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Absolute mean graceful labeling of various m-shadow and m-splitting related graphs

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ABSTRACT

A graph $G = (V, E)$ is said to be an absolute mean graceful graph if there exists a one-to-one function $f : V(G) \to \{0, \pm 1, \pm 2, \ldots, \pm |E(G)|\}$ such that the induced edge-labeling function $f^* : E(G) \to \{1, 2, \ldots, |E(G)|\}$, defined by

$$f^*(xy) = \left\lceil \frac{|f(x) - f(y)|}{2} \right\rceil,$$

is bijective. The labeling function f is called an absolute mean graceful labeling of the graph G. In this paper, we obtain absolute mean graceful labelings for m-splitting and m-shadow graphs of various graphs.

Keywords: absolute mean graceful labeling, m-shadow graph, m-splitting graph

2020 Mathematics Subject Classification: 05C78, 05C76.

1. Introduction

By $G = (V, E)$, we mean a simple, finite, connected, and undirected graph, where V is the set of vertices of G and E is the set of edges of G. For all terminology and notation related to graph theory, we follow Harary [10].

A graph labeling is an assignment of integers to the vertices or edges, or both, subject to certain conditions. For various results and references related to graph labeling, we follow Gallian [8]. In 1967, Rosa [17] introduced the concepts of β-valuation and α-valuation in graph labeling. The β-valuation was later known as graceful labeling due to Golomb [9]. Some applications related to graph labeling can be found in [5, 6, 15, 16].

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Received 14 Feb 2026; Revised 29 Mar 2026; Accepted 04 Apr 2026; Published Online 06 May 2026.

DOI: 10.61091/um127-06

Absolute mean graceful labeling was introduced by Kaneria and Chudasama [11]. They defined absolute mean graceful labeling for the path P_n, cycle C_n, complete bipartite graph $K_{m,n}$, grid graph $P_m \times P_n$, step grid graph St_n, and double step grid graph DSt_n. In [7], the same authors also obtained absolute mean graceful labeling for some graphs created by duplication. Kaneria *et al.* [12] proved that path unions of graphs such as trees, paths P_n, cycles C_n, complete bipartite graphs $K_{m,n}$, grid graphs $P_m \times P_n$, step grid graphs St_n, and double step grid graphs DSt_n are absolute mean graceful graphs. Akbari *et al.* [2, 3] obtained absolute mean graceful labeling for the jewel graph, jellyfish graph, and barycentric subdivision of various graphs. Akbari *et al.* [4] investigated absolute mean graceful labeling for the disjoint union of various graphs. Kaneria and Shah [13, 14] defined absolute mean graceful labeling for graphs, the m-splitting graph of a path and a star, the splitting graph of a bistar, the degree splitting graph of some graphs, and some cycle-related graphs.

In this paper, we investigate absolute mean graceful labeling for some m-splitting and m-shadow-related graphs. We now recall some definitions that are useful for this paper.

Definition 1.1 ([11])**.** A function f is said to be an absolute mean graceful labeling of a graph G with q edges if f is a one-to-one function from $V(G)$ to the set $\{0, \pm 1, \pm 2, \dots, \pm q\}$ such that, when each edge uv of G is assigned the label

$$f^*(uv) = \left\lceil \frac{|f(u) - f(v)|}{2} \right\rceil,$$

the set of resulting edge labels is $\{1, 2, 3, \dots, q\}$. Here, $\lceil\ \rceil$ denotes the ceiling function, which maps any real number x to the smallest integer greater than or equal to x. A graph G that admits an absolute mean graceful labeling is called an absolute mean graceful graph.

Definition 1.2 ([8])**.** Let G be a graph. For each vertex v_i of G, take a new vertex v_i'. Join v_i' to those vertices of G that are adjacent to v_i. The graph thus obtained is called the splitting graph of G, and it is denoted by $S'(G)$.

Definition 1.3 ([8])**.** The shadow graph $D_2(G)$ of a connected graph G is constructed by taking two copies, G_1 and G_2, of G and joining each vertex u of G_1 to the neighbours of the corresponding vertex v in G_2.

Definition 1.4 ([1])**.** Let G be a graph. For each vertex v of G, take new m vertices $v^1, v^2, \dots, v^m$. Join v^i to those vertices of G that are adjacent to v in G. The graph thus obtained is called the m-splitting graph of G, and it is denoted by $Spl_m(G)$.

