}+\lambda_{11}+\lambda_{12}-\lambda_{13}-\lambda_{14}+\lambda_{15}-\lambda_{16}-\lambda_{17}+\lambda_{18}-\lambda_{21}+$
$3\lambda_{23}-\lambda_{25}+\lambda_{26}+\lambda_{27}-\lambda_{28}-\lambda_{29}+\lambda_{34}+\lambda_{39}+\lambda_{40}-\lambda_{41}+\lambda_{42}+\lambda_{43}-\lambda_{44}-\lambda_{45}-\lambda_{46}+\lambda_{47}+\lambda_{48}]g_{23}+[-\lambda_{1}-$
$3\lambda_{2}+\lambda_{7}-\lambda_{8}-\lambda_{9}-\lambda_{10}-\lambda_{11}+\lambda_{12}+\lambda_{13}+\lambda_{14}+\lambda_{15}+\lambda_{16}-\lambda_{17}-\lambda_{18}-\lambda_{19}+3\lambda_{24}+\lambda_{25}+\lambda_{26}-\lambda_{27}-\lambda_{31}-\lambda_{33}+$
$\lambda_{34}+\lambda_{35}+\lambda_{37}-\lambda_{41}-\lambda_{42}-\lambda_{43}-\lambda_{44}+\lambda_{45}+\lambda_{46}+\lambda_{47}+\lambda_{48}]g_{24}+[-\lambda_{1}-\lambda_{2}+\lambda_{3}-\lambda_{4}+\lambda_{5}-\lambda_{6}-\lambda_{7}+\lambda_{16}+\lambda_{17}-$
$3\lambda_{18}+\lambda_{19}+\lambda_{20}-\lambda_{21}+\lambda_{22}-\lambda_{23}+\lambda_{24}+3\lambda_{25}-\lambda_{26}-\lambda_{27}+\lambda_{28}+\lambda_{29}-\lambda_{30}-\lambda_{31}-\lambda_{32}-\lambda_{33}+\lambda_{34}+\lambda_{35}+\lambda_{36}+$
$\lambda_{37}+\lambda_{38}-\lambda_{39}-\lambda_{40}]g_{25}+[-\lambda_{1}-\lambda_{2}-\lambda_{3}+\lambda_{4}-\lambda_{5}+\lambda_{6}-\lambda_{7}+\lambda_{16}-3\lambda_{17}+\lambda_{18}+\lambda_{19}-\lambda_{20}+\lambda_{21}-\lambda_{22}+\lambda_{23}+\lambda_{24}-$
$\lambda_{25}+3\lambda_{26}-\lambda_{27}-\lambda_{28}-\lambda_{29}+\lambda_{30}-\lambda_{31}+\lambda_{32}-\lambda_{33}+\lambda_{34}+\lambda_{35}-\lambda_{36}+\lambda_{37}-\lambda_{38}+\lambda_{39}+\lambda_{40}]g_{26}+[-\lambda_{1}+\lambda_{2}-\lambda_{3}-$
$\lambda_{4}-\lambda_{5}-\lambda_{6}+\lambda_{7}-3\lambda_{16}+\lambda_{17}+\lambda_{18}-\lambda_{19}+\lambda_{20}+\lambda_{21}+\lambda_{22}+\lambda_{23}-\lambda_{24}-\lambda_{25}-\lambda_{26}+3\lambda_{27}-\lambda_{28}-\lambda_{29}-\lambda_{30}+\lambda_{31}-$
$\lambda_{32}+\lambda_{33}+\lambda_{34}-\lambda_{35}+\lambda_{36}-\lambda_{37}+\lambda_{38}+\lambda_{39}+\lambda_{40}]g_{27}+[\lambda_{1}+\lambda_{3}+\lambda_{5}+\lambda_{8}-\lambda_{9}-\lambda_{10}+\lambda_{11}+\lambda_{12}-\lambda_{13}+\lambda_{14}-\lambda_{15}+$
$\lambda_{16}+\lambda_{17}-\lambda_{18}-\lambda_{21}-\lambda_{23}+\lambda_{25}-\lambda_{26}-\lambda_{27}+3\lambda_{28}-\lambda_{29}-\lambda_{34}+\lambda_{39}-3\lambda_{40}+\lambda_{41}-\lambda_{42}+\lambda_{43}-\lambda_{44}-\lambda_{45}+\lambda_{46}+$
$\lambda_{47}-\lambda_{48}]g_{28}+[\lambda_{1}+\lambda_{3}+\lambda_{5}-\lambda_{8}+\lambda_{9}+\lambda_{10}-\lambda_{11}-\lambda_{12}+\lambda_{13}-\lambda_{14}+\lambda_{15}+\lambda_{16}+\lambda_{17}-\lambda_{18}-\lambda_{21}-\lambda_{23}+\lambda_{25}-\lambda_{26}-$
$\lambda_{27}-\lambda_{28}+3\lambda_{29}-\lambda_{34}-3\lambda_{39}+\lambda_{40}-\lambda_{41}+\lambda_{42}-\lambda_{43}+\lambda_{44}+\lambda_{45}-\lambda_{46}-\lambda_{47}+\lambda_{48}]g_{29}+[\lambda_{1}+\lambda_{4}+\lambda_{6}-\lambda_{8}+\lambda_{9}+$
$\lambda_{10}-\lambda_{11}+\lambda_{12}-\lambda_{13}+\lambda_{14}-\lambda_{15}+\lambda_{16}-\lambda_{17}+\lambda_{18}-\lambda_{20}-\lambda_{22}-\lambda_{25}+\lambda_{26}-\lambda_{27}+3\lambda_{30}-\lambda_{32}-\lambda_{34}+\lambda_{36}-3\lambda_{38}+$
$\lambda_{41}-\lambda_{42}+\lambda_{43}-\lambda_{44}+\lambda_{45}-\lambda_{46}-\lambda_{47}+\lambda_{48}]g_{30}+[\lambda_{1}+\lambda_{2}+\lambda_{7}+\lambda_{8}-\lambda_{9}+\lambda_{10}-\lambda_{11}+\lambda_{12}+\lambda_{13}-\lambda_{14}-\lambda_{15}-\lambda_{16}+$
$\lambda_{17}+\lambda_{18}-\lambda_{19}-\lambda_{24}-\lambda_{25}-\lambda_{26}+\lambda_{27}+3\lambda_{31}-\lambda_{33}-\lambda_{34}+\lambda_{35}-3\lambda_{37}+\lambda_{41}+\lambda_{42}-\lambda_{43}-\lambda_{44}+\lambda_{45}-\lambda_{46}+\lambda_{47}-$
$\lambda_{48}]g_{31}+[\lambda_{1}+\lambda_{4}+\lambda_{6}+\lambda_{8}-\lambda_{9}-\lambda_{10}+\lambda_{11}-\lambda_{12}+\lambda_{13}-\lambda_{14}+\lambda_{15}+\lambda_{16}-\lambda_{17}+\lambda_{18}-\lambda_{20}-\lambda_{22}-\lambda_{25}+\lambda_{26}-\lambda_{27}-$
$\lambda_{30}+3\lambda_{32}-\lambda_{34}-3\lambda_{36}+\lambda_{38}-\lambda_{41}+\lambda_{42}-\lambda_{43}+\lambda_{44}-\lambda_{45}+\lambda_{46}+\lambda_{47}-\lambda_{48}]g_{32}+[\lambda_{1}+\lambda_{2}+\lambda_{7}-\lambda_{8}+\lambda_{9}-\lambda_{10}+$
$\lambda_{11}-\lambda_{12}-\lambda_{13}+\lambda_{14}+\lambda_{15}-\lambda_{16}+\lambda_{17}+\lambda_{18}-\lambda_{19}-\lambda_{24}-\lambda_{25}-\lambda_{26}+\lambda_{27}-\lambda_{31}+3\lambda_{33}-\lambda_{34}-3\lambda_{35}+\lambda_{37}-\lambda_{41}-$
$\lambda_{42}+\lambda_{43}+\lambda_{44}-\lambda_{45}+\lambda_{46}-\lambda_{47}+\lambda_{48}]g_{33}+[-3\lambda_{1}-\lambda_{2}-\lambda_{3}-\lambda_{4}-\lambda_{5}-\lambda_{6}-\lambda_{7}-\lambda_{16}-\lambda_{17}-\lambda_{18}+\lambda_{19}+\lambda_{20}+\lambda_{21}+$
$\lambda_{22}+\lambda_{23}+\lambda_{24}+\lambda_{25}+\lambda_{26}+\lambda_{27}-\lambda_{28}-\lambda_{29}-\lambda_{30}-\lambda_{31}-\lambda_{32}-\lambda_{33}+3\lambda_{34}+\lambda_{35}+\lambda_{36}+\lambda_{37}+\lambda_{38}+\lambda_{39}+\lambda_{40}]g_{34}+$
$[-\lambda_{1}-\lambda_{2}-\lambda_{7}+\lambda_{8}-\lambda_{9}+\lambda_{10}-\lambda_{11}+\lambda_{12}+\lambda_{13}-\lambda_{14}-\lambda_{15}+\lambda_{16}-\lambda_{17}-\lambda_{18}+\lambda_{19}+\lambda_{24}+\lambda_{25}+\lambda_{26}-\lambda_{27}+\lambda_{31}-$
$3\lambda_{33}+\lambda_{34}+3\lambda_{35}-\lambda_{37}+\lambda_{41}+\lambda_{42}-\lambda_{43}-\lambda_{44}+\lambda_{45}-\lambda_{46}+\lambda_{47}-\lambda_{48}]g_{35}+[-\lambda_{1}-\lambda_{4}-\lambda_{6}-\lambda_{8}+\lambda_{9}+\lambda_{10}-\lambda_{11}+$
$\lambda_{12}-\lambda_{13}+\lambda_{14}-\lambda_{15}-\lambda_{16}+\lambda_{17}-\lambda_{18}+\lambda_{20}+\lambda_{22}+\lambda_{25}-\lambda_{26}+\lambda_{27}+\lambda_{30}-3\lambda_{32}+\lambda_{34}+3\lambda_{36}-\lambda_{38}+\lambda_{41}-\lambda_{42}+$
$\lambda_{43}-\lambda_{44}+\lambda_{45}-\lambda_{46}-\lambda_{47}+\lambda_{48}]g_{36}+[-\lambda_{1}-\lambda_{2}-\lambda_{7}-\lambda_{8}+\lambda_{9}-\lambda_{10}+\lambda_{11}-\lambda_{12}-\lambda_{13}+\lambda_{14}+\lambda_{15}+\lambda_{16}-\lambda_{17}-$
$\lambda_{18}+\lambda_{19}+\lambda_{24}+\lambda_{25}+\lambda_{26}-\lambda_{27}-3\lambda_{31}+\lambda_{33}+\lambda_{34}-\lambda_{35}+3\lambda_{37}-\lambda_{41}-\lambda_{42}+\lambda_{43}+\lambda_{44}-\lambda_{45}+\lambda_{46}-\lambda_{47}+\lambda_{48}]g_{37}+$
