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Matching-star size Ramsey numbers under connectivity constraint

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ABSTRACT

Given two graphs G_1 and G_2, the size Ramsey number $\hat{r}(G_1, G_2)$ refers to the smallest number of edges in a graph G such that for any red-blue edge-coloring of G, either a red subgraph G_1 or a blue subgraph G_2 is present in G. If we further restrict the host graph G to be connected, we obtain the connected size Ramsey number, denoted as $\hat{r}_c(G_1, G_2)$. Erdős and Faudree (1984) proved that $\hat{r}(nK_2, K_{1,m}) = mn$ for all positive integers m, n. In this paper, we concentrate on the connected analog of this result. Rahadjeng, Baskoro, and Assiyatun (2016) provided the exact values of $\hat{r}_c(nK_2, K_{1,m})$ for $n = 2, 3$. We establish a more general result: for all positive integers m and n with $m \geq (n^2 + 2pn + n - 3)/2$, we have $\hat{r}_c(nK_{1,p}, K_{1,m}) = n(m + p) - 1$. As a corollary, $\hat{r}_c(nK_2, K_{1,m}) = nm + n - 1$ for $m \geq (n^2 + 3n - 3)/2$. We also propose a conjecture for the interested reader.

Keywords: Ramsey number, connected size ramsey number, matching, star

1. Introduction

Given two graphs G_1 and G_2, we define $G \to (G_1, G_2)$ if, for any red-blue edge-coloring of the graph G, G contains either a red subgraph isomorphic to G_1, or a blue subgraph isomorphic to G_2. The term "size" commonly refers to the number of edges in a graph. The concept of the *size Ramsey number* $\hat{r}(G_1, G_2)$ was introduced by Erdős, Faudree, Rousseau, and Schelp [8] in 1978. It represents the smallest positive integer m for which there exists a graph G of size m satisfying $G \to (G_1, G_2)$. Size Ramsey numbers are one of the most well-established topics in graph Ramsey theory, and a comprehensive survey can be found in [9].

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Determining size Ramsey numbers presents a more challenging task compared to (order) Ramsey numbers. A key focus in this field is to find the graphs G for which $\hat{r}(G, G)$ exhibits linear growth with respect to $|V(G)|$. Extensive research has been conducted on this topic if G is a path [1, 6], a tree with bounded maximum degree [5, 13], a cycle [14, 15], and others. On the other hand, numerous studies have aimed to determine the exact values of size Ramsey numbers, as evidenced by a variety of papers addressing this topic [3, 4, 7, 8, 10, 11, 12, 16, 17, 18, 19, 22]. Among them, Erdős and Faudree [7] proved that $\hat{r}(nK_2, K_{1,m}) = mn$ for all positive integers m, n.

Several variants of the size Ramsey number have also been studied. In 2015, Rahadjeng, Baskoro, and Assiyatun [20] investigated a specific variant called the connected size Ramsey number. This variant adds the condition that the host graph G must be connected. More precisely, the *connected size Ramsey number* $\hat{r}_c(G_1, G_2)$ represents the minimum number of edges required for a connected graph G such that $G \to (G_1, G_2)$. It is evident that $\hat{r}(G_1, G_2) \leq \hat{r}_c(G_1, G_2)$, and equality holds when both G_1 and G_2 are connected graphs. However, the latter parameter presents challenges when either G_1 or G_2 is disconnected. Existing research primarily focuses on the connected size Ramsey numbers involving a matching versus a sparse graph, such as a path, a star, or a cycle.

In the context of the connected size Ramsey number of a matching versus a star, Rahadjeng, Baskoro, and Assiyatun [21] derived an upper bound for $\hat{r}_c(nK_2, K_{1,m})$, which is $nm + n - 1$. Here, a matching nK_2 consists of n pairwise non-adjacent edges, and a star $K_{1,m}$ is a tree with m edges sharing a common endpoint. They also demonstrated that this bound is tight when $n = 2$ and 3. On the other hand, when m is small, this upper bound cannot be tight. For instance, Wang, Song, Zhang, and Zhang [23] showed that $\hat{r}_c(nK_2, K_{1,2}) = \lceil (5n - 1)/2 \rceil$. In this paper, we generalize the results of Rahadjeng, Baskoro, and Assiyatun, and determine the exact values of $\hat{r}_c(nK_{1,p}, K_{1,m})$ for larger values of m, as stated by the following theorem. Here, $nK_{1,p}$ is the disjoint union of n copies of $K_{1,p}$, which we call a star matching.