If G has p vertices and q edges, then the graph $Spl_m(G)$ has $(m+1)p$ vertices and $(2m+1)q$ edges.

Definition 1.5 ([1])**.** The m-shadow graph $D_m(G)$ of a connected graph G is constructed by taking m copies $G_1, G_2, \dots, G_m$ of G and joining each vertex u in G_i to the neighbours of the corresponding vertex v in G_j, for $1 \leq i, j \leq m$.

If G has p vertices and q edges, then the graph $D_m(G)$ has mp vertices and m^2q edges.
From the above definitions, the 1-splitting graph is the splitting graph, and the 2-shadow graph is the shadow graph of a given graph G.

Definition 1.6. The bistar $B_{m,n}$ is the graph obtained by joining the center vertices of two stars $K_{1,m}$ and $K_{1,n}$ by an edge.

2. Main Results

Theorem 2.1. *The m-splitting graph of the complete bipartite graph, $Spl_m(K_{r,t})$, is an absolute mean graceful graph for all $m, r, t \geq 1$.*

Proof. Consider the complete bipartite graph $K_{r,t}$ with vertices

$$v_1, v_2, \ldots, v_r, u_1, u_2, \ldots, u_t.$$

Let v_i^j and u_i^j be the vertices corresponding to the vertices v_i and u_i, respectively, for $j = 1, 2, \ldots, m$. Let G be the graph $Spl_m(K_{r,t})$. Then

$$p = |V(G)| = (m+1)(r+t)$$

and

$$q = |E(G)| = (2m+1)rt.$$

Now, consider the labeling function

$$f : V(G) \to \{0, \pm 1, \pm 2, \ldots, \pm q\}$$

defined as follows:

$$\begin{aligned}
f(v_i) &= q - 2t(m+1)(i-1), && \text{for } i = 1, 2, \ldots, r, \\
f(v_i^j) &= f(v_i) - 2mt(r-i) - 2rtj, && \text{for } i = 1, 2, \ldots, r \text{ and } j = 1, 2, \ldots, m, \\
f(u_i) &= -q + 2(mt + i - 1), && \text{for } i = 1, 2, \ldots, t, \\
f(u_i^j) &= f(u_i) - 2tj, && \text{for } i = 1, 2, \ldots, t \text{ and } j = 1, 2, \ldots, m.
\end{aligned}$$

The above-defined function f assigns distinct labels to every vertex of the graph G from the set $\{0, \pm 1, \pm 2, \ldots, \pm q\}$; hence, f is one-to-one. Now consider the induced edge-labeling function f^* defined by

$$f^*(xy) = \left\lceil \frac{|f(x) - f(y)|}{2} \right\rceil$$

for every edge $xy \in E(G)$. Then,

$$f^*(\{v_i^j u_t, v_i^j u_{t-1}, \ldots, v_i^j u_1 : 1 \leq i \leq r,\ 1 \leq j \leq m\}) = \{1, 2, \ldots, mrt\},$$

$$f^*(\{v_r u_t, v_r u_{t-1}, \ldots, v_r u_1\}) = \{mrt+1, mrt+2, \ldots, mrt+t\},$$

$$f^*(\{v_r u_i^j, v_r u_i^j, \ldots, v_r u_i^j : 1 \le i \le r,\ 1 \le j \le m\}) = \{mrt+t+1, mrt+t+2, \ldots, mrt+t+mt\},$$

$$f^*(\{v_{r-1} u_t, v_{r-1} u_{t-1}, \ldots, v_{r-1} u_1\}) = \{mrt+t+mt+1, mrt+t+mt+2, \ldots, mrt+2t+mt\},$$

$$\begin{aligned} &f^*(\{v_{r-1} u_i^j, v_{r-1} u_i^j, \ldots, v_{r-1} u_i^j : 1 \le i \le r,\ 1 \le j \le m\}) \\ &\quad = \{mrt+2t+mt+1, mrt+2t+mt+2, \ldots, mrt+2t+2mt\}, \end{aligned}$$

$$\vdots$$

$$\begin{aligned} f^*(\{v_1 u_t, v_1 u_{t-1}, \ldots, v_1 u_1\}) &= \{mrt+(r-1)t+(r-1)mt+1, \\ &mrt+(r-1)t+(r-1)mt+2, \ldots, mrt+rt+(r-1)mt\}, \end{aligned}$$

$$\begin{aligned} &f^*(\{v_1 u_i^j, v_1 u_i^j, \ldots, v_1 u_i^j : 1 \le i \le r,\ 1 \le j \le m\}) \\ &\quad = \{mrt+rt+(r-1)mt+1, mrt+rt+(r-1)mt+2, \ldots, mrt+rt+rmt = q\}. \end{aligned}$$

