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Weakened Gallai-Ramsey numbers for fans

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ABSTRACT

Let $F_n := K_1 + nK_2$ be a fan of order $2n + 1$. For $1 \leq s < t$, we consider the weakened Gallai-Ramsey number $gr_s^t(F_n)$, defined to be the least $p \in \mathbb{N}$ such that every Gallai t-coloring of K_p contains a subgraph isomorphic to F_n whose edges use at most s colors. Our main results include the evaluations $gr_2^t(F_2) = t+3$, $gr_2^3(F_3) = 9$, and $gr_{2n-1}^t(F_n) = 2n+1$.

Keywords: Ramsey number, Gallai coloring, fan

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1. Introduction

Let nK_2 denote a *matching* of size n, which is defined to be the disjoint union of n edges. Given two graphs G_1 and G_2, the *join* G_1+G_2 is the graph with vertex set $V(G_1)\cup V(G_2)$ and edge set

$$E(G_1) \cup E(G_2) \cup \{xy \mid x \in V(G_1) \text{ and } y \in V(G_2)\}.$$

For $n \in \mathbb{N}$, the *fan* F_n is defined by $F_n := K_1 + nK_2$. From this definiton, we see that $|V(F_n)|= 2n + 1$ and $|E(F_n)|= 3n$. Note that $F_1 = K_3$. In the definition of F_n, the K_1-subgraph is called the *center* of the fan, and the K_2-subgraphs in the matching nK_2 are called the *blades* of the fan. The *path* of order n will be denoted by P_n and is a sequence of n distinct vertices $x_1x_2\cdots x_n$ such that each consecutive pair of vertices form an edge.

A *t-coloring* of a complete graph K_n of order n is a map

$$f : E(K_n) \longrightarrow \{1, 2, \ldots, t\}.$$

For graphs $G_1, G_2, \ldots, G_t$, the *t-color Ramsey number* $r(G_1, G_2, \ldots, G_t)$ is defined to be the least $p \in \mathbb{N}$ such that every t-coloring of K_p contains a monochromatic subgraph

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in color i that is isomorphic to G_i, for some $i \in \{1, 2, \ldots, t\}$. When $G_1 = G_2 = \cdots = G_t$, we shorten the notation to $r^t(G_1)$. Ramsey numbers involving fans have been extensively studied (e.g., see [3], [5], [7], [9], [17], [18], [19], [22], [24], [25], [26], [27]).

A *Gallai t-coloring* of K_n is a t-coloring f of K_n that avoids rainbow triangles. Here, a *rainbow triangle* is a subset $\{x, y, z\} \subseteq V(K_n)$ such that no two of $f(xy)$, $f(xz)$, and $f(yz)$ are equal. For any graph G, the *Gallai-Ramsey number* $gr^t(G)$ is the least $p \in \mathbb{N}$ such that every Gallai t-coloring of K_p contains a monochromatic subgraph that is isomorphic to G. Since every Gallai t-coloring of K_p is a t-coloring of K_p, it follows that $gr^t(G) \leq r^t(G)$ for every graph G. Gallai-Ramsey numbers for fans were studied by Mao, Wang, Magnant, and Schiermeyer [21] in 2023, where they prove that

$$gr^t(F_2) = \begin{cases} 9 & \text{if } t = 2, \\ \frac{83}{2} \cdot 5^{(t-4)/2} + \frac{1}{2} & \text{if } t \geq 4 \text{ is even,} \\ 4 \cdot 5^{(t-1)/2} + 1 & \text{if } t \text{ is odd.} \end{cases}$$

They also showed that for $t \geq 2$,

$$gr^t(F_3) \geq \begin{cases} 14 \cdot 5^{(t-2)/2} - 1 & \text{if } t \text{ is even,} \\ 33 \cdot 5^{(t-3)/2} & \text{if } t \text{ is odd,} \end{cases}$$

and conjectured that these lower bounds are sharp.