Theorem 1.1. *For positive integers m, n, and p with $m \geq (n^2 + 2pn + n - 3)/2$, we have*

$$\hat{r}_c(nK_{1,p}, K_{1,m}) = n(m + p) - 1.$$

When $p = 1$, $nK_{1,p}$ is actually a matching with n edges. We have the following corollary.

Corollary 1.2. *For positive integers m and n with $m \geq (n^2 + 3n - 3)/2$, we have*

$$\hat{r}_c(nK_2, K_{1,m}) = nm + n - 1.$$

It would be intriguing to enhance the lower bound of the parameter m further. Specifically, we propose the following conjecture.

Conjecture 1.3. *There exists a positive constant c such that for all positive integers m, n, and p with $m \geq c(n + p)$, we have*

$$\hat{r}_c(nK_{1,p}, K_{1,m}) = n(m + p) - 1.$$

This conjecture is beyond our ability, and we leave it to interested readers.

2. Preliminaries and methods

A (G_1, G_2)-coloring of G is a red-blue edge-coloring of G such that there is neither a red G_1 nor a blue G_2. Therefore, the statement "$G \not\to (G_1, G_2)$" is equivalent to saying that there exists a (G_1, G_2)-coloring for the graph G. Throughout the discussion, we will use these two equivalent statements interchangeably multiple times. For all other terminologies, please refer to [2].

The following folklore is needed to show the lower bound.

Lemma 2.1. *For any matching M and any vertex cover S of a graph G, we have $|M| \leq |S|$.*

In the second half of this section, let us briefly summarize the proof techniques used in the paper. The main part of the proof lies in determining the lower bound of $\hat{r}_c(nK_{1,p}, K_{1,m})$. We will induct on n and employ the method of minimal counterexample. Suppose that a connected graph G has at most $n(m+p)-2$ edges and satisfies $G \to (nK_{1,p}, K_{1,m})$. Among all graphs G that satisfy the condition, we choose one with the smallest number of edges. We partition the set $V(G)$ into two sets based on their degrees, and then discover some properties of G. We shall find at least $m+p$ edges in G such that when these edges are removed, the graph induced by the remaining edges, denoted as H, is still connected. According to the induction hypothesis, $H \not\to ((n-1)K_{1,p}, K_{1,m})$. Building upon an $((n-1)K_{1,p}, K_{1,m})$-coloring of H, we then color the edges in $E(G) \setminus E(H)$, and obtain an $(nK_{1,p}, K_{1,m})$-coloring of G, leading to a contradiction.

3. Proof of the main theorem

For any red-blue edge-coloring of $K_{1,m+p-1}$, there is either a red copy of $K_{1,p}$ or a blue copy of $K_{1,m}$. Consequently, it is easy to check that $nK_{1,m+p-1} \to (nK_{1,p}, K_{1,m})$. Because $nK_{1,m+p-1}$ has n connected components, we can make this graph connected by adding $n-1$ edges. Let us call this graph G. It has $n(m+p)-1$ edges and $G \to (nK_{1,p}, K_{1,m})$. Hence the upper bound follows.

For the lower bound, let G be an arbitrary connected graph with $n(m+p)-2$ edges. We aim to prove that $G \not\to (nK_{1,p}, K_{1,m})$, where $m \geq (n^2+2pn+n-3)/2$. By induction on n. The base case $n=1$ is straightforward. For each $k \in [n-1]$, assume that any connected graph G with $k(m+p)-2$ edges satisfies $G \not\to (kK_{1,p}, K_{1,m})$. We will now establish the case for $k=n$. Throughout the proof, U denotes the set of vertices with degree at least m in G. Set $t = |U|$ and $V = V(G) \setminus U$.

We will structure our discussion according to the value of t, which naturally leads to three cases in the proof. The first two cases are relatively straightforward to analyze, while our discussion focuses on the last case.

Case 1. $t \leq n-1$.

We color the edges incident to U red and the remaining edges blue. Thus, U forms a vertex cover of the red edges. It is worth noting that the maximum degree induced by the blue edges does not exceed $m-1$. Therefore, this coloring is an $(nK_{1,p}, K_{1,m})$-coloring of G.

Case 2. $t \geq n+1$.