Thus,

$$f^*(E(G)) = \{1, 2, \ldots, q\},$$

where $|E(G)| = q$. Therefore, f^* is bijective. Hence, the function f is an absolute mean graceful labeling of the graph $Spl_m(K_{r,t})$ for all $m, r, t \ge 1$. □

Illustration 2.2. An absolute mean graceful labeling of the graph $Spl_2(K_{2,3})$ is shown in Figure 1.

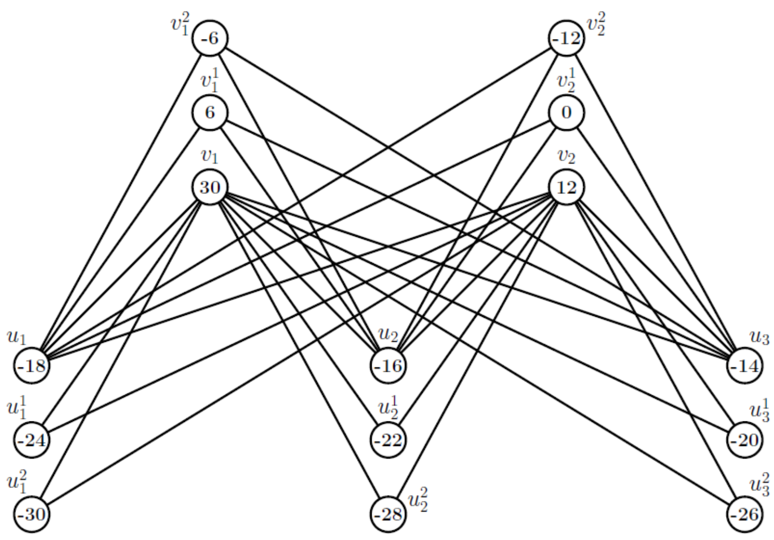

Fig. 1. The graph $Spl_2(K_{2,3})$ with an absolute mean graceful labeling

Theorem 2.3. *The m-splitting graph of the even cycle, $Spl_m(C_{2k})$, is an absolute mean graceful graph for all $m \geq 1$ and $k \geq 2$.*

Proof. Consider the even cycle C_{2k} with vertices $v_1, v_2, \ldots, v_{2k}$. Let v_i^j be the vertices corresponding to each vertex v_i of the graph C_{2k}, for $j = 1, 2, \ldots, m$. Let G be the graph $Spl_m(C_{2k})$. Then

$$p = |V(G)| = 2k(m+1)$$

and

$$q = |E(G)| = 2k(2m+1).$$

Now, consider the labeling function

$$f : V(G) \to \{0, \pm 1, \pm 2, \ldots, \pm q\}$$

defined as follows:

$$f(v_i) = \begin{cases} (-1)^{i+1}(q - 2(i-1)), & \text{for } i = 1, 2, \ldots, k+1, \\ (-1)^{i+1}(q - 1 - 2(2k-i)), & \text{for } i = k+2, k+3, \ldots, 2k, \end{cases}$$

and

$$f(v_i^j) = \begin{cases} f(v_i) - 4kj, & \text{for } i = 1, 3, 5, \ldots, 2k-1, \\ f(v_i) + 4(m+1)k + 4k(j-1), & \text{for } i = 2, 4, 6, \ldots, 2k, \end{cases}$$

for all $j = 1, 2, \ldots, m$.