In this paper, our focus will be on a "weakened" version of $gr^t(F_n)$ in that we will seek out subgraphs isomorphic to F_n that use at most a given number of colors. To be precise, let $1 \leq s < t$ and define the *weakened Gallai-Ramsey number* $gr^t_s(G)$ to be the least $p \in \mathbb{N}$ such that every Gallai t-coloring of K_p contains a subgraph that is isomorphic to G whose edges use at most s colors. This generalizes the concept of a Gallai-Ramsey number in that $gr^t(G) = gr^t_1(G)$. For $s_1 \leq s_2$, the inequality

$$gr^t_{s_2}(G) \leq gr^t_{s_1}(G),$$

guarantees that weakened Gallai-Ramsey numbers exist. At present, weakened Gallai-Ramsey numbers have been studied for complete graphs ([1], [6], [15], and [16]), for cycles ([6] and [15]), and for wheels, books, and complete bipartite graphs [15].

One benefit of working with Gallai t-colorings of K_n (as opposed to t-colorings of K_n) is that the proofs of upper bounds for Gallai-Ramsey numbers can be broken into manageable cases by using the following structural theorem.

Theorem 1.1 ([12])**.** *Every Gallai coloring of a complete graph can be formed by taking a* 2*-colored complete graph with order at least* 2*, and replacing its vertices with Gallai-colored complete graphs.*

The partitioning of the vertex set in the original complete graph into the vertex sets for the Gallai-colored complete graphs is called a *Gallai partition*. The 2-colored complete graph with order at least 2 is called the *base graph* of the Gallai partition, and the Gallai-colored complete graphs that replace the vertices in the base graph are called the *blocks* of the Gallai partition. All edges joining any distinct pair of blocks are necessarily the same color. The following lemma will save us some time when using Theorem 1.1.

Lemma 1.2 ([20]). *If $\mathcal{B}$ is the base graph of a Gallai parition, chosen to have minimal order, then $|V(\mathcal{B})|\neq 3$.*

In Section 2, we prove that

$$gr_2^t(F_2) = t+3, \quad \text{for all } t \geq 3,$$

$$gr_2^3(F_3) = 9,$$

and

$$gr_{2n-1}^t(F_n) = 2n+1, \quad \text{for all } n \geq 2 \text{ and } t \geq 2n.$$

Section 3 concludes with some directions for future work on this topic.

2. Main results

Before we turn to the evaluation of certain weakened Gallai-Ramsey numbers for fans, we recall a couple of known Ramsey numbers. In 1967, Gerencsér and Gyárfás [11] determined the value of the 2-color Ramsey number for paths. They showed that for all $n \geq m \geq 2$,

$$r(P_m, P_n) = n + \left\lfloor \frac{m}{2} \right\rfloor + 1. \tag{1}$$

The Ramsey number for matchings was determined by Cockayne and Lorimer [8] in 1975. They proved that if $n_1, n_2, \ldots, n_t \in \mathbb{N}$ such that $n_1 = \max\{n_1, n_2, \ldots, n_t\}$, then

$$r(n_1K_2, n_2K_2, \ldots, n_tK_2) = n_1 + 1 + \sum_{i=1}^{t}(n_i - 1). \tag{2}$$

In particular, $r^t(nK_2) = n(t+1) - t + 1$. In the following theorem, we determine $gr_2^t(F_2)$.

Theorem 2.1. *For all $t \geq 3$, $gr_2^t(F_2) = t+3$.*

Proof. To prove the lower bound, begin with a K_3 in color 1, introduce a new vertex x_1, and color all edges from x_1 to the K_3 with color 2. Next, introduce vertex x_2 and color all edges from x_2 to the existing graph with color 3. Continue in this manner, sequentially introducing vertices $x_3, x_4, \ldots, x_{t-1}$, coloring all edges from x_i to the existing graph with color $i+1$ (the $t = 3$ case is shown in Figure 1).