$[-\lambda_{1}-\lambda_{4}-\lambda_{6}+\lambda_{8}-\lambda_{9}-\lambda_{10}+\lambda_{11}-\lambda_{12}+\lambda_{13}-\lambda_{14}+\lambda_{15}-\lambda_{16}+\lambda_{17}-\lambda_{18}+\lambda_{20}+\lambda_{22}+\lambda_{25}-\lambda_{26}+\lambda_{27}-3\lambda_{30}+$
$\lambda_{32}+\lambda_{34}-\lambda_{36}+3\lambda_{38}-\lambda_{41}+\lambda_{42}-\lambda_{43}+\lambda_{44}-\lambda_{45}+\lambda_{46}+\lambda_{47}-\lambda_{48}]g_{38}+[-\lambda_{1}-\lambda_{3}-\lambda_{5}+\lambda_{8}-\lambda_{9}-\lambda_{10}+\lambda_{11}+$
$\lambda_{12}-\lambda_{13}+\lambda_{14}-\lambda_{15}-\lambda_{16}-\lambda_{17}+\lambda_{18}+\lambda_{21}+\lambda_{23}-\lambda_{25}+\lambda_{26}+\lambda_{27}+\lambda_{28}-3\lambda_{29}+\lambda_{34}+3\lambda_{39}-\lambda_{40}+\lambda_{41}-\lambda_{42}+$
$\lambda_{43}-\lambda_{44}-\lambda_{45}+\lambda_{46}+\lambda_{47}-\lambda_{48}]g_{39}+[-\lambda_{1}-\lambda_{3}-\lambda_{5}-\lambda_{8}+\lambda_{9}+\lambda_{10}-\lambda_{11}-\lambda_{12}+\lambda_{13}-\lambda_{14}+\lambda_{15}-\lambda_{16}-\lambda_{17}+$
$\lambda_{18}+\lambda_{21}+\lambda_{23}-\lambda_{25}+\lambda_{26}+\lambda_{27}-3\lambda_{28}+\lambda_{29}+\lambda_{34}-\lambda_{39}+3\lambda_{40}-\lambda_{41}+\lambda_{42}-\lambda_{43}+\lambda_{44}+\lambda_{45}-\lambda_{46}-\lambda_{47}+\lambda_{48}]g_{40}+$
$[\lambda_{2}+\lambda_{3}-\lambda_{4}-\lambda_{5}+\lambda_{6}-\lambda_{7}+\lambda_{8}+\lambda_{10}+\lambda_{14}-3\lambda_{15}+\lambda_{19}-\lambda_{20}+\lambda_{21}+\lambda_{22}-\lambda_{23}-\lambda_{24}+\lambda_{28}-\lambda_{29}+\lambda_{30}+\lambda_{31}-\lambda_{32}-$
$\lambda_{33}+\lambda_{35}+\lambda_{36}-\lambda_{37}-\lambda_{38}+\lambda_{39}-\lambda_{40}+3\lambda_{41}-\lambda_{42}-\lambda_{46}-\lambda_{48}]g_{41}+[\lambda_{2}-\lambda_{3}+\lambda_{4}+\lambda_{5}-\lambda_{6}-\lambda_{7}+\lambda_{8}+\lambda_{10}-3\lambda_{14}+$
$\lambda_{15}+\lambda_{19}+\lambda_{20}-\lambda_{21}-\lambda_{22}+\lambda_{23}-\lambda_{24}-\lambda_{28}+\lambda_{29}-\lambda_{30}+\lambda_{31}+\lambda_{32}-\lambda_{33}+\lambda_{35}-\lambda_{36}-\lambda_{37}+\lambda_{38}-\lambda_{39}+\lambda_{40}-\lambda_{41}+$
$3\lambda_{42}-\lambda_{46}-\lambda_{48}]g_{42}+[\lambda_{2}-\lambda_{3}+\lambda_{4}+\lambda_{5}-\lambda_{6}-\lambda_{7}+\lambda_{9}+\lambda_{11}+\lambda_{12}-3\lambda_{13}+\lambda_{19}+\lambda_{20}-\lambda_{21}-\lambda_{22}+\lambda_{23}-\lambda_{24}+\lambda_{28}-$
$\lambda_{29}+\lambda_{30}-\lambda_{31}-\lambda_{32}+\lambda_{33}-\lambda_{35}+\lambda_{36}+\lambda_{37}-\lambda_{38}+\lambda_{39}-\lambda_{40}+3\lambda_{43}-\lambda_{44}-\lambda_{45}-\lambda_{47}]g_{43}+[\lambda_{2}+\lambda_{3}-\lambda_{4}-\lambda_{5}+$
$\lambda_{6}-\lambda_{7}+\lambda_{9}+\lambda_{11}-3\lambda_{12}+\lambda_{13}+\lambda_{19}-\lambda_{20}+\lambda_{21}+\lambda_{22}-\lambda_{23}-\lambda_{24}-\lambda_{28}+\lambda_{29}-\lambda_{30}-\lambda_{31}+\lambda_{32}+\lambda_{33}-\lambda_{35}-\lambda_{36}+$
$\lambda_{37}+\lambda_{38}-\lambda_{39}+\lambda_{40}-\lambda_{43}+3\lambda_{44}-\lambda_{45}-\lambda_{47}]g_{44}+[-\lambda_{2}+\lambda_{3}+\lambda_{4}-\lambda_{5}-\lambda_{6}+\lambda_{7}+\lambda_{9}-3\lambda_{11}+\lambda_{12}+\lambda_{13}-\lambda_{19}+$
$\lambda_{20}+\lambda_{21}-\lambda_{22}-\lambda_{23}+\lambda_{24}-\lambda_{28}+\lambda_{29}+\lambda_{30}+\lambda_{31}-\lambda_{32}-\lambda_{33}+\lambda_{35}+\lambda_{36}-\lambda_{37}-\lambda_{38}-\lambda_{39}+\lambda_{40}-\lambda_{43}-\lambda_{44}+3\lambda_{45}-$
$\lambda_{47}]g_{45}+[-\lambda_{2}+\lambda_{3}+\lambda_{4}-\lambda_{5}-\lambda_{6}+\lambda_{7}+\lambda_{8}-3\lambda_{10}+\lambda_{14}+\lambda_{15}-\lambda_{19}+\lambda_{20}+\lambda_{21}-\lambda_{22}-\lambda_{23}+\lambda_{24}+\lambda_{28}-\lambda_{29}-\lambda_{30}-$
$\lambda_{31}+\lambda_{32}+\lambda_{33}-\lambda_{35}-\lambda_{36}+\lambda_{37}+\lambda_{38}+\lambda_{39}-\lambda_{40}-\lambda_{41}-\lambda_{42}+3\lambda_{46}-\lambda_{48}]g_{46}+[-\lambda_{2}-\lambda_{3}-\lambda_{4}+\lambda_{5}+\lambda_{6}+\lambda_{7}-$
$3\lambda_{9}+\lambda_{11}+\lambda_{12}+\lambda_{13}-\lambda_{19}-\lambda_{20}-\lambda_{21}+\lambda_{22}+\lambda_{23}+\lambda_{24}+\lambda_{28}-\lambda_{29}-\lambda_{30}+\lambda_{31}+\lambda_{32}-\lambda_{33}+\lambda_{35}-\lambda_{36}-\lambda_{37}+\lambda_{38}+$

$\lambda_{39}-\lambda_{40}-\lambda_{43}-\lambda_{44}-\lambda_{45}+3\lambda_{47}]g_{47}+[-\lambda_{2}-\lambda_{3}-\lambda_{4}+\lambda_{5}+\lambda_{6}+\lambda_{7}-3\lambda_{8}+\lambda_{10}+\lambda_{14}+\lambda_{15}-\lambda_{19}-\lambda_{20}-\lambda_{21}+$
$\lambda_{22}+\lambda_{23}+\lambda_{24}-\lambda_{28}+\lambda_{29}+\lambda_{30}-\lambda_{31}-\lambda_{32}+\lambda_{33}-\lambda_{35}+\lambda_{36}+\lambda_{37}-\lambda_{38}-\lambda_{39}+\lambda_{40}-\lambda_{41}-\lambda_{42}-\lambda_{46}+3\lambda_{48}]g_{48}.$

$uv_4 = \frac{1}{48}[3\lambda_{1}-\lambda_{2}-\lambda_{3}-\lambda_{4}-\lambda_{5}-\lambda_{6}-\lambda_{7}-\lambda_{16}-\lambda_{17}-\lambda_{18}-\lambda_{19}-\lambda_{20}-\lambda_{21}-\lambda_{22}-\lambda_{23}-\lambda_{24}-\lambda_{25}-\lambda_{26}-\lambda_{27}+$
$\lambda_{28}+\lambda_{29}+\lambda_{30}+\lambda_{31}+\lambda_{32}+\lambda_{33}+3\lambda_{34}+\lambda_{35}+\lambda_{36}+\lambda_{37}+\lambda_{38}+\lambda_{39}+\lambda_{40}](g_{1}+g_{34})+[-\lambda_{1}+3\lambda_{2}-\lambda_{7}-\lambda_{8}-\lambda_{9}-$
$\lambda_{10}-\lambda_{11}+\lambda_{12}+\lambda_{13}+\lambda_{14}+\lambda_{15}-\lambda_{16}+\lambda_{17}+\lambda_{18}-\lambda_{19}+3\lambda_{24}+\lambda_{25}+\lambda_{26}-\lambda_{27}-\lambda_{31}-\lambda_{33}-\lambda_{34}-\lambda_{35}-\lambda_{37}+\lambda_{41}+$
$\lambda_{42}+\lambda_{43}+\lambda_{44}-\lambda_{45}-\lambda_{46}-\lambda_{47}-\lambda_{48}](g_{2}+g_{24})+[-\lambda_{1}+3\lambda_{3}-\lambda_{5}-\lambda_{8}-\lambda_{9}+\lambda_{10}+\lambda_{11}+\lambda_{12}-\lambda_{13}-\lambda_{14}+\lambda_{15}+$
$\lambda_{16}+\lambda_{17}-\lambda_{18}-\lambda_{21}+3\lambda_{23}-\lambda_{25}+\lambda_{26}+\lambda_{27}-\lambda_{28}-\lambda_{29}-\lambda_{34}-\lambda_{39}-\lambda_{40}+\lambda_{41}-\lambda_{42}-\lambda_{43}+\lambda_{44}+\lambda_{45}+\lambda_{46}-\lambda_{47}-$
$\lambda_{48}](g_{3}+g_{23})+[-\lambda_{1}+3\lambda_{4}-\lambda_{6}-\lambda_{8}-\lambda_{9}+\lambda_{10}+\lambda_{11}-\lambda_{12}+\lambda_{13}+\lambda_{14}-\lambda_{15}+\lambda_{16}-\lambda_{17}+\lambda_{18}-\lambda_{20}+3\lambda_{22}+\lambda_{25}-$
$\lambda_{26}+\lambda_{27}-\lambda_{30}-\lambda_{32}-\lambda_{34}-\lambda_{36}-\lambda_{38}-\lambda_{41}+\lambda_{42}+\lambda_{43}-\lambda_{44}+\lambda_{45}+\lambda_{46}-\lambda_{47}-\lambda_{48}](g_{4}+g_{22})+[-\lambda_{1}-\lambda_{3}+3\lambda_{5}+$