The above-defined function f assigns distinct labels to every vertex of the graph G from the set $\{0, \pm 1, \pm 2, \ldots, \pm q\}$; hence, f is one-to-one. Now consider the induced edge-labeling function f^* defined by

$$f^*(xy) = \left\lceil \frac{|f(x) - f(y)|}{2} \right\rceil$$

for every edge $xy \in E(G)$. Then f^* assigns distinct labels to every edge of the graph G from the set $\{1, 2, \ldots, q\}$ as follows:

$$f^*(v_k v_{k+1}^m) = 1, \quad f^*(v_{k+2} v_{k+1}^m) = 2, \quad f^*(v_k v_{k-1}^m) = 3, \quad f^*(v_{k+1} v_{k+2}^m) = 4, \quad \ldots,$$

$$f^*(v_1 v_2^m) = 2k-1, \quad f^*(v_1 v_{2k}^m) = 2k, \quad f^*(v_{k+1} v_{k+2}^{m-1}) = 2k+1, \quad \ldots,$$

$$f^*(v_1 v_{2k}^{m-1}) = 4k, \quad \ldots, \quad f^*(v_1 v_2) = q-1, \quad f^*(v_1 v_{2k}) = q.$$

Thus, f^* is bijective. Therefore, the function f is an absolute mean graceful labeling of the graph $Spl_m(C_{2k})$ for all $m \geq 1$ and $k \geq 2$. □

Illustration 2.4. An absolute mean graceful labeling of the graph $Spl_3(C_6)$ is shown in Figure 2.

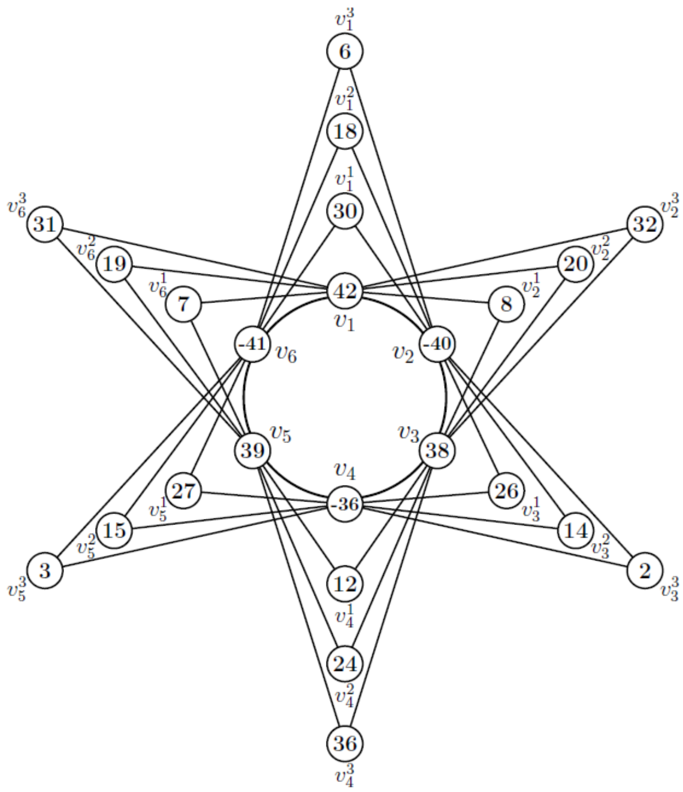

Fig. 2. The graph $Spl_3(C_6)$ with an absolute mean graceful labeling

Theorem 2.5. *The m-shadow graph of the even cycle, $D_m(C_{2k})$, is an absolute mean graceful graph for all $m \geq 2$ and $k \geq 2$.*

Proof. Consider m copies of the graph C_{2k}. Let

$$v_1^j, v_2^j, \ldots, v_{2k}^j$$

be the vertices of the jth copy of the cycle C_{2k}, for $j = 1, 2, \ldots, m$. Let G be the graph $D_m(C_{2k})$. Then

$$p = |V(G)| = 2km$$

and

$$q = |E(G)| = 2km^2.$$

Now, consider the labeling function

$$f : V(G) \to \{0, \pm 1, \pm 2, \ldots, \pm q\}$$

defined as follows:

$$f(v_i^1) = \begin{cases} (-1)^{i+1}(q - 2(i-1)), & \text{for } i = 1, 2, \ldots, k+1, \\ (-1)^{i+1}(q - 1 - 2(2k - i)), & \text{for } i = k+2, k+3, \ldots, 2k, \end{cases}$$

and

$$f(v_i^j) = \begin{cases} f(v_i^1) - 4k(j-1), & \text{for } i = 1, 3, 5, \ldots, 2k-1, \\ f(v_i^1) + 4km(j-1), & \text{for } i = 2, 4, 6, \ldots, 2k, \end{cases}$$

for all $j = 2, 3, \ldots, m$.