The resulting K_{t+2} avoids rainbow triangles. Every F_2-subgraph in which the center of the fan is a vertex in the K_3 in color 1 uses at least three colors on its four edges to the blades of the fan. If an F_2-subgraph has x_i as its center and the edges to its blades are chosen to use at most two colors, then at least one of the blades must use a third color. Hence, no F_2-subgraph exists that uses at most two colors, and it follows that $gr_2^t(F_2) \geq t+3$.

To prove the upper bound, let $\mathcal{B}$ be the base graph, chosen to have minimal order. By Theorem 1.1 and Lemma 1.2, $|V(\mathcal{B})|\geq 2$ and $|V(\mathcal{B})|\neq 3$. If $|V(\mathcal{B})|\geq 5$, then selecting a single vertex from five distinct blocks results in a K_5 whose edges use at most two colors. Such a K_5 contains F_2 as a subgraph. We now handle the cases $|V(\mathcal{B})|= 2$ and $|V(\mathcal{B})|= 4$ separately.

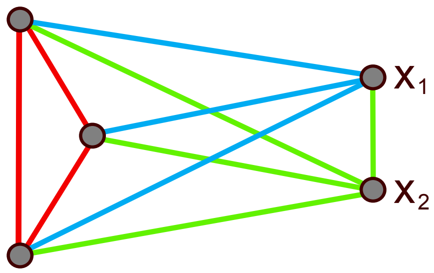

Fig. 1. A Gallai 3-coloring of K_5 that avoids an F_2-subgraph spanned by edges using at most two colors

Case 1. Suppose that $|V(\mathcal{B})|= 2$ and assume that the edges joining the blocks are red (color 1). We consider two subcases.

Subcase 1.1. Suppose that one block only contains a single vertex x. Then the larger block has order $t + 2$. If we view the colors red and blue (colors 1 and 2, respectively) as if they are the same color, then since $gr^{t-1}(2K_2) = t + 2$ by Eq. (2), the larger block contains a red/blue $2K_2$ or a monochromatic copy of $2K_2$ in one of the colors $3, 4, \ldots, t$. In both cases, an F_2 that uses at most two colors (one of which is red) is formed with x as its center and the $2K_2$ forming the blades.

Subcase 1.2. Suppose that both blocks have order at least 2. Note that one block must have order at least 3. Let x_1 and x_2 be in one block and y_1, y_2, and y_3 be in the other block. Since rainbow triangles are avoided, at least two of the edges of the subgraph induced by $\{y_1, y_2, y_3\}$ use the same color. Without loss of generality, suppose that y_1y_2 and y_1y_3 are both the same color. Then an F_2-subgraph is formed with center y_1 and blades x_1y_2 and x_2y_3 whose edges use at most two colors, one of which is red.

Case 2. Suppose that $|V(\mathcal{B})|= 4$ and assume that the edges joining the blocks are red and blue. Denote the vertex sets for the blocks by W, X, Y, and Z. Note that one block must contain at least two vertices. Without loss of generality, assume that $w \in W$, $x \in X$, $y \in Y$, and $z_1, z_2 \in Z$. Then a red/blue F_2 can be formed with center w and blades xz_1 and yz_2.

In both cases, we find that there exists an F_2 whose edges use at most two colors. It follows that $gr^t_2(F_2) \leq t + 3$. □

Now we consider the weakened Gallai-Ramsey number $gr^3_2(F_3)$.

Theorem 2.2. $gr^3_2(F_3) = 9$.

Proof. To prove the lower bound, start by replacing one vertex in a blue K_2 with a green K_5. Call the resulting blue/green coloring of K_6 $\mathcal{G}$. Here, $\mathcal{G}$ is a critical coloring for $r(2K_2, 3K_2)$ (see [8]). Next, take a red K_2 and replace one of its vertices with a blue K_2 and the other vertex with a copy of $\mathcal{G}$ (see Figure 2).