$\lambda_{8}+\lambda_{9}-\lambda_{10}-\lambda_{11}-\lambda_{12}+\lambda_{13}+\lambda_{14}-\lambda_{15}+\lambda_{16}+\lambda_{17}-\lambda_{18}+3\lambda_{21}-\lambda_{23}-\lambda_{25}+\lambda_{26}+\lambda_{27}-\lambda_{28}-\lambda_{29}-\lambda_{34}-\lambda_{39}-$
$\lambda_{40}-\lambda_{41}+\lambda_{42}+\lambda_{43}-\lambda_{44}-\lambda_{45}-\lambda_{46}+\lambda_{47}+\lambda_{48}](g_{5}+g_{21})+[-\lambda_{1}-\lambda_{4}+3\lambda_{6}+\lambda_{8}+\lambda_{9}-\lambda_{10}-\lambda_{11}+\lambda_{12}-\lambda_{13}-$
$\lambda_{14}+\lambda_{15}+\lambda_{16}-\lambda_{17}+\lambda_{18}+3\lambda_{20}-\lambda_{22}+\lambda_{25}-\lambda_{26}+\lambda_{27}-\lambda_{30}-\lambda_{32}-\lambda_{34}-\lambda_{36}-\lambda_{38}+\lambda_{41}-\lambda_{42}-\lambda_{43}+\lambda_{44}-\lambda_{45}-$
$\lambda_{46}+\lambda_{47}+\lambda_{48}](g_{6}+g_{20})+[-\lambda_{1}-\lambda_{2}+3\lambda_{7}+\lambda_{8}+\lambda_{9}+\lambda_{10}+\lambda_{11}-\lambda_{12}-\lambda_{13}-\lambda_{14}-\lambda_{15}-\lambda_{16}+\lambda_{17}+\lambda_{18}+3\lambda_{19}-$
$\lambda_{24}+\lambda_{25}+\lambda_{26}-\lambda_{27}-\lambda_{31}-\lambda_{33}-\lambda_{34}-\lambda_{35}-\lambda_{37}-\lambda_{41}-\lambda_{42}-\lambda_{43}-\lambda_{44}+\lambda_{45}+\lambda_{46}+\lambda_{47}+\lambda_{48}](g_{7}+g_{19})+[-$
$\lambda_{2}-\lambda_{3}-\lambda_{4}+\lambda_{5}+\lambda_{6}+\lambda_{7}+3\lambda_{8}-\lambda_{10}-\lambda_{14}-\lambda_{15}+\lambda_{19}+\lambda_{20}+\lambda_{21}-\lambda_{22}-\lambda_{23}-\lambda_{24}+\lambda_{28}-\lambda_{29}-\lambda_{30}+\lambda_{31}+\lambda_{32}-$
$\lambda_{33}-\lambda_{35}+\lambda_{36}+\lambda_{37}-\lambda_{38}-\lambda_{39}+\lambda_{40}-\lambda_{41}-\lambda_{42}-\lambda_{46}+3\lambda_{48}](g_{8}+g_{48})+[-\lambda_{2}-\lambda_{3}-\lambda_{4}+\lambda_{5}+\lambda_{6}+\lambda_{7}+3\lambda_{9}-$
$\lambda_{11}-\lambda_{12}-\lambda_{13}+\lambda_{19}+\lambda_{20}+\lambda_{21}-\lambda_{22}-\lambda_{23}-\lambda_{24}-\lambda_{28}+\lambda_{29}+\lambda_{30}-\lambda_{31}-\lambda_{32}+\lambda_{33}+\lambda_{35}-\lambda_{36}-\lambda_{37}+\lambda_{38}+\lambda_{39}-$
$\lambda_{40}-\lambda_{43}-\lambda_{44}-\lambda_{45}+3\lambda_{47}](g_{9}+g_{47})+[-\lambda_{2}+\lambda_{3}+\lambda_{4}-\lambda_{5}-\lambda_{6}+\lambda_{7}-\lambda_{8}+3\lambda_{10}-\lambda_{14}-\lambda_{15}+\lambda_{19}-\lambda_{20}-\lambda_{21}+$
$\lambda_{22}+\lambda_{23}-\lambda_{24}-\lambda_{28}+\lambda_{29}+\lambda_{30}+\lambda_{31}-\lambda_{32}-\lambda_{33}-\lambda_{35}-\lambda_{36}+\lambda_{37}+\lambda_{38}+\lambda_{39}-\lambda_{40}-\lambda_{41}-\lambda_{42}+3\lambda_{46}-\lambda_{48}](g_{10}+$
$g_{46})+[-\lambda_{2}+\lambda_{3}+\lambda_{4}-\lambda_{5}-\lambda_{6}+\lambda_{7}-\lambda_{9}+3\lambda_{11}-\lambda_{12}-\lambda_{13}+\lambda_{19}-\lambda_{20}-\lambda_{21}+\lambda_{22}+\lambda_{23}-\lambda_{24}+\lambda_{28}-\lambda_{29}-\lambda_{30}-$
$\lambda_{31}+\lambda_{32}+\lambda_{33}+\lambda_{35}+\lambda_{36}-\lambda_{37}-\lambda_{38}-\lambda_{39}+\lambda_{40}-\lambda_{43}-\lambda_{44}+3\lambda_{45}-\lambda_{47}](g_{11}+g_{45})+[\lambda_{2}+\lambda_{3}-\lambda_{4}-\lambda_{5}+\lambda_{6}-$
$\lambda_{7}-\lambda_{9}-\lambda_{11}+3\lambda_{12}-\lambda_{13}-\lambda_{19}+\lambda_{20}-\lambda_{21}-\lambda_{22}+\lambda_{23}+\lambda_{24}+\lambda_{28}-\lambda_{29}+\lambda_{30}+\lambda_{31}-\lambda_{32}-\lambda_{33}-\lambda_{35}-\lambda_{36}+\lambda_{37}+$
$\lambda_{38}-\lambda_{39}+\lambda_{40}-\lambda_{43}+3\lambda_{44}-\lambda_{45}-\lambda_{47}](g_{12}+g_{44})+[\lambda_{2}-\lambda_{3}+\lambda_{4}+\lambda_{5}-\lambda_{6}-\lambda_{7}-\lambda_{9}-\lambda_{11}-\lambda_{12}+3\lambda_{13}-\lambda_{19}-$
$\lambda_{20}+\lambda_{21}+\lambda_{22}-\lambda_{23}+\lambda_{24}-\lambda_{28}+\lambda_{29}-\lambda_{30}+\lambda_{31}+\lambda_{32}-\lambda_{33}-\lambda_{35}+\lambda_{36}+\lambda_{37}-\lambda_{38}+\lambda_{39}-\lambda_{40}+3\lambda_{43}-\lambda_{44}-\lambda_{45}-$
$\lambda_{47}](g_{13}+g_{43})+[\lambda_{2}-\lambda_{3}+\lambda_{4}+\lambda_{5}-\lambda_{6}-\lambda_{7}-\lambda_{8}-\lambda_{10}+3\lambda_{14}-\lambda_{15}-\lambda_{19}-\lambda_{20}+\lambda_{21}+\lambda_{22}-\lambda_{23}+\lambda_{24}+\lambda_{28}-\lambda_{29}+$
$\lambda_{30}-\lambda_{31}-\lambda_{32}+\lambda_{33}+\lambda_{35}-\lambda_{36}-\lambda_{37}+\lambda_{38}-\lambda_{39}+\lambda_{40}-\lambda_{41}+3\lambda_{42}-\lambda_{46}-\lambda_{48}](g_{14}+g_{42})+[\lambda_{2}+\lambda_{3}-\lambda_{4}-\lambda_{5}+$
$\lambda_{6}-\lambda_{7}-\lambda_{8}-\lambda_{10}-\lambda_{14}+3\lambda_{15}-\lambda_{19}+\lambda_{20}-\lambda_{21}-\lambda_{22}+\lambda_{23}+\lambda_{24}-\lambda_{28}+\lambda_{29}-\lambda_{30}-\lambda_{31}+\lambda_{32}+\lambda_{33}+\lambda_{35}+\lambda_{36}-$
$\lambda_{37}-\lambda_{38}+\lambda_{39}-\lambda_{40}+3\lambda_{41}-\lambda_{42}-\lambda_{46}-\lambda_{48}](g_{15}+g_{41})+[-\lambda_{1}-\lambda_{2}+\lambda_{3}+\lambda_{4}+\lambda_{5}+\lambda_{6}-\lambda_{7}+3\lambda_{16}-\lambda_{17}-\lambda_{18}-$
$\lambda_{19}+\lambda_{20}+\lambda_{21}+\lambda_{22}+\lambda_{23}-\lambda_{24}-\lambda_{25}-\lambda_{26}+3\lambda_{27}-\lambda_{28}-\lambda_{29}-\lambda_{30}+\lambda_{31}-\lambda_{32}+\lambda_{33}-\lambda_{34}+\lambda_{35}-\lambda_{36}+\lambda_{37}-\lambda_{38}-$
$\lambda_{39}-\lambda_{40}](g_{16}+g_{27})+[-\lambda_{1}+\lambda_{2}+\lambda_{3}-\lambda_{4}+\lambda_{5}-\lambda_{6}+\lambda_{7}-\lambda_{16}+3\lambda_{17}-\lambda_{18}+\lambda_{19}-\lambda_{20}+\lambda_{21}-\lambda_{22}+\lambda_{23}+\lambda_{24}-$
$\lambda_{25}+3\lambda_{26}-\lambda_{27}-\lambda_{28}-\lambda_{29}+\lambda_{30}-\lambda_{31}+\lambda_{32}-\lambda_{33}-\lambda_{34}-\lambda_{35}+\lambda_{36}-\lambda_{37}+\lambda_{38}-\lambda_{39}-\lambda_{40}](g_{17}+g_{26})+[-\lambda_{1}+$
$\lambda_{2}-\lambda_{3}+\lambda_{4}-\lambda_{5}+\lambda_{6}+\lambda_{7}-\lambda_{16}-\lambda_{17}+3\lambda_{18}+\lambda_{19}+\lambda_{20}-\lambda_{21}+\lambda_{22}-\lambda_{23}+\lambda_{24}+3\lambda_{25}-\lambda_{26}-\lambda_{27}+\lambda_{28}+\lambda_{29}-$
$\lambda_{30}-\lambda_{31}-\lambda_{32}-\lambda_{33}-\lambda_{34}-\lambda_{35}-\lambda_{36}-\lambda_{37}-\lambda_{38}+\lambda_{39}+\lambda_{40}](g_{18}+g_{25})+[\lambda_{1}-\lambda_{3}-\lambda_{5}+\lambda_{8}-\lambda_{9}-\lambda_{10}+\lambda_{11}+$
$\lambda_{12}-\lambda_{13}+\lambda_{14}-\lambda_{15}-\lambda_{16}-\lambda_{17}+\lambda_{18}-\lambda_{21}-\lambda_{23}+\lambda_{25}-\lambda_{26}-\lambda_{27}+3\lambda_{28}-\lambda_{29}+\lambda_{34}-\lambda_{39}+3\lambda_{40}-\lambda_{41}+\lambda_{42}-$