The above-defined function f assigns distinct labels to every vertex of the graph G from the set $\{0, \pm 1, \pm 2, \ldots, \pm q\}$; hence, f is one-to-one. Now consider the induced edge-labeling function f^* defined by

$$f^*(xy) = \left\lceil \frac{|f(x) - f(y)|}{2} \right\rceil$$

for every edge $xy \in E(G)$. Then f^* assigns distinct labels to every edge of the graph G from the set $\{1, 2, \ldots, q\}$ as follows:

$$f^*(v_{k+1}^m v_k^m) = 1, \quad f^*(v_{k+1}^m v_{k+2}^m) = 2, \quad f^*(v_k^m v_{k-1}^m) = 3, \quad f^*(v_{k+2}^m v_{k+3}^m) = 4, \quad \ldots,$$
$$f^*(v_1^m v_2^m) = 2k-1, \quad f^*(v_1^m v_{2k}^m) = 2k, \quad f^*(v_{k+1}^m v_k^{m-1}) = 2k+1,$$
$$f^*(v_{k+1}^m v_{k+2}^{m-1}) = 2k+2, \quad \ldots, \quad f^*(v_1^1 v_2^1) = q-1, \quad f^*(v_1^1 v_{2k}^1) = q.$$

Thus, f^* is bijective. Therefore, the function f is an absolute mean graceful labeling of the graph $D_m(C_{2k})$ for all $m \geq 2$ and $k \geq 2$. □

Illustration 2.6. An absolute mean graceful labeling of the graph $D_3(C_{10})$ is shown in Figure 3.

Theorem 2.7. *The m-shadow graph of the path, $D_m(P_n)$, is an absolute mean graceful graph for all $m, n \geq 2$.*

Proof. Consider m copies of the graph P_n. Let

$$v_1^j, v_2^j, \ldots, v_n^j$$

be the vertices of the jth copy of P_n, for $j = 1, 2, \ldots, m$. Let G be the graph $D_m(P_n)$. Then

$$p = |V(G)| = mn$$

and

$$q = |E(G)| = m^2(n-1).$$

Now, consider the labeling function

$$f : V(G) \to \{0, \pm 1, \pm 2, \ldots, \pm q\}$$

defined as follows:

$$f(v_i^j) = \begin{cases} q - i + 1 - 2(n-1)(j-1), & \text{if } i \text{ is odd}, \\ -q + i - 1 + 2m(n-1)(j-1), & \text{if } i \text{ is even}, \end{cases}$$

for all $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, m$.

The above-defined function f assigns distinct labels to every vertex of the graph G from the set $\{0, \pm1, \pm2, \ldots, \pm q\}$; hence, f is one-to-one. Now consider the induced edge-labeling function f^* defined by

$$f^*(xy) = \left\lceil \frac{|f(x) - f(y)|}{2} \right\rceil$$

for every edge $xy \in E(G)$. Then f^* assigns distinct labels to every edge of the graph G from the set $\{1, 2, \ldots, q\}$ as follows:

$$f^*(v_n^m v_{n-1}^m) = 1, \quad f^*(v_{n-1}^m v_{n-2}^m) = 2, \quad f^*(v_{n-2}^m v_{n-3}^m) = 3, \quad \ldots,$$
$$f^*(v_2^m v_1^m) = n - 1, \quad \ldots, \quad f^*(v_3^1 v_2^1) = q - 1, \quad f^*(v_2^1 v_1^1) = q.$$

Thus, f^* is bijective. Therefore, the function f is an absolute mean graceful labeling of the graph $D_m(P_n)$ for all $m, n \geq 2$. □