The graph G has size at least $tm - \binom{t}{2}$. This is because the number of edges with at least one end in U is at least $tm - \binom{t}{2}$. Consider the function $f(t) = tm - \binom{t}{2} = -\frac{1}{2}t^2 + (m+\frac{1}{2})t$, which the axis of symmetry is $t = m+1/2$. Note that $t \leq 2n$, since if $t \geq 2n+1$ then G has at least $(2n+1)m/2$

edges, contradicting the assumption that G has $nm+n-2$ edges. Since $n+1 \leq 2n < m+1/2$, the function is minimized by the property of quadratic functions when $t = n+1$. Hence we have

$$\begin{aligned} f(t) &\geq f(n+1) \\ &= (n+1)m - \frac{(n+1)n}{2} \\ &\geq nm + \frac{n^2+2pn+n-3}{2} - \frac{(n+1)n}{2} \\ &= nm + \frac{2pn-3}{2} \\ &> n(m+p)-2. \end{aligned}$$

This is a contradiction since G does not have enough edges.

Case 3. $t = n$.

In this case, we use proof by contradiction. Suppose there exists a connected graph G with at most $n(m+p)-2$ edges such that $G \to (nK_{1,p}, K_{1,m})$. We select G to be a connected graph with the minimum number of edges that satisfies this property. In other words, for any connected graph H with $e(H) < e(G)$, we always have $H \not\to (nK_{1,p}, K_{1,m})$. We will characterize the properties of G through some claims until we finally prove that such a G does not exist.

Claim 1. $G[V]$ is an empty graph.

Proof. Recall that $V = V(G) \setminus U$. Assume that there is an edge e in $G[V]$ such that $G-e$ is still connected. According to the selection of G at the beginning of this case, $G-e \not\to (nK_{1,p}, K_{1,m})$. Hence $G-e$ has an $(nK_{1,p}, K_{1,m})$-coloring. We then color e blue, which does not contribute to a blue star $K_{1,m}$ since the center of any $K_{1,m}$ must be in U. It follows that $G \not\to (nK_{1,p}, K_{1,m})$, a contradiction.

Assume that there is an edge e in $G[V]$ such that G is disconnected after deleting e. Then $G-e$ has two components, written as G_1 and G_2. There exist two positive integers a_1 and a_2 such that

$$(m+p)a_1 - 1 \leq |E(G_1)| \leq (m+p)(a_1+1) - 2,$$

$$(m+p)a_2 - 1 \leq |E(G_2)| \leq (m+p)(a_2+1) - 2.$$

Observe that both a_1 and a_2 are less than n. We have $G_1 \not\to ((a_1+1)K_{1,p}, K_{1,m})$ and $G_2 \not\to ((a_2+1)K_{1,p}, K_{1,m})$ by induction. That is, there exists an $((a_1+1)K_{1,p}, K_{1,m})$-coloring in G_1 and an $((a_2+1)K_{1,p}, K_{1,m})$-coloring in G_2. According to the coloring, there are at most a_1+a_2 copies of red $K_{1,p}$ in $G-e$, and there is no blue $K_{1,m}$. By adding the two inequalities, we obtain that $(m+p)(a_1+a_2)-2 \leq |E(G_1)|+|E(G_2)|$, which together with the fact $|E(G_1)|+|E(G_2)| \leq (m+p)n-3$ implies $a_1+a_2 < n$. Based on the colorings of G_1 and G_2, we color e blue. This is an $(nK_{1,p}, K_{1,m})$-coloring and hence $G \not\to (nK_{1,p}, K_{1,m})$, a contradiciton. □

Claim 2. $G[U]$ is an empty graph.

Proof. The proof of this claim is quite similar to the proof of the previous one. Assuming that an edge $e = u_1u_2$ exists in $G[U]$, we divide the discussion into two cases: when $G-e$ is connected and when $G-e$ is disconnected.

First, we assume that $G-e$ is connected. According to the assumption of G at the beginning of this case, $G-e \nrightarrow (nK_{1,p}, K_{1,m})$. Based on an $(nK_{1,p}, K_{1,m})$-coloring of $G-e$, we proceed to color e red and obtain a coloring of G. If G contains a red $nK_{1,p}$, it must include the edge e, since otherwise $G-e$ would also contain this red $nK_{1,p}$. Due to Claim 1, $U \setminus \{u_1, u_2\}$ is a vertex cover of a red $(n-1)K_{1,p}$. However, combining Lemma 2.1 and the fact that $|U \setminus \{u_1, u_2\}| = n-2$, $U \setminus \{u_1, u_2\}$ cannot be a vertex cover for $(n-1)K_{1,p}$, a contradiction. Thus, this coloring is an $(nK_{1,p}, K_{1,m})$-coloring of G, which contradicts the assumption that $G \to (nK_{1,p}, K_{1,m})$.