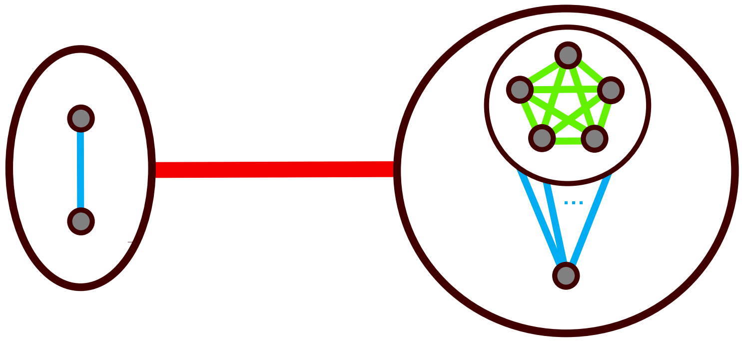

Fig. 2. A Gallai 3-coloring of K_8 that avoids an F_3-subgraph spanned by edges using at most two colors

The resulting Gallai 3-coloring of K_8 does not have a large enough blue/green connected component to have a blue/green F_3. In order to have a red/blue F_3, there would have to be a blue $2K_2$ in the larger block. In order to have a red/green F_3, there would have to be a green $3K_2$ in the larger block. Since this Gallai 3-coloring of K_8 avoids an F_3 whose edges use at most two colors, we obtain the inequality $gr_2^3(F_3) \geq 9$.

To prove the reverse inequality, consider a Gallai 3-coloring of K_9 and let $\mathcal{B}$ be the base graph, chosen to have minimal order. By Theorem 1.1 and Lemma 1.2, it follows that $2 \leq |V(\mathcal{B})| \leq 6$ and $|V(\mathcal{B})| \neq 3$. Consider the following cases.

Case 1. Suppose that $|V(\mathcal{B})| = 2$ and that the edges joining the two blocks are red. Denote the vertex sets for the two blocks by X and Y. We must consider two subcases.

Subcase 1.1. Suppose that one block has order at most 2. Denote its vertex set by X, and denote the vertex set for the larger block by Y. Note that $|Y| \geq 7$. If the subgraph induced by Y does not contain any red edges, then it contains a blue/green K_7, which contains F_3 as a subgraph. So, without loss of generality, assume that y_1y_2 is red, where $y_1, y_2 \in Y$. If the subgraph induced by $Y \setminus \{y_1, y_2\}$ lacks red edges, then $r^2(2K_2) = 5$ (by Eq. (2)) implies that it contains a monochromatic $2K_2$ that is blue or green. This $2K_2$, along with edge y_1y_2 and a vertex $x \in X$, forms an F_3 spanned by edges using at most two colors in which x is the center. So, assume that the subgraph induced by $Y \setminus \{y_1, y_2\}$ does contain a red edge, and without loss of generality, assume that y_3y_4 is one such edge. Then an F_3 that uses at most two colors can be formed with center x and blades y_1y_2, y_3y_4, and y_5y_6, where $y_5, y_6 \in Y \setminus \{y_1, y_2, y_3, y_4\}$.

Subcase 1.2. Suppose that one block has order 3. Denote its vertex set by $X = \{x_1, x_2, x_3\}$ and denote the vertex set for the block of order 6 by $Y = \{y_1, y_2, \ldots, y_6\}$. If the subgraph induced by X contains a red edge (say, x_1x_2 is red), then $r^3(2K_2) = 6$ (by Eq. (2)) implies that the subgraph induced by Y contains a monochromatic $2K_2$. If the edges is this $2K_2$ are given by y_1y_2 and y_3y_4, then an F_3 whose edges use at most two colors is formed with center x_1 and blades y_1y_2, y_3y_4, and x_2y_5. So, assume that the subgraph induced by X is blue/green. At least two of the edges in this blue/green K_3 must be the same color. Without loss of generality, assume that x_1x_2 and x_1x_3 are both blue. If the subgraph induced by Y contains a red or blue edge (say, y_1y_2 is red or blue), then a red/blue F_3 is formed with x_1 as the center and blades y_1y_2, x_2y_3, and x_3y_4. So, assume that the subgraph induced by Y only contains green edges. Then a red/green F_3

is formed with center x_1 and blades y_1y_2, y_3y_4, and y_5y_6.