$\lambda_{43}+\lambda_{44}+\lambda_{45}-\lambda_{46}-\lambda_{47}+\lambda_{48}](g_{28}+g_{40})+[\lambda_{1}-\lambda_{3}-\lambda_{5}-\lambda_{8}+\lambda_{9}+\lambda_{10}-\lambda_{11}-\lambda_{12}+\lambda_{13}-\lambda_{14}+\lambda_{15}-\lambda_{16}-$
$\lambda_{17}+\lambda_{18}-\lambda_{21}-\lambda_{23}+\lambda_{25}-\lambda_{26}-\lambda_{27}-\lambda_{28}+3\lambda_{29}+\lambda_{34}+3\lambda_{39}-\lambda_{40}+\lambda_{41}-\lambda_{42}+\lambda_{43}-\lambda_{44}-\lambda_{45}+\lambda_{46}+\lambda_{47}-$
$\lambda_{48}](g_{29}+g_{39})+[\lambda_{1}-\lambda_{4}-\lambda_{6}-\lambda_{8}+\lambda_{9}+\lambda_{10}-\lambda_{11}+\lambda_{12}-\lambda_{13}+\lambda_{14}-\lambda_{15}-\lambda_{16}+\lambda_{17}-\lambda_{18}-\lambda_{20}-\lambda_{22}-\lambda_{25}+$
$\lambda_{26}-\lambda_{27}+3\lambda_{30}-\lambda_{32}+\lambda_{34}-\lambda_{36}+3\lambda_{38}-\lambda_{41}+\lambda_{42}-\lambda_{43}+\lambda_{44}-\lambda_{45}+\lambda_{46}+\lambda_{47}-\lambda_{48}](g_{30}+g_{38})+[\lambda_{1}-\lambda_{2}-\lambda_{7}+$
$\lambda_{8}-\lambda_{9}+\lambda_{10}-\lambda_{11}+\lambda_{12}+\lambda_{13}-\lambda_{14}-\lambda_{15}+\lambda_{16}-\lambda_{17}-\lambda_{18}-\lambda_{19}-\lambda_{24}-\lambda_{25}-\lambda_{26}+\lambda_{27}+3\lambda_{31}-\lambda_{33}+\lambda_{34}-\lambda_{35}+$
$3\lambda_{37}-\lambda_{41}-\lambda_{42}+\lambda_{43}+\lambda_{44}-\lambda_{45}+\lambda_{46}-\lambda_{47}+\lambda_{48}](g_{31}+g_{37})+[\lambda_{1}-\lambda_{4}-\lambda_{6}+\lambda_{8}-\lambda_{9}-\lambda_{10}+\lambda_{11}-\lambda_{12}+\lambda_{13}-$
$\lambda_{14}+\lambda_{15}-\lambda_{16}+\lambda_{17}-\lambda_{18}-\lambda_{20}-\lambda_{22}-\lambda_{25}+\lambda_{26}-\lambda_{27}-\lambda_{30}+3\lambda_{32}+\lambda_{34}+3\lambda_{36}-\lambda_{38}+\lambda_{41}-\lambda_{42}+\lambda_{43}-\lambda_{44}+$
$\lambda_{45}-\lambda_{46}-\lambda_{47}+\lambda_{48}](g_{32}+g_{36})+[\lambda_{1}-\lambda_{2}-\lambda_{7}-\lambda_{8}+\lambda_{9}-\lambda_{10}+\lambda_{11}-\lambda_{12}-\lambda_{13}+\lambda_{14}+\lambda_{15}+\lambda_{16}-\lambda_{17}-\lambda_{18}-$
$\lambda_{19}-\lambda_{24}-\lambda_{25}-\lambda_{26}+\lambda_{27}-\lambda_{31}+3\lambda_{33}+\lambda_{34}+3\lambda_{35}-\lambda_{37}+\lambda_{41}+\lambda_{42}-\lambda_{43}-\lambda_{44}+\lambda_{45}-\lambda_{46}+\lambda_{47}-\lambda_{48}](g_{33}+g_{35}).$

$uv_5 = \frac{1}{48}[3\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5+\lambda_6+\lambda_7-\lambda_{16}-\lambda_{17}-\lambda_{18}+\lambda_{19}+\lambda_{20}+\lambda_{21}+\lambda_{22}+\lambda_{23}+\lambda_{24}-\lambda_{25}-\lambda_{26}-\lambda_{27}-$
$\lambda_{28}-\lambda_{29}-\lambda_{30}-\lambda_{31}-\lambda_{32}-\lambda_{33}+3\lambda_{34}-\lambda_{35}-\lambda_{36}-\lambda_{37}-\lambda_{38}-\lambda_{39}-\lambda_{40}](g_1+g_{34})+[\lambda_1+3\lambda_2-\lambda_7+\lambda_8+\lambda_9+$
$\lambda_{10}+\lambda_{11}-\lambda_{12}-\lambda_{13}-\lambda_{14}-\lambda_{15}+\lambda_{16}-\lambda_{17}-\lambda_{18}-\lambda_{19}+3\lambda_{24}-\lambda_{25}-\lambda_{26}+\lambda_{27}-\lambda_{31}-\lambda_{33}+\lambda_{34}-\lambda_{35}-\lambda_{37}-\lambda_{41}-$
$\lambda_{42}-\lambda_{43}-\lambda_{44}+\lambda_{45}+\lambda_{46}+\lambda_{47}+\lambda_{48}](g_2+g_{24})+[\lambda_1+3\lambda_3-\lambda_5+\lambda_8+\lambda_9-\lambda_{10}-\lambda_{11}-\lambda_{12}+\lambda_{13}+\lambda_{14}-\lambda_{15}-$
$\lambda_{16}-\lambda_{17}+\lambda_{18}-\lambda_{21}+3\lambda_{23}+\lambda_{25}-\lambda_{26}-\lambda_{27}-\lambda_{28}-\lambda_{29}+\lambda_{34}-\lambda_{39}-\lambda_{40}-\lambda_{41}+\lambda_{42}+\lambda_{43}-\lambda_{44}-\lambda_{45}-\lambda_{46}+\lambda_{47}+$
$\lambda_{48}](g_3+g_{23})+[\lambda_1+3\lambda_4-\lambda_6+\lambda_8+\lambda_9-\lambda_{10}-\lambda_{11}+\lambda_{12}-\lambda_{13}-\lambda_{14}+\lambda_{15}-\lambda_{16}+\lambda_{17}-\lambda_{18}-\lambda_{20}+3\lambda_{22}-\lambda_{25}+$
$\lambda_{26}-\lambda_{27}-\lambda_{30}-\lambda_{32}+\lambda_{34}-\lambda_{36}-\lambda_{38}+\lambda_{41}-\lambda_{42}-\lambda_{43}+\lambda_{44}-\lambda_{45}-\lambda_{46}+\lambda_{47}+\lambda_{48}](g_4+g_{22})+[\lambda_1-\lambda_3+3\lambda_5-$
$\lambda_8-\lambda_9+\lambda_{10}+\lambda_{11}+\lambda_{12}-\lambda_{13}-\lambda_{14}+\lambda_{15}-\lambda_{16}-\lambda_{17}+\lambda_{18}+3\lambda_{21}-\lambda_{23}+\lambda_{25}-\lambda_{26}-\lambda_{27}-\lambda_{28}-\lambda_{29}+\lambda_{34}-\lambda_{39}-$
$\lambda_{40}+\lambda_{41}-\lambda_{42}-\lambda_{43}+\lambda_{44}+\lambda_{45}+\lambda_{46}-\lambda_{47}-\lambda_{48}](g_5+g_{21})+[\lambda_1-\lambda_4+3\lambda_6-\lambda_8-\lambda_9+\lambda_{10}+\lambda_{11}-\lambda_{12}+\lambda_{13}+$
$\lambda_{14}-\lambda_{15}-\lambda_{16}+\lambda_{17}-\lambda_{18}+3\lambda_{20}-\lambda_{22}-\lambda_{25}+\lambda_{26}-\lambda_{27}-\lambda_{30}-\lambda_{32}+\lambda_{34}-\lambda_{36}-\lambda_{38}-\lambda_{41}+\lambda_{42}+\lambda_{43}-\lambda_{44}+\lambda_{45}+$
$\lambda_{46}-\lambda_{47}-\lambda_{48}](g_6+g_{20})+[\lambda_1-\lambda_2+3\lambda_7-\lambda_8-\lambda_9-\lambda_{10}-\lambda_{11}+\lambda_{12}+\lambda_{13}+\lambda_{14}+\lambda_{15}+\lambda_{16}-\lambda_{17}-\lambda_{18}+3\lambda_{19}-$
$\lambda_{24}-\lambda_{25}-\lambda_{26}+\lambda_{27}-\lambda_{31}-\lambda_{33}+\lambda_{34}-\lambda_{35}-\lambda_{37}+\lambda_{41}+\lambda_{42}+\lambda_{43}+\lambda_{44}-\lambda_{45}-\lambda_{46}-\lambda_{47}-\lambda_{48}](g_7+g_{19})+[\lambda_2+$
$\lambda_3+\lambda_4-\lambda_5-\lambda_6-\lambda_7+3\lambda_8-\lambda_{10}-\lambda_{14}-\lambda_{15}-\lambda_{19}-\lambda_{20}-\lambda_{21}+\lambda_{22}+\lambda_{23}+\lambda_{24}-\lambda_{28}+\lambda_{29}+\lambda_{30}-\lambda_{31}-\lambda_{32}+\lambda_{33}+$
$\lambda_{35}-\lambda_{36}-\lambda_{37}+\lambda_{38}+\lambda_{39}-\lambda_{40}-\lambda_{41}-\lambda_{42}-\lambda_{46}+3\lambda_{48}](g_8+g_{48})+[\lambda_2+\lambda_3+\lambda_4-\lambda_5-\lambda_6-\lambda_7+3\lambda_9-\lambda_{11}-\lambda_{12}-$
$\lambda_{13}-\lambda_{19}-\lambda_{20}-\lambda_{21}+\lambda_{22}+\lambda_{23}+\lambda_{24}+\lambda_{28}-\lambda_{29}-\lambda_{30}+\lambda_{31}+\lambda_{32}-\lambda_{33}-\lambda_{35}+\lambda_{36}+\lambda_{37}-\lambda_{38}-\lambda_{39}+\lambda_{40}-\lambda_{43}-$
$\lambda_{44}-\lambda_{45}+3\lambda_{47}](g_9+g_{47})+[\lambda_2-\lambda_3-\lambda_4+\lambda_5+\lambda_6-\lambda_7-\lambda_8+3\lambda_{10}-\lambda_{14}-\lambda_{15}-\lambda_{19}+\lambda_{20}+\lambda_{21}-\lambda_{22}-\lambda_{23}+\lambda_{24}+$
$\lambda_{28}-\lambda_{29}-\lambda_{30}-\lambda_{31}+\lambda_{32}+\lambda_{33}+\lambda_{35}+\lambda_{36}-\lambda_{37}-\lambda_{38}-\lambda_{39}+\lambda_{40}-\lambda_{41}-\lambda_{42}+3\lambda_{46}-\lambda_{48}](g_{10}+g_{46})+[\lambda_2-\lambda_3-$