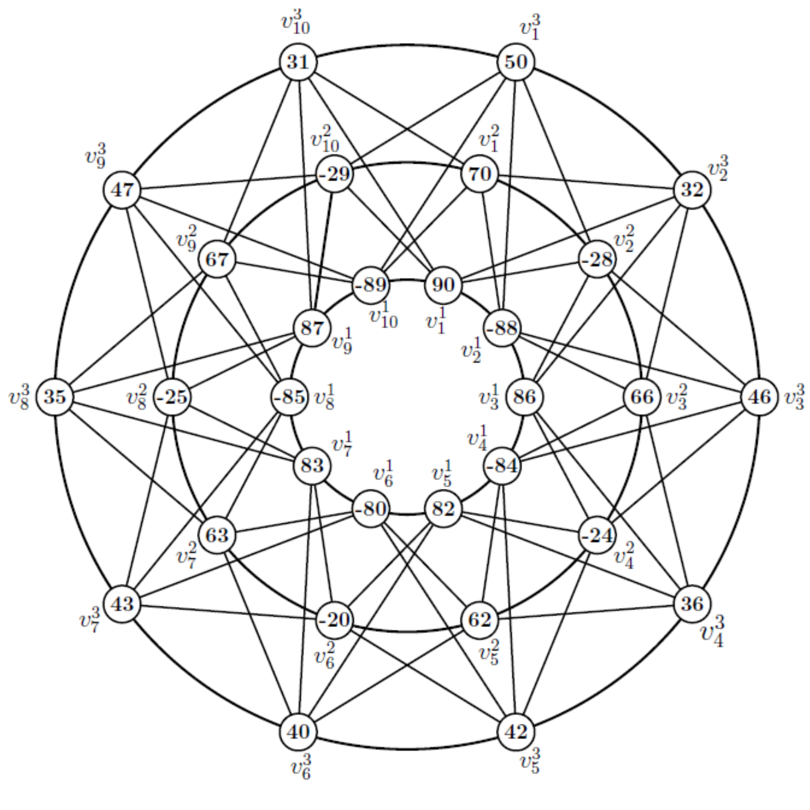

Fig. 3. The graph $D_3(C_{10})$ with an absolute mean graceful labeling

Illustration 2.8. An absolute mean graceful labeling of the graph $D_4(P_7)$ is shown in Figure 4.

Theorem 2.9. *The m-shadow graph of the bistar, $D_m(B_{r,t})$, is an absolute mean graceful graph for all $m, r, t \geq 2$.*

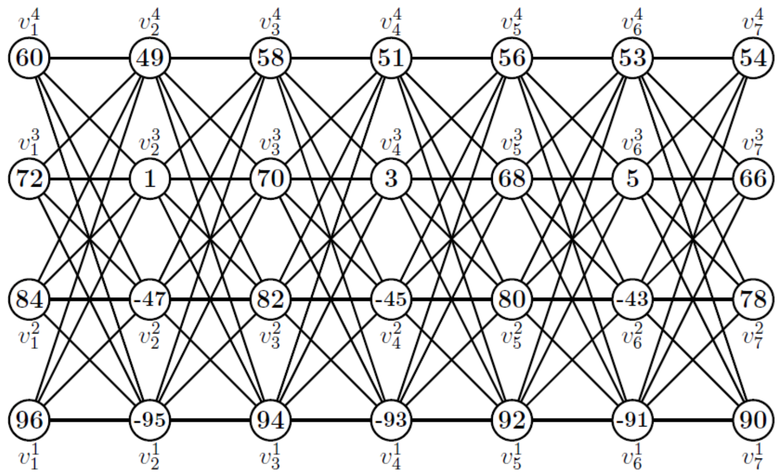

Fig. 4. The graph $D_4(P_7)$ with an absolute mean graceful labeling

Proof. Consider the bistar graph $B_{r,t}$ with vertex set

$$\{u_0, u_1, \ldots, u_r, v_0, v_1, \ldots, v_t\}$$

and edge set

$$\{u_0u_1, u_0u_2, \ldots, u_0u_r, u_0v_0, v_0v_1, v_0v_2, \ldots, v_0v_t\}.$$

Now consider m copies of the graph $B_{r,t}$. Let

$$u_0^j, u_1^j, u_2^j, \ldots, u_r^j, v_0^j, v_1^j, v_2^j, \ldots, v_t^j$$

be the vertices of the jth copy of $B_{r,t}$, for $j = 1, 2, \ldots, m$. Let G be the graph $D_m(B_{r,t})$. Then

$$p = |V(G)| = m(r+t+2)$$

and

$$q = |E(G)| = m^2(r+t+1).$$

Now, consider the labeling function

$$f : V(G) \to \{0, \pm1, \pm2, \ldots, \pm q\}$$

defined as follows:

$$\begin{aligned}
f(u_0^j) &= -q + 2m(j-1)(r+t+1), && \text{for } j = 1, 2, \ldots, m,\\
f(u_i^j) &= q - 2(j-1)(r+t+1) - 2(i-1), && \text{for } i = 1, 2, \ldots, r \text{ and } j = 1, 2, \ldots, m,\\
f(v_0^j) &= q - 2(j-1)(r+t+1) - 2r, && \text{for } j = 1, 2, \ldots, m,\\
f(v_i^j) &= -q + 2m(j-1)(r+t+1) + 2i, && \text{for } i = 1, 2, \ldots, t \text{ and } j = 1, 2, \ldots, m.
\end{aligned}$$