Subcase 1.3. Suppose that one block has order 4 and the other block has order 5. Without loss of generality, assume that $X = \{x_1, x_2, x_3, x_4\}$ and $Y = \{y_1, y_2, y_3, y_4, y_5\}$ are the vertex sets for the blocks. If the subgraph induced by X contains a red edge (say, x_1x_2 is red), then if we view red and blue as the same color, $r^2(2K_2) = 5$ (by Eq. (2)) implies that the subgraph induced by Y contains a red/blue $2K_2$ or a green $2K_2$. If y_1y_2 and y_3y_4 form a red/blue $2K_2$, then a red/blue F_3 is formed with center x_1 and blades y_1y_2, y_3y_4, and x_2y_5. If y_1y_2 and y_3y_4 form a green $2K_2$, then a red/green F_3 can be formed with center x_1 and blades y_1y_2, y_3y_4, and x_2y_5.

So, assume that the subgraph induced by X does not contain any red edges. Since $r^2(P_3) = 3$ (by Eq. (1)), it must contain a monochromatic P_3. Without loss of generality, assume that $x_1x_2x_3$ is a blue P_3. If the subgraph induced by Y contains a red or blue edge (say, y_1y_2 is red or blue), then a red/blue F_3 is formed with center x_2 and blades y_1y_2, x_1y_3, and x_3y_4. So, assume that all of the edges in the subgraph induced by Y are green. Then a red/green F_3 is formed with center y_1 and blades x_1y_2, x_2y_3, and x_3y_4.

Case 2. Suppose that $|V(\mathcal{B})| = 4$ and that the edges in $\mathcal{B}$ are red and blue. Denote the vertex sets for the blocks by W, X, Y, and Z. By the pigeonhole principle, some block must contain at least three vertices, and we assume it is the block with vertex set Z. Consider the following subcases.

Subcase 2.1. Suppose that some block only contains a single vertex (say, $W = \{w\}$). If at least one of X and Y has order at least 2 (say, $x_1x_2 \in X$, $y \in Y$, and $z_1, z_2, z_3 \in Z$), then a red/blue F_3 is formed with center w and blades x_1z_1, x_2z_2, and yz_3. So, assume that X and Y both have order 1 and that $X = \{x\}$, $Y = \{y\}$, and $Z = \{z_1, z_2, \ldots, z_6\}$. If the subgraph induced by Z contains a red or blue edge (say, z_1z_2 is red or blue), then a red/blue F_3 is formed with center w and blades z_1z_2, xz_3, and yz_4. So, assume that the subgraph induced by Z is a green K_6. Then an F_3 using at most two colors (one of which is green) is formed with center w and blades z_1z_2, z_3z_4, and z_5z_6.

Subcase 2.2. Suppose that every block has order at least 2. Without loss of generality, suppose that the vertex sets for the blocks are given by $W = \{w_1, w_2\}$, $X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$, and $Z = \{z_1, z_2, z_3\}$. Then a red/blue F_3 is formed with center w_1 and blades x_1z_1, x_2z_2, and y_1z_3.

Case 3. Suppose that $|V(\mathcal{B})| = 5$ and that the edges in $\mathcal{B}$ are red and blue. Denote the vertex sets for the blocks by V, W, X, Y, and Z. By the pigeonhole principle, some block has order at least 2, and since there are only nine vertices in total, some block has order 1. Without loss of generality, assume that $V = \{v\}$. If some block has order at least 3 (say, $w \in W$, $x \in X$, $y \in Y$, and $z_1, z_2, z_3 \in Z$), then a red/blue F_3 is formed with center v and blades wz_1, xz_2, and yz_3. So, assume that all blocks other than V have order 2. If $w_1, w_2 \in W$, $x_1, x_2 \in X$, $y_1y_2 \in Y$, and $z_1z_2 \in Z$, then a red/blue F_3 is formed with center v and blades w_1x_1, w_2x_2, and y_1z_1.