$\lambda_4+\lambda_5+\lambda_6-\lambda_7-\lambda_9+3\lambda_{11}-\lambda_{12}-\lambda_{13}-\lambda_{19}+\lambda_{20}+\lambda_{21}-\lambda_{22}-\lambda_{23}+\lambda_{24}-\lambda_{28}+\lambda_{29}+\lambda_{30}+\lambda_{31}-\lambda_{32}-\lambda_{33}-$
$\lambda_{35}-\lambda_{36}+\lambda_{37}+\lambda_{38}+\lambda_{39}-\lambda_{40}-\lambda_{43}-\lambda_{44}+3\lambda_{45}-\lambda_{47}](g_{11}+g_{45})+[-\lambda_2-\lambda_3+\lambda_4+\lambda_5-\lambda_6+\lambda_7-\lambda_9-\lambda_{11}+$
$3\lambda_{12}-\lambda_{13}+\lambda_{19}-\lambda_{20}+\lambda_{21}+\lambda_{22}-\lambda_{23}-\lambda_{24}-\lambda_{28}+\lambda_{29}-\lambda_{30}-\lambda_{31}+\lambda_{32}+\lambda_{33}+\lambda_{35}+\lambda_{36}-\lambda_{37}-\lambda_{38}+\lambda_{39}-$
$\lambda_{40}-\lambda_{43}+3\lambda_{44}-\lambda_{45}-\lambda_{47}](g_{12}+g_{44})+-\lambda_2+\lambda_3-\lambda_4-\lambda_5+\lambda_6+\lambda_7-\lambda_9-\lambda_{11}-\lambda_{12}+3\lambda_{13}+\lambda_{19}+\lambda_{20}-\lambda_{21}-$
$\lambda_{22}+\lambda_{23}-\lambda_{24}+\lambda_{28}-\lambda_{29}+\lambda_{30}-\lambda_{31}-\lambda_{32}+\lambda_{33}+\lambda_{35}-\lambda_{36}-\lambda_{37}+\lambda_{38}-\lambda_{39}+\lambda_{40}+3\lambda_{43}-\lambda_{44}-\lambda_{45}-\lambda_{47}](g_{13}+$
$g_{43})+[-\lambda_2+\lambda_3-\lambda_4-\lambda_5+\lambda_6+\lambda_7-\lambda_8-\lambda_{10}+3\lambda_{14}-\lambda_{15}+\lambda_{19}+\lambda_{20}-\lambda_{21}-\lambda_{22}+\lambda_{23}-\lambda_{24}-\lambda_{28}+\lambda_{29}-\lambda_{30}+$
$\lambda_{31}+\lambda_{32}-\lambda_{33}-\lambda_{35}+\lambda_{36}+\lambda_{37}-\lambda_{38}+\lambda_{39}-\lambda_{40}-\lambda_{41}+3\lambda_{42}-\lambda_{46}-\lambda_{48}](g_{14}+g_{42})+[-\lambda_2-\lambda_3+\lambda_4+\lambda_5-\lambda_6+$
$\lambda_7-\lambda_8-\lambda_{10}-\lambda_{14}+3\lambda_{15}+\lambda_{19}-\lambda_{20}+\lambda_{21}+\lambda_{22}-\lambda_{23}-\lambda_{24}+\lambda_{28}-\lambda_{29}+\lambda_{30}+\lambda_{31}-\lambda_{32}-\lambda_{33}-\lambda_{35}-\lambda_{36}+\lambda_{37}+$
$\lambda_{38}-\lambda_{39}+\lambda_{40}+3\lambda_{41}-\lambda_{42}-\lambda_{46}-\lambda_{48}](g_{15}+g_{41})+[-\lambda_1+\lambda_2-\lambda_3-\lambda_4-\lambda_5-\lambda_6+\lambda_7+3\lambda_{16}-\lambda_{17}-\lambda_{18}+\lambda_{19}-$
$\lambda_{20}-\lambda_{21}-\lambda_{22}-\lambda_{23}+\lambda_{24}-\lambda_{25}-\lambda_{26}+3\lambda_{27}+\lambda_{28}+\lambda_{29}+\lambda_{30}-\lambda_{31}+\lambda_{32}-\lambda_{33}-\lambda_{34}-\lambda_{35}+\lambda_{36}-\lambda_{37}+\lambda_{38}+\lambda_{39}+$
$\lambda_{40}](g_{16}+g_{27})+[-\lambda_1-\lambda_2-\lambda_3+\lambda_4-\lambda_5+\lambda_6-\lambda_7-\lambda_{16}+3\lambda_{17}-\lambda_{18}-\lambda_{19}+\lambda_{20}-\lambda_{21}+\lambda_{22}-\lambda_{23}-\lambda_{24}-\lambda_{25}+$
$3\lambda_{26}-\lambda_{27}+\lambda_{28}+\lambda_{29}-\lambda_{30}+\lambda_{31}-\lambda_{32}+\lambda_{33}-\lambda_{34}+\lambda_{35}-\lambda_{36}+\lambda_{37}-\lambda_{38}+\lambda_{39}+\lambda_{40}](g_{17}+g_{26})+[-\lambda_1-\lambda_2+\lambda_3-$
$\lambda_4+\lambda_5-\lambda_6-\lambda_7-\lambda_{16}-\lambda_{17}+3\lambda_{18}-\lambda_{19}-\lambda_{20}+\lambda_{21}-\lambda_{22}+\lambda_{23}-\lambda_{24}+3\lambda_{25}-\lambda_{26}-\lambda_{27}-\lambda_{28}-\lambda_{29}+\lambda_{30}+\lambda_{31}+$
$\lambda_{32}+\lambda_{33}-\lambda_{34}+\lambda_{35}+\lambda_{36}+\lambda_{37}+\lambda_{38}-\lambda_{39}-\lambda_{40}](g_{18}+g_{25})+[-\lambda_1-\lambda_3-\lambda_5-\lambda_8+\lambda_9+\lambda_{10}-\lambda_{11}-\lambda_{12}+\lambda_{13}-$
$\lambda_{14}+\lambda_{15}+\lambda_{16}+\lambda_{17}-\lambda_{18}-\lambda_{21}-\lambda_{23}-\lambda_{25}+\lambda_{26}+\lambda_{27}+3\lambda_{28}-\lambda_{29}-\lambda_{34}-\lambda_{39}+3\lambda_{40}+\lambda_{41}-\lambda_{42}+\lambda_{43}-\lambda_{44}-$
$\lambda_{45}+\lambda_{46}+\lambda_{47}-\lambda_{48}](g_{28}+g_{40})+[-\lambda_1-\lambda_3-\lambda_5+\lambda_8-\lambda_9-\lambda_{10}+\lambda_{11}+\lambda_{12}-\lambda_{13}+\lambda_{14}-\lambda_{15}+\lambda_{16}+\lambda_{17}-\lambda_{18}-$
$\lambda_{21}-\lambda_{23}-\lambda_{25}+\lambda_{26}+\lambda_{27}-\lambda_{28}+3\lambda_{29}-\lambda_{34}+3\lambda_{39}-\lambda_{40}-\lambda_{41}+\lambda_{42}-\lambda_{43}+\lambda_{44}+\lambda_{45}-\lambda_{46}-\lambda_{47}+\lambda_{48}](g_{29}+$
$g_{39})+[-\lambda_1-\lambda_4-\lambda_6+\lambda_8-\lambda_9-\lambda_{10}+\lambda_{11}-\lambda_{12}+\lambda_{13}-\lambda_{14}+\lambda_{15}+\lambda_{16}-\lambda_{17}+\lambda_{18}-\lambda_{20}-\lambda_{22}+\lambda_{25}-\lambda_{26}+\lambda_{27}+$
$3\lambda_{30}-\lambda_{32}-\lambda_{34}-\lambda_{36}+3\lambda_{38}+\lambda_{41}-\lambda_{42}+\lambda_{43}-\lambda_{44}+\lambda_{45}-\lambda_{46}-\lambda_{47}+\lambda_{48}](g_{30}+g_{38})+[-\lambda_1-\lambda_2-\lambda_7-\lambda_8+\lambda_9-$
$\lambda_{10}+\lambda_{11}-\lambda_{12}-\lambda_{13}+\lambda_{14}+\lambda_{15}-\lambda_{16}+\lambda_{17}+\lambda_{18}-\lambda_{19}-\lambda_{24}+\lambda_{25}+\lambda_{26}-\lambda_{27}+3\lambda_{31}-\lambda_{33}-\lambda_{34}-\lambda_{35}+3\lambda_{37}+$
$\lambda_{41}+\lambda_{42}-\lambda_{43}-\lambda_{44}+\lambda_{45}-\lambda_{46}+\lambda_{47}-\lambda_{48}](g_{31}+g_{37})+[-\lambda_1-\lambda_4-\lambda_6-\lambda_8+\lambda_9+\lambda_{10}-\lambda_{11}+\lambda_{12}-\lambda_{13}+\lambda_{14}-$
$\lambda_{15}+\lambda_{16}-\lambda_{17}+\lambda_{18}-\lambda_{20}-\lambda_{22}+\lambda_{25}-\lambda_{26}+\lambda_{27}-\lambda_{30}+3\lambda_{32}-\lambda_{34}+3\lambda_{36}-\lambda_{38}-\lambda_{41}+\lambda_{42}-\lambda_{43}+\lambda_{44}-\lambda_{45}+$
$\lambda_{46}+\lambda_{47}-\lambda_{48}](g_{32}+g_{36})+[-\lambda_1-\lambda_2-\lambda_7+\lambda_8-\lambda_9+\lambda_{10}-\lambda_{11}+\lambda_{12}+\lambda_{13}-\lambda_{14}-\lambda_{15}-\lambda_{16}+\lambda_{17}+\lambda_{18}-\lambda_{19}-$
$\lambda_{24}+\lambda_{25}+\lambda_{26}-\lambda_{27}-\lambda_{31}+3\lambda_{33}-\lambda_{34}+3\lambda_{35}-\lambda_{37}-\lambda_{41}-\lambda_{42}+\lambda_{43}+\lambda_{44}-\lambda_{45}+\lambda_{46}-\lambda_{47}+\lambda_{48}](g_{33}+g_{35}).$

$uv_6 = \frac{1}{48}[3\lambda_1-\lambda_2-\lambda_3-\lambda_4-\lambda_5-\lambda_6-\lambda_7+\lambda_{16}+\lambda_{17}+\lambda_{18}+\lambda_{19}+\lambda_{20}+\lambda_{21}+\lambda_{22}+\lambda_{23}+\lambda_{24}-\lambda_{25}-\lambda_{26}-\lambda_{27}-$
$\lambda_{28}-\lambda_{29}-\lambda_{30}-\lambda_{31}-\lambda_{32}-\lambda_{33}-3\lambda_{34}+\lambda_{35}+\lambda_{36}+\lambda_{37}+\lambda_{38}+\lambda_{39}+\lambda_{40}]g_1+[-\lambda_1+3\lambda_2-\lambda_7-\lambda_8-\lambda_9-\lambda_{10}-$