Second, we assume that $G-e$ is disconnected. Then $G-e$ consists of two components, denoted as G_1 and G_2. There exist two positive integers a_1 and a_2 such that

$$(m+p)a_1 - 1 \leq |E(G_1)| \leq (m+p)(a_1+1) - 2,$$

$$(m+p)a_2 - 1 \leq |E(G_2)| \leq (m+p)(a_2+1) - 2.$$

Note that both a_1 and a_2 are less than n. By induction, we have $G_1 \nrightarrow ((a_1+1)K_{1,p}, K_{1,m})$ and $G_2 \nrightarrow ((a_2+1)K_{1,p}, K_{1,m})$. Consequently, we can find an $((a_1+1)K_{1,p}, K_{1,m})$-coloring in G_1 and an $((a_2+1)K_{1,p}, K_{1,m})$-coloring in G_2. With this coloring, there are at most $a_1 + a_2$ copies of red $K_{1,p}$ in $G-e$, and no blue $K_{1,m}$ is present. By adding the two inequalities, we deduce $(m+p)(a_1+a_2) - 2 \leq |E(G_1)| + |E(G_2)|$. Since $|E(G_1)| + |E(G_2)| \leq (m+p)n - 3$, we have $a_1 + a_2 < n$. Based on the coloring of G_1 and G_2, we color e red. If G has a red $nK_{1,p}$, it must include the edge e, since otherwise $G-e$ would also contain this $K_{1,p}$. By Claim 1, $U \setminus \{u_1, u_2\}$ is a vertex cover of $(n-1)K_{1,p}$. However, combining Lemma 2.1 and the fact that $|U \setminus \{u_1, u_2\}| = n-2$, $U \setminus \{u_1, u_2\}$ cannot be a vertex cover for $(n-1)K_{1,p}$, which leads to a contradiction. Thus, this coloring is an $(nK_{1,p}, K_{1,m})$-coloring of G, contradicting the assumption that $G \to (nK_{1,p}, K_{1,m})$. □

We see that every edge of G has one endpoint in U and the other in V. We have the following claim for $p \geq 2$.

Claim 3. When $p \geq 2$, if G contains a red $nK_{1,p}$ as a subgraph, then each vertex of U is a center of the red $K_{1,p}$'s.

Proof. Proof by contradiction. Let us assume that G contains a red $nK_{1,p}$ as a subgraph, and suppose that some vertex of U is not a center of the $K_{1,p}$'s. Then there exists one of the $K_{1,p}$'s, denoted as F, with its center located in V. According to Claim 1, all the leaves of F are in U. There are $n-p$ vertices in $U \setminus V(F)$. According to the two previous claims, each edge of the remaining $n-1$ copies of the red $K_{1,p}$ has one end in $U \setminus V(F)$. Consequently, the $n-p$ vertices form a vertex cover for $(n-1)K_{1,p}$, which is impossible. This validates our claim. □

In the following, we construct two new graphs, G' and G''. For $u \in U$ and $v \in V$, if the graph G remains connected after deleting the edge uv while ensuring $u, v \in V(G)$, then we delete the edge uv from G and add a new edge uv', where v' is a new vertex. We continue performing this operation on the edges of G until it can no longer be done. As a result, we have constructed a new graph, denoted by G'. For every edge $e = uv$, where $u \in U$ and $v \in V$, according to the above construction, $G' - e$ must be disconnected. Recall that a graph is a tree if and only if it is minimally connected. Thus, G' is a tree.

It is easy to verify from the definition of U and Claim 3 that if G' has an $(nK_{1,p}, K_{1,m})$-coloring, then G has a corresponding $(nK_{1,p}, K_{1,m})$-coloring. Based on the assumption we made at the beginning of this case, namely $G \to (nK_{1,p}, K_{1,m})$, we can conclude that

$$G' \to (nK_{1,p}, K_{1,m}).$$

We still partition $V(G')$ into two parts, U and V. If a vertex in $V(G')$ belongs to the set U in the original graph G, we still assign it to U. Otherwise, we assign it to V.

Now we delete all the leaves from G', and the resulting new graph is denoted as G''. We select two vertices (not necessarily distinct), v_1 and v_2, from V in such a way that the distance between v_1 and v_2 on the tree G'' is maximized. In other words, $d_{G''}(v_1, v_2) = \max\{d_{G''}(v, v') \mid v, v' \in V\}$. From this, we deduce that the vertex v_2 has at least one neighbor that is a leaf. Moreover, due to the maximality of $d_{G''}(v_1, v_2)$, if v_2 has exactly one neighbor that is a leaf, then the degree of v_2 is two.