Case 4. Suppose that $|V(\mathcal{B})| = 6$ and that the edges in $\mathcal{B}$ are red and blue. Denote the vertex sets for the blocks by U, V, W, X, Y, and Z. By the pigeonhole principle, some block must have order at least 2. Without loss of generality, assume that $u \in U$, $v \in V$, $w \in W$, $x \in X$, $y \in Y$, and $z_1, z_2 \in Z$. Then a red/blue F_3 is formed with center u and blades vw, xz_1, and yz_2.

In all cases, we have shown that a Gallai 3-coloring of K_9 contains an F_3 that uses at most two colors. It follows that $gr_2^3(F_3) \leq 9$. □

In the next theorem, we consider a more general case of weakened Gallai-Ramsey numbers for fans.

Theorem 2.3. *For all $n \geq 2$ and $t \geq 2n$, $gr_{2n-1}^t(F_n) = 2n+1$.*

Proof. Since $2n+1$ vertices are required to have an F_n, it follows that $gr_{2n-1}^t(F_n) \geq 2n+1$. We will prove the reverse inequality by induction on $n \geq 2$. For the base case $n = 2$, consider a Gallai t-coloring of K_5 and let $\mathcal{B}$ be the base graph, chosen to have minimal order. Theorem 1.1 and Lemma 1.2 imply that $|V(\mathcal{B})|= 2$ or $|V(\mathcal{B})|= 4$. In the latter case, some block must contain at least two vertices by the pigeonhole principle. If we select two vertices from one block and a single vertex from each of the other three blocks, the subgraph induced by these five vertices is a K_5 (which contains F_2 as a subgraph) that uses at most three colors.

So, assume that $|V(\mathcal{B})|= 2$ with the edges joining the two blocks being red. By the pigeonhole principle, some block must contain at least three vertices. An F_2 that uses at most three colors can then be formed by selecting a single vertex from the smaller block to serve as its center. We then pick four other vertices (at least three of which come from the larger block) and pair them up to form the blades. It follows that $gr_3^t(F_2) \leq 5$.

Assume that $gr_{2n-3}^t(F_{n-1}) \leq 2n-1$ and consider a Gallai t-coloring of K_{2n+1}. By the inductive hypothesis, there exists an F_{n-1} whose edges use at most $2n-3$ colors. If we select two vertices not contained in the F_{n-1} and let them form an n^{th} blade, at most two more colors can be introduced since rainbow triangles are avoided. The resulting F_n is spanned by edges using at most $2n-1$ colors, implying that $gr_{2n-1}^t(F_n) \leq 2n+1$. □

In particular, Theorem 2.3 implies that $gr_3^t(F_2) = 5$ and $gr_5^t(F_3) = 7$. By a theorem of Erdős, Simonovoits, and Sós [10], every Gallai-coloring of K_p uses at most $p-1$ colors. So, the values of $t \geq 2n+1$ in the previous theorem follow from the case $gr_{2n-1}^{2n}(F_n) = 2n+1$.

3. Conclusion

There are many values of t, s, and n that remain to be considered for $gr_s^t(F_n)$. Besides these cases, one can consider analogous problems for generalized fans. A *generalized fan* $F_{t,n}$ is defined by $F_{t,n} := K_1 + nK_t$. It follows that $|V(F_{t,n})|= tn+1$, $|E(F_{t,n})|= \frac{nt(t-1)}{2}$, and $F_n = F_{2,n}$. Ramsey theory involving $F_{t,n}$ was studied in [3], [4], [13], and [14], and [23].

Another related problem is the determination of star-critical weakened Gallai-Ramsey numbers for fans (cf. [2]). Such numbers determine the number of edges that one must add between a vertex and the complete graph $K_{gr_s^t(F_n)-1}$ in order to guarantee the existence of an F_n that uses at most s colors.

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