$\lambda_{11}+\lambda_{12}+\lambda_{13}+\lambda_{14}+\lambda_{15}+\lambda_{16}-\lambda_{17}-\lambda_{18}+\lambda_{19}-3\lambda_{24}+\lambda_{25}+\lambda_{26}-\lambda_{27}+\lambda_{31}+\lambda_{33}+\lambda_{34}-\lambda_{35}-\lambda_{37}-\lambda_{41}-\lambda_{42}-$

$\lambda_{43}-\lambda_{44}+\lambda_{45}+\lambda_{46}+\lambda_{47}+\lambda_{48}]g_2+[-\lambda_1+3\lambda_3-\lambda_5-\lambda_8-\lambda_9+\lambda_{10}+\lambda_{11}+\lambda_{12}-\lambda_{13}-\lambda_{14}+\lambda_{15}-\lambda_{16}-\lambda_{17}+$
$\lambda_{18}+\lambda_{21}-3\lambda_{23}-\lambda_{25}+\lambda_{26}+\lambda_{27}+\lambda_{28}+\lambda_{29}+\lambda_{34}-\lambda_{39}-\lambda_{40}-\lambda_{41}+\lambda_{42}+\lambda_{43}-\lambda_{44}-\lambda_{45}-\lambda_{46}+\lambda_{47}+\lambda_{48}]g_3+$
$[-\lambda_1+3\lambda_4-\lambda_6-\lambda_8-\lambda_9+\lambda_{10}+\lambda_{11}-\lambda_{12}+\lambda_{13}+\lambda_{14}-\lambda_{15}-\lambda_{16}+\lambda_{17}-\lambda_{18}+\lambda_{20}-3\lambda_{22}+\lambda_{25}-\lambda_{26}+\lambda_{27}+\lambda_{30}+$
$\lambda_{32}+\lambda_{34}-\lambda_{36}-\lambda_{38}+\lambda_{41}-\lambda_{42}-\lambda_{43}+\lambda_{44}-\lambda_{45}-\lambda_{46}+\lambda_{47}+\lambda_{48}]g_4+[-\lambda_1-\lambda_3+3\lambda_5+\lambda_8+\lambda_9-\lambda_{10}-\lambda_{11}-$
$\lambda_{12}+\lambda_{13}+\lambda_{14}-\lambda_{15}-\lambda_{16}-\lambda_{17}+\lambda_{18}-3\lambda_{21}+\lambda_{23}-\lambda_{25}+\lambda_{26}+\lambda_{27}+\lambda_{28}+\lambda_{29}+\lambda_{34}-\lambda_{39}-\lambda_{40}+\lambda_{41}-\lambda_{42}-\lambda_{43}+$
$\lambda_{44}+\lambda_{45}+\lambda_{46}-\lambda_{47}-\lambda_{48}]g_5+[-\lambda_1-\lambda_4+3\lambda_6+\lambda_8+\lambda_9-\lambda_{10}-\lambda_{11}+\lambda_{12}-\lambda_{13}-\lambda_{14}+\lambda_{15}-\lambda_{16}+\lambda_{17}-\lambda_{18}-$
$3\lambda_{20}+\lambda_{22}+\lambda_{25}-\lambda_{26}+\lambda_{27}+\lambda_{30}+\lambda_{32}+\lambda_{34}-\lambda_{36}-\lambda_{38}-\lambda_{41}+\lambda_{42}+\lambda_{43}-\lambda_{44}+\lambda_{45}+\lambda_{46}-\lambda_{47}-\lambda_{48}]g_6+[-\lambda_1-$
$\lambda_2+3\lambda_7+\lambda_8+\lambda_9+\lambda_{10}+\lambda_{11}-\lambda_{12}-\lambda_{13}-\lambda_{14}-\lambda_{15}+\lambda_{16}-\lambda_{17}-\lambda_{18}-3\lambda_{19}+\lambda_{24}+\lambda_{25}+\lambda_{26}-\lambda_{27}+\lambda_{31}+\lambda_{33}+$
$\lambda_{34}-\lambda_{35}-\lambda_{37}+\lambda_{41}+\lambda_{42}+\lambda_{43}+\lambda_{44}-\lambda_{45}-\lambda_{46}-\lambda_{47}-\lambda_{48}]g_7+[-\lambda_2-\lambda_3-\lambda_4+\lambda_5+\lambda_6+\lambda_7+3\lambda_8-\lambda_{10}-\lambda_{14}-$
$\lambda_{15}-\lambda_{19}-\lambda_{20}-\lambda_{21}+\lambda_{22}+\lambda_{23}+\lambda_{24}-\lambda_{28}+\lambda_{29}+\lambda_{30}-\lambda_{31}-\lambda_{32}+\lambda_{33}-\lambda_{35}+\lambda_{36}+\lambda_{37}-\lambda_{38}-\lambda_{39}+\lambda_{40}+\lambda_{41}+$
$\lambda_{42}+\lambda_{46}-3\lambda_{48}]g_8+[-\lambda_2-\lambda_3-\lambda_4+\lambda_5+\lambda_6+\lambda_7+3\lambda_9-\lambda_{11}-\lambda_{12}-\lambda_{13}-\lambda_{19}-\lambda_{20}-\lambda_{21}+\lambda_{22}+\lambda_{23}+\lambda_{24}+$
$\lambda_{28}-\lambda_{29}-\lambda_{30}+\lambda_{31}+\lambda_{32}-\lambda_{33}+\lambda_{35}-\lambda_{36}-\lambda_{37}+\lambda_{38}+\lambda_{39}-\lambda_{40}+\lambda_{43}+\lambda_{44}+\lambda_{45}-3\lambda_{47}]g_9+[-\lambda_2+\lambda_3+\lambda_4-$
$\lambda_5-\lambda_6+\lambda_7-\lambda_8+3\lambda_{10}-\lambda_{14}-\lambda_{15}-\lambda_{19}+\lambda_{20}+\lambda_{21}-\lambda_{22}-\lambda_{23}+\lambda_{24}+\lambda_{28}-\lambda_{29}-\lambda_{30}-\lambda_{31}+\lambda_{32}+\lambda_{33}-\lambda_{35}-$
$\lambda_{36}+\lambda_{37}+\lambda_{38}+\lambda_{39}-\lambda_{40}+\lambda_{41}+\lambda_{42}-3\lambda_{46}+\lambda_{48}]g_{10}+[-\lambda_2+\lambda_3+\lambda_4-\lambda_5-\lambda_6+\lambda_7-\lambda_9+3\lambda_{11}-\lambda_{12}-\lambda_{13}-$
$\lambda_{19}+\lambda_{20}+\lambda_{21}-\lambda_{22}-\lambda_{23}+\lambda_{24}-\lambda_{28}+\lambda_{29}+\lambda_{30}+\lambda_{31}-\lambda_{32}-\lambda_{33}+\lambda_{35}+\lambda_{36}-\lambda_{37}-\lambda_{38}-\lambda_{39}+\lambda_{40}+\lambda_{43}+\lambda_{44}-$
$3\lambda_{45}+\lambda_{47}]g_{11}+[\lambda_2+\lambda_3-\lambda_4-\lambda_5+\lambda_6-\lambda_7-\lambda_9-\lambda_{11}+3\lambda_{12}-\lambda_{13}+\lambda_{19}-\lambda_{20}+\lambda_{21}+\lambda_{22}-\lambda_{23}-\lambda_{24}-\lambda_{28}+\lambda_{29}-$
$\lambda_{30}-\lambda_{31}+\lambda_{32}+\lambda_{33}-\lambda_{35}-\lambda_{36}+\lambda_{37}+\lambda_{38}-\lambda_{39}+\lambda_{40}+\lambda_{43}-3\lambda_{44}+\lambda_{45}+\lambda_{47}]g_{12}+[\lambda_2-\lambda_3+\lambda_4+\lambda_5-\lambda_6-$
$\lambda_7-\lambda_9-\lambda_{11}-\lambda_{12}+3\lambda_{13}+\lambda_{19}+\lambda_{20}-\lambda_{21}-\lambda_{22}+\lambda_{23}-\lambda_{24}+\lambda_{28}-\lambda_{29}+\lambda_{30}-\lambda_{31}-\lambda_{32}+\lambda_{33}-\lambda_{35}+\lambda_{36}+\lambda_{37}-$
$\lambda_{38}+\lambda_{39}-\lambda_{40}-3\lambda_{43}+\lambda_{44}+\lambda_{45}+\lambda_{47}]g_{13}+[\lambda_2-\lambda_3+\lambda_4+\lambda_5-\lambda_6-\lambda_7-\lambda_8-\lambda_{10}+3\lambda_{14}-\lambda_{15}+\lambda_{19}+\lambda_{20}-\lambda_{21}-$
$\lambda_{22}+\lambda_{23}-\lambda_{24}-\lambda_{28}+\lambda_{29}-\lambda_{30}+\lambda_{31}+\lambda_{32}-\lambda_{33}+\lambda_{35}-\lambda_{36}-\lambda_{37}+\lambda_{38}-\lambda_{39}+\lambda_{40}+\lambda_{41}-3\lambda_{42}+\lambda_{46}+\lambda_{48}]g_{14}+$
$[\lambda_2+\lambda_3-\lambda_4-\lambda_5+\lambda_6-\lambda_7-\lambda_8-\lambda_{10}-\lambda_{14}+3\lambda_{15}+\lambda_{19}-\lambda_{20}+\lambda_{21}+\lambda_{22}-\lambda_{23}-\lambda_{24}+\lambda_{28}-\lambda_{29}+\lambda_{30}+\lambda_{31}-\lambda_{32}-$
$\lambda_{33}+\lambda_{35}+\lambda_{36}-\lambda_{37}-\lambda_{38}+\lambda_{39}-\lambda_{40}-3\lambda_{41}+\lambda_{42}+\lambda_{46}+\lambda_{48}]g_{15}+[\lambda_1+\lambda_2-\lambda_3-\lambda_4-\lambda_5-\lambda_6+\lambda_7+3\lambda_{16}-\lambda_{17}-$
$\lambda_{18}-\lambda_{19}+\lambda_{20}+\lambda_{21}+\lambda_{22}+\lambda_{23}-\lambda_{24}+\lambda_{25}+\lambda_{26}-3\lambda_{27}-\lambda_{28}-\lambda_{29}-\lambda_{30}+\lambda_{31}-\lambda_{32}+\lambda_{33}-\lambda_{34}-\lambda_{35}+\lambda_{36}-$
$\lambda_{37}+\lambda_{38}+\lambda_{39}+\lambda_{40}]g_{16}+[\lambda_1-\lambda_2-\lambda_3+\lambda_4-\lambda_5+\lambda_6-\lambda_7-\lambda_{16}+3\lambda_{17}-\lambda_{18}+\lambda_{19}-\lambda_{20}+\lambda_{21}-\lambda_{22}+\lambda_{23}+\lambda_{24}+$
$\lambda_{25}-3\lambda_{26}+\lambda_{27}-\lambda_{28}-\lambda_{29}+\lambda_{30}-\lambda_{31}+\lambda_{32}-\lambda_{33}-\lambda_{34}+\lambda_{35}-\lambda_{36}+\lambda_{37}-\lambda_{38}+\lambda_{39}+\lambda_{40}]g_{17}++\lambda_1-\lambda_2+\lambda_3-$