There are two cases in the graph G''. The first case is when v_2 has at least two neighbors that are leaves, denoted as u_1 and u_2. The second case is when v_2 has exactly one neighbor that is a leaf, denoted as u_3, and the degree of v_2 is two. Regarding the degrees of three vertices u_1, u_2, u_3 in the graph G', the following claim holds.

Claim 4. $d_{G'}(u_i) \geq m + p - 1$ for $i \in [3]$. Moreover, either $d_{G'}(u_1) \geq m + p$ or $d_{G'}(u_2) \geq m + p$.

Proof. Suppose there exists $i \in [3]$ such that $d_{G'}(u_i) \leq m + p - 2$. In the graph G', if u_i has at least $m - 1$ leaves, we color the edges between u_i and exactly $m - 1$ leaves as blue, and all other edges in G' as red. On the other hand, if u_i has at most $m - 2$ leaves, we color all edges incident to u_i as blue, and all other edges in G' as red. This graph has at most $m - 1$ blue edges, hence it does not contain a blue $K_{1,m}$. Since u_i is incident to at most $p - 1$ red edges, u_i cannot be the center of a red $K_{1,p}$. According to Claim 3, G' does not contain a red $nK_{1,p}$ for $p \geq 2$. It is easy to verify that the same conclusion holds for $p = 1$. Therefore, $G' \not\to (nK_{1,p}, K_{1,m})$. This contradiction demonstrates that $d_{G'}(u_i) \geq m + p - 1$ for $i \in [3]$.

Suppose $d_{G'}(u_1) = d_{G'}(u_2) = m + p - 1$. In the graph G', we select $m - 1$ leaves of u_1 and color the edges between u_1 and these $m - 1$ leaves as blue. Similarly, we select $m - 1$ leaves of u_2 and color the edges between u_2 and these $m - 1$ leaves as blue. Then, we color all other edges in G' as red. It is noteworthy that there are exactly p red edges incident to u_1 and p red edges incident to u_2. However, since both u_1v_2 and u_2v_2 are red edges, and $nK_{1,p}$ represents the disjoint union of n copies of $K_{1,p}$, it follows that u_1 and u_2 cannot be the centers of different $K_{1,p}$ components in $nK_{1,p}$. According to Claim 3, G' does not contain a red $nK_{1,p}$ for $p \geq 2$. The same conclusion holds for $p = 1$. Hence, we have $G' \not\to (nK_{1,p}, K_{1,m})$, which contradicts our assumption. Consequently, either $d_{G'}(u_1) \geq m + p$ or $d_{G'}(u_2) \geq m + p$. □

If there exists $i \in [3]$ such that $d_{G'}(u_i) \geq m + p$, without loss of generality, assume that $d_{G'}(u_1) \geq m+p$. We delete the vertex u_1 and its adjacent leaves from G', resulting in a connected graph. By the induction hypothesis, $G' \setminus \{u_1\} \not\to ((n-1)K_{1,p}, K_{1,m})$. Building upon the $((n-1)K_{1,p}, K_{1,m})$-coloring of $G' \setminus \{u_1\}$, we color all edges incident to u_1 in red. This gives us an $(nK_{1,p}, K_{1,m})$-coloring of G', which is a contradiction.

According to Claim 4, only one case remains: $d_{G'}(u_3) = m + p - 1$, v_2 is adjacent to u_3, and v_2 has degree two in G'. We remove v_2, u_3, and the adjacent leaves of u_3 from G'. In this way, the resulting graph remains connected. Note that we have removed $m + p$ edges during this process. By the induction hypothesis, $G' \setminus \{v_2, u_3\} \not\to ((n-1)K_{1,p}, K_{1,m})$. Expanding upon the $((n-1)K_{1,p}, K_{1,m})$-coloring of $G' \setminus \{v_2, u_3\}$, we establish an $(nK_{1,p}, K_{1,m})$-coloring of G' as follows: we select $m - 1$ leaves of u_3 and color the edges between u_3 and these $m-1$ leaves as blue. Subsequently, we assign red color to the remaining edges connected to u_3 and v_2. Since u_3 is precisely incident to p red edges, if u_3 is the center of a red $K_{1,p}$, then v_2 must also be a vertex of this $K_{1,p}$ and cannot belong to any other red $K_{1,p}$. This results in an $(nK_{1,p}, K_{1,m})$-coloring of G', which contradicts our assumption. This completes the proof of the main theorem.

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