The above-defined function f assigns distinct labels to every vertex of the graph G from the set $\{0, \pm1, \pm2, \ldots, \pm q\}$; hence, f is one-to-one. Now consider the induced edge-labeling function f^* defined by

$$f^*(xy) = \left\lceil \frac{|f(x) - f(y)|}{2} \right\rceil$$

for every edge $xy \in E(G)$. Then f^* assigns distinct labels to every edge of the graph G from the set $\{1, 2, \ldots, q\}$ as follows:

$$f^*(v_0^m v_t^m) = 1, \quad f^*(v_0^m v_{t-1}^m) = 2, \quad f^*(v_0^m v_{t-2}^m) = 3, \quad \ldots,$$

$$f^*(v_0^m v_1^m) = t, \quad f^*(v_0^m u_0^m) = t+1, \quad f^*(u_0^m u_r^m) = t+2, \quad \ldots,$$

$$f^*(u_0^1 u_2^1) = q-1, \quad f^*(u_0^1 u_1^1) = q.$$

Thus, f^* is bijective. Therefore, the function f is an absolute mean graceful labeling of the graph $D_m(B_{r,t})$ for all $m, r, t \geq 2$. □

Illustration 2.10. An absolute mean graceful labeling of the graph $D_3(B_{4,3})$ is shown in Figure 5.

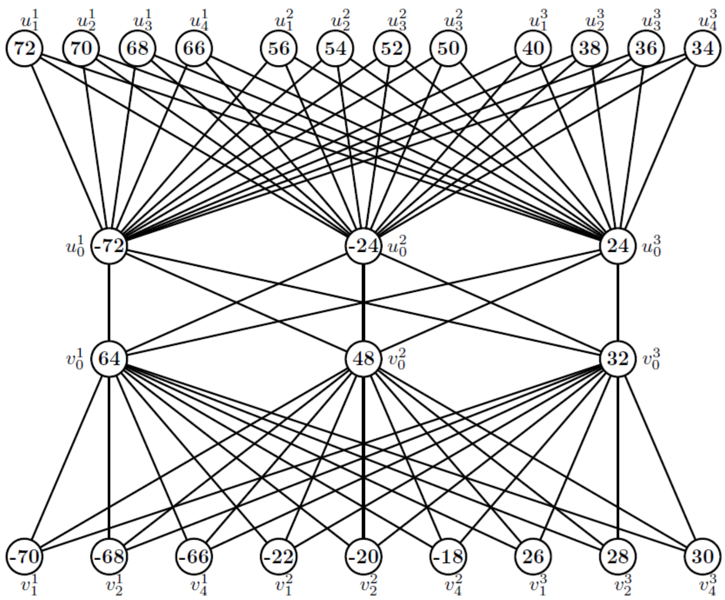

Fig. 5. The graph $D_3(B_{4,3})$ with an absolute mean graceful labeling

3. Concluding Remark

In this paper, we investigated absolute mean graceful labeling for various m-splitting and m-shadow graphs. The illustrations help to clarify the labeling patterns obtained in the main results. In Theorems 2.3 and 2.5, we obtained absolute mean graceful labelings for the m-splitting and m-shadow graphs of even cycles, namely $Spl_m(C_{2k})$ and $D_m(C_{2k})$. By applying a similar pattern, we could not obtain absolute mean graceful labelings for the m-splitting and m-shadow graphs of odd cycles. Therefore, the case of odd cycles is presented as an open problem.

Open Problem

Does there exist an absolute mean graceful labeling for the graphs $Spl_m(C_{2k+1})$ and $D_m(C_{2k+1})$?

Obtaining similar results for different graph families using various graph operations also remains an open area of research.

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