$\lambda_4+\lambda_5-\lambda_6-\lambda_7-\lambda_{16}-\lambda_{17}+3\lambda_{18}+\lambda_{19}+\lambda_{20}-\lambda_{21}+\lambda_{22}-\lambda_{23}+\lambda_{24}-3\lambda_{25}+\lambda_{26}+\lambda_{27}+\lambda_{28}+\lambda_{29}-\lambda_{30}-\lambda_{31}-$
$\lambda_{32}-\lambda_{33}-\lambda_{34}+\lambda_{35}+\lambda_{36}+\lambda_{37}+\lambda_{38}-\lambda_{39}-\lambda_{40}]g_{18}+[\lambda_1+\lambda_2-3\lambda_7-\lambda_8-\lambda_9-\lambda_{10}-\lambda_{11}+\lambda_{12}+\lambda_{13}+\lambda_{14}+$
$\lambda_{15}-\lambda_{16}+\lambda_{17}+\lambda_{18}+3\lambda_{19}-\lambda_{24}-\lambda_{25}-\lambda_{26}+\lambda_{27}-\lambda_{31}-\lambda_{33}-\lambda_{34}+\lambda_{35}+\lambda_{37}-\lambda_{41}-\lambda_{42}-\lambda_{43}-\lambda_{44}+\lambda_{45}+$
$\lambda_{46}+\lambda_{47}+\lambda_{48}]g_{19}+[\lambda_1+\lambda_4-3\lambda_6-\lambda_8-\lambda_9+\lambda_{10}+\lambda_{11}-\lambda_{12}+\lambda_{13}+\lambda_{14}-\lambda_{15}+\lambda_{16}-\lambda_{17}+\lambda_{18}+3\lambda_{20}-\lambda_{22}-$
$\lambda_{25}+\lambda_{26}-\lambda_{27}-\lambda_{30}-\lambda_{32}-\lambda_{34}+\lambda_{36}+\lambda_{38}+\lambda_{41}-\lambda_{42}-\lambda_{43}+\lambda_{44}-\lambda_{45}-\lambda_{46}+\lambda_{47}+\lambda_{48}]g_{20}+[\lambda_1+\lambda_3-3\lambda_5-$
$\lambda_8-\lambda_9+\lambda_{10}+\lambda_{11}+\lambda_{12}-\lambda_{13}-\lambda_{14}+\lambda_{15}+\lambda_{16}+\lambda_{17}-\lambda_{18}+3\lambda_{21}-\lambda_{23}+\lambda_{25}-\lambda_{26}-\lambda_{27}-\lambda_{28}-\lambda_{29}-\lambda_{34}+\lambda_{39}+$
$\lambda_{40}-\lambda_{41}+\lambda_{42}+\lambda_{43}-\lambda_{44}-\lambda_{45}-\lambda_{46}+\lambda_{47}+\lambda_{48}]g_{21}+[\lambda_1-3\lambda_4+\lambda_6+\lambda_8+\lambda_9-\lambda_{10}-\lambda_{11}+\lambda_{12}-\lambda_{13}-\lambda_{14}+$
$\lambda_{15}+\lambda_{16}-\lambda_{17}+\lambda_{18}-\lambda_{20}+3\lambda_{22}-\lambda_{25}+\lambda_{26}-\lambda_{27}-\lambda_{30}-\lambda_{32}-\lambda_{34}+\lambda_{36}+\lambda_{38}-\lambda_{41}+\lambda_{42}+\lambda_{43}-\lambda_{44}+\lambda_{45}+$
$\lambda_{46}-\lambda_{47}-\lambda_{48}]g_{22}+[\lambda_1-3\lambda_3+\lambda_5+\lambda_8+\lambda_9-\lambda_{10}-\lambda_{11}-\lambda_{12}+\lambda_{13}+\lambda_{14}-\lambda_{15}+\lambda_{16}+\lambda_{17}-\lambda_{18}-\lambda_{21}+3\lambda_{23}+$
$\lambda_{25}-\lambda_{26}-\lambda_{27}-\lambda_{28}-\lambda_{29}-\lambda_{34}+\lambda_{39}+\lambda_{40}+\lambda_{41}-\lambda_{42}-\lambda_{43}+\lambda_{44}+\lambda_{45}+\lambda_{46}-\lambda_{47}-\lambda_{48}]g_{23}+[\lambda_1-3\lambda_2+\lambda_7+$
$\lambda_8+\lambda_9+\lambda_{10}+\lambda_{11}-\lambda_{12}-\lambda_{13}-\lambda_{14}-\lambda_{15}-\lambda_{16}+\lambda_{17}+\lambda_{18}-\lambda_{19}+3\lambda_{24}-\lambda_{25}-\lambda_{26}+\lambda_{27}-\lambda_{31}-\lambda_{33}-\lambda_{34}+\lambda_{35}+$
$\lambda_{37}+\lambda_{41}+\lambda_{42}+\lambda_{43}+\lambda_{44}-\lambda_{45}-\lambda_{46}-\lambda_{47}-\lambda_{48}]g_{24}+[-\lambda_1+\lambda_2-\lambda_3+\lambda_4-\lambda_5+\lambda_6+\lambda_7+\lambda_{16}+\lambda_{17}-3\lambda_{18}-$
$\lambda_{19}-\lambda_{20}+\lambda_{21}-\lambda_{22}+\lambda_{23}-\lambda_{24}+3\lambda_{25}-\lambda_{26}-\lambda_{27}-\lambda_{28}-\lambda_{29}+\lambda_{30}+\lambda_{31}+\lambda_{32}+\lambda_{33}+\lambda_{34}-\lambda_{35}-\lambda_{36}-\lambda_{37}-$
$\lambda_{38}+\lambda_{39}+\lambda_{40}]g_{25}+[-\lambda_1+\lambda_2+\lambda_3-\lambda_4+\lambda_5-\lambda_6+\lambda_7+\lambda_{16}-3\lambda_{17}+\lambda_{18}-\lambda_{19}+\lambda_{20}-\lambda_{21}+\lambda_{22}-\lambda_{23}-\lambda_{24}-$
$\lambda_{25}+3\lambda_{26}-\lambda_{27}+\lambda_{28}+\lambda_{29}-\lambda_{30}+\lambda_{31}-\lambda_{32}+\lambda_{33}+\lambda_{34}-\lambda_{35}+\lambda_{36}-\lambda_{37}+\lambda_{38}-\lambda_{39}-\lambda_{40}]g_{26}+[-\lambda_1-\lambda_2+\lambda_3+$
$\lambda_4+\lambda_5+\lambda_6-\lambda_7-3\lambda_{16}+\lambda_{17}+\lambda_{18}+\lambda_{19}-\lambda_{20}-\lambda_{21}-\lambda_{22}-\lambda_{23}+\lambda_{24}-\lambda_{25}-\lambda_{26}+3\lambda_{27}+\lambda_{28}+\lambda_{29}+\lambda_{30}-\lambda_{31}+$
$\lambda_{32}-\lambda_{33}+\lambda_{34}+\lambda_{35}-\lambda_{36}+\lambda_{37}-\lambda_{38}-\lambda_{39}-\lambda_{40}]g_{27}+[-\lambda_1+\lambda_3+\lambda_5-\lambda_8+\lambda_9+\lambda_{10}-\lambda_{11}-\lambda_{12}+\lambda_{13}-\lambda_{14}+$
$\lambda_{15}-\lambda_{16}-\lambda_{17}+\lambda_{18}-\lambda_{21}-\lambda_{23}-\lambda_{25}+\lambda_{26}+\lambda_{27}+3\lambda_{28}-\lambda_{29}+\lambda_{34}+\lambda_{39}-3\lambda_{40}-\lambda_{41}+\lambda_{42}-\lambda_{43}+\lambda_{44}+\lambda_{45}-$
$\lambda_{46}-\lambda_{47}+\lambda_{48}]g_{28}+[-\lambda_1+\lambda_3+\lambda_5+\lambda_8-\lambda_9-\lambda_{10}+\lambda_{11}+\lambda_{12}-\lambda_{13}+\lambda_{14}-\lambda_{15}-\lambda_{16}-\lambda_{17}+\lambda_{18}-\lambda_{21}-\lambda_{23}-$
$\lambda_{25}+\lambda_{26}+\lambda_{27}-\lambda_{28}+3\lambda_{29}+\lambda_{34}-3\lambda_{39}+\lambda_{40}+\lambda_{41}-\lambda_{42}+\lambda_{43}-\lambda_{44}-\lambda_{45}+\lambda_{46}+\lambda_{47}-\lambda_{48}]g_{29}+[-\lambda_1+\lambda_4+$

$\lambda_6+\lambda_8-\lambda_9-\lambda_{10}+\lambda_{11}-\lambda_{12}+\lambda_{13}-\lambda_{14}+\lambda_{15}-\lambda_{16}+\lambda_{17}-\lambda_{18}-\lambda_{20}-\lambda_{22}+\lambda_{25}-\lambda_{26}+\lambda_{27}+3\lambda_{30}-\lambda_{32}+\lambda_{34}+$
$\lambda_{36}-3\lambda_{38}-\lambda_{41}+\lambda_{42}-\lambda_{43}+\lambda_{44}-\lambda_{45}+\lambda_{46}+\lambda_{47}-\lambda_{48}]g_{30}+[-\lambda_1+\lambda_2+\lambda_7-\lambda_8+\lambda_9-\lambda_{10}+\lambda_{11}-\lambda_{12}-\lambda_{13}+$
$\lambda_{14}+\lambda_{15}+\lambda_{16}-\lambda_{17}-\lambda_{18}-\lambda_{19}-\lambda_{24}+\lambda_{25}+\lambda_{26}-\lambda_{27}+3\lambda_{31}-\lambda_{33}+\lambda_{34}+\lambda_{35}-3\lambda_{37}-\lambda_{41}-\lambda_{42}+\lambda_{43}+\lambda_{44}-$
$\lambda_{45}+\lambda_{46}-\lambda_{47}+\lambda_{48}]g_{31}+[-\lambda_1+\lambda_4+\lambda_6-\lambda_8+\lambda_9+\lambda_{10}-\lambda_{11}+\lambda_{12}-\lambda_{13}+\lambda_{14}-\lambda_{15}-\lambda_{16}+\lambda_{17}-\lambda_{18}-\lambda_{20}-$
$\lambda_{22}+\lambda_{25}-\lambda_{26}+\lambda_{27}-\lambda_{30}+3\lambda_{32}+\lambda_{34}-3\lambda_{36}+\lambda_{38}+\lambda_{41}-\lambda_{42}+\lambda_{43}-\lambda_{44}+\lambda_{45}-\lambda_{46}-\lambda_{47}+\lambda_{48}]g_{32}+[-\lambda_1+$
$\lambda_2+\lambda_7+\lambda_8-\lambda_9+\lambda_{10}-\lambda_{11}+\lambda_{12}+\lambda_{13}-\lambda_{14}-\lambda_{15}+\lambda_{16}-\lambda_{17}-\lambda_{18}-\lambda_{19}-\lambda_{24}+\lambda_{25}+\lambda_{26}-\lambda_{27}-\lambda_{31}+3\lambda_{33}+$
$\lambda_{34}-3\lambda_{35}+\lambda_{37}+\lambda_{41}+\lambda_{42}-\lambda_{43}-\lambda_{44}+\lambda_{45}-\lambda_{46}+\lambda_{47}-\lambda_{48}]g_{33}+[-3\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5+\lambda_6+\lambda_7-\lambda_{16}-$
$\lambda_{17}-\lambda_{18}-\lambda_{19}-\lambda_{20}-\lambda_{21}-\lambda_{22}-\lambda_{23}-\lambda_{24}+\lambda_{25}+\lambda_{26}+\lambda_{27}+\lambda_{28}+\lambda_{29}+\lambda_{30}+\lambda_{31}+\lambda_{32}+\lambda_{33}+3\lambda_{34}-\lambda_{35}-\lambda_{36}-$
$\lambda_{37}-\lambda_{38}-\lambda_{39}-\lambda_{40}]g_{34}+[\lambda_1-\lambda_2-\lambda_7-\lambda_8+\lambda_9-\lambda_{10}+\lambda_{11}-\lambda_{12}-\lambda_{13}+\lambda_{14}+\lambda_{15}-\lambda_{16}+\lambda_{17}+\lambda_{18}+\lambda_{19}+\lambda_{24}-$
$\lambda_{25}-\lambda_{26}+\lambda_{27}+\lambda_{31}-3\lambda_{33}-\lambda_{34}+3\lambda_{35}-\lambda_{37}-\lambda_{41}-\lambda_{42}+\lambda_{43}+\lambda_{44}-\lambda_{45}+\lambda_{46}-\lambda_{47}+\lambda_{48}]g_{35}+[\lambda_1-\lambda_4-\lambda_6+$
$\lambda_8-\lambda_9-\lambda_{10}+\lambda_{11}-\lambda_{12}+\lambda_{13}-\lambda_{14}+\lambda_{15}+\lambda_{16}-\lambda_{17}+\lambda_{18}+\lambda_{20}+\lambda_{22}-\lambda_{25}+\lambda_{26}-\lambda_{27}+\lambda_{30}-3\lambda_{32}-\lambda_{34}+3\lambda_{36}-$
$\lambda_{38}-\lambda_{41}+\lambda_{42}-\lambda_{43}+\lambda_{44}-\lambda_{45}+\lambda_{46}+\lambda_{47}-\lambda_{48}]g_{36}+[\lambda_1-\lambda_2-\lambda_7+\lambda_8-\lambda_9+\lambda_{10}-\lambda_{11}+\lambda_{12}+\lambda_{13}-\lambda_{14}-\lambda_{15}-$
$\lambda_{16}+\lambda_{17}+\lambda_{18}+\lambda_{19}+\lambda_{24}-\lambda_{25}-\lambda_{26}+\lambda_{27}-3\lambda_{31}+\lambda_{33}-\lambda_{34}-\lambda_{35}+3\lambda_{37}+\lambda_{41}+\lambda_{42}-\lambda_{43}-\lambda_{44}+\lambda_{45}-\lambda_{46}+$
$\lambda_{47}-\lambda_{48}]g_{37}+[\lambda_1-\lambda_4-\lambda_6-\lambda_8+\lambda_9+\lambda_{10}-\lambda_{11}+\lambda_{12}-\lambda_{13}+\lambda_{14}-\lambda_{15}+\lambda_{16}-\lambda_{17}+\lambda_{18}+\lambda_{20}+\lambda_{22}-\lambda_{25}+\lambda_{26}-$
$\lambda_{27}-3\lambda_{30}+\lambda_{32}-\lambda_{34}-\lambda_{36}+3\lambda_{38}+\lambda_{41}-\lambda_{42}+\lambda_{43}-\lambda_{44}+\lambda_{45}-\lambda_{46}-\lambda_{47}+\lambda_{48}]g_{38}+[\lambda_1-\lambda_3-\lambda_5-\lambda_8+\lambda_9+$
$\lambda_{10}-\lambda_{11}-\lambda_{12}+\lambda_{13}-\lambda_{14}+\lambda_{15}+\lambda_{16}+\lambda_{17}-\lambda_{18}+\lambda_{21}+\lambda_{23}+\lambda_{25}-\lambda_{26}-\lambda_{27}+\lambda_{28}-3\lambda_{29}-\lambda_{34}+3\lambda_{39}-\lambda_{40}-$
$\lambda_{41}+\lambda_{42}-\lambda_{43}+\lambda_{44}+\lambda_{45}-\lambda_{46}-\lambda_{47}+\lambda_{48}]g_{39}+[\lambda_1-\lambda_3-\lambda_5+\lambda_8-\lambda_9-\lambda_{10}+\lambda_{11}+\lambda_{12}-\lambda_{13}+\lambda_{14}-\lambda_{15}+\lambda_{16}+$
$\lambda_{17}-\lambda_{18}+\lambda_{21}+\lambda_{23}+\lambda_{25}-\lambda_{26}-\lambda_{27}-3\lambda_{28}+\lambda_{29}-\lambda_{34}-\lambda_{39}+3\lambda_{40}+\lambda_{41}-\lambda_{42}+\lambda_{43}-\lambda_{44}-\lambda_{45}+\lambda_{46}+\lambda_{47}-$
$\lambda_{48}]g_{40}+[-\lambda_2-\lambda_3+\lambda_4+\lambda_5-\lambda_6+\lambda_7+\lambda_8+\lambda_{10}+\lambda_{14}-3\lambda_{15}-\lambda_{19}+\lambda_{20}-\lambda_{21}-\lambda_{22}+\lambda_{23}+\lambda_{24}-\lambda_{28}+\lambda_{29}-\lambda_{30}-$
$\lambda_{31}+\lambda_{32}+\lambda_{33}-\lambda_{35}-\lambda_{36}+\lambda_{37}+\lambda_{38}-\lambda_{39}+\lambda_{40}+3\lambda_{41}-\lambda_{42}-\lambda_{46}-\lambda_{48}]g_{41}+[-\lambda_2+\lambda_3-\lambda_4-\lambda_5+\lambda_6+\lambda_7+\lambda_8+$
$\lambda_{10}-3\lambda_{14}+\lambda_{15}-\lambda_{19}-\lambda_{20}+\lambda_{21}+\lambda_{22}-\lambda_{23}+\lambda_{24}+\lambda_{28}-\lambda_{29}+\lambda_{30}-\lambda_{31}-\lambda_{32}+\lambda_{33}-\lambda_{35}+\lambda_{36}+\lambda_{37}-\lambda_{38}+\lambda_{39}-$
$\lambda_{40}-\lambda_{41}+3\lambda_{42}-\lambda_{46}-\lambda_{48}]g_{42}+[-\lambda_2+\lambda_3-\lambda_4-\lambda_5+\lambda_6+\lambda_7+\lambda_9+\lambda_{11}+\lambda_{12}-3\lambda_{13}-\lambda_{19}-\lambda_{20}+\lambda_{21}+\lambda_{22}-$
$\lambda_{23}+\lambda_{24}-\lambda_{28}+\lambda_{29}-\lambda_{30}+\lambda_{31}+\lambda_{32}-\lambda_{33}+\lambda_{35}-\lambda_{36}-\lambda_{37}+\lambda_{38}-\lambda_{39}+\lambda_{40}+3\lambda_{43}-\lambda_{44}-\lambda_{45}-\lambda_{47}]g_{43}+[-$
$\lambda_2-\lambda_3+\lambda_4+\lambda_5-\lambda_6+\lambda_7+\lambda_9+\lambda_{11}-3\lambda_{12}+\lambda_{13}-\lambda_{19}+\lambda_{20}-\lambda_{21}-\lambda_{22}+\lambda_{23}+\lambda_{24}+\lambda_{28}-\lambda_{29}+\lambda_{30}+\lambda_{31}-\lambda_{32}-$
$\lambda_{33}+\lambda_{35}+\lambda_{36}-\lambda_{37}-\lambda_{38}+\lambda_{39}-\lambda_{40}-\lambda_{43}+3\lambda_{44}-\lambda_{45}-\lambda_{47}]g_{44}+[\lambda_2-\lambda_3-\lambda_4+\lambda_5+\lambda_6-\lambda_7+\lambda_9-3\lambda_{11}+\lambda_{12}+$
$\lambda_{13}+\lambda_{19}-\lambda_{20}-\lambda_{21}+\lambda_{22}+\lambda_{23}-\lambda_{24}+\lambda_{28}-\lambda_{29}-\lambda_{30}-\lambda_{31}+\lambda_{32}+\lambda_{33}-\lambda_{35}-\lambda_{36}+\lambda_{37}+\lambda_{38}+\lambda_{39}-\lambda_{40}-\lambda_{43}-$
$\lambda_{44}+3\lambda_{45}-\lambda_{47}]g_{45}+[\lambda_2-\lambda_3-\lambda_4+\lambda_5+\lambda_6-\lambda_7+\lambda_8-3\lambda_{10}+\lambda_{14}+\lambda_{15}+\lambda_{19}-\lambda_{20}-\lambda_{21}+\lambda_{22}+\lambda_{23}-\lambda_{24}-\lambda_{28}+$
$\lambda_{29}+\lambda_{30}+\lambda_{31}-\lambda_{32}-\lambda_{33}+\lambda_{35}+\lambda_{36}-\lambda_{37}-\lambda_{38}-\lambda_{39}+\lambda_{40}-\lambda_{41}-\lambda_{42}+3\lambda_{46}-\lambda_{48}]g_{46}+[\lambda_2+\lambda_3+\lambda_4-\lambda_5-\lambda_6-$
$\lambda_7-3\lambda_9+\lambda_{11}+\lambda_{12}+\lambda_{13}+\lambda_{19}+\lambda_{20}+\lambda_{21}-\lambda_{22}-\lambda_{23}-\lambda_{24}-\lambda_{28}+\lambda_{29}+\lambda_{30}-\lambda_{31}-\lambda_{32}+\lambda_{33}-\lambda_{35}+\lambda_{36}+\lambda_{37}-$
$\lambda_{38}-\lambda_{39}+\lambda_{40}-\lambda_{43}-\lambda_{44}-\lambda_{45}+3\lambda_{47}]g_{47}+[\lambda_2+\lambda_3+\lambda_4-\lambda_5-\lambda_6-\lambda_7-3\lambda_8+\lambda_{10}+\lambda_{14}+\lambda_{15}+\lambda_{19}+\lambda_{20}+\lambda_{21}-$
$\lambda_{22}-\lambda_{23}-\lambda_{24}+\lambda_{28}-\lambda_{29}-\lambda_{30}+\lambda_{31}+\lambda_{32}-\lambda_{33}+\lambda_{35}-\lambda_{36}-\lambda_{37}+\lambda_{38}+\lambda_{39}-\lambda_{40}-\lambda_{41}-\lambda_{42}-\lambda_{46}+3\lambda_{48}]g_{48}.$